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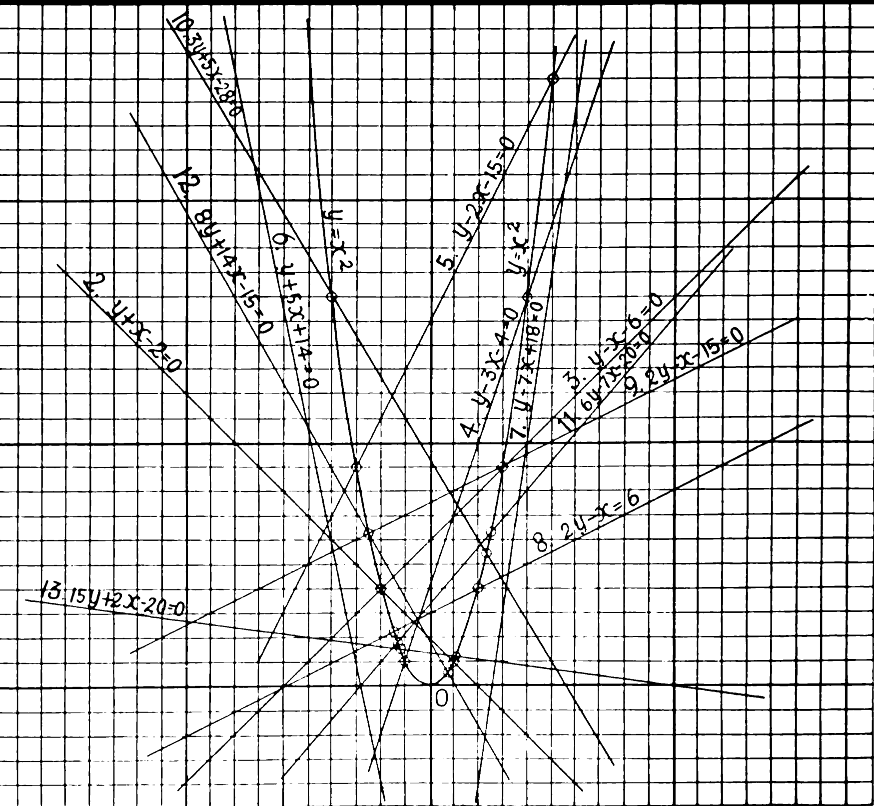

Key to Standard algebra and Standard algebra-revised

William James Milne

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KEY

TO

STANDARD ALGEBRA

AND

STANDARD ALGEBRA—REVISED

BY

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CONTENTS

NOTE. — When a problem in the "Standard Algebra — Revised" is the same as one in the "Standard Algebra," the solution is not repeated, but reference is made to the page of the Key where the solution is given for the "Standard Algebra."

Thus, to find the solution to examples on p. 20 of "Standard Algebra — Revised," turn first to the "Key to Standard Algebra — Revised" (pp. 433–544). On p. 433 where "Page 20" appears as a center heading, the reference under 2–17 and 19–30 shows that these examples are solved on p. 3 of the Key. As Ex. 54, p. 21, of "Standard Algebra — Revised" does not appear in the "Standard Algebra," the solution is given in full on p. 433 of the Key.

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E. P. 3

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KEY TO STANDARD ALGEBRA

DEFINITIONS AND NOTATION

Page 20

2. $10\,a = 10 \cdot 5 = 50.$

3. $2\,ab = 2 \cdot 5 \cdot 3 = 30.$

4. $3\,cm = 3 \cdot 10 \cdot 4 = 120.$

5. $6\,bc = 6 \cdot 3 \cdot 10 = 180.$

6. $5\,m^2 = 5 \cdot 4 \cdot 4 = 80.$

7. $2\,a^2b = 2 \cdot 5 \cdot 5 \cdot 3 = 150.$

8. $3\,bm^3 = 3 \cdot 3 \cdot 4 \cdot 4 \cdot 4 = 576.$

9. $4\,a^3b = 4 \cdot 5 \cdot 5 \cdot 5 \cdot 3 = 1500.$

10. $am^4 = 5 \cdot 4 \cdot 4 \cdot 4 \cdot 4 = 1280.$

11. $(ab)^2 = (ab)(ab) = (5 \cdot 3)(5 \cdot 3) = 15 \cdot 15 = 225.$

12. $a^2b^2 = aabb = 5 \cdot 5 \cdot 3 \cdot 3 = 225.$

13. $a^bc = 5^3 \cdot 10 = 5 \cdot 5 \cdot 5 \cdot 10 = 1250.$

14. $\frac{1}{3}\,ab^2 = \frac{1}{3} \cdot 5 \cdot 3 \cdot 3 = 15.$

15. $\frac{1}{2}\,bm = \frac{1}{2} \cdot 3 \cdot 4 = 6.$

16. $\frac{1}{5}\,abc = \frac{1}{5} \cdot 5 \cdot 3 \cdot 10 = 30.$

17. $3\,b^2cm^2 = 3 \cdot 3 \cdot 3 \cdot 10 \cdot 4 \cdot 4 = 4320.$

19. $\sqrt{2\,ab} = \sqrt{2 \cdot 4 \cdot 2} = \sqrt{4 \cdot 4} = 4.$

20. $7\,b^2r = 7 \cdot 2 \cdot 2 \cdot 0 = 0.$

21. $\sqrt{as^2} = \sqrt{4 \cdot 5 \cdot 5} = \sqrt{2 \cdot 2 \cdot 5 \cdot 5} = \sqrt{10 \cdot 10} = 10.$

22. $\sqrt{7\,r^3s} = \sqrt{7 \cdot 0 \cdot 0 \cdot 0 \cdot 5} = \sqrt{0} = 0.$

23. $3\,s^bb^a = 3 \cdot 5^2 \cdot 2^4 = 3 \cdot 5 \cdot 5 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 1200.$

24. $\sqrt[3]{8\,ab} = \sqrt[3]{8 \cdot 4 \cdot 2} = \sqrt[3]{2 \cdot 2 \cdot 2 \times 2 \cdot 2 \cdot 2} = 2 \times 2 = 4.$

25. $3\,a\sqrt{b^2s^4} = 3 \cdot 4\sqrt{2^25^4} = 12\sqrt{2 \cdot 2 \times 5 \cdot 5 \times 5 \cdot 5} = 12(2 \times 5 \times 5) = 600.$

26. $\frac{3}{8}\,a^3bs = \frac{3}{8} \cdot 4 \cdot 4 \cdot 4 \cdot 2 \cdot 5 = 240.$

27. $.8\,s\sqrt{a^2b^4} = .8 \cdot 5\sqrt{4 \cdot 4 \cdot 2 \cdot 2 \cdot 2 \cdot 2} = 4\,(4 \cdot 2 \cdot 2) = 64.$

28. $6^b\sqrt{s^2b^a} = 6^2\sqrt{5^2 \cdot 2^4} = 6 \cdot 6\sqrt{5 \cdot 5 \times 2 \cdot 2 \times 2 \cdot 2} = 6 \cdot 6(5 \cdot 2 \cdot 2) = 720.$

29. $2^a\,b^3s^2r^4 = 2^4 \cdot 2^3 \cdot 5^2 \cdot 0^4 = 0.$

30. $\sqrt[3]{9^ar^sb^6} = \sqrt[3]{9^4 \cdot 0^5 \cdot 2^6} = \sqrt[3]{0} = 0.$

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31. $\frac{3\,a^2b}{sb} = \frac{3 \cdot 4 \cdot 4 \cdot 2}{5 \cdot 2} = \frac{96}{10} = 9\frac{3}{5}.$

32. $\frac{bs^4r}{abs} = \frac{2 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 0}{4 \cdot 2 \cdot 5} = \frac{0}{40} = 0.$

33. $\frac{6\,a^sb^a}{b^{2s}a^4} = \frac{6 \cdot 4^5 \cdot 2^4}{2^{10} \cdot 4^4} = \frac{6 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 2 \cdot 2 \cdot 2 \cdot 2}{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 4 \cdot 4 \cdot 4 \cdot 4} = \frac{3}{8}.$

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34. $\dfrac{24\,a^b b^a s^a}{6\,a^3b^3s^3}=\dfrac{24\cdot 4^2\cdot 2^5\cdot 5^4}{6\cdot 4^3\cdot 2^3\cdot 5^3}=\dfrac{24\cdot 4\cdot 4\cdot 2\cdot 2\cdot 2\cdot 2\cdot 2\cdot 5\cdot 5\cdot 5\cdot 5}{6\cdot 4\cdot 4\cdot 4\cdot 2\cdot 2\cdot 2\cdot 5\cdot 5\cdot 5}=20.$

36. $a^2+b^2=5\cdot 5+3\cdot 3=25+9=34.$

37. $(a+b)^2=(5+3)^2=8^2=8\cdot 8=64.$

38. $\dfrac{m+2\,n}{m-2\,n}=\dfrac{4+2}{4-2}=\dfrac{6}{2}=3.$

39. $(n-1)^5=(1-1)^5=0^5=0.$

40. $n^5-1=1^5-1=1-1=0.$

41. $m+\dfrac{2\,n}{m-2\,n}=4+\dfrac{2}{4-2}=4+\dfrac{2}{2}=5.$

42. $m^{a-b}=4^{5-3}=4^2=16.$

43. $(bm)^{a-b}=(3\cdot 4)^{5-3}=12^2=144.$

44. $\dfrac{a^3-b^3}{a-b}=\dfrac{5^3-3^3}{5-3}=\dfrac{5\cdot 5\cdot 5-3\cdot 3\cdot 3}{2}=\dfrac{125-27}{2}=\dfrac{98}{2}=49.$

45. $ab-bn+mb^2\div 3\,mn^2=5\cdot 3-3\cdot 1+4\cdot 3\cdot 3\div 3\cdot 4\cdot 1\cdot 1$
$=15-3+36\div 12=15-3+3=15.$

46. $(ab-bn+mb^2)\div 3\,mn^2=(5\cdot 3-3\cdot 1+4\cdot 3\cdot 3)\div 3\cdot 4\cdot 1\cdot 1$
$=(15-3+36)\div 12=48\div 12=4.$

47. $2^a\,m^2n^2-abmn\div 4\,bn-m^3n^7=2^5\cdot 4^2\cdot 1^2-5\cdot 3\cdot 4\cdot 1\div 4\cdot 3\cdot 1-4^3\cdot 1^7$
$=512-60\div 12-64=512-5-64=443.$

48. $\frac{1}{2}m+3\,a^2b-\frac{2}{3}b^2m^2-8\,a=\frac{1}{2}\cdot 4+3\cdot 5\cdot 5\cdot 3-\frac{2}{3}\cdot 3\cdot 3\cdot 4\cdot 4-8\cdot 5$
$=2+225-96-40=91.$

49. $\frac{3}{5}a^2m+\frac{1}{2}m^2n-\frac{2}{9}ab^3-n^4=\frac{3}{5}\cdot 5^2\cdot 4+\frac{1}{2}\cdot 4^2\cdot 1-\frac{2}{9}\cdot 5\cdot 3^3-1^4$
$=60+8-30-1=37.$

50. $ambn^2-\frac{3}{4}b^2m+\frac{5}{8}m^2n^3-\frac{1}{5}m^3=5\cdot 4\cdot 3\cdot 1^2-\frac{3}{4}\cdot 3^2\cdot 4+\frac{5}{8}\cdot 4^2\cdot 1^3-\frac{1}{5}\cdot 4^3$
$=60-27+10-12\frac{4}{5}=30\frac{1}{5}.$

51. When $x=2$, $\quad 2\,x+3\,x=2\cdot 2+3\cdot 2=4+6=10,$
and $\quad 5\,x=5\cdot 2=10;$
hence, $\quad 2\,x+3\,x=5\,x.$
When $x=3$, $\quad 2\,x+3\,x=2\cdot 3+3\cdot 3=6+9=15,$
and $\quad 5\,x=5\cdot 3=15;$
hence, $\quad 2\,x+3\,x=5\,x.$

52. When $m=5$, $a=4$, $b=3$,
$m(a+b)=5(4+3)=5\cdot 7=35,$
and $\quad ma+mb=5\cdot 4+5\cdot 3=20+15=35;$
hence, $\quad m(a+b)=ma+mb.$

53. When $a=3$ and $b=2$,
$(a+b)^2=(3+2)^2=5^2=25,$
and $\quad a^2+2\,ab+b^2=3^2+2\cdot 3\cdot 2+2^2=9+12+4=25;$
hence, $\quad (a+b)^2=a^2+2\,ab+b^2.$

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55. $rx+yz+rs-xz=2\cdot 6+3\cdot 0+2\cdot 10-6\cdot 0=12+0+20-0=32.$

56. $sx^2-r^2s+xyz-xy^2=10\cdot 6^2-2^2\cdot 10+6\cdot 3\cdot 0-6\cdot 3^2$
$=360-40+0-54=266.$

57. $12\,z^3+r^3y^2+5\,xy\div 3\,s=12\cdot 0^3+2^3\cdot 3^2+5\cdot 6\cdot 3\div 3\cdot 10$
$=0+72+90\div 30=72+3=75.$

58. $\frac{1}{3}xr^2-\frac{1}{2}y^4z+\frac{3}{5}s^2=\frac{1}{3}\cdot 6\cdot 2^2-\frac{1}{2}\cdot 3^4\cdot 0+\frac{3}{5}\cdot 10^2=8-0+60=68.$

59. $4^2sy^2\div\frac{5}{8}x^2r^2-\frac{11}{12}x^3y^2z=4^2\cdot 10\cdot 3^2\div\frac{5}{8}\cdot 6^2\cdot 2^2-\frac{11}{12}\cdot 6^3\cdot 3^2\cdot 0$
$=1440\div 90-0=16.$

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60. $5xy^4 - y\sqrt{r^2s^2} + \frac{5}{6}xsz = 5\cdot 6\cdot 3^4 - 3\sqrt{2^2\cdot 10^2} + \frac{5}{6}\cdot 6\cdot 10\cdot 0$
$= 2430 - 3(2\cdot 10) + 0 = 2430 - 60 = 2370.$

61. $x^2+y^2+z^2-2xy+2xz-2yz = 6^2+3^2+0^2-2\cdot 6\cdot 3+2\cdot 6\cdot 0-2\cdot 3\cdot 0$
$= 36 + 9 + 0 - 36 + 0 - 0 = 9.$

62. $(x-y)^2 + 2(x-y)(r+s) + (r+s)^2$
$= (6-3)^2 + 2(6-3)(2+10) + (2+10)^2$
$= 3^2 + 2\cdot 3\cdot 12 + 12^2 = 9 + 72 + 144 = 225.$

63. $5x + 3rs \div 2x \times 7y + 14x^r + \sqrt{xyz}$
$= 5\cdot 6 + 3\cdot 2\cdot 10 \div 2\cdot 6 \times 7\cdot 3 + 14\cdot 6^2 + \sqrt{6\cdot 3\cdot 0}$
$= 30 + 60 \div 12 \times 21 + 504 + 0 = 30 + 5 \times 21 + 504$
$= 30 + 105 + 504 = 639.$

---

1. $s = vt = 48\cdot 30 = 1440$, the number of feet.

2. $s = vt = 8.2\cdot 11 = 90.2$, the number of miles.

3. $s = vt = 44\cdot 75 = 3300$, the number of feet.

4. $s = \frac{1}{2}gt^2 = \frac{1}{2}\cdot 32.16\cdot 5^2 = 402$, the number of feet.

5. $s = \frac{1}{2}gt^2 = \frac{1}{2}\cdot 32.16\cdot 4^2 = 257.28$, the number of feet.
$= \frac{1}{2}\cdot 9.8\cdot 4^2 = 78.4$, the number of meters.

6. $s = \frac{1}{2}gt^2 = \frac{1}{2}\cdot 9.8\cdot 3^2 = 44.1$, the number of meters.

SUBTRACTION

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18.
$$\begin{aligned}
&4a + b - \{x + 4a + b - 2y - (x+y)\}\\
&= 4a + b - \{x + 4a + b - 2y - x - y\}\\
&= 4a + b - \{4a + b - 3y\}\\
&= 4a + b - 4a - b + 3y = 3y.
\end{aligned}$$

19.
$$\begin{aligned}
&ab - \{ab + ac - a - (2a - ac) + (2a - 2ac)\}\\
&= ab - \{ab + ac - a - 2a + ac + 2a - 2ac\}\\
&= ab - \{ab - a\}\\
&= ab - ab + a = a.
\end{aligned}$$

20.
$$\begin{aligned}
&a + [y - \{5 + 4a - (6y + 3)\} - (7y - 4a - 1)]\\
&= a + [y - \{5 + 4a - 6y - 3\} - 7y + 4a + 1]\\
&= a + [-6y - \{2 + 4a - 6y\} + 4a + 1]\\
&= a + [-6y - 2 - 4a + 6y + 4a + 1]\\
&= a + [-1] = a - 1.
\end{aligned}$$

21.
$$\begin{aligned}
&4m - [p + 3n - (m+n) + 3 - (6p - 3n - 5m)]\\
&= 4m - [p + 3n - m - n + 3 - 6p + 3n + 5m]\\
&= 4m - [-5p + 5n + 4m + 3]\\
&= 4m + 5p - 5n - 4m - 3 = 5p - 5n - 3.
\end{aligned}$$

22.
$$\begin{aligned}
&a + 2b + (14a - 5b) - \{6a + 6b - (5a - \overline{4a - 4b})\}\\
&= a + 2b + 14a - 5b - \{6a + 6b - (5a - 4a + 4b)\}\\
&= 15a - 3b - \{6a + 6b - (a + 4b)\}\\
&= 15a - 3b - \{6a + 6b - a - 4b\}\\
&= 15a - 3b - \{5a + 2b\}\\
&= 15a - 3b - 5a - 2b = 10a - 5b.
\end{aligned}$$

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23.
$$\begin{aligned}
&12a - \{[4 - 3b - (6b + 3c)] + b - 8 - (5a - 2b - 6)\}\\
&= 12a - \{[4 - 3b - 6b - 3c] + b - 8 - 5a + 2b + 6\}\\
&= 12a - \{4 - 3b - 6b - 3c + b - 8 - 5a + 2b + 6\}\\
&= 12a - \{2 - 6b - 3c - 5a\}\\
&= 12a - 2 + 6b + 3c + 5a\\
&= 17a + 6b + 3c - 2.
\end{aligned}$$

24.
$$\begin{aligned}
&a + b - \{-[a + b - (c + x)] - [3a - (c - x + a) - b] + 4a\}\\
&= a + b - \{-[a + b - c - x] - [3a - c + x - a - b] + 4a\}\\
&= a + b - \{-a - b + c + x - 2a + c - x + b + 4a\}\\
&= a + b - \{a + 2c\}\\
&= a + b - a - 2c = b - 2c.
\end{aligned}$$

25.
$$\begin{aligned}
&x^3 - [x^2 - (1 - x)] - \{1 + [x^2 - (1 - x) + x^3]\}\\
&= x^3 - x^2 + (1 - x) - 1 - [x^2 - (1 - x) + x^3]\\
&= x^3 - x^2 + 1 - x - 1 - x^2 + (1 - x) - x^3\\
&= -2x^2 - x + (1 - x)\\
&= -2x^2 - x + 1 - x = 1 - 2x - 2x^2.
\end{aligned}$$

26.
$$\begin{aligned}
&4 - \{[5y - (3 - \overline{2x - 2})] - [x + (5y - \overline{x + 3})]\}\\
&= 4 - [5y - (3 - \overline{2x - 2})] + [x + (5y - \overline{x + 3})]\\
&= 4 - 5y + (3 - \overline{2x - 2}) + x + (5y - \overline{x + 3})\\
&= 4 - 5y + 3 - \overline{2x - 2} + x + 5y - \overline{x + 3}\\
&= 7 - \overline{2x - 2} + x - \overline{x + 3}\\
&= 7 - 2x + 2 + x - x - 3 = 6 - 2x.
\end{aligned}$$

27.
$$\begin{aligned}
&ab - \{5 + x - (b + c - ab + x)\} + [x - (b - c - 7)]\\
&= ab - \{5 + x - b - c + ab - x\} + [x - b + c + 7]\\
&= ab - 5 + b + c - ab + x - b + c + 7 = 2 + 2c + x.
\end{aligned}$$

28.
$$\begin{aligned}
&-\{3ax - [5xy - 3z] + z - (4xy + [6z + 7ax] + 3z)\}\\
&= -\{3ax - 5xy + 3z + z - (4xy + 6z + 7ax + 3z)\}\\
&= -\{3ax - 5xy + 4z - 4xy - 9z - 7ax\}\\
&= -\{-4ax - 9xy - 5z\} = 4ax + 9xy + 5z.
\end{aligned}$$

29.
$$\begin{aligned}
&1 - x - \{1 - x - [1 - x - (1 - x) - (x - 1)] - x + 1\}\\
&= 1 - x - \{1 - x - [1 - x - 1 + x - x + 1] - x + 1\}\\
&= 1 - x - \{1 - x - [-x + 1] - x + 1\}\\
&= 1 - x - \{1 - x + x - 1 - x + 1\}\\
&= 1 - x - \{-x + 1\} = 1 - x + x - 1 = 0.
\end{aligned}$$

30.
$$\begin{aligned}
&1 - x - \{1 - [x - 1 + (x - 1) - (1 - x) - x] + 1 - x\}\\
&= 1 - x - \{1 - [x - 1 + x - 1 - 1 + x - x] + 1 - x\}\\
&= 1 - x - \{1 - 2x + 3 + 1 - x\}\\
&= 1 - x - 5 + 3x = 2x - 4.
\end{aligned}$$

31.
$$\begin{aligned}
&a - (b - c) - [a - \{b - c - (b + c - a) + (a - b) + (c - a)\}]\\
&= a - b + c - [a - \{b - c - b - c + a + a - b + c - a\}]\\
&= a - b + c - [a + b + c - a] = a - b + c - b - c = a - 2b.
\end{aligned}$$

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13. Let x = number of dollars brown skin cost.
Then, $3x$ = number of dollars dark skin cost.

$\therefore 3x - x = 400.$

Solving, $x = 200,$
and $3x = 600.$

Hence, the brown skin cost \$200 and the dark skin \$600.

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14. Let $x =$ number of feet above ground.
Then, $172 - x =$ number of feet below ground.
$\therefore x = 16(172 - x) + 2.$
Solving, $x = 162$, the number of feet above ground.

15. Let $x =$ number of pounds wagon weighs.
Then, $4200 - x =$ number of pounds coal weighs.
But $1800 + x =$ number of pounds coal weighs.
$\therefore 1800 + x = 4200 - x.$
Solving, $x = 1200,$
and $4200 - x = 3000.$
Hence, the wagon weighs 1200 pounds and the coal 3000 pounds.

16. Let $x =$ number of cents it costs to mine ton of coal.
Then, $190 - x =$ number of cents it costs to ship ton of coal.
But $x + 10 =$ number of cents it costs to ship ton of coal.
$\therefore x + 10 = 190 - x.$
Solving, $x = 90.$
Hence, it costs \$.90 per ton to mine coal.

17. Let $x =$ number of cents in profit per ton.
Then, $528 - x =$ number of cents each ton cost.
But $22 + x =$ number of cents each ton cost.
$\therefore 22 + x = 528 - x.$
Solving, $x = 253$, number of cents, profit per ton.
Hence, the profit per ton is \$2.53.

18. Let $x =$ number of steamers.
Then, $1025 - x =$ number of sailing vessels.
But $\frac{1}{4}x + 25 =$ number of sailing vessels.
$\therefore \frac{1}{4}x + 25 = 1025 - x.$
Solving, $x = 800,$
and $1025 - x = 225.$
Hence, there were 800 steamers and 225 sailing vessels.

19. Let $x =$ number of feet in width of steamship.
Then, $9x - 2 =$ number of feet in length of steamship.
But $790 =$ number of feet in length of steamship.
$\therefore 9x - 2 = 790.$
Solving, $x = 88$, number of feet in width of vessel.

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20. Let $x =$ number of feet in height of American falls.
Then, $8 + \frac{10}{11}x =$ number of feet in height of Canadian falls.
But $158 =$ number of feet in height of Canadian falls.
$\therefore 8 + \frac{10}{11}x = 158.$
Solving, $x = 165$, number of feet in height of American falls.

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21. Let $x =$ number of feet in length of bridge.
Then, $\frac{3}{5}x - 180 =$ number of feet in width of river.
But $1800 =$ number of feet in width of river.
$\therefore \frac{3}{5}x - 180 = 1800.$
Solving, $x = 3300$, number of feet in length of bridge.

22. Let $x =$ number of boxes of apples.
Then, $24{,}470 - x =$ number of barrels.
But $15x - 170 =$ number of barrels.
$\therefore 15x - 170 = 24{,}470 - x.$
Solving, $x = 1540,$
and $24{,}470 - x = 22{,}930.$
Hence, there were 22,930 barrels and 1540 boxes.

23. Let $x =$ number of 16-candle power lamps.
Then, $2x =$ number of 20-candle power lamps.
$\therefore 16x + 40x = 224.$
Solving, $x = 4,$
and $2x = 8.$
Hence, there were 4 16-candle power and 8 20-candle power.

24. Let $x =$ number of bales India produced.
Then, $50{,}000 + \frac{7}{2}x =$ number of bales United States produced.
But $16{,}700{,}000 - x =$ number of bales United States produced.
$\therefore 50{,}000 + \frac{7}{2}x = 16{,}700{,}000 - x.$
Solving, $x = 3{,}700{,}000,$
and $50{,}000 + \frac{7}{2}x = 13{,}000{,}000.$
Hence, India produced 3,700,000 bales, and United States 13,000,000 bales.

25. Let $x =$ number of gallons per minute small pump delivered.
Then, $4x =$ number of gallons per minute large pump delivered.
$\therefore 4x + 8x = 4800.$
Solving, $x = 400,$
and $4x = 1600.$
Hence, small pump delivered 400 gallons, large pump 1600 gallons.

26. Let $x =$ number of dollars 1 mile of macadam road cost.
Then, $\frac{2}{5}x =$ number of dollars 1 mile of gravel road cost.
$\therefore 4x - \frac{16}{5}x = 2400.$
Solving, $x = 3000,$
and $\frac{2}{5}x = 1200.$
Hence, macadam road cost \$3000 and gravel road \$1200 per mile.

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27. Let $x =$ number of dollars for duty on figs.

Then, $\frac{5}{4}x =$ number of dollars for duty on raisins.

and $\frac{1}{4}x =$ number of dollars for duty on dates.

$$\therefore x = 50 - \tfrac{5}{4}x - \tfrac{1}{4}x.$$

Solving, $x = 20,$

$\frac{5}{4}x = 25,$

and $\frac{1}{4}x = 5.$

Hence, duty on figs was \$20, on raisins was \$25, and on dates \$5.

REVIEW

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14. $a - (e + b) - (c + d) - (e - d + b + c)$
$= 1 - (5 + 2) - (3 + 4) - (5 - 4 + 2 + 3) = 1 - 7 - 7 - 6 = -19.$

15. $3\,ab^2 - 2\,bc^3 - (d^ce^a - ac^2) + 8\,be^2$
$= 3 \cdot 1 \cdot 2^2 - 2 \cdot 2 \cdot 3^3 - (4^3 \cdot 5 - 1 \cdot 3^2) + 8 \cdot 2 \cdot 5^2$
$= 12 - 108 - 311 + 400 = -7.$

16. $5\,ac + b\sqrt{d} + a + 2\,(b - a)(e - d) + bce$
$= 5 \cdot 1 \cdot 3 + 2\sqrt{4} + 1 + 2\,(2 - 1)(5 - 4) + 2 \cdot 3 \cdot 5$
$= 15 + 4 + 1 + 2 + 30 = 15 + 4 + 2 + 30 = 51.$

17. $\sqrt{2\,edb + 4\,e} - \sqrt[3]{9\,a^cc^4} - 2\,c^2d - (abe - abcde)$
$= \sqrt{2 \cdot 5 \cdot 4 \cdot 2 + 4 \cdot 5} - \sqrt[3]{9 \cdot 1^3 \cdot 3^4} - 2 \cdot 3^2 \cdot 4$
$- (1 \cdot 2 \cdot 5 - 1 \cdot 2 \cdot 3 \cdot 4 \cdot 5)$
$= \sqrt{80 + 20} - \sqrt[3]{3 \cdot 3 \times 3 \cdot 3 \times 3 \cdot 3} - 72 - (-110)$
$= 10 - 3 \cdot 3 - 72 + 110 = 10 - 9 - 72 + 110 = 39.$

MULTIPLICATION

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8. $-2(x^2y - xy^2) - 5(xy^2 - x^2y)$
$= -2\,x^2y + 2\,xy^2 - 5\,xy^2 + 5\,x^2y = 3\,x^2y - 3\,xy^2.$

9. $(3\,a - 2)(2\,a - 3) - 6(a - 2)(a - 1)$
$= 6\,a^2 - 13\,a + 6 - 6\,a^2 + 18\,a - 12 = 5\,a - 6.$

10. $8\,x^3 - (4\,x^2 - 2\,xy + y^2)(2\,x + y) = 8\,x^3 - (8\,x^3 + y^3)$
$= 8\,x^3 - 8\,x^3 - y^3 = -y^3.$

11. $(3\,m - 1)(m + 2) - 3\,m(m + 3) + 2(m + 1)$
$= 3\,m^2 + 5\,m - 2 - 3\,m^2 - 9\,m + 2\,m + 2 = -2\,m.$

12. $(a - b)^2 - 2(a^2 - b^2) - 2\,a(-a - b) - 4\,b^2$
$= a^2 - 2\,ab + b^2 - 2\,a^2 + 2\,b^2 + 2\,a^2 + 2\,ab - 4\,b^2 = a^2 - b^2.$

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13. $4(ax - bx + cx - dx) - 3(ax + bx - cx - dx)$
$= 4ax - 4bx + 4cx - 4dx - 3ax - 3bx + 3cx + 3dx$
$= ax - 7bx + 7cx - dx.$

14. $(x+1)(x+2) - 2(x-1)(x-2) + 4(x+2)(x+3)$
$= x^2 + 3x + 2 - 2x^2 + 6x - 4 + 4x^2 + 20x + 24$
$= 3x^2 + 29x + 22.$

15. $(x^2 + 2xy + y^2)(x^2 - 2xy + y^2) - (x^2 + y^2)(x^2 + y^2)$
$= x^4 - 2x^2y^2 + y^4 - x^4 - 2x^2y^2 - y^4 = -4x^2y^2.$

16. $b^4 + (a^2 - ab + b^2)(a^2 + b^2) - (a^3 - b^3)(a + 2b)$
$= b^4 + a^4 - a^3b + 2a^2b^2 - ab^3 + b^4 - a^4 + ab^3 - 2a^3b + 2b^4$
$= 4b^4 - 3a^3b + 2a^2b^2.$

17. $y^3 - [2x^3 - xy(x-y) - y^3] + 2(x-y)(x^2 + xy + y^2)$
$= y^3 - [2x^3 - x^2y + xy^2 - y^3] + 2x^3 - 2y^3$
$= y^3 - 2x^3 + x^2y - xy^2 + y^3 + 2x^3 - 2y^3 = x^2y - xy^2.$

---

2. $m(x-y) + z^2 = 6(3+4) + 0 = 42.$

3. $z + m^2 - (y^3 - 1) = 0 + 36 - (-64 - 1) = 36 + 65 = 101.$

4. $x^2 - y^2 - m^2 + n^2 = 3^2 - (-4)^2 - 6^2 + 2^2 = 9 - 16 - 36 + 4 = -39.$

5. $(x+y)(m-n) + 3z = (3-4)(6-2) + 0 = -4.$

6. $(m+x)^2 - (n-y)^2 - y^4 = (6+3)^2 - (2+4)^2 - (-4)^4$
$= 81 - 36 - 256 = -211.$

7. $xyz - n(x-m)^3 - (nx)^3 = 3(-4)0 - 2(3-6)^3 - (2 \cdot 3)^3$
$= 0 + 54 - 216 = -162.$

8. $3m(m-n) + 4n(y-x) - 7(y+z)$
$= 3 \cdot 6(6-2) + 4 \cdot 2(-4-3) - 7(-4+0) = 72 - 56 + 28 = 44.$

9. $\frac{1}{2}(y - 2n) - \frac{3}{4}(n - 2y)(3y - 4n) = \frac{1}{2}(-4-4) - \frac{3}{4}(2+8)(-12-8)$
$= -4 + 150 = 146.$

10. $x^2y^2(m-n)^2(m+n) + (m+n)^2(m-n)$
$= 9 \cdot 16(6-2)^2(6+2) + (6+2)^2(6-2) = 18{,}432 + 256 = 18{,}688.$

11. $(x-y)^2 - xy(x-y)(x+y)(x^2+y^2)$
$= (3+4)^2 - 3(-4)(3+4)(3-4)(9+16) = 49 - 2100 = -2051.$

12. $3m(x - y - n)^2 - (y - n - x)(n - x - y)$
$= 18(3 + 4 - 2)^2 - (-4 - 2 - 3)(2 - 3 + 4) = 450 + 27 = 477.$

13. $(2x + y)^n - (x^2 - 2y)^x - (m+n)^2(x + y + z)^3$
$= (6-4)^2 - (9+8)^3 - (6+2)^2(3 - 4 + 0)^3 = 4 - 4913 + 64 = -4845.$

14. $(x + y + z)^2 - xy(y + z - x)(x + z - y) - z(x + y - z)$
$= (3 - 4 + 0)^2 - 3(-4)(-4 + 0 - 3)(3 + 0 + 4) - 0(3 - 4 - 0)$
$= 1 - 588 - 0 = -587.$

15. $(m + n + x)^n - (m + n - x)^n - (m - n + x)^n(-m + n + x)^n$
$= (6 + 2 + 3)^2 - (6 + 2 - 3)^2 - (6 - 2 + 3)^2(-6 + 2 + 3)^2$
$= 121 - 25 - 49 = 47.$

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16. $(a-b+c)^2=(1-2+3)^2=4.$

$$a^2+b^2+c^2-2ab+2ac-2bc=1+2^2+3^2-2\cdot1\cdot2+2\cdot1\cdot3-2\cdot2\cdot3$$
$$=1+4+9-4+6-12=4.$$
$$\therefore (a-b+c)^2=a^2+b^2+c^2-2ab+2ac-2bc.$$
$$(a-b+c)^2=(4-2-1)^2=1.$$
$$a^2+b^2+c^2-2ab+2ac-2bc$$
$$=4^2+2^2+(-1)^2-2\cdot4\cdot2+2\cdot4(-1)-2\cdot2(-1)$$
$$=16+4+1-16-8+4=1.$$
$$\therefore (a-b+c)^2=a^2+b^2+c^2-2ab+2ac-2bc.$$

17. Let $a=1,\ b=2,\ c=4.$

$$(a+b)(b+c)(c+a)+abc=(1+2)(2+4)(4+1)+1\cdot2\cdot4=98.$$
$$(a+b+c)(ab+bc+ac)=(1+2+4)(1\cdot2+2\cdot4+1\cdot4)=98.$$
$$\therefore (a+b)(b+c)(c+a)+abc=(a+b+c)(ab+bc+ac).$$

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21. $(-5n-b)(-5n+b)=(-5n)^2-(b)^2=25n^2-b^2.$

23. $(-4+3a)(-4-3a)=(-4)^2-(3a)^2=16-9a^2.$

24. $(x^{m-1}+y^{n+1})(x^{m-1}-y^{n+1})=(x^{m-1})^2-(y^{n+1})^2=x^{2m-2}-y^{2n+2}.$

25. $(3x^m+7y^n)(3x^m-7y^n)=(3x^m)^2-(7y^n)^2=9x^{2m}-49y^{2n}.$

26. $(5a^3b^2+2x^s)(5a^3b^2-2x^s)=(5a^3b^2)^2-(2x^s)^2=25a^6b^4-4x^{2s}.$

28. $(a+x-y)(a-x+y)=[a+(x-y)][a-(x-y)]$
$=a^2-(x-y)^2=a^2-x^2+2xy-y^2.$

29. $(x+c-d)(x-c+d)=[x+(c-d)][x-(c-d)]$
$=x^2-(c-d)^2=x^2-c^2+2cd-d^2.$

30. $(r+p-q)(r-p+q)=[r+(p-q)][r-(p-q)]$
$=r^2-(p-q)^2=r^2-p^2+2pq-q^2.$

31. $(r+p+q)(r-p-q)=[r+(p+q)][r-(p+q)]$
$=r^2-(p+q)^2=r^2-p^2-2pq-q^2.$

32. $(x+b+n)(x-b-n)=[x+(b+n)][x-(b+n)]$
$=x^2-(b+n)^2=x^2-b^2-2bn-n^2.$

33. $(y+c+d)(y+c-d)=[(y+c)+d][(y+c)-d]$
$=(y+c)^2-d^2=y^2+2cy+c^2-d^2.$

34. $(a+x+y)(a+x-y)=[(a+x)+y][(a+x)-y]$
$=(a+x)^2-y^2=a^2+2ax+x^2-y^2.$

35. $(x^2+2x+1)(x^2+2x-1)=[(x^2+2x)+1][(x^2+2x)-1]$
$=(x^2+2x)^2-1^2=x^4+4x^3+4x^2-1.$

36. $(x^2+2x-1)(x^2-2x+1)=[x^2+(2x-1)][x^2-(2x-1)]$
$=(x^2)^2-(2x-1)^2=x^4-(4x^2-4x+1)=x^4-4x^2+4x-1.$

37. $(x^2+3x-2)(x^2-3x+2)=[x^2+(3x-2)][x^2-(3x-2)]$
$=(x^2)^2-(3x-2)^2=x^4-(9x^2-12x+4)=x^4-9x^2+12x-4.$

38. $(m^4-2m^2+1)(m^4+2m^2+1)=[(m^4+1)-2m^2][(m^4+1)+2m^2]$
$=(m^4+1)^2-(2m^2)^2=m^8+2m^4+1-4m^4=m^8-2m^4+1.$

39. $(2x+3y-4z)(2x+3y+4z)=[(2x+3y)-4z][(2x+3y)+4z]$
$=(2x+3y)^2-(4z)^2=4x^2+12xy+9y^2-16z^2.$

40. $(2x^2-xy+3y^2)(2x^2+xy-3y^2)$
$=[2x^2-(xy-3y^2)][2x^2+(xy-3y^2)]=(2x^2)^2-(xy-3y^2)^2$
$=4x^4-(x^2y^2-6xy^3+9y^4)=4x^4-x^2y^2+6xy^3-9y^4.$

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41. $(x^2 + xy + y^2)(x^2 - xy + y^2) = [(x^2 + y^2) + xy][(x^2 + y^2) - xy]$
$= (x^2 + y^2)^2 - (xy)^2 = x^4 + 2x^2y^2 + y^4 - x^2y^2 = x^4 + x^2y^2 + y^4.$

42. $[(a + b) + (c + d)][(a + b) - (c + d)] = (a + b)^2 - (c + d)^2$
$= a^2 + 2ab + b^2 - c^2 - 2cd - d^2.$

43. $(a + b + x + y)(a + b - x - y) = (a + b)^2 - (x + y)^2$
$= a^2 + 2ab + b^2 - x^2 - 2xy - y^2.$

44. $(a + b + m - n)(a + b - m + n) = (a + b)^2 - (m - n)^2$
$= a^2 + 2ab + b^2 - m^2 + 2mn - n^2.$

45. $(x - m + y - n)(x - m - y + n) = (x - m)^2 - (y - n)^2$
$= x^2 - 2mx + m^2 - y^2 + 2ny - n^2.$

46. $(p - q + r + s)(p - q - r - s) = (p - q)^2 - (r + s)^2$
$= p^2 - 2pq + q^2 - r^2 - 2rs - s^2.$

47. $(a - m - b - n)(a + m - b + n) = (a - b - m - n)(a - b + m + n)$
$= (a - b)^2 - (m + n)^2$
$= a^2 - 2ab + b^2 - m^2 - 2mn - n^2.$

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29. $(\overline{x + y} - 1)(\overline{x + y} + 2) = (x + y)^2 + 1(x + y) - 2$
$= x^2 + 2xy + y^2 + x + y - 2.$

30. $(\overline{x - y} - 2)(\overline{x - y} - 8) = (x - y)^2 - 10(x - y) + 16$
$= x^2 - 2xy + y^2 - 10x + 10y + 16.$

31. $(\overline{x^2 + x} - 1)(\overline{x^2 + x} + 3) = (x^2 + x)^2 + 2(x^2 + x) - 3$
$= x^4 + 2x^3 + x^2 + 2x^2 + 2x - 3$
$= x^4 + 2x^3 + 3x^2 + 2x - 3.$

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19. Let $x =$ number of bunches exported to United States.
Then, $16{,}000{,}000 - x =$ number of bunches exported to other countries.
$x - 600{,}000 = 10(16{,}000{,}000 - x).$
Solving, $x = 14{,}600{,}000.$
Hence, there were 14,600,000 bunches exported to United States.

20. Let $x =$ number of cents fuel cost per dollar of income.
Then, $60 - x =$ number of cents expended for other things.
$\frac{1}{3}(60 - x) - 5 =$ number of cents fuel cost per dollar of income.
$\therefore \frac{1}{3}(60 - x) - 5 = x.$
Solving, $x = 11\frac{1}{4}.$
Hence, fuel cost $11\frac{1}{4}$¢ per dollar of income.

21. Let $x =$ number of dollars devoted to education.
Then, $x - 6{,}500{,}000 =$ number of dollars devoted to police protection.
$98{,}100{,}000 = 3(x + x - 6{,}500{,}000) + 6{,}600{,}000.$
Solving, $x = 18{,}500{,}000,$
and $x - 6{,}500{,}000 = 12{,}000{,}000.$
Hence, \$18,500,000 was devoted to education, and \$12,000,000 was devoted to police protection.

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22. Let $x =$ number of cents cherries cost per pound in 8-lb. boxes.

Then, $x - 2 =$ number of cents cherries cost per pound in 5-lb. boxes.

$\therefore 8x =$ number of cents paid for an 8-lb. box,

and $5(x-2) =$ number of cents paid for a 5-lb. box.

But $4x - 2 =$ number of cents paid for a 5-lb. box.

$\therefore 5(x-2) = 4x - 2.$

Solving, $x = 8,$

and $x - 2 = 6.$

Hence, cherries in 8-lb. boxes were 8¢ per pound,

and cherries in 5-lb. boxes were 6¢ per pound.

23. Let $x =$ number of feet in width of floor.

Then, $x + 4 =$ number of feet in length of floor.

$x(x+4) =$ number of square feet in area of floor.

$x(x+4) - 112 =$ number of square feet in area of rug.

$(x-4)(x+4-4) =$ number of square feet in area of rug.

$\therefore (x-4)(x+4-4) = x(x+4) - 112.$

Solving, $x = 14.$

Then, $x(x+4) = 252,$

and $x(x+4) - 112 = 140.$

Hence, area of rug is 140 sq. ft., and area of floor is 252 sq. ft.

24. Let $x =$ number of hundred violets sold in December.

Then, $240 - x =$ number of hundred violets sold in January.

$2x =$ number of dollars received for violets in December.

Then, $405 - 2x =$ number of dollars received for violets in January.

But $\frac{3}{2}(240 - x) =$ number of dollars received for violets in January.

$\therefore 405 - 2x = \frac{3}{2}(240 - x).$

Solving, $x = 90,$

and $240 - x = 150.$

Hence, 90 hundred violets were sold in Dec. and 150 hundred in Jan.

25. Let $x =$ number of cents paid for second class ticket.

Then, $415 + x =$ number of cents paid for first class ticket.

$8x =$ number of cents paid for eight second class.

$3(415 + x) =$ number of cents paid for three first class.

But $2[3(415 + x)] - 570 =$ number of cents paid for eight second class.

$\therefore 8x = 2[3(415 + x)] - 570.$

Solving, $x = 960,$

and $415 + x = 1375.$

Hence, price of second class ticket is $9.60,

and price of first class ticket is $13.75.

26. Let $x =$ number of tons put on board per hour.

By 1st condition, $\frac{25}{2}(x - 63) =$ number of tons put on board.

By 2d condition, $10(x - 13) =$ number of tons put on board.

$\therefore \frac{25}{2}(x - 63) = 10(x - 13).$

Solving, $x = 263,$ number of tons put on board per hour.

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DIVISION

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18.

$$
\begin{array}{l|l}
ax^3 - a^2x^2 - bx^2 + b^2 & ax - b \\
\underline{ax^3 \qquad\quad - bx^2} & \overline{x^2 - ax - b} \\
\quad - a^2x^2 \qquad + b^2 & \\
\quad \underline{- a^2x^2 + abx} & \\
\qquad\quad - abx + b^2 & \\
\qquad\quad \underline{- abx + b^2} &
\end{array}
$$

TEST. — Let $a = 1$, $b = 1$, $x = 5$. Then, the dividend becomes $125 - 25 - 25 + 1$, or 76; the divisor $5 - 1$, or 4; and the quotient $25 - 5 - 1$, or 19. Since $76 \div 4 = 19$, it may be assumed that the quotient is correct.

20.

$$
\begin{array}{l|l}
a^4 - 4\,a^3x + 6\,a^2x^2 - 4\,ax^3 + x^4 & a^2 - 2\,ax + x^2 \\
\underline{a^4 - 2\,a^3x + \quad a^2x^2} & \overline{a^2 - 2\,ax + x^2} \\
\quad - 2\,a^3x + 5\,a^2x^2 - 4\,ax^3 & \\
\quad \underline{- 2\,a^3x + 4\,a^2x^2 - 2\,ax^3} & \\
\qquad\qquad a^2x^2 - 2\,ax^3 + x^4 & \\
\qquad\qquad \underline{a^2x^2 - 2\,ax^3 + x^4} &
\end{array}
$$

TEST. — Let $a = 1$, $x = 10$. Then, the dividend becomes $1 - 40 + 600 - 4000 + 10{,}000$, or 6561; the divisor $1 - 20 + 100$, or 81; and the quotient also 81. Since $6561 \div 81 = 81$, it may be assumed that the quotient is correct.

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23.

$$
\begin{array}{l|l}
x^4 + 81 & x - 3 \\
\underline{x^4 - 3\,x^3} & \overline{x^3 + 3\,x^2 + 9\,x + 27 + \dfrac{162}{x - 3}} \\
\qquad 3\,x^3 & \\
\qquad \underline{3\,x^3 - 9\,x^2} & \\
\qquad\qquad 9\,x^2 & \\
\qquad\qquad \underline{9\,x^2 - 27\,x} & \\
\qquad\qquad\qquad 27\,x + 81 & \\
\qquad\qquad\qquad \underline{27\,x - 81} & \\
\qquad\qquad\qquad\qquad 162 &
\end{array}
$$

24.

$$
\begin{array}{l|l}
x^5 + 32 & x + 2 \\
\underline{x^5 + 2\,x^4} & \overline{x^4 - 2\,x^3 + 4\,x^2 - 8\,x + 16} \\
\quad - 2\,x^4 & \\
\quad \underline{- 2\,x^4 - 4\,x^3} & \\
\qquad\quad 4\,x^3 & \\
\qquad\quad \underline{4\,x^3 + 8\,x^2} & \\
\qquad\qquad - 8\,x^2 & \\
\qquad\qquad \underline{- 8\,x^2 - 16\,x} & \\
\qquad\qquad\qquad 16\,x + 32 & \\
\qquad\qquad\qquad \underline{16\,x + 32} &
\end{array}
$$

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25.

$$\begin{array}{l|l}
x^6 - y^6 & x^2 + y^2 \\
\underline{x^6 + x^4y^2} & x^4 - x^2y^2 + y^4 + \dfrac{-2y^6}{x^2+y^2} \\
\quad -x^4y^2 & \\
\quad \underline{-x^4y^2 - x^2y^4} & \\
\qquad\qquad x^2y^4 - y^6 & \\
\qquad\qquad \underline{x^2y^4 + y^6} & \\
\qquad\qquad\quad -2y^6 &
\end{array}$$

26.

$$\begin{array}{l|l}
a^7 + b^7 & a + b \\
\underline{a^7 + a^6b} & a^6 - a^5b + a^4b^2 - a^3b^3 + a^2b^4 - ab^5 + b^6 \\
\quad -a^6b & \\
\quad \underline{-a^6b - a^5b^2} & \\
\qquad\quad a^5b^2 & \\
\qquad\quad \underline{a^5b^2 + a^4b^3} & \\
\qquad\qquad -a^4b^3 & \\
\qquad\qquad \underline{-a^4b^3 - a^3b^4} & \\
\qquad\qquad\qquad a^3b^4 & \\
\qquad\qquad\qquad \underline{a^3b^4 + a^2b^5} & \\
\qquad\qquad\qquad\quad -a^2b^5 & \\
\qquad\qquad\qquad\quad \underline{-a^2b^5 - ab^6} & \\
\qquad\qquad\qquad\qquad\quad ab^6 + b^7 & \\
\qquad\qquad\qquad\qquad\quad \underline{ab^6 + b^7} &
\end{array}$$

27.

$$\begin{array}{l|l}
m^5 - n^5 & m + n \\
\underline{m^5 + m^4n} & m^4 - m^3n + m^2n^2 - mn^3 + n^4 + \dfrac{-2n^5}{m+n} \\
\quad -m^4n & \\
\quad \underline{-m^4n - m^3n^2} & \\
\qquad\quad m^3n^2 & \\
\qquad\quad \underline{m^3n^2 + m^2n^3} & \\
\qquad\qquad -m^2n^3 & \\
\qquad\qquad \underline{-m^2n^3 - mn^4} & \\
\qquad\qquad\qquad mn^4 - n^5 & \\
\qquad\qquad\qquad \underline{mn^4 + n^5} & \\
\qquad\qquad\qquad\quad -2n^5 &
\end{array}$$

28.

$$\begin{array}{l|l}
m^5 + n^5 & m + n \\
\underline{m^5 + m^4n} & m^4 - m^3n + m^2n^2 - mn^3 + n^4 \\
\quad -m^4n & \\
\quad \underline{-m^4n - m^3n^2} & \\
\qquad\quad m^3n^2 & \\
\qquad\quad \underline{m^3n^2 + m^2n^3} & \\
\qquad\qquad -m^2n^3 & \\
\qquad\qquad \underline{-m^2n^3 - mn^4} & \\
\qquad\qquad\qquad mn^4 + n^5 & \\
\qquad\qquad\qquad \underline{mn^4 + n^5} &
\end{array}$$

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29.
$$
\begin{array}{l|l}
a^6 + 5a^5 - a^3 + 2a + 3 & a - 1 \\
\hline
\underline{a^6 - a^5} & a^5 + 6a^4 + 6a^3 + 5a^2 + 5a + 7 + \dfrac{10}{a-1} \\
\quad 6a^5 & \\
\quad \underline{6a^5 - 6a^4} & \\
\qquad 6a^4 - a^3 & \\
\qquad \underline{6a^4 - 6a^3} & \\
\qquad\quad 5a^3 & \\
\qquad\quad \underline{5a^3 - 5a^2} & \\
\qquad\qquad 5a^2 + 2a & \\
\qquad\qquad \underline{5a^2 - 5a} & \\
\qquad\qquad\quad 7a + 3 & \\
\qquad\qquad\quad \underline{7a - 7} & \\
\qquad\qquad\qquad 10 &
\end{array}
$$

30.
$$
\begin{array}{l|l}
x^7 + 2x^6 - 2x^4 + 2x^2 - 1 & x + 1 \\
\hline
\underline{x^7 + x^6} & x^6 + x^5 - x^4 - x^3 + x^2 + x - 1 \\
\quad x^6 & \\
\quad \underline{x^6 + x^5} & \\
\qquad -x^5 - 2x^4 & \\
\qquad \underline{-x^5 - x^4} & \\
\qquad\quad -x^4 & \\
\qquad\quad \underline{-x^4 - x^3} & \\
\qquad\qquad x^3 + 2x^2 & \\
\qquad\qquad \underline{x^3 + x^2} & \\
\qquad\qquad\quad x^2 & \\
\qquad\qquad\quad \underline{x^2 + x} & \\
\qquad\qquad\qquad -x - 1 & \\
\qquad\qquad\qquad \underline{-x - 1} &
\end{array}
$$

31.
$$
\begin{array}{l|l}
2x^8 - x^7 + x^5 + 2x^4 - x^2 + 5 & x + 1 \\
\hline
\underline{2x^8 + 2x^7} & 2x^7 - 3x^6 + 3x^5 - 2x^4 + 4x^3 \\
\quad -3x^7 & -4x^2 + 3x - 3 + \dfrac{8}{x+1} \\
\quad \underline{-3x^7 - 3x^6} & \\
\qquad 3x^6 + x^5 & \\
\qquad \underline{3x^6 + 3x^5} & \\
\qquad\quad -2x^5 + 2x^4 & \\
\qquad\quad \underline{-2x^5 - 2x^4} & \\
\qquad\qquad 4x^4 & \\
\qquad\qquad \underline{4x^4 + 4x^3} & \\
\qquad\qquad\quad -4x^3 - x^2 & \\
\qquad\qquad\quad \underline{-4x^3 - 4x^2} & \\
\qquad\qquad\qquad 3x^2 & \\
\qquad\qquad\qquad \underline{3x^2 + 3x} & \\
\qquad\qquad\qquad\quad -3x + 5 & \\
\qquad\qquad\qquad\quad \underline{-3x - 3} & \\
\qquad\qquad\qquad\qquad 8 &
\end{array}
$$

32.
$$\begin{array}{r|l}
y^5+3y^4+5y^3+3y^2+3y+5 & y+1 \\ \cline{2-2}
y^5+y^4 & y^4+2y^3+3y^2+3+\dfrac{2}{y+1} \\ \cline{1-1}
2y^4+5y^3 & \\
2y^4+2y^3 & \\ \cline{1-1}
3y^3+3y^2 & \\
3y^3+3y^2 & \\ \cline{1-1}
3y+5 & \\
3y+3 & \\ \cline{1-1}
2 &
\end{array}$$

33.
$$\begin{array}{r|l}
2n^5-4n^4-3n^3+7n^2-3n+2 & n-2 \\ \cline{2-2}
2n^5-4n^4 & 2n^4-3n^2+n-1 \\ \cline{1-1}
-3n^3+7n^2 & \\
-3n^3+6n^2 & \\ \cline{1-1}
n^2-3n & \\
n^2-2n & \\ \cline{1-1}
-\quad n+2 & \\
-\quad n+2 & \\ \cline{1-1}
\end{array}$$

34.
$$\begin{array}{r|l}
1 & 1+x \\ \cline{2-2}
1+x & 1-x+x^2-x^3+x^4 \\ \cline{1-1}
-x & \\
-x-x^2 & \\ \cline{1-1}
x^2 & \\
x^2+x^3 & \\ \cline{1-1}
-x^3 & \\
-x^3-x^4 & \\ \cline{1-1}
x^4 & \\
x^4+x^5 & \\ \cline{1-1}
\text{Remainder,}\quad -x^5 &
\end{array}$$

35.
$$\begin{array}{r|l}
1 & 1-x \\ \cline{2-2}
1-x & 1+x+x^2+x^3+x^4 \\ \cline{1-1}
x & \\
x-x^2 & \\ \cline{1-1}
x^2 & \\
x^2-x^3 & \\ \cline{1-1}
x^3 & \\
x^3-x^4 & \\ \cline{1-1}
x^4 & \\
x^4-x^5 & \\ \cline{1-1}
\text{Remainder,}\quad x^5 &
\end{array}$$

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36.

$$
\begin{array}{l|l}
a^3 - 6a^2 + 12a - 8 - b^3 & a - 2 - b \\
\underline{a^3 - 2a^2 - a^2b} & \overline{a^2 - 4a + 4 + ab - 2b + b^2} \\
\quad - 4a^2 + 12a + a^2b - 8 & \\
\quad \underline{- 4a^2 + 8a + 4ab} & \\
\qquad 4a - 8 + a^2b - 4ab & \\
\qquad \underline{4a - 8 - 4b} & \\
\qquad\qquad a^2b - 4ab + 4b & \\
\qquad\qquad \underline{a^2b - 2ab - ab^2} & \\
\qquad\qquad\quad - 2ab + 4b + ab^2 & \\
\qquad\qquad\quad \underline{- 2ab + 4b + 2b^2} & \\
\qquad\qquad\qquad ab^2 - 2b^2 - b^3 & \\
\qquad\qquad\qquad \underline{ab^2 - 2b^2 - b^3} &
\end{array}
$$

37.

$$
\begin{array}{l|l}
y^5 + 32x^5 & y^4 - 2xy^3 + 4x^2y^2 - 8x^3y + 16x^4 \\
\underline{y^5 - 2xy^4 + 4x^2y^3 - 8x^3y^2 + 16x^4y} & \overline{y + 2x} \\
\quad 2xy^4 - 4x^2y^3 + 8x^3y^2 - 16x^4y + 32x^5 & \\
\quad \underline{2xy^4 - 4x^2y^3 + 8x^3y^2 - 16x^4y + 32x^5} &
\end{array}
$$

38.

$$
\begin{array}{l|l}
x^3 - 3xyz + y^3 + z^3 & x + y + z \\
\underline{x^3 + x^2y + x^2z} & \overline{x^2 - xy - xz + y^2 - yz + z^2} \\
\quad - x^2y - x^2z - 3xyz & \\
\quad \underline{- x^2y - xy^2 - xyz} & \\
\quad - x^2z + xy^2 - 2xyz & \\
\quad \underline{- x^2z - xyz - xz^2} & \\
\qquad xy^2 - xyz + xz^2 + y^3 & \\
\qquad \underline{xy^2 + y^2z + y^3} & \\
\qquad\quad - xyz + xz^2 - y^2z & \\
\qquad\quad \underline{- xyz - y^2z - yz^2} & \\
\qquad\qquad xz^2 + yz^2 + z^3 & \\
\qquad\qquad \underline{xz^2 + yz^2 + z^3} &
\end{array}
$$

39.

$$
\begin{array}{l|l}
m^3 + 3m^2n + 3mn^2 + n^3 + x^3 & m + n + x \\
\underline{m^3 + m^2n + m^2x} & \overline{m^2 + 2mn + n^2 - mx - nx + x^2} \\
\quad 2m^2n + 3mn^2 - m^2x + n^3 & \\
\quad \underline{2m^2n + 2mn^2 + 2mnx} & \\
\qquad mn^2 + n^3 - m^2x - 2mnx & \\
\qquad \underline{mn^2 + n^3 + n^2x} & \\
\qquad\quad - m^2x - 2mnx - n^2x & \\
\qquad\quad \underline{- m^2x - mnx - mx^2} & \\
\qquad\qquad - mnx - n^2x + mx^2 & \\
\qquad\qquad \underline{- mnx - n^2x - nx^2} & \\
\qquad\qquad\quad mx^2 + nx^2 + x^3 & \\
\qquad\qquad\quad \underline{mx^2 + nx^2 + x^3} &
\end{array}
$$

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40.
$$\begin{array}{l|l}
a^3 - 2a^2c + 4ac^2 - ax^2 - 4c^2x + 2cx^2 & a - x \\ \cline{2-2}
\underline{a^3 - a^2x} & a^2 + ax - 2ac - 2cx + 4c^2 \\
\quad a^2x & \\
\quad \underline{a^2x \qquad\qquad - ax^2} & \\
\quad -2a^2c + 4ac^2 \qquad - 4c^2x & \\
\quad \underline{-2a^2c + 2acx} & \\
\qquad -2acx + 4ac^2 - 4c^2x + 2cx^2 & \\
\qquad \underline{-2acx \qquad\qquad\qquad + 2cx^2} & \\
\qquad\qquad 4ac^2 - 4c^2x & \\
\qquad\qquad \underline{4ac^2 - 4c^2x} &
\end{array}$$

41.
$$\begin{array}{l|l}
a^3 + 3abc - b^3 + c^3 & a^2 + ab - ac + b^2 + bc + c^2 \\ \cline{2-2}
\underline{a^3 + a^2b - a^2c + ab^2 + abc + ac^2} & a - b + c \\
\quad -a^2b + a^2c - ab^2 + 2abc - ac^2 - b^3 + c^3 & \\
\quad \underline{-a^2b \qquad - ab^2 + abc \qquad - b^3 - b^2c - bc^2} & \\
\qquad a^2c \qquad + abc - ac^2 + b^2c + bc^2 + c^3 & \\
\qquad \underline{a^2c \qquad + abc - ac^2 + b^2c + bc^2 + c^3} &
\end{array}$$

42.
$$\begin{array}{l|l}
\frac{9}{16}m^4 - \frac{3}{4}m^3 - \frac{7}{4}m^2 + \frac{4}{3}m + \frac{16}{9} & \frac{3}{2}m^2 - m - \frac{4}{3} \\ \cline{2-2}
\underline{\frac{9}{16}m^4 - \frac{3}{8}m^3 - m^2} & \frac{3}{8}m^2 - \frac{1}{4}m - \frac{2}{3} \\
\quad -\frac{3}{8}m^3 - \frac{3}{4}m^2 + \frac{4}{3}m & \\
\quad \underline{-\frac{3}{8}m^3 + \frac{1}{4}m^2 + \frac{8}{12}m} & \\
\qquad -m^2 + \frac{2}{3}m + \frac{16}{9} & \\
\qquad \underline{-m^2 + \frac{2}{3}m + \frac{16}{9}} &
\end{array}$$

43.
$$\begin{array}{l|l}
\frac{9}{8}x^4 - \frac{3}{2}ax^3 + \frac{1}{2}a^2x^2 - \frac{2}{9}a^4 & \frac{3}{4}x^2 - \frac{1}{2}ax + \frac{1}{3}a^2 \\ \cline{2-2}
\underline{\frac{9}{8}x^4 - \frac{3}{4}ax^3 + \frac{1}{2}a^2x^2} & \frac{3}{2}x^2 - ax - \frac{2}{3}a^2 \\
\quad -\frac{3}{4}ax^3 \qquad\qquad - \frac{2}{9}a^4 & \\
\quad \underline{-\frac{3}{4}ax^3 + \frac{1}{2}a^2x^2 - \frac{1}{3}a^3x} & \\
\qquad -\frac{1}{2}a^2x^2 + \frac{1}{3}a^3x - \frac{2}{9}a^4 & \\
\qquad \underline{-\frac{1}{2}a^2x^2 + \frac{1}{3}a^3x - \frac{2}{9}a^4} &
\end{array}$$

44.
$$\begin{array}{l|l}
\frac{1}{8}x^3 - \frac{1}{2}xyz + \frac{1}{27}y^3 + z^3 & \frac{1}{2}x + \frac{1}{3}y + z \\ \cline{2-2}
\underline{\frac{1}{8}x^3 + \frac{1}{12}x^2y + \frac{1}{4}x^2z} & \frac{1}{4}x^2 - \frac{1}{6}xy - \frac{1}{2}xz + \frac{1}{9}y^2 - \frac{1}{3}yz + z^2 \\
\quad -\frac{1}{12}x^2y - \frac{1}{4}x^2z - \frac{1}{2}xyz & \\
\quad \underline{-\frac{1}{12}x^2y - \frac{1}{18}xy^2 - \frac{1}{6}xyz} & \\
\quad -\frac{1}{4}x^2z + \frac{1}{18}xy^2 - \frac{1}{3}xyz & \\
\quad \underline{-\frac{1}{4}x^2z \qquad\qquad - \frac{1}{6}xyz - \frac{1}{2}xz^2} & \\
\qquad \frac{1}{18}xy^2 - \frac{1}{6}xyz + \frac{1}{2}xz^2 + \frac{1}{27}y^3 & \\
\qquad \underline{\frac{1}{18}xy^2 \qquad\qquad + \frac{1}{9}y^2z + \frac{1}{27}y^3} & \\
\qquad\quad -\frac{1}{6}xyz + \frac{1}{2}xz^2 - \frac{1}{9}y^2z & \\
\qquad\quad \underline{-\frac{1}{6}xyz \qquad\qquad - \frac{1}{9}y^2z - \frac{1}{3}yz^2} & \\
\qquad\qquad \frac{1}{2}xz^2 \qquad\qquad + \frac{1}{3}yz^2 + z^3 & \\
\qquad\qquad \underline{\frac{1}{2}xz^2 \qquad\qquad + \frac{1}{3}yz^2 + z^3} &
\end{array}$$

45.
$$\begin{array}{l|l}
r^{2n} + 11r^n + 30 & r^n + 6 \\ \cline{2-2}
\underline{r^{2n} + 6r^n} & r^n + 5 \\
\qquad 5r^n + 30 & \\
\qquad \underline{5r^n + 30} &
\end{array}$$

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46.

$$\begin{array}{l|l}
x^{3n-3} + y^{3n+3} & x^{n-1} + y^{n+1} \\ \cline{2-2}
\underline{x^{3n-3} + x^{2n-2}y^{n+1}} & x^{2n-2} - x^{n-1}y^{n+1} + y^{2n+2} \\
\qquad - x^{2n-2}y^{n+1} & \\
\qquad \underline{- x^{2n-2}y^{n+1} - x^{n-1}y^{2n+2}} & \\
\qquad\qquad\qquad x^{n-1}y^{2n+2} + y^{3n+3} & \\
\qquad\qquad\qquad \underline{x^{n-1}y^{2n+2} + y^{3n+3}} &
\end{array}$$

47.

$$\begin{array}{l|l}
x^n + y^n & x + y \\ \cline{2-2}
\underline{x^n + x^{n-1}y} & x^{n-1} - x^{n-2}y + x^{n-3}y^2 - x^{n-4}y^3 + x^{n-5}y^4 \\
\quad - x^{n-1}y & \\
\quad \underline{- x^{n-1}y - x^{n-2}y^2} & \\
\qquad\qquad x^{n-2}y^2 & \\
\qquad\qquad \underline{x^{n-2}y^2 + x^{n-3}y^3} & \\
\qquad\qquad\qquad - x^{n-3}y^3 & \\
\qquad\qquad\qquad \underline{- x^{n-3}y^3 - x^{n-4}y^4} & \\
\qquad\qquad\qquad\qquad x^{n-4}y^4 & \\
\qquad\qquad\qquad\qquad \underline{x^{n-4}y^4 + x^{n-5}y^5} & \\
\text{Remainder,} \qquad\qquad\qquad\qquad - x^{n-5}y^5 + y^n &
\end{array}$$

48.

$$\begin{array}{l|l}
2 - 3n^x + 13n^{2x} + 23n^{3x} - 11n^{4x} + 6n^{5x} & 2 + 3n^x \\ \cline{2-2}
\underline{2 + 3n^x} & 1 - 3n^x + 11n^{2x} - 5n^{3x} + 2n^{4x} \\
\quad - 6n^x + 13n^{2x} & \\
\quad \underline{- 6n^x - 9n^{2x}} & \\
\qquad\qquad 22n^{2x} + 23n^{3x} & \\
\qquad\qquad \underline{22n^{2x} + 33n^{3x}} & \\
\qquad\qquad\qquad - 10n^{3x} - 11n^{4x} & \\
\qquad\qquad\qquad \underline{- 10n^{3x} - 15n^{4x}} & \\
\qquad\qquad\qquad\qquad 4n^{4x} + 6n^{5x} & \\
\qquad\qquad\qquad\qquad \underline{4n^{4x} + 6n^{5x}} &
\end{array}$$

49.

$$\begin{array}{l|l}
a^{p+3} + a^{p+1} + a^{p-1} & a^{p+1} + a^p + a^{p-1} \\ \cline{2-2}
\underline{a^{p+3} + a^{p+2} + a^{p+1}} & a^2 - a + 1 \\
\quad - a^{p+2} + a^{p-1} & \\
\quad \underline{- a^{p+2} - a^{p+1} - a^p} & \\
\qquad\qquad a^{p+1} + a^p + a^{p-1} & \\
\qquad\qquad \underline{a^{p+1} + a^p + a^{p-1}} &
\end{array}$$

50.

$$\begin{array}{l|l}
x^{2r+1}y^{2s} + 2x^{2r+3}y^{2s+1} + x^{2r+5}y^{2s+2} & x^ry^{s-1} + x^{r+2}y^s \\ \cline{2-2}
\underline{x^{2r+1}y^{2s} + x^{2r+3}y^{2s+1}} & x^{r+1}y^{s+1} + x^{r+3}y^{s+2} \\
\qquad\qquad x^{2r+3}y^{2s+1} + x^{2r+5}y^{2s+2} & \\
\qquad\qquad \underline{x^{2r+3}y^{2s+1} + x^{2r+5}y^{2s+2}} &
\end{array}$$

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51. $$\begin{array}{l|l}
6a^{2m}+5a^{2m-1}-10a^{2m-2}+20a^{2m-3}-16a^{2m-4} & 2a^m+3a^{m-1}-4a^{m-2} \\ \hline
6a^{2m}+9a^{2m-1}-12a^{2m-2} & 3a^m-2a^{m-1}+4a^{m-2} \\ \hline
\quad -4a^{2m-1}+2a^{2m-2}+20a^{2m-3} & \\
\quad -4a^{2m-1}-6a^{2m-2}+8a^{2m-3} & \\ \hline
\qquad\qquad 8a^{2m-2}+12a^{2m-3}-16a^{2m-4} & \\
\qquad\qquad 8a^{2m-2}+12a^{2m-3}-16a^{2m-4} & \\ \hline
\end{array}$$

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53. $$\begin{array}{l|l}
1+0+0+0+0+0+0+8+7 & 1+2+1 \\ \hline
1+2+1 & 1-2+3-4+5-6+7 \\ \hline
\quad -2-1 & =x^6-2x^5+3x^4-4x^3+5x^2-6x+7 \\
\quad -2-4-2 & \\ \hline
\qquad 3+2 & \\
\qquad 3+6+3 & \\ \hline
\qquad\quad -4-3 & \\
\qquad\quad -4-8-4 & \\ \hline
\qquad\qquad 5+4 & \\
\qquad\qquad 5+10+5 & \\ \hline
\qquad\qquad\quad -6-5+8 & \\
\qquad\qquad\quad -6-12-6 & \\ \hline
\qquad\qquad\qquad 7+14+7 & \\
\qquad\qquad\qquad 7+14+7 & \\ \hline
\end{array}$$

54. $$\begin{array}{l|l}
1+0+0+0+0+38+12 & 1+2 \\ \hline
1+2 & 1-2+4-8+16+6 \\ \hline
\quad -2 & =a^5-2a^4+4a^3-8a^2+16a+6 \\
\quad -2-4 & \\ \hline
\qquad 4 & \\
\qquad 4+8 & \\ \hline
\qquad\quad -8 & \\
\qquad\quad -8-16 & \\ \hline
\qquad\qquad 16+38 & \\
\qquad\qquad 16+32 & \\ \hline
\qquad\qquad\quad 6+12 & \\
\qquad\qquad\quad 6+12 & \\ \hline
\end{array}$$

55. $$\begin{array}{l|l}
1+0+0+0-19-6 & 1+2 \\ \hline
1+2 & 1-2+4-8-3 \\ \hline
\quad -2 & =m^4-2m^3+4m^2-8m-3 \\
\quad -2-4 & \\ \hline
\qquad 4 & \\
\qquad 4+8 & \\ \hline
\qquad\quad -8-19 & \\
\qquad\quad -8-16 & \\ \hline
\qquad\qquad -3-6 & \\
\qquad\qquad -3-6 & \\ \hline
\end{array}$$

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56. $1+0+0+0+0+0-64 \;\big|\; \dfrac{1-2}{1+2+4+8+16+32}$

$= z^5+2z^4+4z^3+8z^2+16z+32.$

$$\begin{array}{l}
\underline{1-2} \\
2 \\
\underline{2-4} \\
4 \\
\underline{4-8} \\
8 \\
\underline{8-16} \\
16 \\
\underline{16-32} \\
32-64 \\
\underline{32-64}
\end{array}$$

57. $1+0+0+0+0+243 \;\big|\; \dfrac{1+3}{1-3+9-27+81}$

$= n^4-3n^3+9n^2-27n+81.$

$$\begin{array}{l}
\underline{1+3} \\
-3 \\
\underline{-3-9} \\
9 \\
\underline{9+27} \\
-27 \\
\underline{-27-81} \\
81+243 \\
\underline{81+243}
\end{array}$$

58. $1+0+0+0-256 \;\big|\; \dfrac{1+4}{1-4+16-64}$

$= a^3-4a^2+16a-64.$

$$\begin{array}{l}
\underline{1+4} \\
-4 \\
\underline{-4-16} \\
16 \\
\underline{16+64} \\
-64-256 \\
\underline{-64-256}
\end{array}$$

59. Rearranging terms, divisor $= a^2-3a+5$.

$1+0+0+0+27-9-10 \;\big|\; \dfrac{1-3+5}{1+3+4-3-2}$

$= a^4+3a^3+4a^2-3a-2.$

$$\begin{array}{l}
\underline{1-3+5} \\
3-5 \\
\underline{3-9+15} \\
4-15+27 \\
\underline{4-12+20} \\
-3+7-9 \\
\underline{-3+9-15} \\
-2+6-10 \\
\underline{-2+6-10}
\end{array}$$

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60. Rearranging terms, dividend $= 21x^4 - 29x^3 - 8x^2 + 6x + 4$.

$$\begin{array}{l|l}
21-29-8+6+4 & \underline{3-2} \\
\underline{21-14} & \underline{7-5-6-2} \\
\quad -15-8 & =7x^3-5x^2-6x-2. \\
\quad \underline{-15+10} & \\
\qquad -18+6 & \\
\qquad \underline{-18+12} & \\
\qquad\quad -6+4 & \\
\qquad\quad \underline{-6+4} &
\end{array}$$

61. Rearranging terms, dividend $= 2x^4 - 11x^3 + 16x^2 - 12x + 9$.

$$\begin{array}{l|l}
2-11+16-12+9 & \underline{2-3} \\
\underline{2-3} & \underline{1-4+2-3} \\
\quad -8+16 & =x^3-4x^2+2x-3. \\
\quad \underline{-8+12} & \\
\qquad 4-12 & \\
\qquad \underline{4-6} & \\
\qquad\quad -6+9 & \\
\qquad\quad \underline{-6+9} &
\end{array}$$

62. Rearranging terms, dividend $= 30x^4 - 62x^3 + 60x^2 - 36x + 8$.

$$\begin{array}{l|l}
30-62+60-36+8 & \underline{5-2} \\
\underline{30-12} & \underline{6-10+8-4} \\
\quad -50+60 & =6x^3-10x^2+8x-4. \\
\quad \underline{-50+20} & \\
\qquad 40-36 & \\
\qquad \underline{40-16} & \\
\qquad\quad -20+8 & \\
\qquad\quad \underline{-20+8} &
\end{array}$$

63.

$$\begin{array}{l|l}
1+0-2+0-1+0-10-36 & \underline{1-2} \\
\underline{1-2} & \underline{1+2+2+4+7+14+18} \\
\quad 2-2 & =x^6+2x^5+2x^4+4x^3+7x^2+14x+18. \\
\quad \underline{2-4} & \\
\qquad 2 & \\
\qquad \underline{2-4} & \\
\qquad\quad 4-1 & \\
\qquad\quad \underline{4-8} & \\
\qquad\qquad 7 & \\
\qquad\qquad \underline{7-14} & \\
\qquad\qquad\quad 14-10 & \\
\qquad\qquad\quad \underline{14-28} & \\
\qquad\qquad\qquad 18-36 & \\
\qquad\qquad\qquad \underline{18-36} &
\end{array}$$

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64. Rearranging terms, dividend $= y^4 - y^3 - 10y^2 + 7y + 15.$

$$\begin{array}{l|l}
1 - 1 - 10 + 7 + 15 & 1 - 2 - 3 \\
\underline{1 - 2 - 3} & \overline{1 + 1 - 5} \\
\phantom{1 - {}} 1 - 7 + 7 & = y^2 + y - 5. \\
\phantom{1 - {}} \underline{1 - 2 - 3} & \\
\phantom{1 - 1 {}} - 5 + 10 + 15 & \\
\phantom{1 - 1 {}} \underline{- 5 + 10 + 15} &
\end{array}$$

65. Rearranging terms, dividend $= 2x^4 + 7x^3 - 27x^2 - 8x + 16.$

$$\begin{array}{l|l}
2 + 7 - 27 - 8 + 16 & 1 + 5 - 4 \\
\underline{2 + 10 - 8} & \overline{2 - 3 - 4} \\
 - 3 - 19 - 8 & = 2x^2 - 3x - 4. \\
 \underline{- 3 - 15 + 12} & \\
 - 4 - 20 + 16 & \\
 \underline{- 4 - 20 + 16} &
\end{array}$$

66. Rearranging terms, divisor $= 4x^2 + 2x + 2.$

$$\begin{array}{l|l}
28 + 6 + 6 - 6 - 2 & 4 + 2 + 2 \\
\underline{28 + 14 + 14} & \overline{7 - 2 - 1} \\
 - 8 - 8 - 6 & = 7x^2 - 2x - 1. \\
 \underline{- 8 - 4 - 4} & \\
 - 4 - 2 - 2 & \\
 \underline{- 4 - 2 - 2} &
\end{array}$$

67. Rearranging terms, dividend $= 3v^4 - 20v^3 + 25v^2 + 16v - 6.$

$$\begin{array}{l|l}
3 - 20 + 25 + 16 - 6 & 3 - 8 + 2 \\
\underline{3 - 8 + 2} & \overline{1 - 4 - 3} \\
 - 12 + 23 + 16 & = v^2 - 4v - 3. \\
 \underline{- 12 + 32 - 8} & \\
 - 9 + 24 - 6 & \\
 \underline{- 9 + 24 - 6} &
\end{array}$$

68. Rearranging terms, dividend $= 6x^4 - 23x^3 + 30x^2 - 18x + 4.$

$$\begin{array}{l|l}
6 - 23 + 30 - 18 + 4 & 2 - 5 + 2 \\
\underline{6 - 15 + 6} & \overline{3 - 4 + 2} \\
 - 8 + 24 - 18 & = 3x^2 - 4x + 2. \\
 \underline{- 8 + 20 - 8} & \\
 4 - 10 + 4 & \\
 \underline{4 - 10 + 4} &
\end{array}$$

69. Rearranging terms, dividend $= 24x^4 + 32x^3 - 16x^2 - 25x - 4.$

$$\begin{array}{l|l}
24 + 32 - 16 - 25 - 4 & 6 - 1 - 4 \\
\underline{24 - 4 - 16} & \overline{4 + 6 + 1} \\
\phantom{24 + {}} 36 + 0 - 25 & = 4x^2 + 6x + 1. \\
\phantom{24 + {}} \underline{36 - 6 - 24} & \\
\phantom{24 + 32 - 16 {}} 6 - 1 - 4 & \\
\phantom{24 + 32 - 16 {}} \underline{6 - 1 - 4} &
\end{array}$$

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70.

$$\begin{array}{l|l}
1-2+\frac{1}{12}+\frac{2}{5}+\frac{1}{15}+\frac{5}{4} & 1-\frac{3}{2} \\
\cline{2-2}
\underline{1-\frac{3}{2}} & 1-\frac{1}{2}-\frac{2}{3}-\frac{3}{5}-\frac{5}{6} \\
\cline{2-2}
\quad -\frac{1}{2}+\frac{1}{12} & =t^4-\frac{1}{2}t^3-\frac{2}{3}t^2-\frac{3}{5}t-\frac{5}{6}. \\
\quad \underline{-\frac{1}{2}+\frac{3}{4}} & \\
\qquad -\frac{2}{3}+\frac{2}{5} & \\
\qquad \underline{-\frac{2}{3}+1} & \\
\qquad\quad -\frac{3}{5}+\frac{1}{15} & \\
\qquad\quad \underline{-\frac{3}{5}+\frac{9}{10}} & \\
\qquad\qquad -\frac{5}{6}+\frac{5}{4} & \\
\qquad\qquad \underline{-\frac{5}{6}+\frac{5}{4}} &
\end{array}$$

71.

$$\begin{array}{l|l}
1-\frac{5}{4}+\frac{29}{24}-\frac{31}{24}+\frac{5}{6}-\frac{1}{4} & 1-\frac{3}{4} \\
\cline{2-2}
\underline{1-\frac{3}{4}} & 1-\frac{1}{2}+\frac{5}{6}-\frac{2}{3}+\frac{1}{3} \\
\cline{2-2}
\quad -\frac{1}{2}+\frac{29}{24} & =a^4-\frac{1}{2}a^3+\frac{5}{6}a^2-\frac{2}{3}a+\frac{1}{3}. \\
\quad \underline{-\frac{1}{2}+\frac{3}{8}} & \\
\qquad \frac{5}{6}-\frac{31}{24} & \\
\qquad \underline{\frac{5}{6}-\frac{15}{24}} & \\
\qquad\quad -\frac{2}{3}+\frac{5}{6} & \\
\qquad\quad \underline{-\frac{2}{3}+\frac{1}{2}} & \\
\qquad\qquad \frac{1}{3}-\frac{1}{4} & \\
\qquad\qquad \underline{\frac{1}{3}-\frac{1}{4}} &
\end{array}$$

72.

$$\begin{array}{l|l}
1-\frac{1}{6}+\frac{5}{6}-\frac{13}{36}+\frac{37}{36}-\frac{17}{36}+\frac{7}{24} & 1-\frac{2}{3}+\frac{1}{2} \\
\cline{2-2}
\underline{1-\frac{2}{3}+\frac{1}{2}} & 1+\frac{1}{2}+\frac{2}{3}-\frac{1}{6}+\frac{7}{12} \\
\cline{2-2}
\quad \frac{1}{2}+\frac{1}{3}-\frac{13}{36} & =z^4+\frac{1}{2}z^3+\frac{2}{3}z^2-\frac{1}{6}z+\frac{7}{12}. \\
\quad \underline{\frac{1}{2}-\frac{1}{3}+\frac{1}{4}} & \\
\qquad \frac{2}{3}-\frac{11}{18}+\frac{37}{36} & \\
\qquad \underline{\frac{2}{3}-\frac{4}{9}+\frac{1}{3}} & \\
\qquad\quad -\frac{1}{6}+\frac{25}{36}-\frac{17}{36} & \\
\qquad\quad \underline{-\frac{1}{6}+\frac{1}{9}-\frac{1}{12}} & \\
\qquad\qquad \frac{7}{12}-\frac{7}{18}+\frac{7}{24} & \\
\qquad\qquad \underline{\frac{7}{12}-\frac{7}{18}+\frac{7}{24}} &
\end{array}$$

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23. Since $a^6+b^6=(a^2)^3+(b^2)^3$, it is divisible by a^2+b^2 (Prin. 4).
24. x^7+a^7 is divisible by $x+a$ (Prin. 4).
25. Since $a^{10}+b^{10}=(a^2)^5+(b^2)^5$, it is divisible by a^2+b^2 (Prin. 4).
26. Since $a^{10}+b^5=(a^2)^5+b^5$, it is divisible by a^2+b (Prin. 4).
27. Since $a^{12}+b^{12}=(a^4)^3+(b^4)^3$, it is divisible by a^4+b^4 (Prin. 4).
28. Since $a^3-27=a^3-3^3$, it is divisible by $a-3$ (Prin. 1).
29. Since $a^6-27=(a^2)^3-3^3$, it is divisible by a^2-3 (Prin. 1).
30. As the difference of the same powers of a and b, and also of a^2 and b^2, a^4-b^4 is divisible by $a-b$ and by a^2-b^2 (Prin. 1).
As the difference of the same even powers of a and b, and also of a^2 and b^2, a^4-b^4 is divisible by $a+b$ and by a^2+b^2 (Prin. 2).

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31. $a^6 - 1$ is divisible by $a - 1$ (Prin. 1) and by $a + 1$ (Prin. 2).
Since $a^6 - 1 = (a^2)^3 - 1^3$, $a^6 - 1$ is divisible by $a^2 - 1$ (Prin. 1).
Since $a^6 - 1 = (a^3)^2 - 1^2$, $a^6 - 1$ is divisible by $a^3 - 1$ (Prin. 1).
Since $a^6 - 1 = (a^3)^2 - 1^2$, $a^6 - 1$ is divisible by $a^3 + 1$ (Prin. 2).

32. $a^8 - b^8$ is divisible by $a - b$ (Prin. 1) and by $a + b$ (Prin. 2).
Since $a^8 - b^8 = (a^2)^4 - (b^2)^4$, $a^8 - b^8$ is divisible by $a^2 - b^2$ (Prin. 1) and by $a^2 + b^2$ (Prin. 2).
Since $a^8 - b^8 = (a^4)^2 - (b^4)^2$, $a^8 - b^8$ is divisible by $a^4 - b^4$ (Prin. 1) and by $a^4 + b^4$ (Prin. 2).

33. $a^{10} - b^{10}$ is divisible by $a - b$ (Prin. 1) and by $a + b$ (Prin. 2).
Since $a^{10} - b^{10} = (a^2)^5 - (b^2)^5$, $a^{10} - b^{10}$ is divisible by $a^2 - b^2$ (Prin. 1).
Since $a^{10} - b^{10} = (a^5)^2 - (b^5)^2$, $a^{10} - b^{10}$ is divisible by $a^5 - b^5$ (Prin. 1) and by $a^5 + b^5$ (Prin. 2).

34. $a^{16} - b^{16}$ is divisible by $a - b$ (Prin. 1) and by $a + b$ (Prin. 2).
Since $a^{16} - b^{16} = (a^2)^8 - (b^2)^8$, $a^{16} - b^{16}$ is divisible by $a^2 - b^2$ (Prin. 1) and by $a^2 + b^2$ (Prin. 2).
Since $a^{16} - b^{16} = (a^4)^4 - (b^4)^4$, $a^{16} - b^{16}$ is divisible by $a^4 - b^4$ (Prin. 1) and by $a^4 + b^4$ (Prin. 2).
Since $a^{16} - b^{16} = (a^8)^2 - (b^8)^2$, $a^{16} - b^{16}$ is divisible by $a^8 - b^8$ (Prin. 1) and by $a^8 + b^8$ (Prin. 2).

35. $a^{12} - b^{12}$ is divisible by $a - b$ (Prin. 1) and by $a + b$ (Prin. 2).
Since $a^{12} - b^{12} = (a^2)^6 - (b^2)^6$, $a^{12} - b^{12}$ is divisible by $a^2 - b^2$ (Prin. 1) and by $a^2 + b^2$ (Prin. 2).
Since $a^{12} - b^{12} = (a^3)^4 - (b^3)^4$, $a^{12} - b^{12}$ is divisible by $a^3 - b^3$ (Prin. 1) and by $a^3 + b^3$ (Prin. 2).
Since $a^{12} - b^{12} = (a^4)^3 - (b^4)^3$, $a^{12} - b^{12}$ is divisible by $a^4 - b^4$ (Prin. 1).
Since $a^{12} - b^{12} = (a^6)^2 - (b^6)^2$, $a^{12} - b^{12}$ is divisible by $a^6 - b^6$ (Prin. 1) and by $a^6 + b^6$ (Prin. 2).

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18. Let $2x =$ number of votes A received.
Then, $x + 500 =$ number of votes B received.
By the 1st condition, $2x + (x + 500) = 8000$;
$$\therefore x = 2500,$$
whence, $2x = 5000$,
and $x + 500 = 3000$.
Hence, A received 5000 votes and B 3000 votes.
VERIFICATION. — 1st condition : $5000 + 3000 = 8000$.
2d condition : 3000 is 500 more than $\frac{1}{2}$ of 5000.

19. Let $3x =$ number of dollars A had.
Then, $x + 10 =$ number of dollars B had.
By the 1st condition, $3x - (x + 10) = 40$.
$$3x - x - 10 = 40;$$
$$\therefore x = 25,$$
$$3x = 75,$$
and $x + 10 = 35$.
Hence, A had \$75 and B \$35.
VERIFICATION. — 1st condition : \$75 is \$40 more than \$35.
2d condition : \$35 is \$10 more than $\frac{1}{3}$ of \$75.

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20. Let $x =$ number of years in A's age.

Then, $x + 2 = 2(x - 2)$.

$x + 2 = 2x - 4$;

$\therefore x = 6$.

Hence, A is 6 years old.

VERIFICATION. $(6 + 2)$ years $= 2$ times $(6 - 2)$ years.

21. Let $x =$ number of hours occupied in making the trip.

Then, $x - 3 =$ number of hours the first rode,

and $x - 1 =$ number of hours the second rode.

Since the first wheelman rode 10 miles an hour, the number of miles from A to B may be expressed by $10(x - 3)$; since the second rode 8 miles an hour, the distance may be expressed also by $8(x - 1)$. Equating these two expressions, $10(x - 3) = 8(x - 1)$,

or $10x - 30 = 8x - 8$;

$\therefore x = 11$, the number of hours,

whence, $10(x - 3) = 80$, the number of miles.

VERIFICATION. — 1st condition: The first wheelman makes the trip of 80 miles by riding 10 miles an hour for 8 hours and resting 3 hours.

2d condition: The second wheelman makes the trip of 80 miles by riding 8 miles an hour for 10 hours and resting 1 hour.

3d condition: They make the trip in equal times, the first in $(8 + 3)$ hours, the second in $(10 + 1)$ hours.

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23. Let $x =$ number of words in the message.

Then, $x - 10 =$ number of words in excess of 10 words.

$\therefore 25 + 2(x - 10) = 75$.

Solving, $x = 35$, the number of words.

VERIFICATION. 25¢ for 10 words + 50¢ for 25 additional words = 75¢.

24. Let $x =$ number of minutes conversation lasted.

Then, $x - 3 =$ number of minutes in excess of 3 minutes.

$\therefore 50 + 15(x - 3) = 125$.

Solving, $x = 8$, number of minutes conversation lasted.

VERIFICATION. 50¢ for 3 minutes + 75¢ for 5 additional minutes = \$1.25.

25. Let $x =$ number of words in the message.

Then, $x - 10 =$ number of words in excess of 10 words.

$60 + 4(x - 10) =$ number of cents message costs at day rate.

$40 + 3(x - 10) =$ number of cents message costs at night rate.

$\therefore 60 + 4(x - 10) = 40 + 3(x - 10) + 25$.

Solving, $x = 15$, number of words in the message.

VERIFICATION. — 1st condition: 60¢ + 20¢ = 80¢ cost of message at day rate.

2d condition: 40¢ + 15¢ = 55¢ cost of message at night rate.

3d condition: 55¢ is 25¢ less than 80¢.

26. Let $x =$ number representing first part.

Then, $24 - x =$ number representing second part.

By condition, $x + 3 = 2(24 - x)$.

Solving, $x = 15$, and $24 - x = 9$.

VERIFICATION. 15 is 3 less than twice 9.

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27. Let $x =$ number representing first part.
Then, $52 - x =$ number representing second part.
By condition, $2x = 3(52 - x) + 4.$
Solving, $x = 32,$
and $52 - x = 20.$
VERIFICATION. 2 times 32 is 4 greater than 3 times 20.

28. Let $x =$ number of apples at 5 cents each.
Then, $17 - x =$ number of apples at 3 cents each.
By the 1st condition, $5x + 3(17 - x) = 61,$
$$5x + 51 - 3x = 61;$$
$$\therefore x = 5,$$
and $17 - x = 12.$
Hence, Mary bought 5 apples at 5 cents each and 12 apples at 3 cents each.
VERIFICATION. — 1st condition: 5 apples + 12 apples = 17 apples.
2d condition: 5 times 5 cents + 12 times 3 cents = 61 cents.

29. Let $x =$ number of years in son's age.
Then, $2x =$ number of years in father's age.
By the 2d condition, $3(x - a) = 2x - a,$
or $3x - 3a = 2x - a;$
$$\therefore x = 2a,$$
and $2x = 4a.$
Hence, George is $2a$ years old and his father $4a$ years old.
VERIFICATION. — 1st condition: $2a$ years is $\frac{1}{2}$ of $4a$ years.
2d condition: $(2a - a)$ years is $\frac{1}{3}$ of $(4a - a)$ years.

30. Let $x =$ number of feet in width of rug.
Then, $x + 3 =$ number of feet in length of rug,
$x + 4 =$ number of feet in width of floor,
and $x + 7 =$ number of feet in length of floor.
By the 3d condition,
$$(x + 4)(x + 7) - x(x + 3) = 172.$$
$$x^2 + 11x + 28 - x^2 - 3x = 172;$$
$$\therefore x = 18,$$
whence, $x + 4 = 22,$
and $x + 7 = 25.$
Hence, the floor is 25 feet long and 22 feet wide.
VERIFICATION. — 1st condition: The rug is 21 feet long and 18 feet wide.
2d condition: The floor is 25 feet long and 22 feet wide.
3d condition: (25×22) sq. ft. $-$ (21×18) sq. ft. $=$ 172 sq. ft.

REVIEW

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5.
$$x^5 - (x^5 - 5x^4y + 10x^3y^2 - 10x^2y^3 + 5xy^4 - y^5)$$
$$= x^5 - x^5 + 5x^4y - 10x^3y^2 + 10x^2y^3 - 5xy^4 + y^5$$
$$= 5x^4y - 10x^3y^2 + 10x^2y^3 - 5xy^4 + y^5.$$

6.
$$\tfrac{1}{2}a - \tfrac{5}{8}x - (\tfrac{3}{4}a - \tfrac{1}{2}x) - (3b - \tfrac{11}{4}x - \tfrac{2}{3}a) + \tfrac{1}{6}a$$
$$= \tfrac{1}{2}a - \tfrac{5}{8}x - \tfrac{3}{4}a + \tfrac{1}{2}x - 3b + \tfrac{11}{4}x + \tfrac{2}{3}a + \tfrac{1}{6}a$$
$$= \tfrac{7}{12}a + \tfrac{29}{12}x - 3b.$$

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7. $x^2-(2xy-y^2)-(x^2+xy-y^2)-x^2-\overline{2xy-y^2}+5y^2$
$=x^2-2xy+y^2-x^2-xy+y^2-x^2-2xy+y^2+5y^2$
$=-x^2-5xy+8y^2.$

8. $m+2\{2m-[n+3p-(4p-3n)-5n+2m]-7p\}$
$=m+2\{2m-[n+3p-4p+3n-5n+2m]-7p\}$
$=m+2\{2m-n-3p+4p-3n+5n-2m-7p\}$
$=m+4m-2n-6p+8p-6n+10n-4m-14p$
$=m+2n-12p.$

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20. Rearranging terms, dividend $=x^5-2x^4+2x^3+12x^2-x-8$.

$$
\begin{array}{l|l}
1-2+2+12-1-8 & 1+1 \\ \cline{2-2}
\underline{1+1} & 1-3+5+7-8 \\ \cline{2-2}
\quad -3+2 & =x^4-3x^3+5x^2+7x-8 \\
\quad \underline{-3-3} & \\
\qquad\quad 5+12 & \\
\qquad\quad \underline{5+\ \ 5} & \\
\qquad\qquad 7-1 & \\
\qquad\qquad \underline{7+7} & \\
\qquad\qquad\ -8-8 & \\
\qquad\qquad\ \underline{-8-8} &
\end{array}
$$

21. Rearranging terms, dividend $=x^4-4x^3+5x^2-4x+1$; divisor $=x^2-3x+1$.

$$
\begin{array}{l|l}
1-4+5-4+1 & 1-3+1 \\ \cline{2-2}
\underline{1-3+1} & 1-1+1 \\ \cline{2-2}
\ -1+4-4 & =x^2-x+1. \\
\ \underline{-1+3-1} & \\
\qquad 1-3+1 & \\
\qquad \underline{1-3+1} &
\end{array}
$$

22. Rearranging terms, divisor $=a^3-2a^2+4a-3$.

$$
\begin{array}{l|l}
1+0+0+0+0-12-1+12 & 1-2+4-3 \\ \cline{2-2}
\underline{1-2+4-3} & 1+2+0-5-4 \\ \cline{2-2}
\quad 2-4+3+\ 0 & =a^4+2a^3-5a-4. \\
\quad \underline{2-4+8-\ 6} & \\
\qquad -5+\ 6-12-\ 1 & \\
\qquad \underline{-5+10-20+15} & \\
\qquad\quad -\ 4+\ 8-16+12 & \\
\qquad\quad \underline{-\ 4+\ 8-16+12} &
\end{array}
$$

23. $1-\{1-[x^2-3-(2x-4)^2+3x^2+1]-(x-4)^2\}-1$
$=1-\{1-[x^2-3-4x^2+16x-16+3x^2+1]-x^2+8x-16\}-1$
$=1-\{1-x^2+3+4x^2-16x+16-3x^2-1-x^2+8x-16\}-1$
$=1-1+x^2-3-4x^2+16x-16+3x^2+1+x^2-8x+16-1=x^2+8x-3.$

24. $x-\{5x-[6x-(7x-\overline{8x-9x})-10x]+11x\}+9x$
$=x-\{5x-[6x-(7x+x)-10x]+11x\}+9x$
$=10x-\{16x-[6x-8x-10x]\}$
$=10x-\{16x+12x\}=10x-28x=-18x.$

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25. $1 - \{-[-(1 - x) - 1] - 1\} - \{x - (5 - 3x) - 7 + x\}$

$$\begin{aligned} &= 1 - \{-[-1 + x - 1] - 1\} - \{2x - 7 - 5 + 3x\} \\ &= 1 - \{1 - x + 1 - 1\} - 5x + 12 \\ &= 13 - 1 + x - 5x = 12 - 4x. \end{aligned}$$

33. $(a - b)(a + b)(a^2 + b^2) = (a^2 - b^2)(a^2 + b^2) = a^4 - b^4.$

34. $(1 - x)(1 + x)(1 + x^2)(1 + x^4) = (1 - x^2)(1 + x^2)(1 + x^4)$
$= (1 - x^4)(1 + x^4) = 1 - x^8.$

35. $(1 - x)(1 + x)(1 - x)(1 + x) = (1 - x^2)(1 - x^2) = 1 - 2x^2 + x^4.$

36. $(a^3 + 3a^2y + 3ay^2)(a^2 - 2ay + y^2)$
$= a^5 - 2a^4y + a^3y^2 + 3a^4y - 6a^3y^2 + 3a^2y^3 + 3a^3y^2 - 6a^2y^3 + 3ay^4$
$= a^5 + a^4y - 2a^3y^2 - 3a^2y^3 + 3ay^4.$

37. $(x^{2n} + 2x^ny^n + y^{2n})(x^{2n} - 2x^ny^n + y^{2n})$
$= (x^{2n} + y^{2n} + 2x^ny^n)(x^{2n} + y^{2n} - 2x^ny^n) = (x^{2n} + y^{2n})^2 - (2x^ny^n)^2$
$= x^{4n} + 2x^{2n}y^{2n} + y^{4n} - 4x^{2n}y^{2n} = x^{4n} - 2x^{2n}y^{2n} + y^{4n}.$

38. $(\frac{1}{4}x^2 + \frac{1}{3}xy + \frac{1}{9}y^2)(\frac{1}{4}x^2 - \frac{1}{3}xy + \frac{1}{9}y^2)$
$= (\frac{1}{4}x^2 + \frac{1}{9}y^2 + \frac{1}{3}xy)(\frac{1}{4}x^2 + \frac{1}{9}y^2 - \frac{1}{3}xy) = (\frac{1}{4}x^2 + \frac{1}{9}y^2)^2 - (\frac{1}{3}xy)^2$
$= \frac{1}{16}x^4 + \frac{1}{18}x^2y^2 + \frac{1}{81}y^4 - \frac{1}{9}x^2y^2 = \frac{1}{16}x^4 - \frac{1}{18}x^2y^2 + \frac{1}{81}y^4.$

39.

$$\begin{array}{rrrrr}
.2 & -\ .8 & +\ 1.6 & & \\
.1 & +\ .4 & +\ \ .8 & & \\
\hline
.02 & -\ .08 & +\ .16 & & \\
 & .08 & -\ .32 & +\ .64 & \\
 & & .16 & -\ .64 & +\ 1.28 \\
\hline
.02 & & & & 1.28
\end{array}$$

$= .02a^4 + 1.28.$

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45.

$$\begin{array}{l|l}
b^8 - 10b^2 - 5b + 4 & b^3 - 2b^2 + 3b - 1 \\
\underline{b^8 - 2b^7 + 3b^6 - b^5} & \overline{b^5 + 2b^4 + b^3 - 3b^2 - 7b - 4} \\
\quad 2b^7 - 3b^6 + b^5 & \\
\quad \underline{2b^7 - 4b^6 + 6b^5 - 2b^4} & \\
\qquad b^6 - 5b^5 + 2b^4 & \\
\qquad \underline{b^6 - 2b^5 + 3b^4 - b^3} & \\
\qquad\quad -3b^5 - b^4 + b^3 - 10b^2 & \\
\qquad\quad \underline{-3b^5 + 6b^4 - 9b^3 + 3b^2} & \\
\qquad\qquad -7b^4 + 10b^3 - 13b^2 - 5b & \\
\qquad\qquad \underline{-7b^4 + 14b^3 - 21b^2 + 7b} & \\
\qquad\qquad\quad -4b^3 + 8b^2 - 12b + 4 & \\
\qquad\qquad\quad \underline{-4b^3 + 8b^2 - 12b + 4} &
\end{array}$$

TEST. — Let $b = 1$. Then, the dividend becomes $1 - 10 - 5 + 4$, or -10; the divisor $1 - 2 + 3 - 1$, or 1; and the quotient $1 + 2 + 1 - 3 - 7 - 4$, or -10. Since $-10 \div 1 = -10$, it may be assumed that the quotient is correct.

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46.

$$\begin{array}{l|l}
m^{10} - 6m^3 + 5m - 2 & m^4 + 2m^3 - 3m - 2 \\ \cline{2-2}
\underline{m^{10} + 2m^9 - 3m^7 - 2m^6} & m^6 - 2m^5 + 4m^4 - 5m^3 + 6m^2 - 4m + 1 \\
\quad - 2m^9 + 3m^7 + 2m^6 & \\
\quad \underline{- 2m^9 - 4m^8 + 6m^6 + 4m^5} & \\
\qquad 4m^8 + 3m^7 - 4m^6 - 4m^5 & \\
\qquad \underline{4m^8 + 8m^7 - 12m^5 - 8m^4} & \\
\qquad\quad - 5m^7 - 4m^6 + 8m^5 + 8m^4 - 6m^3 & \\
\qquad\quad \underline{- 5m^7 - 10m^6 + 15m^4 + 10m^3} & \\
\qquad\qquad 6m^6 + 8m^5 - 7m^4 - 16m^3 & \\
\qquad\qquad \underline{6m^6 + 12m^5 - 18m^3 - 12m^2} & \\
\qquad\qquad\quad - 4m^5 - 7m^4 + 2m^3 + 12m^2 + 5m & \\
\qquad\qquad\quad \underline{- 4m^5 - 8m^4 + 12m^2 + 8m} & \\
\qquad\qquad\qquad m^4 + 2m^3 - 3m - 2 & \\
\qquad\qquad\qquad \underline{m^4 + 2m^3 - 3m - 2} &
\end{array}$$

TEST. — Let $m = 1$. Then, the dividend becomes $1 - 6 + 5 - 2$, or -2; the divisor $1 + 2 - 3 - 2$, or -2; and the quotient $1 - 2 + 4 - 5 + 6 - 4 + 1$, or 1. Since $-2 \div -2 = 1$, it may be assumed that the quotient is correct.

47.

$$\begin{array}{l|l}
a^7 - 160a^4 + 127a^3 - 100a^2 - 20a + 16 & a^3 - 6a^2 + 5a - 4 \\ \cline{2-2}
\underline{a^7 - 6a^6 + 5a^5 - 4a^4} & a^4 + 6a^3 + 31a^2 - 4 \\
6a^6 - 5a^5 - 156a^4 + 127a^3 & \\
\underline{6a^6 - 36a^5 + 30a^4 - 24a^3} & \\
\quad 31a^5 - 186a^4 + 151a^3 - 100a^2 & \\
\quad \underline{31a^5 - 186a^4 + 155a^3 - 124a^2} & \\
\qquad - 4a^3 + 24a^2 - 20a + 16 & \\
\qquad \underline{- 4a^3 + 24a^2 - 20a + 16} &
\end{array}$$

TEST. — Let $a = 1$. Then, the dividend becomes $1 - 160 + 127 - 100 - 20 + 16$, or -136; the divisor $1 - 6 + 5 - 4$, or -4; and the quotient $1 + 6 + 31 - 4$, or 34. Since $-136 \div -4 = 34$, it may be assumed that the quotient is correct.

48.

$$\begin{array}{l|l}
b^{10} + 29b^4 - 170b^3 - 61b^2 + 210b - 22 & b^4 + 2b^2 - 5b - 11 \\ \cline{2-2}
\underline{b^{10} + 2b^8 - 5b^7 - 11b^6} & b^6 - 2b^4 + 5b^3 + 15b^2 - 20b + 2 \\
\quad - 2b^8 + 5b^7 + 11b^6 + 29b^4 & \\
\quad \underline{- 2b^8 - 4b^6 + 10b^5 + 22b^4} & \\
\qquad 5b^7 + 15b^6 - 10b^5 + 7b^4 - 170b^3 & \\
\qquad \underline{5b^7 + 10b^5 - 25b^4 - 55b^3} & \\
\qquad\quad 15b^6 - 20b^5 + 32b^4 - 115b^3 - 61b^2 & \\
\qquad\quad \underline{15b^6 + 30b^4 - 75b^3 - 165b^2} & \\
\qquad\qquad - 20b^5 + 2b^4 - 40b^3 + 104b^2 + 210b & \\
\qquad\qquad \underline{- 20b^5 - 40b^3 + 100b^2 + 220b} & \\
\qquad\qquad\quad 2b^4 + 4b^2 - 10b - 22 & \\
\qquad\qquad\quad \underline{2b^4 + 4b^2 - 10b - 22} &
\end{array}$$

TEST. — Let $b = 1$. Then, the dividend becomes $1 + 29 - 170 - 61 + 210 - 22$, or -13; the divisor $1 + 2 - 5 - 11$, or -13; and the quotient $1 - 2 + 5 + 15 - 20 + 2$, or 1. Since $-13 \div -13 = 1$, it may be assumed that the quotient is correct.

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49.
$$\begin{aligned} & a-(2b+5a)(6b-3a)-2b-6[3a^2-4ab-2b^2] \\ &= a-(12b^2+24ab-15a^2)-2b-18a^2+24ab+12b^2 \\ &= a-12b^2-24ab+15a^2-2b-18a^2+24ab+12b^2 \\ &= a-3a^2-2b. \end{aligned}$$

50.
$$\begin{aligned} & x-\{3y+[4x-2(y+3x)-3y]^2-(5y+2x)^2-8y\} \\ &= x-\{3y+[4x-2y-6x-3y]^2-(25y^2+20xy+4x^2)-8y\} \\ &= x-\{3y+4x^2+20xy+25y^2-25y^2-20xy-4x^2-8y\} \\ &= x-3y-4x^2-20xy-25y^2+25y^2+20xy+4x^2+8y \\ &= x+5y. \end{aligned}$$

51.
$$\begin{aligned} & (a^2+ab-b^2)^2-(a^2-ab-b^2)^2-4ab(a^2-b^2) \\ &= a^4+a^2b^2+b^4+2a^3b-2a^2b^2-2ab^3-(a^4+a^2b^2 \\ &\qquad +b^4-2a^3b-2a^2b^2+2ab^3)-4a^3b+4ab^3 \\ &= a^4+a^2b^2+b^4+2a^3b-2a^2b^2-2ab^3-a^4-a^2b^2-b^4+2a^3b \\ &\qquad +2a^2b^2-2ab^3-4a^3b+4ab^3=0. \end{aligned}$$

52.
$$\begin{aligned} & a^2-[-b^2+b(5b-3a)-\{-\overline{3ab+a^2}-b(a-\overline{2a+2b})\}] \\ &= a^2-[-b^2+5b^2-3ab-\{-3ab-a^2-b(a-2a-2b)\}] \\ &= a^2-[-b^2+5b^2-3ab-\{-3ab-a^2-ab+2ab+2b^2\}] \\ &= a^2-[-b^2+5b^2-3ab+3ab+a^2+ab-2ab-2b^2] \\ &= a^2+b^2-5b^2+3ab-3ab-a^2-ab+2ab+2b^2=ab-2b^2. \end{aligned}$$

63. By notation, $\quad a^5=a\cdot a\cdot a\cdot a\cdot a,$

and $\quad a^6=a\cdot a\cdot a\cdot a\cdot a\cdot a;$

$$\begin{aligned} \therefore a^5\cdot a^6 &= (a\cdot a\cdot a\cdot a\cdot a)(a\cdot a\cdot a\cdot a\cdot a\cdot a) \\ &= a\cdot a\cdot a\cdot a\cdot a\cdot a\cdot a\cdot a\cdot a\cdot a\cdot a \end{aligned}$$

by notation, $\quad = a^{11}.$

By notation, $\quad a^{11}=a\cdot a\cdot a\cdot a\cdot a\cdot a\cdot a\cdot a\cdot a\cdot a\cdot a,$

and $\quad a^6=a\cdot a\cdot a\cdot a\cdot a\cdot a;$

$$\therefore \frac{a^{11}}{a^6}=\frac{a\cdot a\cdot a\cdot a\cdot a\cdot a\cdot a\cdot a\cdot a\cdot a\cdot a}{a\cdot a\cdot a\cdot a\cdot a\cdot a}.$$

Then, $\quad a^{11}\div a^6=a\cdot a\cdot a\cdot a\cdot a,$

by notation, $\quad = a^5.$

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78.
$$\begin{aligned} & (x+y)(x-y)(x^2+y^2)(x^4+y^4)(x^8+y^8) \\ &= (x^2-y^2)(x^2+y^2)(x^4+y^4)(x^8+y^8) \\ &= (x^4-y^4)(x^4+y^4)(x^8+y^8) \\ &= (x^8-y^8)(x^8+y^8) \\ &= x^{16}-y^{16}. \end{aligned}$$

79.
$$\begin{aligned} & (m^8+1)(m^4+1)(m^2+1)(m+1)(m-1) \\ &= (m^8+1)(m^4+1)(m^2+1)(m^2-1) \\ &= (m^8+1)(m^4+1)(m^4-1) \\ &= (m^8+1)(m^8-1) \\ &= m^{16}-1. \end{aligned}$$

80.
$$\begin{aligned} & (16x^4+1)(4x^2+1)(2x+1)(2x-1) \\ &= (16x^4+1)(4x^2+1)(4x^2-1) \\ &= (16x^4+1)(16x^4-1) \\ &= 256x^8-1. \end{aligned}$$

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83. Divisor: $x^{a+1} + y^a - z^{a-1}$; Quotient: $x^{a+1} - 2y^a + z^{a-1}$

$$\begin{array}{l}
x^{2a+2} - x^{a+1}y^a - 2y^{2a} + 3y^a z^{a-1} - z^{2a-2} \\
\underline{x^{2a+2} + x^{a+1}y^a - x^{a+1}z^{a-1}} \\
\quad -2x^{a+1}y^a - 2y^{2a} + x^{a+1}z^{a-1} + 3y^a z^{a-1} - z^{2a-2} \\
\quad \underline{-2x^{a+1}y^a - 2y^{2a} \qquad\qquad + 2y^a z^{a-1}} \\
\qquad\qquad x^{a+1}z^{a-1} + y^a z^{a-1} - z^{2a-2} \\
\qquad\qquad \underline{x^{a+1}z^{a-1} + y^a z^{a-1} - z^{2a-2}}
\end{array}$$

84. Divisor: $a^3 + \frac{1}{2}a^2y - \frac{1}{4}ay^2 + \frac{1}{8}y^3$; Quotient: $6a^4 - 3a^3y + 3a^2y^2 - 3ay^3 + \frac{21}{8}y^4$

$$\begin{array}{l}
6a^7 + \frac{39}{16}a^2y^5 - \frac{33}{32}ay^6 + \frac{21}{64}y^7 \\
\underline{6a^7 + 3a^6y - \frac{3}{2}a^5y^2 + \frac{3}{4}a^4y^3} \\
\quad -3a^6y + \frac{3}{2}a^5y^2 - \frac{3}{4}a^4y^3 \\
\quad \underline{-3a^6y - \frac{3}{2}a^5y^2 + \frac{3}{4}a^4y^3 - \frac{3}{8}a^3y^4} \\
\qquad 3a^5y^2 - \frac{3}{2}a^4y^3 + \frac{3}{8}a^3y^4 + \frac{39}{16}a^2y^5 \\
\qquad \underline{3a^5y^2 + \frac{3}{2}a^4y^3 - \frac{3}{4}a^3y^4 + \frac{3}{8}a^2y^5} \\
\qquad\quad -3a^4y^3 + \frac{9}{8}a^3y^4 + \frac{33}{16}a^2y^5 - \frac{33}{32}ay^6 \\
\qquad\quad \underline{-3a^4y^3 - \frac{3}{2}a^3y^4 + \frac{3}{4}a^2y^5 - \frac{3}{8}ay^6} \\
\qquad\qquad \frac{21}{8}a^3y^4 + \frac{21}{16}a^2y^5 - \frac{21}{32}ay^6 + \frac{21}{64}y^7 \\
\qquad\qquad \underline{\frac{21}{8}a^3y^4 + \frac{21}{16}a^2y^5 - \frac{21}{32}ay^6 + \frac{21}{64}y^7}
\end{array}$$

85. Divisor, $ac - b^2 + cd - d^2$ Quotient, $a - b - c$

$$\begin{array}{l}
a^2c - ab^2 + acd - ad^2 - abc + b^3 - bcd + bd^2 - ac^2 + cb^2 - c^2d + cd^2 \\
\underline{a^2c - ab^2 + acd - ad^2} \\
\qquad \underline{-abc + b^3 - bcd + bd^2} \\
\qquad\qquad \underline{-ac^2 + cb^2 - c^2d + cd^2}
\end{array}$$

FACTORING

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4. $am - an + mx - nx = a(m-n) + x(m-n) = (a+x)(m-n)$.
5. $bc - bd + cx - dx = b(c-d) + x(c-d) = (b+x)(c-d)$.
6. $pq - px - rq + rx = p(q-x) - r(q-x) = (p-r)(q-x)$.
7. $ay - by - ab + b^2 = y(a-b) - b(a-b) = (y-b)(a-b)$.
8. $x^2 - xy - 5x + 5y = x(x-y) - 5(x-y) = (x-5)(x-y)$.
9. $b^2 - bc + ab - ac = b(b-c) + a(b-c) = (b+a)(b-c)$.
10. $x^2 + xy - ax - ay = x(x+y) - a(x+y) = (x+y)(x-a)$.
11. $c^2 - 4c + ac - 4a = c(c-4) + a(c-4) = (c+a)(c-4)$.
12. $2x - y + 4x^2 - 2xy = (2x-y) + 2x(2x-y) = (2x-y)(1+2x)$.
13. $1 - m + n - mn = (1-m) + n(1-m) = (1+n)(1-m)$.
14. $2p + q + 6p^2 + 3pq = (2p+q) + 3p(2p+q) = (1+3p)(2p+q)$.
15. $ar - rs - ab + bs = r(a-s) - b(a-s) = (r-b)(a-s)$.
16. $x^3 + x^2 + x + 1 = x^2(x+1) + (x+1) = (x+1)(x^2+1)$.
17. $y^3 + y^2 - 3y - 3 = y^2(y+1) - 3(y+1) = (y^2-3)(y+1)$.
18. $x^5 + x^3 + x^2y + y = x^3(x^2+1) + y(x^2+1) = (x^3+y)(x^2+1)$.
19. $2 - 2n - n^2 + n^3 = 2(1-n) - n^2(1-n) = (2-n^2)(1-n)$.
20. $x^2 - x - a + ax = x^2 - x + ax - a$
$\qquad = x(x-1) + a(x-1) = (x+a)(x-1)$.

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21. $3x^3 - 15x + 10y - 2x^2y = 3x^3 - 15x - 2x^2y + 10y$
$= 3x(x^2-5) - 2y(x^2-5) = (3x-2y)(x^2-5)$

22. $12a^3 - 8ab - 3a^4 + 2a^2b = 4a(3a^2 - 2b) - a^2(3a^2 - 2b)$
$= (4a - a^2)(3a^2 - 2b)$
$= a(4 - a)(3a^2 - 2b).$

23. $3m^2n - 9mn^2 + am - 3an = 3mn(m - 3n) + a(m - 3n)$
$= (3mn + a)(m - 3n).$

24. $15ab^2 - 9b^2c - 35ab + 21bc = 3b^2(5a - 3c) - 7b(5a - 3c)$
$= (3b^2 - 7b)(5a - 3c)$
$= b(3b - 7)(5a - 3c).$

25. $16ax + 12ay - 8bx - 6by = 4a(4x + 3y) - 2b(4x + 3y)$
$= (4a - 2b)(4x + 3y)$
$= 2(2a - b)(4x + 3y).$

26. $ax^2 - ax - axy + ay + x - 1 = ax(x - 1) - ay(x - 1) + (x - 1)$
$= (ax - ay + 1)(x - 1).$

27. $xy + x - 3y^2 - 3y - 4y - 4 = x(y + 1) - 3y(y + 1) - 4(y + 1)$
$= (x - 3y - 4)(y + 1).$

28. $ax - a - bx + b - cx + c = a(x - 1) - b(x - 1) - c(x - 1)$
$= (a - b - c)(x - 1).$

29. $mx - nx - x - my + ny + y = x(m - n - 1) - y(m - n - 1)$
$= (x - y)(m - n - 1).$

30. $bx^2 - b - xy - y + yx^2 - bx = bx^2 + yx^2 - bx - xy - b - y$
$= x^2(b + y) - x(b + y) - (b + y)$
$= (x^2 - x - 1)(b + y).$

31. $m^2 + mn + mn + n^2 + m + n = m(m + n) + n(m + n) + (m + n)$
$= (m + n + 1)(m + n).$

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31. $x^2 + 2x(x - y) + (x - y)^2 = (x + x - y)(x + x - y)$
$= (2x - y)(2x - y).$

32. $t^2 - 4t(t - 1) + 4(t - 1)^2 = [t - 2(t - 1)][t - 2(t - 1)]$
$= (t - 2t + 2)(t - 2t + 2)$
$= (2 - t)(2 - t).$

33. $(r + s)^2 - 4(r + s) + 4 = [(r + s) - 2][(r + s) - 2]$
$= (r + s - 2)(r + s - 2).$

34. $c^2 - 6c(a - c) + 9(a - c)^2 = [c - 3(a - c)][c - 3(a - c)]$
$= [c - 3a + 3c][c - 3a + 3c]$
$= (4c - 3a)(4c - 3a).$

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35. $16 - 24(t - l) + 9(t - l)^2 = [4 - 3(t - l)][4 - 3(t - l)]$
$= (4 - 3t + 3l)(4 - 3t + 3l).$

36. $14(x - y) + (x - y)^2 + 49 = (x - y)^2 + 14(x - y) + 49$
$= (x - y + 7)(x - y + 7).$

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37. $(a+b)^2 - 2(a+b)(b+c) + (b+c)^2$
$= [(a+b) - (b+c)][(a+b) - (b+c)]$
$= (a+b-b-c)(a+b-b-c) = (a-c)(a-c).$

38. $(a-2x)^2 + 4(a-2x)(2x-b) + 4(2x-b)^2$
$= [a-2x+2(2x-b)][a-2x+2(2x-b)]$
$= (a-2x+4x-2b)(a-2x+4x-2b)$
$= (a+2x-2b)(a+2x-2b).$

39. $16(a-x)^2 + 32(a-x)(x+b) + 16(x+b)^2$
$= 16[(a-x)^2 + 2(a-x)(x+b) + (x+b)^2]$
$= 16(a-x+x+b)(a-x+x+b)$
$= 16(a+b)(a+b).$

40. $(a+3b)^2 - 4(a+3b)(3b-2c) + 4(3b-2c)^2$
$= [(a+3b) - 2(3b-2c)][(a+3b) - 2(3b-2c)]$
$= (a+3b-6b+4c)(a+3b-6b+4c)$
$= (a-3b+4c)(a-3b+4c).$

41. $(x^2+x+1)^2 + 2(x+1)(x^2+x+1) + (x+1)^2$
$= (x^2+x+1+x+1)(x^2+x+1+x+1)$
$= (x^2+2x+2)(x^2+2x+2).$

42. $(a+b+c)^2 + 2(a+b-c)(a+b+c) + (a+b-c)^2$
$= (a+b+c+a+b-c)(a+b+c+a+b-c)$
$= 2(a+b)\cdot 2(a+b) = 4(a+b)(a+b).$

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10. $x^4 - 81 = (x^2+9)(x^2-9) = (x^2+9)(x+3)(x-3).$

12. $a^{16} - b^8 = (a^8+b^4)(a^8-b^4) = (a^8+b^4)(a^4+b^2)(a^4-b^2)$
$= (a^8+b^4)(a^4+b^2)(a^2+b)(a^2-b).$

15. $m^4 - 16n^4 = (m^2+4n^2)(m^2-4n^2)$
$= (m^2+4n^2)(m+2n)(m-2n).$

18. $36a^4 - 225 = 9(2a^2+5)(2a^2-5).$

19. $121b^2 - a^2c^2 = (11b+ac)(11b-ac).$

21. $64x^2 - 625y^2 = (8x+25y)(8x-25y).$

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23. $400x^2 - 36y^2 = 4(10x+3y)(10x-3y).$

24. $144m^2 - 16n^2 = 16(9m^2-n^2) = 16(3m+n)(3m-n).$

25. $x^4y^4 - 256 = (x^2y^2+16)(x^2y^2-16)$
$= (x^2y^2+16)(xy+4)(xy-4).$

26. $2a^8 - 2b^8 = 2(a^8-b^8) = 2(a^4+b^4)(a^4-b^4)$
$= 2(a^4+b^4)(a^2+b^2)(a^2-b^2)$
$= 2(a^4+b^4)(a^2+b^2)(a+b)(a-b).$

27. $4m^4 - 4b^4 = 4(m^4-b^4) = 4(m^2+b^2)(m^2-b^2)$
$= 4(m^2+b^2)(m+b)(m-b).$

28. $3x^4 - 3y^6 = 3(x^4-y^6) = 3(x^2+y^3)(x^2-y^8).$

29. $5x^8 - 5 = 5(x^8-1) = 5(x^4+1)(x^4-1)$
$= 5(x^4+1)(x^2+1)(x^2-1)$
$= 5(x^4+1)(x^2+1)(x+1)(x-1).$

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30. $3a^5 - 3a = 3a(a^4 - 1) = 3a(a^2+1)(a^2-1)$
$= 3a(a^2+1)(a+1)(a-1).$

31. $x^3 - xy^2 = x(x^2 - y^2) = x(x+y)(x-y).$

32. $x^2 - .01 = (x + .1)(x - .1).$

33. $a^4 - \frac{1}{16} = (a^2 + \frac{1}{4})(a^2 - \frac{1}{4}) = (a^2 + \frac{1}{4})(a + \frac{1}{2})(a - \frac{1}{2}).$

35. $x^{2n-2} - y^{4m} = (x^{n-1} + y^{2m})(x^{n-1} - y^{2m}).$

36. $x^{2n+1} - xy^{2n} = x(x^{2n} - y^{2n}) = x(x^n + y^n)(x^n - y^n).$

38. $a^2 - (b+c)^2 = (a+b+c)(a-b-c).$

39. $b^2 - (2a+b)^2 = (b+2a+b)(b-2a-b)$
$= (2a+2b)(-2a) = -4a(a+b).$

40. $a^2 - (a+b)^2 = (a+a+b)(a-a-b)$
$= (2a+b)(-b) = -b(2a+b).$

41. $4c^2 - (b+c)^2 = (2c+b+c)(2c-b-c)$
$= (3c+b)(c-b).$

42. $9b^2 - (a-x)^2 = (3b+a-x)(3b-a+x).$

43. $9a^2 - (2a-5)^2 = (3a+2a-5)(3a-2a+5)$
$= (5a-5)(a+5) = 5(a-1)(a+5).$

44. $x^4 - (3x^2 - 2y)^2 = (x^2 + 3x^2 - 2y)(x^2 - 3x^2 + 2y)$
$= 2(2x^2 - y) \cdot 2(y - x^2)$
$= 4(2x^2 - y)(y - x^2).$

45. $49a^2 - (5a - 4b)^2 = (7a + 5a - 4b)(7a - 5a + 4b)$
$= 4(3a - b) \cdot 2(a + 2b)$
$= 8(3a - b)(a + 2b).$

47. $(2a+3b)^2 - (a+b)^2 = (2a+3b+a+b)(2a+3b-a-b)$
$= (3a+4b)(a+2b).$

48. $(5a-3b)^2 - (a-b)^2 = (5a-3b+a-b)(5a-3b-a+b)$
$= 2(3a-2b) \cdot 2(2a-b)$
$= 4(3a-2b)(2a-b).$

49. $(2x+5)^2 - (5-3x)^2 = (2x+5+5-3x)(2x+5-5+3x)$
$= (10-x)(5x) = 5x(10-x).$

50. $(a-2b)^2 - (a-5)^2 = (a-2b+a-5)(a-2b-a+5)$
$= (2a-2b-5)(5-2b).$

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51. $(2x-3y)^2 - (3y+z)^2$
$= (2x-3y+3y+z)(2x-3y-3y-z)$
$= (2x+z)(2x-6y-z).$

52. $(5b-4c)^2 - (3a-2c)^2$
$= (5b-4c+3a-2c)(5b-4c-3a+2c)$
$= (5b-6c+3a)(5b-2c-3a).$

53. $(4x-3y)^2 - (2x-3a)^2$
$= (4x-3y+2x-3a)(4x-3y-2x+3a)$
$= 3(2x-y-a)(2x-3y+3a).$

54. $(9x+6y)^2 - (4x-3y)^2$
$= (9x+6y+4x-3y)(9x+6y-4x+3y)$
$= (13x+3y)(5x+9y).$

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55. $(x^3+x^2)^2-(2x+2)^2$
$=(x^3+x^2+2x+2)(x^3+x^2-2x-2)$
$=[x^2(x+1)+2(x+1)][x^2(x+1)-2(x+1)]$
$=(x^2+2)(x+1)(x^2-2)(x+1).$

56. $(a+b+c)^2-(a-b-c)^2$
$=(a+b+c+a-b-c)(a+b+c-a+b+c)$
$=2a\cdot 2(b+c)=4a(b+c).$

59. $a^2-2ax+x^2-n^2=(a-x)^2-n^2=(a-x+n)(a-x-n).$

60. $b^2+2by+y^2-n^2=(b+y)^2-n^2=(b+y+n)(b+y-n).$

61. $1-4q+4q^2-a^2=(1-2q)^2-a^2=(1-2q+a)(1-2q-a).$

62. $r^2-2rx+x^2-16t^2=(r-x)^2-(4t)^2=(r-x+4t)(r-x-4t).$

63. $9a^2b-6ab^2+b^3-4bc^2=b(9a^2-6ab+b^2-4c^2)$
$=b[(3a-b)^2-(2c)^2]=b(3a-b+2c)(3a-b-2c).$

64. $c^2-a^2-b^2-2ab=c^2-(a^2+2ab+b^2)=c^2-(a+b)^2$
$=(c+a+b)(c-a-b).$

65. $b^2-x^2-y^2+2xy=b^2-(x^2-2xy+y^2)=b^2-(x-y)^2$
$=(b+x-y)(b-x+y).$

66. $4c^2-x^2-y^2-2xy=4c^2-(x^2+2xy+y^2)$
$=(2c)^2-(x+y)^2=(2c+x+y)(2c-x-y).$

67. $9c^2-x^2-y^2+2xy=9c^2-(x^2-2xy+y^2)=(3c)^2-(x-y)^2$
$=(3c+x-y)(3c-x+y).$

68. $x^3-a^2x-4b^2x-4abx=x(x^2-a^2-4ab-4b^2)$
$=x[x^2-(a+2b)^2]=x(x+a+2b)(x-a-2b).$

69. $bc^2-9a^2b-b^3-6ab^2=b(c^2-9a^2-6ab-b^2)$
$=b[c^2-(3a+b)^2]=b(c+3a+b)(c-3a-b).$

70. $ab^2-4a^3-12a^2c-9ac^2=a(b^2-4a^2-12ac-9c^2)$
$=a[b^2-(2a+3c)^2]=a(b+2a+3c)(b-2a-3c).$

71. $a^2-2ab+b^2-c^2+2cd-d^2=(a-b)^2-(c-d)^2$
$=(a-b+c-d)(a-b-c+d).$

72. $x^2-2xy+y^2-m^2+10m-25=(x-y)^2-(m-5)^2$
$=(x-y+m-5)(x-y-m+5).$

73. $4x^2+9-12x+10mn-m^2-25n^2$
$=4x^2-12x+9-m^2+10mn-25n^2=(2x-3)^2-(m-5n)^2$
$=(2x-3+m-5n)(2x-3-m+5n).$

74. $x^2-a^2+y^2-b^2+2xy-2ab=x^2+2xy+y^2-a^2-2ab-b^2$
$=(x+y)^2-(a+b)^2=(x+y+a+b)(x+y-a-b).$

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16. $+6a^2=(+2a)\times(+3a)$ and $+5a=(+2a)+(+3a)$;
$\therefore x^2+5ax+6a^2=(x+2a)(x+3a).$

17. $+5a^2=(-a)\times(-5a)$ and $(-a)+(-5a)=-6a$;
$\therefore x^2-6ax+5a^2=(x-a)(x-5a).$

18. $-12b^2=(+2b)\times(-6b)$ and $-4b=(+2b)+(-6b)$;
$\therefore y^2-4by-12b^2=(y+2b)(y-6b).$

19. $-28n^2=(+4n)\times(-7n)$ and $-3n=(+4n)+(-7n)$;
$\therefore y^2-3ny-28n^2=(y+4n)(y-7n).$

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20. $-2a^2n^2 = (+an) \times (-2an)$ and $-an = (+an) + (-2an)$;
$\therefore z^2 - anz - 2a^2n^2 = (z + an)(z - 2an)$.

21. $+90c^2 = (+9c) \times (+10c)$ and $+19c = (+9c) + (+10c)$;
$\therefore x^4 + 19cx^2 + 90c^2 = (x^2 + 9c)(x^2 + 10c)$.

22. $+20a^2 = (+2a) \times (+10a)$ and $+12a = (+2a) + (+10a)$;
$\therefore x^6 + 12ax^3 + 20a^2 = (x^3 + 2a)(x^3 + 10a)$.

23. $+24b^4 = (-3b^2) \times (-8b^2)$ and $-11b^2 = (-3b^2) + (-8b^2)$;
$\therefore x^{10} - 11b^2x^5 + 24b^4 = (x^5 - 3b^2)(x^5 - 8b^2)$.

24. $5nx^2 - 55nx + 150n = 5n(x^2 - 11x + 30) = 5n(x-5)(x-6)$.

25. $3a^2bx^2 - 3a^2bx - 6a^2b = 3a^2b(x^2 - x - 2) = 3a^2b(x+1)(x-2)$.

26. $4ax + 2ax^2 - 48a = 2a(x^2 + 2x - 24) = 2a(x+6)(x-4)$.

27. $11a^2x - 55ax + 66x = 11x(a^2 - 5a + 6) = 11x(a-2)(a-3)$.

28. $20bx + 10b^2 - 630x^2 = 10(b^2 + 2bx - 63x^2) = 10(b+9x)(b-7x)$.

29. $x^2 - (c+d)x + cd = x^2 + (-c-d)x + (-c)(-d) = (x-c)(x-d)$.

30. $x^2 - (a-d)x - ad = x^2 + (-a+d)x + (-a)(d) = (x-a)(x+d)$.

31. $x^2 - 2(a-n)x - 4an = x^2 + (-2a+2n)x + (-2a)(2n)$
$= (x-2a)(x+2n)$.

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2.

First factor, try	$5x+2$,	$5x-2$,	$5x+1$,	$5x-1$.
Second factor, try	$x-1$,	$x+1$,	$x-2$,	$x+2$.
Products, 2d terms,	$-3x$,	$+3x$,	$-9x$,	$+9x$.

$\therefore 5x^2 + 9x - 2 = (5x-1)(x+2)$.

3.

First factor, try	$2x+4$,	$2x-4$,	$2x+6$,	$2x-6$,	$2x-3$,	$2x+3$,	…
Second factor, try	$x-3$,	$x+3$,	$x-2$,	$x+2$,	$x+4$,	$x-4$,	…
Products, 2d terms,	$-2x$,	$2x$,	$2x$,	$-2x$,	$5x$,	$-5x$,	…

$\therefore 2x^2 - 5x - 12 = (2x+3)(x-4)$.

4.

First factor, try	$3x-1$,	$3x-2$, …
Second factor, try	$x-10$,	$x-5$, …
Products, 2d terms,	$-31x$,	$-17x$, …

$\therefore 3x^2 - 17x + 10 = (3x-2)(x-5)$.

5.

First factor, try	$3x+2$,	$3x-2$,	$3x-3$,	$3x+3$, …
Second factor, try	$x-3$,	$x+3$,	$x+2$,	$x-2$, …
Products, 2d terms,	$-7x$,	$+7x$,	$+3x$,	$-3x$, …

$\therefore 3x^2 - 7x - 6 = (3x+2)(x-3)$.

6.

First factor, try	$6x-3$,	$6x-2$,	$3x-3$,	$3x-2$.
Second factor, try	$x-2$,	$x-3$,	$2x-2$,	$2x-3$.
Products, 2d terms,	$-15x$,	$-20x$,	$-12x$,	$-13x$.

$\therefore 6x^2 - 13x + 6 = (3x-2)(2x-3)$.

7.

First factor, try	$6x+5$,	$6x-5$,	$6x+7$,	$6x-7$,	$3x+5$, …
Second factor, try	$x-7$,	$x+7$,	$x-5$,	$x+5$,	$2x-7$, …
Products, 2d terms,	$-37x$,	$+37x$,	$-23x$,	$+23x$,	$-11x$.

$\therefore 6x^2 - 11x - 35 = (3x+5)(2x-7)$.

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11. First factor, try $2x+1$, $2x-1$, $2x+3$, $2x-3$, $2x+5$, $2x-5$.
Second factor, try $\underline{x-15}$, $\underline{x+15}$, $\underline{x-5}$, $\underline{x+5}$, $\underline{x-3}$, $\underline{x+3}$.
Products, 2d terms, $-29x$, $+29x$, $-7x$, $+7x$, $-x$, $+x$.
$\therefore 2x^2+x-15=(2x-5)(x+3)$.

12. $9x^2-42x+40=(3x)^2-14(3x)+40$
Put m for $3x$, $=m^2-14m+40$
$=(m-4)(m-10)$
Put $3x$ for m, $=(3x-4)(3x-10)$.

13. First factor, try $5x+1$, $5x+2$, $5x+3$, ...
Second factor, try $\underline{x+6}$, $\underline{x+3}$, $\underline{x+2}$, ...
Products, 2d terms, $+31x$, $+17x$, $+13x$, ...
$\therefore 5x^2+13x+6=(5x+3)(x+2)$.

14. $25x^2+15x+2=(5x)^2+3(5x)+2$
Put m for $5x$, $=m^2+3m+2$
$=(m+1)(m+2)$
Put $5x$ for m, $=(5x+1)(5x+2)$.

15. $16x^2+20x-66=(4x)^2+5(4x)-66$
Put m for $4x$, $=m^2+5m-66$
$=(m-6)(m+11)$
Put $4x$ for m, $=(4x-6)(4x+11)$
$=2(2x-3)(4x+11)$.

16. $36x^2-48x-20=4(9x^2-12x-5)$
Put m for $3x$, $=4(m^2-4m-5)$
$=4(m+1)(m-5)$
Put $3x$ for m, $=4(3x+1)(3x-5)$.

17. Since $\sqrt{9x^2}$, or $3x$, is not exactly contained in $43x$, the factors of $9x^2$ are not $3x$ and $3x$, and hence are $9x$ and x.
First factor, try $9x-1$, $9x-2$, ...
Second factor, try $\underline{x+10}$, $\underline{x+5}$, ...
Products, 2d terms, $+89x$, $+43x$, ...
$\therefore 9x^2+43x-10=(9x-2)(x+5)$.

18. $25x^2+25x-24=(5x)^2+5(5x)-24$
Put m for $5x$, $=m^2+5m-24$
$=(m+8)(m-3)$
Put $5x$ for m, $=(5x+8)(5x-3)$.

19. $49x^2-42x-55=(7x)^2-6(7x)-55$
Put m for $7x$, $=m^2-6m-55$
$=(m+5)(m-11)$
Put $7x$ for m, $=(7x+5)(7x-11)$.

20. $$16x^2+50x-21=\frac{64x^2+200x-84}{4}=\frac{(8x)^2+25(8x)-84}{4}$$
$$=\frac{(8x+28)(8x-3)}{4}=(2x+7)(8x-3).$$

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21. $$9x^4 - 10x^2 - 16 = \frac{81x^4 - 90x^2 - 144}{9} = \frac{(9x^2)^2 - 10(9x^2) - 144}{9}$$
$$= \frac{(9x^2 - 18)(9x^2 + 8)}{9} = (x^2 - 2)(9x^2 + 8).$$

22. $$27b^4 - 3b^2 - 14 = \frac{81b^4 - 9b^2 - 42}{3} = \frac{(9b^2)^2 - 1(9b^2) - 42}{3}$$
$$= \frac{(9b^2 + 6)(9b^2 - 7)}{3} = (3b^2 + 2)(9b^2 - 7).$$

23. $10x^6 - 2x^3 - 44 = 2(5x^6 - x^3 - 22)$.

$$\begin{array}{lll} \text{To factor } 5x^6 - x^3 - 22, \text{ try} & 5x^3 - 22, & 5x^3 - 11, \ldots \\ \text{multiplied by} & \underline{x^3 + \;\, 1,} & \underline{x^3 + \;\, 2,} \ldots \\ \text{Products, 2d terms,} & -17x^3, & -x^3, \ldots \end{array}$$

$\therefore 10x^6 - 2x^3 - 44 = 2(x^3 + 2)(5x^3 - 11)$.

24.
$$\begin{array}{lll} \text{First factor, try} & 2x + y, & 2x + 2y. \\ \text{Second factor, try} & \underline{x + 2y,} & \underline{x + y.} \\ \text{Products, 2d terms,} & +5xy, & +4xy. \end{array}$$

$\therefore 2x^2 + 5xy + 2y^2 = (2x + y)(x + 2y)$.

25.
$$\begin{array}{lll} \text{First factor, try} & 2x + y, & 2x - y. \\ \text{Second factor, try} & \underline{x - 2y,} & \underline{x + 2y.} \\ \text{Products, 2d terms,} & -3xy, & +3xy. \end{array}$$

$\therefore 2x^2 + 3xy - 2y^2 = (2x - y)(x + 2y)$.

26.
$$\begin{array}{lll} \text{First factor, try} & 3x - y, & 3x - 3y. \\ \text{Second factor, try} & \underline{x - 3y,} & \underline{x - y.} \\ \text{Products, 2d terms,} & -10xy, & -6xy. \end{array}$$

$\therefore 3x^2 - 10xy + 3y^2 = (3x - y)(x - 3y)$.

27. $$15x^2 - 14x - 8 = \frac{225x^2 - 210x - 120}{15} = \frac{(15x)^2 - 14(15x) - 120}{15}$$
$$= \frac{(15x + 6)(15x - 20)}{15} = \frac{3(5x + 2) \cdot 5(3x - 4)}{15}$$
$$= (5x + 2)(3x - 4).$$

28. $$15x^2 + 17x - 4 = \frac{225x^2 + 255x - 60}{15} = \frac{(15x)^2 + 17(15x) - 60}{15}$$
$$= \frac{(15x - 3)(15x + 20)}{15} = \frac{3(5x - 1) \cdot 5(3x + 4)}{15}$$
$$= (5x - 1)(3x + 4).$$

29. $$21a^2 - a - 10 = \frac{441a^2 - 21a - 210}{21} = \frac{(21a)^2 - 1(21a) - 210}{21}$$
$$= \frac{(21a + 14)(21a - 15)}{21} = \frac{7(3a + 2) \cdot 3(7a - 5)}{21}$$
$$= (3a + 2)(7a - 5).$$

30. $$18x^2 - 3x - 36 = \frac{36x^2 - 6x - 72}{2} = \frac{(6x)^2 - 1(6x) - 72}{2}$$
$$= \frac{(6x + 8)(6x - 9)}{2} = \frac{2(3x + 4) \cdot 3(2x - 3)}{2}$$
$$= 3(3x + 4)(2x - 3).$$

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9. $x - x^4 = x(1 - x^3) = x(1 - x)(1 + x + x^2).$

10. $v^7 + 27\,v = v(v^6 + 27) = v[(v^2)^3 + (3)^3]$
$= v(v^2 + 3)(v^4 - 3\,v^2 + 9).$

11. $a^3b^3 - c^3d^3 = (ab - cd)(a^2b^2 + abcd + c^2d^2).$

12. $r^6 + 64\,s^3 = (r^2)^3 + (4\,s)^3 = (r^2 + 4\,s)(r^4 - 4\,r^2s + 16\,s^2).$

13. $x^3y^6z^9 - 216 = (xy^2z^3)^3 - (6)^3 = (xy^2z^3 - 6)(x^2y^4z^6 + 6\,xy^2z^3 + 36).$

14. $343\,n^3 + 1000 = (7\,n)^3 + (10)^3 = (7\,n + 10)(49\,n^2 - 70\,n + 100).$

15. $r^{3x} - 729\,s^{3x} = (r^x)^3 - (9\,s^x)^3 = (r^x - 9\,s^x)(r^{2x} + 9\,r^xs^x + 81\,s^{2x}).$

16. $512\,x^{6n} + 64\,y^{3n} = 64\,[(2\,x^{2n})^3 + (y^n)^3]$
$= 64\,(2\,x^{2n} + y^n)(4\,x^{4n} - 2\,x^{2n}y^n + y^{2n}).$

17. $1 + (a + b)^3 = [1 + (a + b)][1 - (a + b) + (a + b)^2]$
$= (1 + a + b)(1 - a - b + a^2 + 2\,ab + b^2).$

18. $(x - y)^3 - 8 = [(x - y) - 2][(x - y)^2 + 2(x - y) + 4]$
$= (x - y - 2)(x^2 - 2\,xy + y^2 + 2\,x - 2\,y + 4).$

19. $8\,(m + n)^3 + 125\,n^3 = [2(m + n)]^3 + (5\,n)^3$
$= [2(m + n) + 5\,n][4(m + n)^2 - 2(m + n)(5\,n) + (5\,n)^2]$
$= (2\,m + 7\,n)(4\,m^2 + 8\,mn + 4\,n^2 - 10\,mn - 10\,n^2 + 25\,n^2)$
$= (2\,m + 7\,n)(4\,m^2 - 2\,mn + 19\,n^2).$

20. $(x - y)^3 - (x + y)^3$
$= [(x - y) - (x + y)][(x - y)^2 + (x - y)(x + y) + (x + y)^2]$
$= (-2\,y)(x^2 - 2\,xy + y^2 + x^2 - y^2 + x^2 + 2\,xy + y^2)$
$= -2\,y(3\,x^2 + y^2).$

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2. $x^6 - y^6 = (x^3)^2 - (y^3)^2$
§ 152, $= (x^3 + y^3)(x^3 - y^3)$
§ 159, $= (x + y)(x^2 - xy + y^2)(x - y)(x^2 + xy + y^2).$

3. $x^6 - 1 = (x^3)^2 - (1)^2$
§ 152, $= (x^3 + 1)(x^3 - 1)$, or $(x^3 + 1^3)(x^3 - 1^3)$,
§ 159, $= (x + 1)(x^2 - x + 1)(x - 1)(x^2 + x + 1).$

4. $a^8 - b^8 = (a^4 + b^4)(a^4 - b^4)$
§ 152, $= (a^4 + b^4)(a^2 + b^2)(a^2 - b^2)$
§ 152, $= (a^4 + b^4)(a^2 + b^2)(a + b)(a - b).$

5. $x^4 - 16 = (x^2 + 4)(x^2 - 4)$
§ 152, $= (x^2 + 4)(x + 2)(x - 2).$

6. $x^4 - 81 = (x^2 + 9)(x^2 - 9)$
§ 152, $= (x^2 + 9)(x + 3)(x - 3).$

7. $a^4 - 625 = (a^2 + 25)(a^2 - 25)$
§ 152, $= (a^2 + 25)(a + 5)(a - 5).$

8. $1 - b^6 = (1)^2 - (b^3)^2$
§ 152, $= (1 + b^3)(1 - b^3)$, or $(1^3 + b^3)(1^3 - b^3)$,
§ 159, $= (1 + b)(1 - b + b^2)(1 - b)(1 + b + b^2).$

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9. $64 - y^6 = (8)^2 - (y^3)^2$

§ 152, $= (8 + y^3)(8 - y^3)$, or $(2^3 + y^3)(2^3 - y^3)$,

§ 159, $= (2 + y)(4 - 2y + y^2)(2 - y)(4 + 2y + y^2)$.

10. $1 - x^8 = 1^2 - (x^4)^2$

§ 152, $= (1 + x^4)(1 - x^4)$

§ 152, $= (1 + x^4)(1 + x^2)(1 - x^2)$

$= (1 + x^4)(1 + x^2)(1 + x)(1 - x)$.

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9. $x^3 - 9x^2 + 23x - 15$

Substituting 1 for x, $= 1 - 9 + 23 - 15 = 0$; $\therefore x - 1$ is a factor,

and $x^3 - 9x^2 + 23x - 15 = (x - 1)(x^2 - 8x + 15)$

§ 154, $= (x - 1)(x - 3)(x - 5)$.

10. $x^3 - 13x^2 + 47x - 35$

Substituting 1 for x, $= 1 - 13 + 47 - 35 = 0$; $\therefore x - 1$ is a factor,

and $x^3 - 13x^2 + 47x - 35 = (x - 1)(x^2 - 12x + 35)$

§ 154, $= (x - 1)(x - 5)(x - 7)$.

11. $x^3 - 14x^2 + 35x - 22$

Substituting 1 for x, $= 1 - 14 + 35 - 22 = 0$; $\therefore x - 1$ is a factor,

and $x^3 - 14x^2 + 35x - 22 = (x - 1)(x^2 - 13x + 22)$

§ 154, $= (x - 1)(x - 2)(x - 11)$.

12. $x^3 - 4x^2 - 7x + 10$

Substituting 1 for x, $= 1 - 4 - 7 + 10 = 0$; $\therefore x - 1$ is a factor,

and $x^3 - 4x^2 - 7x + 10 = (x - 1)(x^2 - 3x - 10)$

§ 154, $= (x - 1)(x + 2)(x - 5)$.

13. $x^3 - 6x^2 - 9x + 14$

Substituting 1 for x, $= 1 - 6 - 9 + 14 = 0$; $\therefore x - 1$ is a factor,

and $x^3 - 6x^2 - 9x + 14 = (x - 1)(x^2 - 5x - 14)$

§ 154, $= (x - 1)(x + 2)(x - 7)$.

14. $x^3 - 12x^2 + 41x - 30$

Substituting 1 for x, $= 1 - 12 + 41 - 30 = 0$; $\therefore x - 1$ is a factor,

and $x^3 - 12x^2 + 41x - 30 = (x - 1)(x^2 - 11x + 30)$

§ 154, $= (x - 1)(x - 5)(x - 6)$.

15. $x^3 - 11x^2 + 31x - 21$

Substituting 1 for x, $= 1 - 11 + 31 - 21 = 0$; $\therefore x - 1$ is a factor,

and $x^3 - 11x^2 + 31x - 21 = (x - 1)(x^2 - 10x + 21)$

§ 154, $= (x - 1)(x - 3)(x - 7)$.

16. $x^3 - 10x^2 + 29x - 20$

Substituting 1 for x, $= 1 - 10 + 29 - 20 = 0$; $\therefore x - 1$ is a factor,

and $x^3 - 10x^2 + 29x - 20 = (x - 1)(x^2 - 9x + 20)$

§ 154, $= (x - 1)(x - 4)(x - 5)$.

17. $x^3 - 16x^2 + 71x - 56$.

Substituting 1 for x, $= 1 - 16 + 71 - 56 = 0$; $\therefore x - 1$ is a factor,

and $x^3 - 16x^2 + 71x - 56 = (x - 1)(x^2 - 15x + 56)$

§ 154, $= (x - 1)(x - 7)(x - 8)$.

18. $x^3 - 57x + 56$

Substituting 1 for x, $= 1 - 57 + 56 = 0$; $\therefore x - 1$ is a factor,

and $x^3 - 57x + 56 = (x - 1)(x^2 + x - 56)$

§ 154, $= (x - 1)(x - 7)(x + 8)$.

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19. $x^3 - 21xy^2 + 20y^3$

Substituting y for x, $= y^3 - 21y^3 + 20y^3 = 0$; $\therefore x - y$ is a factor,

and $x^3 - 21xy^2 + 20y^3 = (x - y)(x^2 + xy - 20y^2)$
$= (x - y)(x - 4y)(x + 5y)$.

20. $x^3 - 31xy^2 - 30y^3$

Substituting $(-y)$ for x, $= -y^3 + 31y^3 - 30y^3 = 0$; $\therefore (x + y)$ is a factor,

and $x^3 - 31xy^2 - 30y^3 = (x + y)(x^2 - xy - 30y^2)$
$= (x + y)(x + 5y)(x - 6y)$.

21. $x^3 - 13xy^2 + 12y^3$

Substituting y for x, $= y^3 - 13y^3 + 12y^3 = 0$; $\therefore x - y$ is a factor,

and $x^3 - 13xy^2 + 12y^3 = (x - y)(x^2 + xy - 12y^2)$
$= (x - y)(x - 3y)(x + 4y)$.

22. $x^3 - 7x + 6$

Substituting 1 for x, $= 1 - 7 + 6 = 0$; $\therefore x - 1$ is a factor,

and $x^3 - 7x + 6 = (x - 1)(x^2 + x - 6)$

§ 154, $= (x - 1)(x - 2)(x + 3)$.

23. $x^3 - 19x + 30$

Substituting 2 for x, $= 8 - 38 + 30 = 0$; $\therefore x - 2$ is a factor,

and $x^3 - 19x + 30 = (x - 2)(x^2 + 2x - 15)$

§ 154, $= (x - 2)(x - 3)(x + 5)$.

24. $x^3 - 67x - 126$

Substituting -2 for x, $= -8 + 134 - 126 = 0$; $\therefore x + 2$ is a factor,

and $x^3 - 67x - 126 = (x + 2)(x^2 - 2x - 63)$
$= (x + 2)(x + 7)(x - 9)$.

25. $x^3 - 39x - 70$

Substituting -2 for x, $= -8 + 78 - 70 = 0$; $\therefore x + 2$ is a factor,

and $x^3 - 39x - 70 = (x + 2)(x^2 - 2x - 35)$

§ 154, $= (x + 2)(x + 5)(x - 7)$.

26. $a^3 + 4a^2 - 11a - 30$

Substituting -2 for a, $= -8 + 16 + 22 - 30 = 0$; $\therefore a + 2$ is a factor,

and $a^3 + 4a^2 - 11a - 30 = (a + 2)(a^2 + 2a - 15)$

§ 154, $= (a + 2)(a - 3)(a + 5)$.

27. $a^3 + 9a^2 + 26a + 24$

Substituting -2 for a, $= -8 + 36 - 52 + 24 = 0$; $\therefore a + 2$ is a factor,

and $a^3 + 9a^2 + 26a + 24 = (a + 2)(a^2 + 7a + 12)$

§ 154, $= (a + 2)(a + 3)(a + 4)$.

28. $m^3 - 6m^2 - m + 30$

Substituting -2 for m, $= -8 - 24 + 2 + 30 = 0$; $\therefore m + 2$ is a factor,

and $m^3 - 6m^2 - m + 30 = (m + 2)(m^2 - 8m + 15)$

§ 154, $= (m + 2)(m - 3)(m - 5)$.

29. $b^3 - 5b^2 - 29b + 105$

Substituting 3 for b, $= 27 - 45 - 87 + 105 = 0$; $\therefore b - 3$ is a factor,

and $b^3 - 5b^2 - 29b + 105 = (b - 3)(b^2 - 2b - 35)$

§ 154, $= (b - 3)(b + 5)(b - 7)$.

30. $a^3 + 10a^2 - 17a - 66$

Substituting -2 for a, $= -8 + 40 + 34 - 66 = 0$; $\therefore a + 2$ is a factor,

and $a^3 + 10a^2 - 17a - 66 = (a + 2)(a^2 + 8a - 33)$

§ 154, $= (a + 2)(a - 3)(a + 11)$.

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31. $m^3 + 7m^2 + 2m - 40$

Substituting 2 for m, $= 8 + 28 + 4 - 40 = 0$; $\therefore m - 2$ is a factor,

and $m^3 + 7m^2 + 2m - 40 = (m-2)(m^2 + 9m + 20)$

§ 154, $= (m-2)(m+4)(m+5)$.

32. $b^3 + 16b^2 + 73b + 90$

Substituting -2 for b, $= -8 + 64 - 146 + 90 = 0$; $\therefore b+2$ is a factor,

and $b^3 + 16b^2 + 73b + 90 = (b+2)(b^2 + 14b + 45)$

§ 154, $= (b+2)(b+5)(b+9)$.

33. $n^3 + 12n^2 + 41n + 42$

Substituting -2 for n, $= -8 + 48 - 82 + 42 = 0$; $n + 2$ is a factor,

and $n^3 + 12n^2 + 41n + 42 = (n+2)(n^2 + 10n + 21)$

§ 154, $= (n+2)(n+3)(n+7)$.

34. $x^4 - 15x^2 + 10x + 24$

Substituting -1 for x, $= 1 - 15 - 10 + 24 = 0$.

Substituting 2 for x, $= 16 - 60 + 20 + 24 = 0$.

Hence, $x + 1$ and $x - 2$ are factors.

$\therefore x^4 - 15x^2 + 10x + 24 = (x+1)(x-2)(x^2 + x - 12)$

§ 154, $= (x+1)(x-2)(x-3)(x+4)$.

35. $x^4 - 25x^2 + 60x - 36$

Substituting 1 for x, $= 1 - 25 + 60 - 36 = 0$.

Substituting 2 for x, $= 16 - 100 + 120 - 36 = 0$.

Hence, $x - 1$ and $x - 2$ are factors.

$\therefore x^4 - 25x^2 + 60x - 36 = (x-1)(x-2)(x^2 + 3x - 18)$

§ 154, $= (x-1)(x-2)(x-3)(x+6)$.

36. $x^4 + 13x^2 - 54x + 40$

Substituting 1 for x, $= 1 + 13 - 54 + 40 = 0$.

Substituting 2 for x, $= 16 + 52 - 108 + 40 = 0$.

Hence, $x - 1$ and $x - 2$ are factors.

$\therefore x^4 + 13x^2 - 54x + 40 = (x-1)(x-2)(x^2 + 3x + 20)$.

37. $x^4 + 22x^2 + 27x - 50$

Substituting 1 for x, $= 1 + 22 + 27 - 50 = 0$.

Substituting -2 for x, $= 16 + 88 - 54 - 50 = 0$.

Hence, $x - 1$ and $x + 2$ are factors.

$\therefore x^4 + 22x^2 + 27x - 50 = (x-1)(x+2)(x^2 - x + 25)$.

38. $x^4 - 9x^2y^2 - 4xy^3 + 12y^4$

Substituting y for x, $= y^4 - 9y^4 - 4y^4 + 12y^4 = 0$.

Substituting $-2y$ for x, $= 16y^4 - 36y^4 + 8y^4 + 12y^4 = 0$.

Hence, $x - y$ and $x + 2y$ are factors.

$\therefore x^4 - 9x^2y^2 - 4xy^3 + 12y^4 = (x-y)(x+2y)(x^2 - xy - 6y^2)$

§ 154, $= (x-y)(x+2y)(x+2y)(x-3y)$.

39. $x^4 - 9x^2y^2 + 12xy^3 - 4y^4$

Substituting y for x, $= y^4 - 9y^4 + 12y^4 - 4y^4 = 0$.

Substituting $2y$ for x, $= 16y^4 - 36y^4 + 24y^4 - 4y^4 = 0$.

Hence, $x - y$ and $x - 2y$ are factors.

$\therefore x^4 - 9x^2y^2 + 12xy^3 - 4y^4 = (x-y)(x-2y)(x^2 + 3xy - 2y^2)$.

40. $x^4 - x^3 - 7x^2 + x + 6$

Substituting 1 for x, $= 1 - 1 - 7 + 1 + 6 = 0$.

Substituting -1 for x, $= 1 + 1 - 7 - 1 + 6 = 0$.

Hence, $x - 1$ and $x + 1$ are factors.

$\therefore x^4 - x^3 - 7x^2 + x + 6 = (x-1)(x+1)(x^2 - x - 6)$

§ 154, $= (x-1)(x+1)(x+2)(x-3)$.

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41. $x^4 - 9x^3 + 21x^2 + x - 30$

Substituting -1 for x, $= 1 + 9 + 21 - 1 - 30 = 0.$

Substituting 2 for x, $= 16 - 72 + 84 + 2 - 30 = 0.$

Hence, $x + 1$ and $x - 2$ are factors.

$\therefore x^4 - 9x^3 + 21x^2 + x - 30 = (x + 1)(x - 2)(x^2 - 8x + 15)$

§ 154, $= (x + 1)(x - 2)(x - 3)(x - 5).$

42. $x^4 + 8x^3 + 14x^2 - 8x - 15$

Substituting -1 for x, $= 1 - 8 + 14 + 8 - 15 = 0.$

Substituting 1 for x, $= 1 + 8 + 14 - 8 - 15 = 0.$

Hence, $x + 1$ and $x - 1$ are factors.

$\therefore x^4 + 8x^3 + 14x^2 - 8x - 15 = (x + 1)(x - 1)(x^2 + 8x + 15)$

§ 154, $= (x + 1)(x - 1)(x + 3)(x + 5).$

43. $x^5 - 4x^4 + 19x^2 - 28x + 12$

Substituting 1 for x, $= 1 - 4 + 19 - 28 + 12 = 0.$

Substituting 2 for x, $= 32 - 64 + 76 - 56 + 12 = 0.$

Hence, $x - 1$ and $x - 2$ are factors. Removing these factors by division, the quotient is $x^3 - x^2 - 5x + 6.$

Substituting 2 for x, $x^3 - x^2 - 5x + 6 = 8 - 4 - 10 + 6 = 0.$

Hence, $x - 2$ is a factor of $x^3 - x^2 - 5x + 6$, and the other factor is $x^2 + x - 3.$

$\therefore x^5 - 4x^4 + 19x^2 - 28x + 12 = (x - 1)(x - 2)(x - 2)(x^2 + x - 3).$

44. $x^5 - 18x^3 + 30x^2 - 19x + 30$

Substituting 2 for x, $= 32 - 144 + 120 - 38 + 30 = 0.$

Substituting 3 for x, $= 243 - 486 + 270 - 57 + 30 = 0.$

Hence, $x - 2$ and $x - 3$ are factors. Removing these factors by division, the remaining factor is found to be $x^3 + 5x^2 + x + 5.$

Substituting -5 for x, $x^3 + 5x^2 + x + 5 = -125 + 125 - 5 + 5 = 0.$

Hence, $x + 5$ is a factor, and the other factor is $x^2 + 1.$

$\therefore x^5 - 18x^3 + 30x^2 - 19x + 30 = (x - 2)(x - 3)(x + 5)(x^2 + 1).$

45. $x^5 - 10x^4 + 40x^3 - 80x^2 + 80x - 32$

Substituting 2 for x, $= 32 - 160 + 320 - 320 + 160 - 32 = 0.$

Hence, $x - 2$ is a factor. Removing this factor by division, the quotient is $x^4 - 8x^3 + 24x^2 - 32x + 16.$

Substituting 2 for x, $= 16 - 64 + 96 - 64 + 16 = 0.$

Hence, $x - 2$ is a factor. Removing this factor by division, the quotient is $x^3 - 6x^2 + 12x - 8.$

Substituting 2 for x, $= 8 - 24 + 24 - 8 = 0.$

Hence, $x - 2$ is a factor, and the other factor is $x^2 - 4x + 4$, which is equal to $(x - 2)(x - 2).$

$\therefore x^5 - 10x^4 + 40x^3 - 80x^2 + 80x - 32 = (x - 2)(x - 2)(x - 2)(x - 2)(x - 2).$

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2. $9x^2 + 4y^2 + 25z^2 - 12xy + 30xz - 20yz$

Sol. ex. 1, $= (3x)^2 + (-2y)^2 + (5z)^2 + 2(3x)(-2y) + 2(3x)(5z) + 2(-2y)(5z)$

§ 111, $= (3x - 2y + 5z)(3x - 2y + 5z).$

3. $25m^2 + 36n^2 + p^2 - 60mn - 10mp + 12np$

Sol. ex. 1, $= (5m)^2 + (-6n)^2 + (-p)^2 + 2(5m)(-6n) + 2(5m)(-p) + 2(-6n)(-p)$

§ 111, $= (5m - 6n - p)(5m - 6n - p).$

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4. $a^2 + 16x^4 + 36y^2 - 8ax^2 + 12ay - 48x^2y$

Sol. ex. 1, $= (a)^2 + (-4x^2)^2 + (6y)^2 + 2(a)(-4x^2) + 2(a)(6y) + 2(-4x^2)(6y)$

§ 111, $= (a - 4x^2 + 6y)(a - 4x^2 + 6y).$

5. $x^2 + 4a^2 + b^2 + y^2 + 4ax - 2bx + 2xy - 4ab + 4ay - 2by$

Sol. ex. 1, $= (x)^2 + (2a)^2 + (-b)^2 + (y)^2 + 2(x)(2a) + 2(x)(-b) + 2(x)(y) + 2(2a)(-b) + 2(2a)(y) + 2(-b)(y)$

§ 111, $= (x + 2a - b + y)(x + 2a - b + y).$

6. $m^2 + 4n^2 + a^2 + 9 - 4mn - 2am + 6m + 4an - 12n - 6a$

Sol. ex. 1, $= (m)^2 + (-2n)^2 + (-a)^2 + (3)^2 + 2(m)(-2n) + 2(m)(-a) + 2(m)(3) + 2(-2n)(-a) + 2(-2n)(3) + 2(-a)(3)$

§ 111, $= (m - 2n - a + 3)(m - 2n - a + 3).$

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4. $x^4 + x^2y^2 + y^4 = x^4 + 2x^2y^2 + y^4 - x^2y^2$
$= (x^2 + y^2)^2 - (xy)^2$
$= (x^2 + xy + y^2)(x^2 - xy + y^2).$

5. $a^8 + a^4b^4 + b^8 = a^8 + 2a^4b^4 + b^8 - a^4b^4$
$= (a^4 + b^4)^2 - (a^2b^2)^2$
$= (a^4 + a^2b^2 + b^4)(a^4 - a^2b^2 + b^4)$

Ex. 1, $= (a^2 + ab + b^2)(a^2 - ab + b^2)(a^4 - a^2b^2 + b^4).$

6. $9x^4 + 20x^2y^2 + 16y^4 = 9x^4 + 24x^2y^2 + 16y^4 - 4x^2y^2$
$= (3x^2 + 4y^2)^2 - (2xy)^2$
$= (3x^2 + 2xy + 4y^2)(3x^2 - 2xy + 4y^2).$

7. $4a^4 + 11a^2b^2 + 9b^4 = 4a^4 + 12a^2b^2 + 9b^4 - a^2b^2$
$= (2a^2 + 3b^2)^2 - (ab)^2$
$= (2a^2 + ab + 3b^2)(2a^2 - ab + 3b^2).$

8. $16a^4 - 17a^2x^2 + x^4 = 16a^4 + 8a^2x^2 + x^4 - 25a^2x^2$
$= (4a^2 + 5ax + x^2)(4a^2 - 5ax + x^2)$
$= (4a + x)(a + x)(4a - x)(a - x).$

9. $25x^4 - 29x^2y^2 + 4y^4 = 25x^4 + 20x^2y^2 + 4y^4 - 49x^2y^2$
$= (5x^2 + 7xy + 2y^2)(5x^2 - 7xy + 2y^2)$
$= (5x + 2y)(x + y)(5x - 2y)(x - y).$

10. $x^4 + x^2 + 1 = x^4 + 2x^2 + 1 - x^2 = (x^2 + 1)^2 - x^2$
$= (x^2 + x + 1)(x^2 - x + 1).$

11. $n^8 + n^4 + 1 = n^8 + 2n^4 + 1 - n^4 = (n^4 + 1)^2 - (n^2)^2$
$= (n^4 + n^2 + 1)(n^4 - n^2 + 1)$
$= (n^2 + n + 1)(n^2 - n + 1)(n^4 - n^2 + 1).$

12. $16x^4 + 4x^2y^2 + y^4 = 16x^4 + 8x^2y^2 + y^4 - 4x^2y^2$
$= (4x^2 + y^2)^2 - (2xy)^2$
$= (4x^2 + 2xy + y^2)(4x^2 - 2xy + y^2).$

13. $a^4b^4 - 21a^2b^2 + 36 = a^4b^4 - 12a^2b^2 + 36 - 9a^2b^2$
$= (a^2b^2 - 6)^2 - (3ab)^2$
$= (a^2b^2 + 3ab - 6)(a^2b^2 - 3ab - 6).$

14. $25a^4 - 14a^2b^4 + b^8 = 25a^4 - 10a^2b^4 + b^8 - 4a^2b^4$
$= (5a^2 - b^4)^2 - (2ab^2)^2$
$= (5a^2 + 2ab^2 - b^4)(5a^2 - 2ab^2 - b^4).$

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15. $9a^4 + 26a^2b^2 + 25b^4 = 9a^4 + 30a^2b^2 + 25b^4 - 4a^2b^2$
$= (3a^2 + 5b^2)^2 - (2ab)^2$
$= (3a^2 + 2ab + 5b^2)(3a^2 - 2ab + 5b^2).$

16. $b^4 + 64 = b^4 + 16b^2 + 64 - 16b^2 = (b^2+8)^2 - (4b)^2$
$= (b^2 + 4b + 8)(b^2 - 4b + 8).$

17. $a^4 + 4b^4 = a^4 + 4a^2b^2 + 4b^4 - 4a^2b^2$
$= (a^2 + 2b^2)^2 - (2ab)^2$
$= (a^2 + 2ab + 2b^2)(a^2 - 2ab + 2b^2).$

18. $m^8 + 4 = m^8 + 4m^4 + 4 - 4m^4$
$= (m^4 + 2)^2 - (2m^2)^2$
$= (m^4 + 2m^2 + 2)(m^4 - 2m^2 + 2).$

19. $a^4 + 324 = a^4 + 36a^2 + 324 - 36a^2$
$= (a^2 + 18)^2 - (6a)^2$
$= (a^2 + 6a + 18)(a^2 - 6a + 18).$

20. $a^8 - 16 = (a^4 + 4)(a^4 - 4)$
$= (a^4 + 4a^2 + 4 - 4a^2)(a^4 - 4)$
$= [(a^2 + 2)^2 - (2a)^2](a^2 + 2)(a^2 - 2)$
$= (a^2+2a+2)(a^2-2a+2)(a^2+2)(a^2-2).$

21. $m^5 + 4mn^4 = m(m^4 + 4n^4)$
$= m(m^4 + 4m^2n^2 + 4n^4 - 4m^2n^2)$
$= m[(m^2 + 2n^2)^2 - (2mn)^2]$
$= m(m^2 + 2mn + 2n^2)(m^2 - 2mn + 2n^2).$

22. $x^4 + 64y^4 = x^4 + 16x^2y^2 + 64y^4 - 16x^2y^2$
$= (x^2 + 8y^2)^2 - (4xy)^2$
$= (x^2 + 4xy + 8y^2)(x^2 - 4xy + 8y^2).$

23. $4a^4 + 81 = 4a^4 + 36a^2 + 81 - 36a^2$
$= (2a^2 + 9)^2 - (6a)^2$
$= (2a^2 + 6a + 9)(2a^2 - 6a + 9).$

24. $x^5y^2 + 4xy^2 = xy^2(x^4 + 4)$
$= xy^2(x^4 + 4x^2 + 4 - 4x^2)$
$= xy^2[(x^2 + 2)^2 - (2x)^2]$
$= xy^2(x^2 + 2x + 2)(x^2 - 2x + 2).$

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2. $a^2 + 2ab + b^2 + 8ac + 8bc + 15c^2$
$= (a+b)^2 + 8c(a+b) + 3c \cdot 5c = (a + b + 3c)(a + b + 5c).$

3. $x^2 - 6xy + 9y^2 + 6xz - 18yz + 5z^2$
$= (x-3y)^2 + 6z(x-3y) + z \cdot 5z = (x-3y+z)(x-3y+5z).$

4. $m^2 + n^2 - 2mn + 7mp - 7np - 30p^2$
$= (m - n)^2 + 7p(m - n) + (-3p)(+10p)$
$= (m - n + 10p)(m - n - 3p).$

5. $16n^2 + 55 - 64n - 16m + m^2 + 8mn$
$= m^2 + 8mn + 16n^2 - 16m - 64n + 55$
$= (m + 4n)^2 - 16(m + 4n) + (-5)(-11)$
$= (m + 4n - 5)(m + 4n - 11).$

6. $9m^4 + k^2 - 30 + 39m^2 + 13k + 6m^2k$
$= 9m^4 + 6m^2k + k^2 + 39m^2 + 13k - 30$
$= (3m^2 + k)^2 + 13(3m^2 + k) + (-2)(+15)$
$= (3m^2 + k - 2)(3m^2 + k + 15).$

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7. $25a^2 + y^2 + 10x^2 + 10ay - 35ax - 7xy$
$= 25a^2 + 10ay + y^2 - 35ax - 7xy + 10x^2$
$= (5a + y)^2 - 7x(5a + y) + (-2x)(-5x)$
$= (5a + y - 5x)(5a + y - 2x).$

8. $a^2 + b^2 + c^2 + 2ab + 2ac + 2bc + 5a + 5b + 5c + 6$
$= (a + b + c)^2 + 5(a + b + c) + 2 \cdot 3$
$= (a + b + c + 2)(a + b + c + 3).$

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1. $y^4 - 1 = (y^2 + 1)(y^2 - 1) = (y^2 + 1)(y + 1)(y - 1).$

2. $1 - x^8 = (1 + x^4)(1 - x^4) = (1 + x^4)(1 + x^2)(1 + x)(1 - x).$

3. $x^{10} - 1 = (x^5 + 1)(x^5 - 1)$
$= (x + 1)(x^4 - x^3 + x^2 - x + 1)(x - 1)(x^4 + x^3 + x^2 + x + 1).$

4. $x^6 - 1 = (x^3 + 1)(x^3 - 1) = (x + 1)(x^2 - x + 1)(x - 1)(x^2 + x + 1).$

5. $a - a^7 = a(1 - a^6) = a(1 + a^3)(1 - a^3)$
$= a(1 + a)(1 - a + a^2)(1 - a)(1 + a + a^2).$

6. $b^7 + b = b(b^6 + 1) = b(b^2 + 1)(b^4 - b^2 + 1).$

7. $p^4 + 4 = p^4 + 4p^2 + 4 - 4p^2 = (p^2 + 2)^2 - (2p)^2$
$= (p^2 + 2p + 2)(p^2 - 2p + 2).$

8. $1 + x^{12} = 1^3 + (x^4)^3 = (1 + x^4)(1 - x^4 + x^8).$

9. $y - a^4y = y(1 - a^4) = y(1 + a^2)(1 + a)(1 - a).$

10. $x^2y - y^3 = y(x^2 - y^2) = y(x + y)(x - y).$

11. $a^{13} - ab^{12} = a(a^{12} - b^{12}) = a(a^6 + b^6)(a^6 - b^6)$
$= a(a^2 + b^2)(a^4 - a^2b^2 + b^4)(a^3 + b^3)(a^3 - b^3)$
$= a(a^2 + b^2)(a^4 - a^2b^2 + b^4)(a + b)(a^2 - ab + b^2)(a - b)(a^2 + ab + b^2).$

12. $a^4 - 256 = (a^2 + 16)(a^2 - 16) = (a^2 + 16)(a + 4)(a - 4).$

13. $64 - 2y^5 = 2(32 - y^5) = 2(2^5 - y^5)$
$= 2(2 - y)(16 + 8y + 4y^2 + 2y^3 + y^4).$

14. $7n^7 - 7n = 7n(n^6 - 1) = 7n(n^3 + 1)(n^3 - 1)$
$= 7n(n + 1)(n^2 - n + 1)(n - 1)(n^2 + n + 1).$

15. $4x^4 - 4x = 4x(x^3 - 1) = 4x(x - 1)(x^2 + x + 1).$

16. $7y^4 - 175 = 7(y^4 - 25) = 7(y^2 + 5)(y^2 - 5).$

17. $8 - 27a^3x^3 = 2^3 - (3ax)^3$
$= (2 - 3ax)[2^2 + 2 \cdot 3ax + (3ax)^2]$
$= (2 - 3ax)(4 + 6ax + 9a^2x^2).$

18. $32x - 2x^3 = 2x(16 - x^2) = 2x(4 + x)(4 - x).$

19. $6b^4 + 24 = 6(b^4 + 4) = 6(b^4 + 4b^2 + 4 - 4b^2)$
$= 6[(b^2 + 2)^2 - (2b)^2]$
$= 6(b^2 + 2b + 2)(b^2 - 2b + 2).$

20. $a^5 + 27a^2 = a^2(a^3 + 27) = a^2(a + 3)(a^2 - 3a + 9).$

21. $b^2 - 196 = (b + 14)(b - 14).$

22. $450 - 2a^2 = 2(225 - a^2) = 2(15 + a)(15 - a).$

23. $4m^3 + .004 = .004(1000m^3 + 1) = .004[(10m)^3 + 1^3]$
$= .004(10m + 1)(100m^2 - 10m + 1).$

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24. $$125 - 8x^6 = 5^3 - (2x^2)^3 = (5 - 2x^2)[5^2 + 5 \cdot 2x^2 + (2x^2)^2]$$
$$= (5 - 2x^2)(25 + 10x^2 + 4x^4).$$

25. $$x^2 - xy - 132y^2 = x^2 + (+11y - 12y)x + (+11y)(-12y)$$
$$= (x + 11y)(x - 12y).$$

26. $$ax^2 - 3ax - 4a = a(x^2 - 3x - 4) = a(x + 1)(x - 4).$$

27. $$x^3 + 5x^2 - 6x = x(x^2 + 5x - 6) = x(x - 1)(x + 6).$$

28. $$3x^2 + 30x + 27 = 3(x^2 + 10x + 9) = 3(x + 1)(x + 9).$$

29. $$128a^2 - 250a^5 = 2a^2(64 - 125a^3) = 2a^2[4^3 - (5a)^3]$$
$$= 2a^2(4 - 5a)[4^2 + 4 \cdot 5a + (5a)^2]$$
$$= 2a^2(4 - 5a)(16 + 20a + 25a^2).$$

30. $$5x^{10} + 10x^5 - 15 = 5(x^{10} + 2x^5 - 3) = 5(x^5 - 1)(x^5 + 3)$$
$$= 5(x - 1)(x^4 + x^3 + x^2 + x + 1)(x^5 + 3).$$

31. $$6x^2 - 19x + 15 = 6x^2 - 9x - 10x + 15$$
$$= 3x(2x - 3) - 5(2x - 3)$$
$$= (2x - 3)(3x - 5).$$

32. $$x^{2n} + 2x^ny^p + y^{2p} = (x^n)^2 + 2 \cdot x^n \cdot y^p + (y^p)^2$$
$$= (x^n + y^p)(x^n + y^p).$$

33. $$7x^2 - 77xy - 84y^2 = 7(x^2 - 11xy - 12y^2) = 7(x + y)(x - 12y).$$

35. $$9x^2 - 24xy + 16y^2 = (3x)^2 - 2(3x)(4y) + (4y)^2$$
$$= (3x - 4y)(3x - 4y).$$

39. $$10a^2c + 33ac - 7c = c(10a^2 + 33a - 7)$$
$$= \frac{c(100a^2 + 330a - 70)}{10} = \frac{c[(10a)^2 + 33(10a) + (-2)(+35)]}{10}$$
$$= \frac{c(10a - 2)(10a + 35)}{10} = \frac{c \cdot 2(5a - 1) \cdot 5(2a + 7)}{10} = c(5a-1)(2a+7).$$

40. $$60ny^2 - 61ny - 56n = n(60y^2 - 61y - 56)$$
$$= \frac{n(3600y^2 - 61 \cdot 60y - 7 \cdot 8 \cdot 5 \cdot 12)}{60}$$
$$= \frac{n[(60y)^2 - 61(60y) + 35(-96)]}{60}$$
$$= \frac{n(60y + 35)(60y - 96)}{60}$$
$$= \frac{n \cdot 5(12y + 7) \cdot 12(5y - 8)}{60}$$
$$= n(12y + 7)(5y - 8).$$

41. $$25x^2 + 60xy + 36y^2 = (5x)^2 + 2(5x)(6y) + (6y)^2$$
$$= (5x + 6y)(5x + 6y).$$

42. $$6ax^2 + 5axy - 6ay^2 = a(6x^2 + 5xy - 6y^2)$$
By trial,
$$= a(3x - 2y)(2x + 3y).$$

43. $$169x^4 - 26ax^3 + a^2x^2 = x^2(169x^2 - 26ax + a^2)$$
$$= x^2[(13x)^2 - 2(13x)a + a^2]$$
$$= x^2(13x - a)(13x - a).$$

44. $$a^4c^4 + a^2b^2c^2 + b^4 = a^4c^4 + 2a^2b^2c^2 + b^4 - a^2b^2c^2$$
$$= (a^2c^2 + b^2)^2 - (abc)^2$$
$$= (a^2c^2 + abc + b^2)(a^2c^2 - abc + b^2).$$

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45. $$16x^4+4x^2y^2+y^4=16x^4+8x^2y^2+y^4-4x^2y^2$$
$$=(4x^2+y^2)^2-(2xy)^2$$
$$=(4x^2+2xy+y^2)(4x^2-2xy+y^2).$$

46. $$b^4c-13b^2c+42c=c(b^4-13b^2+42)$$
$$=c(b^2-7)(b^2-6).$$

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47. $$17x^2+25x-18=\frac{289x^2+25\cdot17x-306}{17}=\frac{(17x)^2+25(17x)-9\cdot34}{17}$$
$$=\frac{(17x-9)(17x+34)}{17}=(17x-9)(x+2).$$

48. First factor, try $5x-\ y$, $\quad 5x-5y$.
Second factor, try $\underline{x-5y}$, $\quad \underline{x-\ y}$.
Products, 2d terms, $-26xy$, $\quad -10xy$.
$\therefore\ 5x^2-26xy+5y^2=(5x-y)(x-5y).$

50. $$8a^2-21ab-9b^2=-(9b^2+21ab-8a^2)$$
$$=-[(3b)^2+7a(3b)+(-a)(+8a)]$$
$$=-(3b-a)(3b+8a)$$
$$=(a-3b)(8a+3b).$$

51. $$60a^2+8ax-3x^2$$
$$=\frac{900a^2+120ax-45x^2}{15}=\frac{(30a)^2+4x(30a)+(-5x)(+9x)}{15}$$
$$=\frac{(30a-5x)(30a+9x)}{15}=\frac{5(6a-x)\cdot3(10a+3x)}{15}$$
$$=(6a-x)(10a+3x).$$

52. $$30x^2-37x-77=\frac{900x^2-37(30x)-77\cdot30}{30}=\frac{(30x)^2-37(30x)-70\cdot33}{30}$$
$$=\frac{(30x+33)(30x-70)}{30}=\frac{3(10x+11)\cdot10(3x-7)}{30}$$
$$=(10x+11)(3x-7).$$

53. $$2x^3+28x^2+66x=2x(x^2+14x+33)$$
$$=2x(x+3)(x+11).$$

54. $$a^2+b^2-c^2-2ab=a^2-2ab+b^2-c^2=(a-b)^2-c^2$$
$$=(a-b+c)(a-b-c).$$

55. $$ax^2+10ax-39a=a(x^2+10x-39)$$
$$=a(x-3)(x+13).$$

56. $$n^4+n^2a^2b^4+a^4b^8=n^4+2n^2a^2b^4+a^4b^8-n^2a^2b^4$$
$$=(n^2+a^2b^4)^2-(nab^2)^2$$
$$=(n^2+nab^2+a^2b^4)(n^2-nab^2+a^2b^4).$$

57. $$a^2z^4+a^2z^2+a^2=a^2(z^4+z^2+1)$$
$$=a^2(z^4+2z^2+1-z^2)$$
$$=a^2(z^2+z+1)(z^2-z+1).$$

60. $$b^8+b^4y^2+y^4=b^8+2b^4y^2+y^4-b^4y^2$$
$$=(b^4+b^2y+y^2)(b^4-b^2y+y^2).$$

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61. $x^7 - 2x^6 + x = x(x^6 - 2x^5 + 1).$

Substituting 1 for x, $x^6 - 2x^5 + 1 = 1 - 2 + 1 = 0.$

Hence, $x - 1$ is a factor.

$\therefore\ x^7 - 2x^6 + x = x(x-1)(x^5 - x^4 - x^3 - x^2 - x - 1).$

62. $x^3 + x^2y - 41xy^2 - 105y^3$

Substituting $-3y$ for x, $= -27y^3 + 9y^3 + 123y^3 - 105y^3 = 0.$

Hence, $x + 3y$ is a factor.

$\therefore\ x^3 + x^2y - 41xy^2 - 105y^3 = (x+3y)(x^2 - 2xy - 35y^2)$

§ 154, $= (x+3y)(x+5y)(x-7y).$

63. $x^2 - cx + 2dx - 2cd = x(x-c) + 2d(x-c)$
$= (x-c)(x+2d).$

64. $x^3y + 4x^2y - 31xy - 70y = y(x^3 + 4x^2 - 31x - 70)$

§ 164, $= y(x+2)(x^2 + 2x - 35)$

§ 154, $= y(x+2)(x-5)(x+7).$

65. $x^2 - 3ax + 4bx - 12ab = x(x-3a) + 4b(x-3a)$
$= (x-3a)(x+4b).$

66. $ax^3 - 9ax^2 + 26ax - 24a = a(x^3 - 9x^2 + 26x - 24)$

§ 164, $= a(x-2)(x^2 - 7x + 12)$

§ 154, $= a(x-2)(x-3)(x-4).$

67. $12ax - 8bx - 9ay + 6by = 4x(3a - 2b) - 3y(3a - 2b)$
$= (4x - 3y)(3a - 2b).$

68. $25x^2 - 9y^2 - 24yz - 16z^2 = 25x^2 - (9y^2 + 24yz + 16z^2)$
$= (5x)^2 - (3y + 4z)^2$
$= (5x + 3y + 4z)(5x - 3y - 4z).$

69. $x^2 - z^2 + y^2 - a^2 - 2xy + 2az = x^2 - 2xy + y^2 - (a^2 - 2az + z^2)$
$= (x-y)^2 - (a-z)^2$
$= (x - y + a - z)(x - y - a + z).$

70. $2b^2m - 3ab^2 + 2bmx - 3abx = b^2(2m - 3a) + bx(2m - 3a)$
$= b(b+x)(2m - 3a).$

71. $a^2 + b^2 + c^2 - 2ab - 2ac + 2bc = a^2 - 2ab + b^2 - 2ac + 2bc + c^2$
$= (a-b)^2 - 2c(a-b) + c^2$
$= (a - b - c)(a - b - c).$

72. $x^3y + 14x^2y + 43xy + 30y = y(x^3 + 14x^2 + 43x + 30)$

§ 164, $= y(x+1)(x^2 + 13x + 30)$

§ 154, $= y(x+1)(x+3)(x+10).$

73. $x^3y - 15x^2y + 38xy - 24y = y(x^3 - 15x^2 + 38x - 24)$

§ 164, $= y(x-1)(x^2 - 14x + 24)$

§ 154, $= y(x-1)(x-2)(x-12).$

74. $abx^3 + 3abx^2 - abx - 3ab = ab(x^3 + 3x^2 - x - 3)$
$= ab[x^2(x+3) - (x+3)]$
$= ab(x^2 - 1)(x+3)$
$= ab(x+1)(x-1)(x+3).$

75. $3bmx + 2bm - 3anx - 2an = bm(3x+2) - an(3x+2)$
$= (3x+2)(bm - an).$

76. $20ax^3 - 28ax^2 + 5a^2x - 7a^2 = 4ax^2(5x-7) + a^2(5x-7)$
$= a(4x^2 + a)(5x - 7).$

77. $x^2 + 9y^2 + 25z^2 - 6xy - 10xz + 30yz$
$= (x)^2 + (-3y)^2 + (-5z)^2 + 2x(-3y) + 2x(-5z) + 2(-3y)(-5z)$
$= (x - 3y - 5z)(x - 3y - 5z).$

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78. $9x^2 + y^2 + 16z^2 - 6xy - 8yz + 24xz$
$= (3x)^2 + (-y)^2 + (4z)^2 + 2(3x)(-y) + 2(3x)(4z) + 2(-y)(4z)$
$= (3x - y + 4z)(3x - y + 4z).$

79. $x^2y^2z^2 + a^2b^2 + 1 + 2abxyz + 2xyz + 2ab$
$= (xyz)^2 + (ab)^2 + (1)^2 + 2(xyz)(ab) + 2(xyz)(1) + 2(ab)(1)$
$= (xyz + ab + 1)(xyz + ab + 1).$

80. $a^2b^2 + b^2c^2 + c^2d^2 - 2ab^2c + 2abcd - 2bc^2d$
$= (ab)^2 + (-bc)^2 + (cd)^2 + 2(ab)(-bc) + 2(ab)(cd) + 2(-bc)(cd)$
$= (ab - bc + cd)(ab - bc + cd).$

81. $x^8 + n^4x^4 + n^8 + 2n^2x^6 + 2n^4x^4 + 2n^6x^2$
$= (x^4)^2 + (n^2x^2)^2 + (n^4)^2 + 2(x^4)(n^2x^2) + 2(x^4)(n^4) + 2(n^2x^2)(n^4)$
$= (x^4 + n^2x^2 + n^4)(x^4 + n^2x^2 + n^4)$
$= (x^4 + 2n^2x^2 + n^4 - n^2x^2)(x^4 + 2n^2x^2 + n^4 - n^2x^2)$
$= [(x^2 + n^2)^2 - (nx)^2][(x^2 + n^2)^2 - (nx)^2]$
$= (x^2 + xn + n^2)(x^2 - xn + n^2)(x^2 + xn + n^2)(x^2 - xn + n^2).$

82. $a^2b^2x^2 - a^2b^2 - b^2x^2 + b^2 - a^2x^2 + a^2 + x^2 - 1$
$= a^2b^2(x^2 - 1) - b^2(x^2 - 1) - a^2(x^2 - 1) + (x^2 - 1)$
$= (a^2b^2 - b^2 - a^2 + 1)(x^2 - 1)$
$= [b^2(a^2 - 1) - (a^2 - 1)](x^2 - 1)$
$= (b^2 - 1)(a^2 - 1)(x^2 - 1)$
$= (a + 1)(a - 1)(b + 1)(b - 1)(x + 1)(x - 1).$

83. $(a + b)^6 - 1$
$= [(a + b)^3 + 1][(a + b)^3 - 1]$
$= (a+b+1)[(a+b)^2 - (a+b)1 + 1^2](a+b-1)[(a+b)^2 + (a+b)1 + 1^2]$
$= (a+b+1)(a^2+2ab+b^2-a-b+1)(a+b-1)(a^2+2ab+b^2+a+b+1).$

84. Substituting 1 for a, $a^3 - 2a^2 + 1 = 1 - 2 + 1 = 0$;
$\therefore a^3 - 2a^2 + 1 = (a - 1)(a^2 - a - 1).$

85. Substituting 2 for b, $b^3 - 4b^2 + 8 = 8 - 16 + 8 = 0$;
$\therefore b^3 - 4b^2 + 8 = (b - 2)(b^2 - 2b - 4).$

86. Substituting 5 for x, $x^3 - 10x^2 + 125 = 125 - 250 + 125 = 0$;
$\therefore x^3 - 10x^2 + 125 = (x - 5)(x^2 - 5x - 25).$

87. $8x^4 - 6x^2 - 35 = \dfrac{16x^4 - 12x^2 - 70}{2} = \dfrac{(4x^2)^2 - 3(4x^2) - 70}{2}$
$= \dfrac{(4x^2 + 7)(4x^2 - 10)}{2} = (4x^2 + 7)(2x^2 - 5).$

88. $3x^6 + 96x = 3x(x^5 + 32) = 3x(x^5 + 2^5)$
$= 3x(x + 2)(x^4 - 2x^3 + 4x^2 - 8x + 16).$

89. $(a - 2)^3 + (a - 1)^3$
$= [(a - 2) + (a - 1)][(a - 2)^2 - (a - 2)(a - 1) + (a - 1)^2]$
$= (a - 2 + a - 1)(a^2 - 4a + 4 - a^2 + 3a - 2 + a^2 - 2a + 1)$
$= (2a - 3)(a^2 - 3a + 3).$

90. $12x^3 + 3x^2 - 8x - 2 = 3x^2(4x + 1) - 2(4x + 1)$
$= (4x + 1)(3x^2 - 2).$

91. $2x^2 + 10x + ax + 5a = 2x(x + 5) + a(x + 5)$
$= (x + 5)(2x + a).$

92. $x^3 + 5x^2 - 29x - 105$
§ 164, $= (x + 3)(x^2 + 2x - 35)$
§ 154, $= (x + 3)(x - 5)(x + 7).$

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93. $a^2b^2 - 4\,abx - 4\,x + 2\,ab + 4\,x^2$
$$= a^2b^2 - 4\,abx + 4\,x^2 + 2\,ab - 4\,x$$
$$= (ab - 2\,x)^2 + 2\,(ab - 2\,x)$$
$$= (ab - 2\,x + 2)(ab - 2\,x).$$

94. $(a + b)^2(x - y) - (a + b)(x^2 - y^2).$

$(a + b)(x - y)$ is a monomial factor. Hence, § 146, finding its coefficient by dividing the given polynomial by $(a + b)(x - y)$,
$$(a + b)^2(x - y) - (a + b)(x^2 - y^2) = (a + b)(x - y)\,[(a + b) - (x + y)]$$
$$= (a + b)(x - y)(a + b - x - y).$$

95. $1 - x^2 + abx^2 + bx^3 - bx - ab = 1 - ab - bx - x^2 + abx^2 + bx^3$
$$= (1 - ab - bx) - x^2\,(1 - ab - bx)$$
$$= (1 - x^2)(1 - ab - bx)$$
$$= (1 + x)(1 - x)(1 - ab - bx).$$

96. $x^2 - x^3 + x^2y - xy + x^2y - xy^2 = x(x - y - x^2 + xy + xy - y^2)$
$$= x\,[(x - y) - x\,(x - y) + y(x - y)]$$
$$= x\,(x - y)(1 - x + y).$$

98. $x^3 + 15\,x^2 + 75\,x + 125 = x^3 + 5^3 + 15\,x(x + 5)$
$$= (x + 5)(x^2 - 5\,x + 25) + 15\,x\,(x + 5)$$
$$= (x^2 - 5\,x + 25 + 15\,x)(x + 5)$$
$$= (x^2 + 10\,x + 25)(x + 5)$$
$$= (x + 5)(x + 5)(x + 5).$$

99. $4\,(ab + cd)^2 - (a^2 + b^2 - c^2 - d^2)^2$
$$= (2\,ab + 2\,cd)^2 - (a^2 + b^2 - c^2 - d^2)^2$$
$$= (2\,ab + 2\,cd + a^2 + b^2 - c^2 - d^2)(2\,ab + 2\,cd - a^2 - b^2 + c^2 + d^2)$$
$$= [a^2 + 2\,ab + b^2 - (c^2 - 2\,cd + d^2)]\,[c^2 + 2\,cd + d^2 - (a^2 - 2\,ab + b^2)]$$
$$= (a + b + c - d)(a + b - c + d)(c + d + a - b)(c + d - a + b)$$
$$= (a + b + c - d)(a + b - c + d)(a - b + c + d)(-a + b + c + d).$$

100. $x^{3n} - a^{3b} = (x^n)^3 - (a^b)^3 = (x^n - a^b)(x^{2n} + x^na^b + a^{2b}).$

101. $(a^2 + b^2 - c^2)^2 - 4\,a^2b^2 = (a^2 + b^2 - c^2 + 2\,ab)(a^2 + b^2 - c^2 - 2\,ab)$
$$= (a^2 + 2\,ab + b^2 - c^2)(a^2 - 2\,ab + b^2 - c^2)$$
$$= (a + b + c)(a + b - c)(a - b + c)(a - b - c).$$

103. $x^3 - xy - x^2y + y^2 = x\,(x^2 - y) - y\,(x^2 - y)$
$$= (x - y)(x^2 - y).$$

104. $x^4 - 4\,x^2y^2 + 2\,x^3 - 16\,y^3 = x^2(x^2 - 4\,y^2) + 2\,(x^3 - 8\,y^3)$
$$= x^2(x + 2\,y)(x - 2\,y) + 2\,(x - 2\,y)(x^2 + 2\,xy + 4\,y^2)$$
$$= (x - 2\,y)[x^2\,(x + 2\,y) + 2\,(x^2 + 2\,xy + 4\,y^2)]$$
$$= (x - 2\,y)(x^3 + 2\,x^2y + 2\,x^2 + 4\,xy + 8\,y^2).$$

105. $a^4 - b^4 - (a + b)(a - b) = (a^2 + b^2)(a + b)(a - b) - (a + b)(a - b)$
$$= (a + b)(a - b)(a^2 + b^2 - 1).$$

106. $x^3 - 6\,x^2 + 12\,x - 8 = x^3 - 8 - 6\,x^2 + 12\,x$
$$= (x - 2)(x^2 + 2\,x + 4) - 6\,x\,(x - 2)$$
$$= (x^2 + 2\,x + 4 - 6\,x)(x - 2)$$
$$= (x^2 - 4\,x + 4)(x - 2)$$
$$= (x - 2)(x - 2)(x - 2).$$

107. $1000\,x^3 - 27\,y^3 = (10\,x)^3 - (3\,y)^3$
$$= (10\,x - 3\,y)(100\,x^2 + 30\,xy + 9\,y^2).$$

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108. $(a+x)^4 - x^4 = [(a+x)^2 + x^2][(a+x)^2 - x^2]$
$= (a^2+2ax+2x^2)[(a+x)+x][(a+x)-x]$
$= (a^2+2ax+2x^2)(a+2x)a$
$= a(a+2x)(a^2+2ax+2x^2).$

109. $1+(x+1)^3 = 1^3 + (x+1)^3$
§ 159, $= (1+x+1)[1-1(x+1)+(x+1)^2]$
$= (x+2)(x^2+x+1).$

110. $ab - bx^n + x^ny^m - ay^m = b(a-x^n) - y^m(a-x^n) = (b-y^m)(a-x^n).$

111. $x^9 + 4x = x(x^8+4)$
$= x(x^8+4x^4+4-4x^4)$
$= x(x^4+2x^2+2)(x^4-2x^2+2).$

112. $x^5 - x^2 - x^4 + x^3 = x^5 + x^3 - (x^4+x^2)$
$= x^3(x^2+1) - x^2(x^2+1)$
$= x^2(x-1)(x^2+1).$

113. $(a+b)^4 - (b-c)^4$
$= [(a+b)^2 + (b-c)^2][(a+b)^2 - (b-c)^2]$
$= [(a+b)^2 + (b-c)^2](a+b+b-c)(a+b-b+c)$
$= (a^2+2ab+b^2+b^2-2bc+c^2)(a+2b-c)(a+c)$
$= (a^2+2ab+2b^2-2bc+c^2)(a+2b-c)(a+c).$

114. $3ab(a+b) + a^3 + b^3 = 3ab(a+b) + (a^2-ab+b^2)(a+b)$
$= (a^2+2ab+b^2)(a+b)$
$= (a+b)(a+b)(a+b).$

115. $(x+y)^3 + (x-y)^3$
§ 159, $= [(x+y)+(x-y)][(x+y)^2 - (x+y)(x-y) + (x-y)^2]$
$= 2x(x^2+2xy+y^2-x^2+y^2+x^2-2xy+y^2)$
$= 2x(x^2+3y^2).$

116. $a^3 - (a+b)^3$
§ 159, $= [a-(a+b)][a^2+a(a+b)+(a+b)^2]$
$= -b(3a^2+3ab+b^2).$

117. $x^4 - 119x^2y^2 + y^4 = x^4 + 2x^2y^2 + y^4 - 121x^2y^2$
$= (x^2+y^2)^2 - (11xy)^2$
$= (x^2+11xy+y^2)(x^2-11xy+y^2).$

118. $m^3 + m^2 - mn - mn^2 = m(m^2+m-n-n^2)$
$= m(m^2-n^2+m-n)$
$= m[(m+n)(m-n)+(m-n)]$
$= m(m-n)(m+n+1).$

119. $(x^2-y^2)^2 - (x^2-xy)^2$
$= (x+y)(x-y)\cdot(x+y)(x-y) - x(x-y)\cdot x(x-y)$
$= (x-y)(x-y)[(x+y)(x+y) - x^2]$
$= (x-y)(x-y)(x+y+x)(x+y-x)$
$= y(x-y)(x-y)(2x+y).$

120. $x^6 - y^6 - 3x^2y^2(x^2-y^2)$
$= (x^2-y^2)(x^4+x^2y^2+y^4) - 3x^2y^2(x^2-y^2)$
$= (x^4+x^2y^2+y^4-3x^2y^2)(x^2-y^2)$
$= (x^4-2x^2y^2+y^4)(x^2-y^2)$
$= (x^2-y^2)(x^2-y^2)(x^2-y^2)$
$= (x+y)(x-y)(x+y)(x-y)(x+y)(x-y).$

121. $(x^2+6x+9)^2 - (x^2+5x+6)^2$
$= (x+3)(x+3)(x+3)(x+3) - (x+2)(x+3)(x+2)(x+3)$
$= (x+3)(x+3)[x^2+6x+9-(x^2+4x+4)]$
$= (x+3)(x+3)(2x+5).$

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122. $2 - 3b + 3ab - 2a + 4a^2 - 6a^2b$
$= 2 - 3b - (2a - 3ab) + (4a^2 - 6a^2b)$
$= (2 - 3b) - a(2 - 3b) + 2a^2(2 - 3b)$
$= (2 - 3b)(1 - a + 2a^2).$

123. Substituting 2 for x, $32 - x^5 = 32 - 32 = 0$;
$\therefore x - 2$ and consequently $-1(x - 2)$, or $2 - x$, is a factor.
Dividing by $2 - x$, $32 - x^5 = (2 - x)(16 + 8x + 4x^2 + 2x^3 + x^4)$.

124. Substituting -1 for x, $16 + 5x - 11x^2 = 16 - 5 - 11 = 0$;
$\therefore x + 1$ is one factor.
Dividing by $x + 1$, or $1 + x$, $16 + 5x - 11x^2 = (1 + x)(16 - 11x)$.

125. Substituting a for x, $x^n - a^n = a^n - a^n = 0$;
$\therefore x - a$ is a factor of $x^n - a^n$.
Substituting $-a$ for x, $x^n - a^n = (-a)^n - a^n$
If n is odd, $= -a^n - a^n = -2a^n$;
$\therefore x + a$ is not a factor of $x^n - a^n$ in this case. Hence, $x - a$ is the only rational binomial factor of the first degree.
Dividing by $x - a$, $x^n - a^n = (x - a)(x^{n-1} + x^{n-2}a + x^{n-3}a^2 + \cdots + a^{n-1})$.

126. Substituting $-r$ for x, $x^n + r^n = (-r)^n + r^n$
If n is odd, $= -r^n + r^n = 0$;
$\therefore x + r$ is a factor of $x^n + r^n$, n being odd.
Substituting r for x, $x^n + r^n = r^n + r^n = 2r^n$;
$\therefore x + r$ is the only rational binomial factor of the first degree.
Dividing by $x + r$, $x^n + r^n = (x + r)(x^{n-1} - x^{n-2}r + x^{n-3}r^2 - \cdots + r^{n-1})$.

127. $x^3 - 6bx^2 + 12b^2x - 8b^3$
Substituting $2b$ for x, $= 8b^3 - 24b^3 + 24b^3 - 8b^3 = 0$;
$\therefore x - 2b$ is one factor.
Dividing by $x - 2b$, $= (x - 2b)(x^2 - 4bx + 4b^2)$
Substituting $2b$ for x, $= (x - 2b)(4b^2 - 8b^2 + 4b^2) = 0$
Dividing by $x - 2b$, $= (x - 2b)(x - 2b)(x - 2b)$.

128. Substituting $+a$ for x, $x^n + a^n = (+a)^n + a^n = 2a^n$. (1)
Substituting $-a$ for x, $x^n + a^n = (-a)^n + a^n$
If n is odd, $= -a^n + a^n = 0$. (2)
If n is even, $= +a^n + a^n = 2a^n$. (3)
By (2), $x^3 + a^3$, $x^5 + a^5$, $x^7 + a^7$, $x^9 + a^9$, $x^{11} + a^{11}$, $x^{13} + a^{13}$, $x^{15} + a^{15}$, $x^{17} + a^{17}$, and $x^{19} + a^{19}$ have the binomial factor $x + a$.
By (3) and (1), $x^2 + a^2$, $x^4 + a^4$, $x^6 + a^6$, $x^8 + a^8$. $x^{10} + a^{10}$, $x^{12} + a^{12}$, $x^{14} + a^{14}$, $x^{16} + a^{16}$, $x^{18} + a^{18}$, and $x^{20} + a^{20}$ have neither $x + a$ nor $x - a$ for a binomial factor.

Testing the latter for other binomial factors,

$x^6 + a^6 = (x^2)^3 + (a^2)^3 = 0$, if $x^2 = -a^2$; $\therefore x^2 + a^2$ is a factor;
$x^{10} + a^{10} = (x^2)^5 + (a^2)^5 = 0$, if $x^2 = -a^2$; $\therefore x^2 + a^2$ is a factor;
$x^{12} + a^{12} = (x^4)^3 + (a^4)^3 = 0$, if $x^4 = -a^4$; $\therefore x^4 + a^4$ is a factor;
$x^{14} + a^{14} = (x^2)^7 + (a^2)^7 = 0$, if $x^2 = -a^2$; $\therefore x^2 + a^2$ is a factor;
$x^{18} + a^{18} = (x^2)^9 + (a^2)^9 = 0$, if $x^2 = -a^2$; $\therefore x^2 + a^2$ is a factor;
$x^{20} + a^{20} = (x^4)^5 + (a^4)^5 = 0$, if $x^4 = -a^4$; $\therefore x^4 + a^4$ is a factor.

The above sums of like even powers of x and a have binomial factors because they can be written as sums of like *odd* powers of two other numbers, as x^2 and a^2, or x^4 and a^4; but $x^2 + a^2$, $x^4 + a^4$, $x^8 + a^8$, and $x^{16} + a^{16}$ cannot be so expressed, and hence have no rational binomial factors.

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93. $x^3 - 15x^2 + 71x - 105 = 0.$

Factoring by the factor theorem, $(x-3)(x-5)(x-7) = 0.$

$\therefore x - 3 = 0$ or $x - 5 = 0$ or $x - 7 = 0;$

whence, $x = 3$ or 5 or $7.$

94. $x^3 + 10x^2 + 11x - 70 = 0.$

Factoring by the factor theorem, $(x-2)(x+5)(x+7) = 0.$

$\therefore x - 2 = 0$ or $x + 5 = 0$ or $x + 7 = 0;$

whence, $x = 2$ or -5 or $-7.$

95. $x^3 - 12x + 16 = 0.$

Factoring by the factor theorem, $(x-2)(x-2)(x+4) = 0.$

$\therefore x - 2 = 0$ or $x - 2 = 0$ or $x + 4 = 0;$

whence, $x = 2$ or 2 or $-4.$

96. $x^3 - 19x - 30 = 0.$

Factoring by the factor theorem, $(x+2)(x+3)(x-5) = 0.$

$\therefore x + 2 = 0$ or $x + 3 = 0$ or $x - 5 = 0;$

whence, $x = -2$ or -3 or $5.$

97. $x^4 + x^3 - 21x^2 - x + 20 = 0.$

Factoring by the factor theorem, $(x-1)(x+1)(x-4)(x+5) = 0.$

$\therefore x - 1 = 0$ or $x + 1 = 0$ or $x - 4 = 0$ or $x + 5 = 0;$

whence, $x = 1$ or -1 or 4 or $-5.$

98. $x^4 - 7x^3 + x^2 + 63x - 90 = 0.$

Factoring by the factor theorem, $(x-2)(x-3)(x+3)(x-5) = 0.$

$\therefore x - 2 = 0$ or $x - 3 = 0$ or $x + 3 = 0$ or $x - 5 = 0;$

whence, $x = 2$ or 3 or -3 or $5.$

99. $x^5 - 11x^4 + 45x^3 - 85x^2 + 74x - 24 = 0.$

By the factor theorem, $(x-1)(x-1)(x-2)(x-3)(x-4) = 0.$

$\therefore x - 1 = 0$ or $x - 1 = 0$ or $x - 2 = 0$ or $x - 3 = 0$ or $x - 4 = 0;$

whence, $x = 1$ or 1 or 2 or 3 or $4.$

HIGHEST COMMON FACTOR

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30.
$$\begin{array}{l} x^4 + xy^3 = x(x^3 + y^3) = x(x+y)(x^2 - xy + y^2) \\ \underline{x^2y + xy^2 = xy(x+y) = x \cdot y(x+y)} \\ \therefore \text{H. C. F.} = x(x+y). \end{array}$$

31.
$$\begin{array}{l} pq^4 + p^6q = pq(q^3 + p^5) = pq(q^3 + p^5) \\ \underline{qp^3 + q^2p^2 = p^2q(p+q) = p \cdot pq(p+q)} \\ \therefore \text{H C. F.} = pq. \end{array}$$

32.
$$\begin{array}{l} 17\,abc^3d^5 - 51\,a^3bc^4d^4 = 17\,abc^3d^4(d - 3\,a^2c) \\ \underline{abc^2d^2 - 3\,a^3bc^3d = abc^2d\,(d - 3\,a^2c)} \\ \therefore \text{H. C. F.} = abc^2d\,(d - 3\,a^2c). \end{array}$$

33.
$$\begin{array}{l} 38\,xyz - 95\,x^3yz^2 = xyz(38 - 95\,x^2z) \\ \underline{34\,xy^2z - 85\,x^2yz^2 = xyz(34\,y - 85\,xz)} \\ \therefore \text{H. C. F.} = xyz. \end{array}$$

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34.
$$\begin{aligned} x^7y + xy^4 &= xy(x^6+y^3) = xy(x^2+y)(x^4-x^2y+y^2)\\ 2x^5y - 2x^3y^2 + 2xy^3 &= 2xy(x^4-x^2y+y^2) = 2\cdot xy(x^4-x^2y+y^2)\\ \therefore \text{H.C.F.} &= xy(x^4-x^2y+y^2). \end{aligned}$$

35.
$$\begin{aligned} 6r^7 + 10r^6s - 4r^5s^2 &= 2r^5(3r^2+5rs-2s^2) = 2r^5(3r-s)(r+2s)\\ 2r^7 + 2r^6s - 4r^5s^2 &= 2r^5(r^2+rs-2s^2) = 2r^5(r-s)(r+2s)\\ \therefore \text{H.C.F.} &= 2r^5(r+2s). \end{aligned}$$

36.
$$\begin{aligned} x^4 - x^3 - 2x^2 &= x^2(x^2-x-2) = x^2(x-2)(x+1)\\ x^4 - 2x^3 - 3x^2 &= x^2(x^2-2x-3) = x^2(x-3)(x+1)\\ x^4 - 3x^3 - 4x^2 &= x^2(x^2-3x-4) = x^2(x-4)(x+1)\\ \therefore \text{H.C.F} &= x^2(x+1). \end{aligned}$$

37.
$$\begin{aligned} 3m^3n^3 - 3mn^4 &= 3mn^3(m^2-n)\\ 6m^4n^3 + 6m^3n^2 - 6m^2n^3 - 6mn^4 &= 2\cdot 3mn^2(m^3n+m^2-mn-n^2)\\ \therefore \text{H.C.F.} &= 3mn^2. \end{aligned}$$

38.
$$\begin{aligned} 7l^3t^3+35l^2t^3+42lt^3 &= 7lt^3(l^2+5l+6) = 7lt^3(l+3)(l+2)\\ 7l^4t^3+21l^3t^3-28l^2t^3-84lt^3 &= 7lt^3(l^3+3l^2-4l-12) = 7lt^3(l+3)(l+2)(l-2)\\ \therefore \text{H.C.F.} &= 7lt^3(l+3)(l+2). \end{aligned}$$

39.
$$\begin{aligned} x^2+a^2-b^2+2ax &= (x+a)^2-b^2 = (x+a+b)(x+a-b)\\ x^2-a^2+b^2+2bx &= (x+b)^2-a^2 = (x+b+a)(x+b-a)\\ x^2-a^2-b^2-2ab &= x^2-(a+b)^2 = (x+a+b)(x-a-b)\\ \therefore \text{H.C.F.} &= (x+a+b). \end{aligned}$$

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40.
$$\begin{aligned} x^2-6x+5 &= (x-1)(x-5)\\ x^3-5x^2+7x-3 &= (x-1)(x-1)(x-3)\\ \therefore \text{H.C.F.} &= x-1. \end{aligned}$$

41.
$$\begin{aligned} x^2-4 &= (x+2)(x-2)\\ x^3-10x^2+31x-30 &= (x-2)(x-3)(x-5)\\ \therefore \text{H.C.F.} &= x-2. \end{aligned}$$

42.
$$\begin{aligned} x^3-4x+3 &= (x-1)(x^2+x-3)\\ x^3+x^2-37x+35 &= (x-1)(x-5)(x+7)\\ \therefore \text{H.C.F.} &= x-1. \end{aligned}$$

43.
$$\begin{aligned} 3x^4-12x^2 &= 3x^2(x^2-4) = 3x\cdot x(x+2)(x-2)\\ 6x^4+30x^3-96x^2+24x &= 6x(x^3+5x^2-16x+4) = 3x\cdot 2(x-2)(x^2+7x-2)\\ \therefore \text{H.C.F.} &= 3x(x-2). \end{aligned}$$

44.
$$\begin{aligned} a^4b - a^2b^3 &= a^2b(a^2-b^2) = a\cdot ab(a+b)(a-b)\\ a^4b+2a^3b^2+2a^2b^3+ab^4 &= ab(a^3+2a^2b+2ab^2+b^3) = ab(a+b)(a^2+ab+b^2)\\ \therefore \text{H.C.F.} &= ab(a+b). \end{aligned}$$

45.
$$\begin{aligned} 9-n^2 &= -(n^2-9) = -(n+3)(n-3)\\ n^2-n-6 &= (n-3)(n+2) = (n+2)(n-3)\\ \therefore \text{H.C.F.} &= (n-3). \end{aligned}$$

46.
$$\begin{aligned} 1-x^2 &= -(x^2-1) = -(x+1)(x-1)\\ x^3-6x^2-9x+14 &= (x-1)(x+2)(x-7)\\ \therefore \text{H.C.F.} &= (x-1). \end{aligned}$$

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47.

$4 - a^2 = -(a^2 - 4) \quad = -(a+2)(a-2) \quad = -(a+2)(a-2)$

$a^4 + a^3 - 10a^2 - 4a + 24 = (a-2)(a+2)(a^2+a-6)$

$= (a-2)(a+2)(a+3)(a-2)$

$\therefore$ H. C. F. $= (a+2)(a-2) = (a^2-4)$.

48. $(9-x^2)^2 = (x^2-9)(x^2-9) = (x+3)(x-3)(x+3)(x-3)$

$x^4 + 5x^3 - 3x^2 - 45x - 54 = (x+2)(x+3)(x+3)(x-3)$

$\therefore$ H. C. F. $= (x+3)(x+3)(x-3) = (x+3)^2(x-3)$.

49. $(4-c^2)^2 = (c^2-4)(c^2-4) = (c+2)(c-2)(c+2)(c-2)$

$c^3 + 9c^2 + 26c + 24 = (c+2)(c+3)(c+4)$

$\therefore$ H. C. F. $= (c+2)$.

50. $(x-x^2)^3 = (x-x^2)(x-x^2)(x-x^2) = x^3(1-x)(1-x)(1-x)$

$= x^3(1-x)^3$

$(x^2-1)^3 = -(1-x^2)(1-x^2)(1-x^2) \quad = -(1+x)^3(1-x)^3$

$(1-x)^3 \quad = (1-x)^3$

$\therefore$ H.C.F. $= (1-x)^3$.

51. $(1-y^4)^2 = (1+y^2)(1-y^2)(1+y^2)(1-y^2)$

$= (1+y^2)^2(1+y)^2(1-y)^2$

$(y+1)^2(1-y)^2(y^3-7y+6) = (1+y)^2(1-y)^2(y-1)(y+3)(y-2)$

$= -(1+y)^2(1-y)^3(y+3)(y-2)$.

$\therefore$ H. C. F. $= (1+y)^2(1-y)^2$.

52. $xy - y^2 = y(x-y) \quad = y(x-y)$

$-(y^3 - x^2y) = (x^2y - y^3) = y(x^2-y^2) = y(x+y)(x-y)$

$x^2y - xy^2 = xy(x-y) \quad = xy(x-y)$

$\therefore$ H. C. F. $= y(x-y)$.

53. $16 - s^4 = (4+s^2)(4-s^2) = (4+s^2)(2+s)(2-s)$

$2s - s^2 \quad = s(2-s)$

$s^2 - 4s + 4 = 4 - 4s + s^2 \quad = (2-s)(2-s)$

$\therefore$ H. C. F. $= (2-s)$.

54. $y^4 - x^4 = (y^2+x^2)(y+x)(y-x)$

$x^5 + y^5 = (x+y)(x^4 - x^3y + x^2y^2 - xy^3 + y^4)$

$y^2 + 2yx + x^2 = (x+y)(x+y)$

$\therefore$ H. C. F. $= (x+y)$.

55. $x^2 - (y+z)^2 = (x+y+z)(x-y-z) = (x+y+z)(x-y-z)$

$(y-x)^2 - z^2 = (y-x+z)(y-x-z) = (x-y+z)(x-y-z)$

$y^2 - (x-z)^2 = (y+x-z)(y-x+z) = (-x-y+z)(x-y-z)$

$\therefore$ H. C. F. $= (x-y-z)$.

56. $(y-x)^2(n-m)^3 = -(m-n)(m-n)(m-n)(y-x)(y-x)$

$(x^2y - y^3)(m^2n - 2mn^2 + n^3) = -y(y+x)(y-x)(n)(m-n)(m-n)$

$\therefore$ H. C. F. $= (m-n)^2(y-x)$.

57. $(m+2)(m^2-9) = (m+2)(m+3)(m-3)$

$m^4 - 3m^3 + 3m^3n + 3m^2n^2 - 9m^2n + mn^3 - 9mn^2 - 3n^3$

$= (m-3)(m+n)^3$

$\therefore$ H. C. F. $= (m-3)$.

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58.
$$\begin{aligned} 6x^2-3x-45 &= 3(x-3)(2x+5)\\ 9x^2-33x+18 &= 3(x-3)(3x-2)\\ 6x^3-3x^2-39x-18 &= 3(x-3)(2x+1)(x+2) \end{aligned}$$
$$\therefore \text{H. C. F.} = 3(x-3).$$

59.
$$\begin{aligned} 2x^4-x^3-x^2 &= x^2(2x+1)(x-1) = x^2(2x+1)(x-1)\\ 2x^2+x-3 &= (2x+3)(x-1) = (2x+3)(x-1)\\ x^3-x^2-x+1 &= x^2(x-1)-(x-1) = (x+1)(x-1)(x-1) \end{aligned}$$
$$\therefore \text{H. C. F.} = (x-1).$$

60.
$$\begin{aligned} x^4-4x^3+2x^2+x+6 &= (x-2)(x-3)(x^2+x+1)\\ 2x^3-9x^2+7x+6 &= (x-2)(x-3)(2x+1)\\ x^2-5x+6 &= (x-2)(x-3) \end{aligned}$$
$$\therefore \text{H. C. F.} = (x-2)(x-3).$$

61.
$$\begin{aligned} s^3-8 &= (s-2)(s^2+2s+4)\\ s^3+s^2+2s-4 &= (s-1)(s^2+2s+4)\\ s^4+2s^3-s^2-10s-20 &= (s^2+2s+4)(s^2-5) \end{aligned}$$
$$\therefore \text{H. C. F.} = (s^2+2s+4).$$

62.
$$\begin{aligned} x^5+y^5 &= (x+y)(x^4-x^3y+x^2y^2-xy^3+y^4)\\ x^5-2x^4y+2x^3y^2-2x^2y^3+2xy^4-y^5 &= (x-y)(x^4-x^3y+x^2y^2-xy^3+y^4) \end{aligned}$$
$$\therefore \text{H. C. F.} = (x^4-x^3y+x^2y^2-xy^3+y^4).$$

LOWEST COMMON MULTIPLE

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26.
$$\begin{aligned} x^2-y^2 &= (x+y)(x-y)\\ x^4+x^2y^2+y^4 &= (x^2+xy+y^2)(x^2-xy+y^2)\\ x^3+y^3 &= (x+y)(x^2-xy+y^2)\\ x^2+xy+y^2 &= x^2+xy+y^2 \end{aligned}$$
$$\begin{aligned} \therefore \text{L. C. M.} &= (x+y)(x-y)(x^2+xy+y^2)(x^2-xy+y^2)\\ &= (x^3+y^3)(x^3-y^3) = x^6-y^6. \end{aligned}$$

27.
$$\begin{aligned} x^3+x^2y+xy^2+y^3 &= (x^2+y^2)(x+y)\\ x^3-x^2y+xy^2-y^3 &= (x^2+y^2)(x-y) \end{aligned}$$
$$\therefore \text{L. C. M.} = (x^2+y^2)(x+y)(x-y) = x^4-y^4.$$

28.
$$\begin{aligned} a^2+4a+4 &= (a+2)^2\\ a^2-4 &= (a+2)(a-2)\\ 4-a^2 &= -(a+2)(a-2)\\ a^4-16 &= (a^2+4)(a+2)(a-2) \end{aligned}$$
$$\therefore \text{L. C. M.} = (a+2)^2(a-2)(a^2+4) = (a^4-16)(a+2).$$

29.
$$\begin{aligned} a^2-(b+c)^2 &= (a+b+c)(a-b-c)\\ b^2-(c+a)^2 &= (b+c+a)(b-c-a)\\ c^2-(a+b)^2 &= (c+a+b)(c-a-b) \end{aligned}$$
$$\therefore \text{L. C. M.} = (a+b+c)(a-b-c)(b-c-a)(c-a-b).$$

30.
$$\begin{aligned} m-n &= (m-n)\\ (m^2-n^2)^2 &= (m+n)^2(m-n)^2\\ (m+n)^3 &= (m+n)(m+n)(m+n) \end{aligned}$$
$$\therefore \text{L. C.M.} = (m+n)^3(m-n)^2 = (m^2-n^2)^2(m+n).$$

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31.
$$\begin{aligned} a^6 - b^3 &= (a^2 - b)(a^4 + a^2b + b^2) \\ a^8 + a^4b^2 + b^4 &= (a^4 + a^2b + b^2)(a^4 - a^2b + b^2) \\ \hline \therefore \text{L. C. M.} &= (a^2 - b)(a^4 + a^2b + b^2)(a^4 - a^2b + b^2) \\ &= (a^2 - b)(a^8 + a^4b^2 + b^4). \end{aligned}$$

32.
$$\begin{aligned} x^6 + y^6 &= (x^2 + y^2)(x^4 - x^2y^2 + y^4) \\ a^2x^2 - b^2y^2 + a^2y^2 - b^2x^2 &= (a^2 - b^2)(x^2 + y^2) \\ \hline \therefore \text{L. C. M.} &= (x^2 + y^2)(x^4 - x^2y^2 + y^4)(a^2 - b^2) \\ &= (a^2 - b^2)(x^6 + y^6). \end{aligned}$$

33.
$$\begin{aligned} a^4 - a^2 + 1 &= a^4 - a^2 + 1 \\ a^6 + 1 &= (a^2 + 1)(a^4 - a^2 + 1) \\ a^4 + a^2 + 1 &= a^4 + a^2 + 1 \\ a^2 - 1 &= a^2 - 1 \\ \hline \therefore \text{L. C. M.} &= (a^4 - a^2 + 1)(a^2 + 1)(a^4 + a^2 + 1)(a^2 - 1) \\ &= (a^6 + 1)(a^6 - 1) = a^{12} - 1. \end{aligned}$$

34.
$$\begin{aligned} 2(ax^2 - x^3)^2 &= 2(ax^2 - x^3)(ax^2 - x^3) &&= 2x^4(a - x)^2 \\ 3x(a^2x - x^3)^3 &= 3x(a^2x - x^3)(a^2x - x^3)(a^2x - x^3) &&= 3x^4(a + x)^3(a - x)^3 \\ 6(a^2x^2 - a^4) &= -6(a^4 - a^2x^2) &&= -6a^2(a + x)(a - x) \\ \hline \therefore \text{L. C. M.} &= 6a^2x^4(a + x)^3(a - x)^3 = 6a^2x^4(a^2 - x^2)^3. \end{aligned}$$

35.
$$\begin{aligned} (yz^2 - xyz)^2 &= y^2z^2(z - x)^2 \\ y^2(xz^2 - x^3) &= xy^2(z + x)(z - x) \\ x^2z^2 + 2xz^3 + z^4 &= z^2(x + z)^2 \\ \hline \therefore \text{L. C. M.} &= xy^2z^2(z - x)^2(x + z)^2. \end{aligned}$$

36.
$$\begin{aligned} x^3 - 6x^2 + 11x - 6 &= (x - 1)(x - 2)(x - 3) \\ x^3 - 9x^2 + 26x - 24 &= (x - 2)(x - 3)(x - 4) \\ \hline \therefore \text{L. C. M.} &= (x - 1)(x - 2)(x - 3)(x - 4). \end{aligned}$$

37.
$$\begin{aligned} x^3 - 5x^2 - 4x + 20 &= (x - 2)(x + 2)(x - 5) \\ x^3 + 2x^2 - 25x - 50 &= (x + 2)(x - 5)(x + 5) \\ \hline \therefore \text{L. C. M.} &= (x - 2)(x + 2)(x - 5)(x + 5) \\ &= (x^2 - 4)(x^2 - 25). \end{aligned}$$

38.
$$\begin{aligned} x^3 - 4x^2 + 5x - 2 &= (x - 1)(x - 1)(x - 2) \\ x^3 - 8x^2 + 21x - 18 &= (x - 2)(x - 3)^2 \\ \hline \therefore \text{L. C. M.} &= (x - 1)^2(x - 2)(x - 3)^2. \end{aligned}$$

39.
$$\begin{aligned} x^3 + 5x^2 + 7x + 3 &= (x + 1)(x + 1)(x + 3) \\ x^3 - 7x^2 - 5x + 75 &= (x - 5)(x - 5)(x + 3) \\ \hline \therefore \text{L. C. M.} &= (x + 1)^2(x + 3)(x - 5)^2. \end{aligned}$$

40.
$$\begin{aligned} x^3 + 2x^2 - 4x - 8 &= (x - 2)(x + 2)^2 \\ x^3 - x^2 - 8x + 12 &= (x - 2)(x - 2)(x + 3) \\ x^3 + 4x^2 - 3x - 18 &= (x - 2)(x + 3)^2 \\ \hline \therefore \text{L. C. M.} &= (x - 2)^2(x + 2)^2(x + 3)^2. \end{aligned}$$

FRACTIONS

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28. $$\frac{a^2 - b^2}{a^2 + 2ab + b^2} = \frac{(a + b)(a - b)}{(a + b)(a + b)} = \frac{a - b}{a + b}.$$

29. $$\frac{a^2 - 2ab + b^2}{a^2 - b^2} = \frac{(a - b)(a - b)}{(a + b)(a - b)} = \frac{a - b}{a + b}.$$

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30. $\dfrac{4a^2-9x^2}{8a^3+27x^3}=\dfrac{(2a+3x)(2a-3x)}{(2a+3x)(4a^2-6ax+9x^2)}=\dfrac{2a-3x}{4a^2-6ax+9x^2}.$

31. $\dfrac{3a^2+3ab}{a^4+ab^3}=\dfrac{3a(a+b)}{a(a+b)(a^2-ab+b^2)}=\dfrac{3}{a^2-ab+b^2}.$

32. $\dfrac{3x^2y-6xy}{x^4y-8xy}=\dfrac{3xy(x-2)}{xy(x-2)(x^2+2x+4)}=\dfrac{3}{x^2+2x+4}.$

33. $\dfrac{3a^2b-3b^3}{2a^3b-2b^4}=\dfrac{3b(a+b)(a-b)}{2b(a-b)(a^2+ab+b^2)}=\dfrac{3(a+b)}{2(a^2+ab+b^2)}.$

34. $\dfrac{2x^2y^2-8y^4}{4x^3y-32y^4}=\dfrac{2y^2(x+2y)(x-2y)}{4y(x-2y)(x^2+2xy+4y^2)}=\dfrac{y(x+2y)}{2(x^2+2xy+4y^2)}.$

35. $\dfrac{10nx+10ny}{25nx^2-25ny^2}=\dfrac{10n(x+y)}{25n(x+y)(x-y)}=\dfrac{2}{5(x-y)}.$

36. $\dfrac{x^{n+2}-x^n}{x^{n+3}-x^n}=\dfrac{x^n(x+1)(x-1)}{x^n(x-1)(x^2+x+1)}=\dfrac{x+1}{x^2+x+1}.$

37. $\dfrac{a^{n+4}-a^ny^4}{a^{n+3}+a^{n+1}y^2}=\dfrac{a^n(a^2+y^2)(a^2-y^2)}{a^{n+1}(a^2+y^2)}=\dfrac{a^2-y^2}{a}.$

38. $\dfrac{x^4y+x^2y^3+y^5}{x^6-y^6}=\dfrac{y(x^4+x^2y^2+y^4)}{(x^2-y^2)(x^4+x^2y^2+y^4)}=\dfrac{y}{x^2-y^2}.$

39. $\dfrac{x^4y-x^2y^3+y^5}{x^6+y^6}=\dfrac{y(x^4-x^2y^2+y^4)}{(x^2+y^2)(x^4-x^2y^2+y^4)}=\dfrac{y}{x^2+y^2}.$

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40. $\dfrac{a^2-11a+24}{a^2-a-6}=\dfrac{(a-3)(a-8)}{(a-3)(a+2)}=\dfrac{a-8}{a+2}.$

41. $\dfrac{x^3-6x^2+5x}{x^3+2x^2-35x}=\dfrac{x(x-1)(x-5)}{x(x+7)(x-5)}=\dfrac{x-1}{x+7}.$

42. $\dfrac{7x-2x^2-3}{2x^2+7x-4}=\dfrac{(3-x)(2x-1)}{(x+4)(2x-1)}=\dfrac{3-x}{x+4}.$

43. $\dfrac{a(a+2b)^4}{b(a^2-4b^2)^2}=\dfrac{a(a+2b)(a+2b)(a+2b)(a+2b)}{b(a+2b)(a-2b)(a+2b)(a-2b)}=\dfrac{a(a+2b)^2}{b(a-2b)^2}.$

44. $\dfrac{a^3+2a^2b+ab^2}{a^5-2a^3b^2+ab^4}=\dfrac{a(a+b)(a+b)}{a(a+b)(a-b)(a+b)(a-b)}=\dfrac{1}{(a-b)^2}.$

45. $\dfrac{x^2-2x^4+x^6}{x^6-x^2}=\dfrac{x^2(1-x^2)(1-x^2)}{-x^2(1+x^2)(1-x^2)}=\dfrac{1-x^2}{-(1+x^2)}=\dfrac{x^2-1}{x^2+1}.$

46. $\dfrac{x^3+5x^2-6x}{2x^2-2}=\dfrac{x(x+6)(x-1)}{2(x+1)(x-1)}=\dfrac{x(x+6)}{2(x+1)}.$

47. $\dfrac{x^3-7x+6}{x^4-10x^2+9}=\dfrac{(x-1)(x-2)(x+3)}{(x+1)(x-1)(x+3)(x-3)}=\dfrac{x-2}{x^2-2x-3}.$

48. $\dfrac{20-21x+x^3}{x^4-26x^2+25}=\dfrac{(x+5)(x-4)(x-1)}{(x+5)(x-5)(x+1)(x-1)}=\dfrac{x-4}{x^2-4x-5}.$

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49. $\dfrac{x^3+3x^2+3x+1}{4+4x-x^2-x^3}=\dfrac{(x+1)(x+1)(x+1)}{-(x+1)(x+2)(x-2)}=\dfrac{x^2+2x+1}{4-x^2}.$

50. $\dfrac{a^3-3a^2b+3ab^2-b^3}{3ab^2-3a^2b}=\dfrac{(a-b)(a-b)(a-b)}{-3ab(a-b)}$

$$=-\frac{a^2-2ab+b^2}{3ab}$$

$$=\frac{-a^2+2ab-b^2}{3ab}.$$

51. $\dfrac{3a^2+4ax-4x^2}{9a^2-12ax+4x^2}=\dfrac{(3a-2x)(a+2x)}{(3a-2x)(3a-2x)}=\dfrac{a+2x}{3a-2x}.$

52. $\dfrac{2ax-ay-4bx+2by}{4ax-2ay-2bx+by}=\dfrac{(a-2b)(2x-y)}{(2a-b)(2x-y)}=\dfrac{a-2b}{2a-b}.$

53. $\dfrac{9x^3-13a^2x-4a^3}{3bx+3xy-4ab-4ay}=\dfrac{(3x+a)(x+a)(3x-4a)}{(3x-4a)(b+y)}$

$$=\frac{3x^2+4ax+a^2}{b+y}.$$

54. $\dfrac{m-m^2-n+mn}{m-mn+n^2-n}=\dfrac{(m-n)(1-m)}{(m-n)(1-n)}=\dfrac{1-m}{1-n}.$

55. $\dfrac{am-an-m+n}{am-an+m-n}=\dfrac{(a-1)(m-n)}{(a+1)(m-n)}=\dfrac{a-1}{a+1}.$

56. $$\frac{x^3+5x^2-9x-45}{x^3+3x^2-25x-75}.$$

Factoring the numerator by the factor theorem, we have

$$x^3+5x^2-9x-45=(x^2+8x+15)(x-3).$$

By trial, it is found that $x^2+8x+15$ is a factor also of the denominator, whose other factor is $x-5$.

$$\therefore\ \frac{x^3+5x^2-9x-45}{x^3+3x^2-25x-75}=\frac{(x^2+8x+15)(x-3)}{(x^2+8x+15)(x-5)}=\frac{x-3}{x-5}.$$

57. $$\frac{x^3+2x^2-23x-60}{x^3-11x^2-10x+200}.$$

Factoring the numerator by the factor theorem, we have

$$x^3+2x^2-23x-60=(x^2-x-20)(x+3).$$

By trial, it is found that x^2-x-20 is a factor also of the denominator, whose other factor is $x-10$.

$$\therefore\ \frac{x^3+2x^2-23x-60}{x^3-11x^2-10x+200}=\frac{(x^2-x-20)(x+3)}{(x^2-x-20)(x-10)}=\frac{x+3}{x-10}.$$

58. $$\frac{3x^3-7x^2+4}{5x^3-17x^2+16x-4}.$$

Factoring the numerator by the factor theorem, we have

$$3x^3-7x^2+4=(x^2-3x+2)(3x+2).$$

By trial, it is found that x^2-3x+2 is a factor also of the denominator, whose other factor is $5x-2$.

$$\therefore\ \frac{3x^3-7x^2+4}{5x^3-17x^2+16x-4}=\frac{(x^2-3x+2)(3x+2)}{(x^2-3x+2)(5x-2)}=\frac{3x+2}{5x-2}.$$

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59. $$\frac{x^3 - 6x^2y + 2xy^2 + 3y^3}{5y^3 + 2xy^2 - 6x^2y - x^3}.$$

By § 179, $x - y$ is found to be the H.C.F. of numerator and denominator.

$$\therefore \frac{x^3-6x^2y+2xy^2+3y^3}{5y^3+2xy^2-6x^2y-x^3} = \frac{(x-y)(x^2-5xy-3y^2)}{-(x-y)(x^2+7xy+5y^2)} = \frac{-x^2+5xy+3y^2}{x^2+7xy+5y^2}.$$

60. $$\frac{a^2 + b^2 + 2c^2 + 2ab + 3ac + 3bc}{a^2 + b^2 + c^2 + 2ab + 2ac + 2bc}.$$

Factoring numerator and denominator, we have

§ 168, $a^2 + b^2 + 2c^2 + 2ab + 3ac + 3bc = (a + b + c)(a + b + 2c)$,

and, § 166, $a^2 + b^2 + c^2 + 2ab + 2ac + 2bc = (a + b + c)(a + b + c)$.

$$\therefore \frac{a^2+b^2+2c^2+2ab+3ac+3bc}{a^2+b^2+c^2+2ab+2ac+2bc} = \frac{(a+b+c)(a+b+2c)}{(a+b+c)(a+b+c)} = \frac{a+b+2c}{a+b+c}.$$

61. $$\frac{a^2 + b^2 + c^2 + 2ab - 2ac - 2bc}{a^2 + b^2 - c^2 + 2ab}.$$

Factoring numerator and denominator, we have

§ 166, $a^2 + b^2 + c^2 + 2ab - 2ac - 2bc = (a + b - c)(a + b - c)$,

and, § 153, $a^2 + b^2 - c^2 + 2ab = (a + b - c)(a + b + c)$.

$$\therefore \frac{a^2+b^2+c^2+2ab-2ac-2bc}{a^2+b^2-c^2+2ab} = \frac{(a+b-c)(a+b-c)}{(a+b-c)(a+b+c)} = \frac{a+b-c}{a+b+c}.$$

62. $$\frac{a^2 + b^2 + c^2 - 2ab - 2ac + 2bc}{a^2 + b^2 + 5c^2 - 2ab - 6ac + 6bc}.$$

Factoring numerator and denominator, we have

§ 166, $a^2 + b^2 + c^2 - 2ab - 2ac - 2bc = (a - b - c)(a - b - c)$,

and, § 168, $a^2 + b^2 + 5c^2 - 2ab - 6ac + 6bc = (a - b - c)(a - b - 5c)$.

$$\therefore \frac{a^2+b^2+c^2-2ab-2ac+2bc}{a^2+b^2+5c^2-2ab-6ac+6bc} = \frac{(a-b-c)(a-b-c)}{(a-b-c)(a-b-5c)} = \frac{a-b-c}{a-b-5c}.$$

63. $$\frac{4a^2 + 9b^2 + 16c^2 + 12ab + 16ac + 24bc}{4a^2 - 9b^2 + 16c^2 + 16ac}.$$

Factoring numerator and denominator, we have

§ 166, $4a^2 + 9b^2 + 16c^2 + 12ab + 16ac + 24bc$
$= (2a + 3b + 4c)(2a + 3b + 4c)$,

and, § 153, $4a^2 - 9b^2 + 16c^2 + 16ac = (2a + 3b + 4c)(2a - 3b + 4c)$.

$$\therefore \frac{4a^2+9b^2+16c^2+12ab+16ac+24bc}{4a^2-9b^2+16c^2+16ac} = \frac{(2a+3b+4c)(2a+3b+4c)}{(2a+3b+4c)(2a-3b+4c)}$$
$$= \frac{2a + 3b + 4c}{2a - 3b + 4c}.$$

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7. The L.C.D. is $a(m + n)$.

$$\therefore \frac{m-n}{a} = \frac{(m-n)(m+n)}{a(m+n)} = \frac{m^2-n^2}{a(m+n)},$$

$$2 = \frac{2 \cdot a(m+n)}{a(m+n)} = \frac{2am + 2an}{a(m+n)},$$

and $$\frac{a}{m+n} = \frac{a \cdot a}{a(m+n)} = \frac{a^2}{a(m+n)}.$$

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8. The L.C.D. is $x^2 - 1$.

$$\therefore \frac{x^2}{x^2-1} = \frac{x^2}{x^2-1},$$

$$\frac{x}{x+1} = \frac{x(x-1)}{(x+1)(x-1)} = \frac{x^2-x}{x^2-1},$$

and
$$\frac{x}{x-1} = \frac{x(x+1)}{(x-1)(x+1)} = \frac{x^2+x}{x^2-1}.$$

9. The L.C.D. is $a^4 - 16$.

$$\therefore \frac{a^3}{a^4-16} = \frac{a^3}{a^4-16},$$

$$\frac{a}{a^2+4} = \frac{a(a^2-4)}{(a^2+4)(a^2-4)} = \frac{a(a^2-4)}{a^4-16},$$

$$\frac{2a}{4-a^2} = \frac{2a(-a^2-4)}{(4-a^2)(-a^2-4)} = -\frac{2a^3+8a}{a^4-16}.$$

10. The L.C.D. is $a^2 - b^2$.

$$\therefore \frac{4a}{a-b} = \frac{4a(a+b)}{(a-b)(a+b)} = \frac{4a^2+4ab}{a^2-b^2},$$

$$\frac{3b}{b+a} = \frac{3b(a-b)}{(b+a)(a-b)} = \frac{3ab-3b^2}{a^2-b^2},$$

and
$$\frac{1}{a^2-b^2} = \frac{1}{a^2-b^2}.$$

11. The L.C.D. is $(x-1)(x+2)(x+5)$.

$$\therefore \frac{1}{x^2+7x+10} = \frac{1}{(x+2)(x+5)} = \frac{x-1}{(x-1)(x+2)(x+5)},$$

$$\frac{1}{x^2+x-2} = \frac{1}{(x-1)(x+2)} = \frac{x+5}{(x-1)(x+2)(x+5)},$$

and
$$\frac{1}{x^2+4x-5} = \frac{1}{(x-1)(x+5)} = \frac{x+2}{(x-1)(x+2)(x+5)}.$$

12. The L.C.D. is $(a-1)(a-3)(a-5)$.

$$\therefore \frac{a+5}{a^2-4a+3} = \frac{a+5}{(a-1)(a-3)} = \frac{(a+5)(a-5)}{(a-1)(a-3)(a-5)} = \frac{a^2-25}{(a-1)(a-3)(a-5)},$$

$$\frac{a-2}{a^2-8a+15} = \frac{a-2}{(a-3)(a-5)} = \frac{(a-2)(a-1)}{(a-1)(a-3)(a-5)} = \frac{a^2-3a+2}{(a-1)(a-3)(a-5)},$$

and
$$\frac{a+1}{a^2-6a+5} = \frac{a+1}{(a-1)(a-5)} = \frac{(a+1)(a-3)}{(a-1)(a-3)(a-5)} = \frac{a^2-2a-3}{(a-1)(a-3)(a-5)}.$$

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11. $$\frac{2x+1}{3} + \frac{x-2}{4} - \frac{x-3}{6} + \frac{5-x}{2} = \frac{8x+4+3x-6-(2x-6)+30-6x}{12}$$

$$= \frac{8x+4+3x-6-2x+6+30-6x}{12} = \frac{3x+34}{12}$$

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12. $\frac{x-2}{6}-\frac{x-4}{9}+\frac{2-3x}{4}-\frac{2x+1}{12}=\frac{6x-12-(4x-16)+18-27x-(6x+3)}{36}$

$=\frac{6x-12-4x+16+18-27x-6x-3}{36}=\frac{19-31x}{36}.$

13. $\frac{x-1}{3}-\frac{x-2}{18}-\frac{4x-3}{27}+\frac{1-x}{6}=\frac{18x-18-(3x-6)-(8x-6+9-9x}{54}$

$=\frac{18x-18-3x+6-8x+6+9-9x}{54}=\frac{3-2x}{54}.$

14. $\frac{2-6x}{5}+\frac{4x-1}{2}-\frac{5x-3}{6}-\frac{1-x}{3}$

$=\frac{12-36x+60x-15-(25x-15)-(10-10x)}{30}$

$=\frac{12-36x+60x-15-25x+15-10+10x}{30}=\frac{9x+2}{30}.$

15. $\frac{x+3}{4}-\frac{x-2}{5}+\frac{x-4}{10}-\frac{x+3}{6}=\frac{15x+45-(12x-24)+6x-24-(10x+30)}{60}$

$=\frac{15x+45-12x+24+6x-24-10x-30}{60}=\frac{15-x}{60}.$

16. $\frac{1-2a}{5}+\frac{2a-1}{4}-\frac{2a-a^2+1}{8}=\frac{8-16a+20a-10-(10a-5a^2+5)}{40}$

$=\frac{8-16a+20a-10-10a+5a^2-5}{40}=\frac{5a^2-6a-7}{40}.$

17. $\frac{3+x-x^2}{4}-\frac{1-x+x^2}{6}-\frac{1-2x-2x^2}{3}$

$=\frac{9+3x-3x^2-(2-2x+2x^2)-(4-8x-8x^2)}{12}$

$=\frac{9+3x-3x^2-2+2x-2x^2-4+8x+8x^2}{12}=\frac{3+13x+3x^2}{12}.$

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27. $\frac{a-b}{ab}+\frac{b-c}{bc}=\frac{ac-bc}{abc}+\frac{ab-ac}{abc}=\frac{ab-bc}{abc}=\frac{a-c}{ac}.$

28. $\frac{a+b}{a-b}+\frac{a-b}{a+b}=\frac{a^2+2ab+b^2}{a^2-b^2}+\frac{a^2-2ab+b^2}{a^2-b^2}=\frac{2a^2+2b^2}{a^2-b^2}.$

29. $\frac{b-c}{bc}-\frac{a-c}{ac}=\frac{ab-ac-(ab-bc)}{abc}=\frac{ab-ac-ab+bc}{abc}=\frac{bc-ac}{abc}=\frac{b-a}{ab}.$

30. $\frac{a+b}{a-b}-\frac{a-b}{a+b}=\frac{a^2+2ab+b^2-(a^2-2ab+b^2)}{a^2-b^2}$

$=\frac{a^2+2ab+b^2-a^2+2ab-b^2}{a^2-b^2}=\frac{4ab}{a^2-b^2}.$

31. $x+y-\frac{x^2+y^2}{x-y}=\frac{x^2-y^2-(x^2+y^2)}{x-y}=\frac{x^2-y^2-x^2-y^2}{x-y}=\frac{-2y^2}{x-y}=\frac{2y^2}{y-x}.$

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32. $\dfrac{x}{x-2}-\dfrac{x-2}{x+2}=\dfrac{x^2+2x-(x^2-4x+4)}{x^2-4}=\dfrac{x^2+2x-x^2+4x-4}{x^2-4}=\dfrac{6x-4}{x^2-4}.$

34. $x+1+\dfrac{x^3-3}{x-1}=\dfrac{x^2-1}{x-1}+\dfrac{x^3-3}{x-1}=\dfrac{x^3+x^2-4}{x-1}.$

35. $m-\dfrac{m^2+n^2}{m-n}+n=m+n-\dfrac{m^2+n^2}{m-n}=\dfrac{m^2-n^2-(m^2+n^2)}{m-n}$

$=\dfrac{m^2-n^2-m^2-n^2}{m-n}=\dfrac{-2n^2}{m-n}=\dfrac{2n^2}{n-m}.$

36. $1-\dfrac{ax-bx+ab}{x^2}=\dfrac{x^2-(ax-bx+ab)}{x^2}=\dfrac{x^2-ax+bx-ab}{x^2}.$

37. $\dfrac{1}{x}+1+\dfrac{2x}{1+x}-2=\dfrac{1}{x}+\dfrac{2x}{1+x}-1$

$=\dfrac{1+x+2x^2-(x+x^2)}{x+x^2}=\dfrac{1+x+2x^2-x-x^2}{x+x^2}=\dfrac{1+x^2}{x+x^2}.$

38. $2a-3b-\dfrac{4a^2+9b^2}{2a+3b}=\dfrac{4a^2-9b^2-(4a^2+9b^2)}{2a+3b}$

$=\dfrac{4a^2-9b^2-4a^2-9b^2}{2a+3b}=\dfrac{-18b^2}{2a+3b}.$

39. $3a-2x-\dfrac{8a^2-4x^2}{3a+2x}=\dfrac{9a^2-4x^2-(8a^2-4x^2)}{3a+2x}$

$=\dfrac{9a^2-4x^2-8a^2+4x^2}{3a+2x}=\dfrac{a^2}{3a+2x}.$

40. $\dfrac{1}{x-1}-\dfrac{1}{x+1}-\dfrac{2}{x^2}=\dfrac{(x+1)(x^2)-(x-1)(x^2)-2(x-1)(x+1)}{(x-1)(x+1)(x^2)}$

$=\dfrac{x^3+x^2-x^3+x^2-2x^2+2}{x^4-x^2}=\dfrac{2}{x^2(x^2-1)}.$

41. $\dfrac{1}{a+b}-\dfrac{1}{a-b}+\dfrac{2a}{a^2-b^2}=\dfrac{(a-b)-(a+b)+2a}{a^2-b^2}$

$=\dfrac{a-b-a-b+2a}{a^2-b^2}=\dfrac{2a-2b}{a^2-b^2}=\dfrac{2}{a+b}.$

42. $\dfrac{a+x}{a-x}+\dfrac{a-x}{a+x}+\dfrac{4ax}{a^2-x^2}=\dfrac{(a+x)^2+(a-x)^2+4ax}{a^2-x^2}$

$=\dfrac{a^2+2ax+x^2+a^2-2ax+x^2+4ax}{a^2-x^2}$

$=\dfrac{2a^2+4ax+2x^2}{a^2-x^2}=\dfrac{2(a+x)^2}{a^2-x^2}=\dfrac{2(a+x)}{a-x}.$

43. $\dfrac{a+1}{a^2+a+1}+\dfrac{a-1}{a^2-a+1}=\dfrac{a^3+1}{a^4+a^2+1}+\dfrac{a^3-1}{a^4+a^2+1}=\dfrac{2a^3}{a^4+a^2+1}.$

44. $3x+\dfrac{5}{ax}-\left(2x+\dfrac{3}{ax}\right)=3x+\dfrac{5}{ax}-2x-\dfrac{3}{ax}=x+\dfrac{2}{ax}=\dfrac{ax^2+2}{ax}.$

45. $\dfrac{a-b}{2(a+b)}+\dfrac{a^2+b^2}{a^2-b^2}-\dfrac{a}{a-b}=\dfrac{a^2-2ab+b^2+2a^2+2b^2-(2a^2+2ab)}{2(a^2-b^2)}$

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$$= \frac{a^2 - 2ab + b^2 + 2a^2 + 2b^2 - 2a^2 - 2ab}{2(a^2 - b^2)}$$

$$= \frac{a^2 - 4ab + 3b^2}{2(a^2 - b^2)} = \frac{(a-b)(a-3b)}{2(a+b)(a-b)} = \frac{a-3b}{2(a+b)}.$$

46. $$\frac{a+33}{a^2-9} - \frac{6}{a-3} + \frac{10}{a+3} = \frac{(a+33) - 6(a+3) + 10(a-3)}{a^2-9}$$

$$= \frac{a + 33 - 6a - 18 + 10a - 30}{a^2 - 9}$$

$$= \frac{5a - 15}{a^2 - 9} = \frac{5(a-3)}{(a+3)(a-3)} = \frac{5}{a+3}.$$

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47. $$\frac{a}{a-2} - \frac{a-2}{a+2} + \frac{3}{4-a^2} = \frac{a}{a-2} - \frac{a-2}{a+2} + \frac{-3}{a^2-4}$$

$$= \frac{a^2 + 2a - (a^2 - 4a + 4) - 3}{a^2 - 4}$$

$$= \frac{a^2 + 2a - a^2 + 4a - 4 - 3}{a^2 - 4} = \frac{6a - 7}{a^2 - 4}.$$

48. $$\frac{a+1}{a-1} + \frac{2}{a+1} + \frac{4a}{1-a^2} = \frac{a+1}{a-1} + \frac{2}{a+1} + \frac{-4a}{a^2-1}$$

$$= \frac{a^2 + 2a + 1 + 2a - 2 - 4a}{a^2 - 1} = \frac{a^2 - 1}{a^2 - 1} = 1.$$

49. $$\frac{5x+2}{x^2-4} + \frac{2}{x-2} - \frac{3}{2-x} = \frac{5x+2}{x^2-4} + \frac{2}{x-2} + \frac{3}{x-2}$$

$$= \frac{5x+2}{x^2-4} + \frac{5}{x-2} = \frac{10x+12}{x^2-4}.$$

50. $$\frac{x(a+x)}{a-x} - \frac{3ax - x^2}{x-a} + 4a = \frac{x(a+x)}{a-x} + \frac{3ax - x^2}{a-x} + 4a$$

$$= \frac{ax + x^2 + 3ax - x^2 + 4a^2 - 4ax}{a-x} = \frac{4a^2}{a-x}.$$

51. $$\frac{1}{a^3+8} - \frac{1}{8-a^3} + \frac{1}{4-a^2} = \frac{(8-a^3) - (8+a^3) + (16 + 4a^2 + a^4)}{64 - a^6}$$

$$= \frac{8 - a^3 - 8 - a^3 + 16 + 4a^2 + a^4}{64 - a^6} = \frac{16 + 4a^2 - 2a^3 + a^4}{64 - a^6}.$$

52. $$\frac{5(x-3)}{x^2-x-2} - \frac{2(x+2)}{x^2+4x+3} - \frac{x-1}{6-x-x^2}$$

$$= \frac{5(x-3)}{(x+1)(x-2)} - \frac{2(x+2)}{(x+1)(x+3)} + \frac{x-1}{(x-2)(x+3)}$$

$$= \frac{5(x-3)(x+3) - 2(x+2)(x-2) + (x-1)(x+1)}{(x+1)(x-2)(x+3)}$$

$$= \frac{5x^2 - 45 - 2x^2 + 8 + x^2 - 1}{(x+1)(x-2)(x+3)} = \frac{4x^2 - 38}{(x+1)(x-2)(x+3)}.$$

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54. $$\frac{a^2+2ab+b^2}{a^2+b^2}-1+\frac{2ab}{a^2-b^2}=1+\frac{2ab}{a^2+b^2}-1+\frac{2ab}{a^2-b^2}$$
$$=\frac{2ab}{a^2+b^2}+\frac{2ab}{a^2-b^2}=\frac{2a^3b-2ab^3+2a^3b+2ab^3}{a^4-b^4}=\frac{4a^3b}{a^4-b^4}.$$

55. $$\frac{a^2+3ab+2b^2}{a^2+3ab-4b^2}-\frac{a^2-13b^2}{a^2-16b^2}=\left(1+\frac{6b^2}{a^2+3ab-4b^2}\right)-\left(1+\frac{3b^2}{a^2-16b^2}\right)$$
$$=\frac{6b^2}{a^2+3ab-4b^2}-\frac{3b^2}{a^2-16b^2}$$
$$=\frac{6b^2}{(a+4b)(a-b)}-\frac{3b^2}{(a+4b)(a-4b)}$$
$$=\frac{6ab^2-24b^3-(3ab^2-3b^3)}{(a+4b)(a-4b)(a-b)}=\frac{6ab^2-24b^3-3ab^2+3b^3}{(a+4b)(a-4b)(a-b)}$$
$$=\frac{3ab^2-21b^3}{(a+4b)(a-4b)(a-b)}=\frac{3ab^2-21b^3}{(a^2-16b^2)(a-b)}.$$

56.
$$\frac{x+1}{x-1}+\frac{x-1}{x+1}-\frac{x+2}{x-2}-\frac{x-2}{x+2}=1+\frac{2}{x-1}+1-\frac{2}{x+1}-\left(1+\frac{4}{x-2}\right)-\left(1-\frac{4}{x+2}\right)$$
$$=\frac{2}{x-1}-\frac{2}{x+1}-\frac{4}{x-2}+\frac{4}{x+2}$$
$$=\frac{2(x+1)(x^2-4)-2(x-1)(x^2-4)-4(x+2)(x^2-1)+4(x-2)(x^2-1)}{(x^2-1)(x^2-4)}$$
$$=\frac{(2x+2-2x+2)(x^2-4)+(-4x-8+4x-8)(x^2-1)}{(x^2-1)(x^2-4)}$$
$$=\frac{4(x^2-4)-16(x^2-1)}{(x^2-1)(x^2-4)}$$
$$=\frac{4x^2-16-16x^2+16}{(x^2-1)(x^2-4)}=\frac{-12x^2}{(x^2-1)(x^2-4)}.$$

57.
$$\frac{x+3}{x-3}-\frac{x-3}{x+3}+\frac{x+4}{x-4}-\frac{x-4}{x+4}=1+\frac{6}{x-3}-\left(1-\frac{6}{x+3}\right)+1+\frac{8}{x-4}-\left(1-\frac{8}{x+4}\right)$$
$$=\frac{6}{x-3}+\frac{6}{x+3}+\frac{8}{x-4}+\frac{8}{x+4}$$
$$=\frac{6(x+3)(x^2-16)+6(x-3)(x^2-16)+8(x+4)(x^2-9)+8(x-4)(x^2-9)}{(x^2-9)(x^2-16)}$$
$$=\frac{(6x+18+6x-18)(x^2-16)+(8x+32+8x-32)(x^2-9)}{(x^2-9)(x^2-16)}$$
$$=\frac{12x(x^2-16)+16x(x^2-9)}{(x^2-9)(x^2-16)}$$
$$=\frac{12x^3-192x+16x^3-144x}{(x^2-9)(x^2-16)}=\frac{28x^3-336x}{(x^2-9)(x^2-16)}.$$

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59. $$\frac{a+b}{a-b}-\frac{a-b}{a+b}-\frac{4ab}{a^2+b^2}+\frac{8ab^3}{a^4+b^4}=\frac{4ab}{a^2-b^2}-\frac{4ab}{a^2+b^2}+\frac{8ab^3}{a^4+b^4}$$
$$=\frac{8ab^3}{a^4-b^4}+\frac{8ab^3}{a^4+b^4}=\frac{16a^5b^3}{a^8-b^8}.$$

60. $$\frac{1}{a-b}-\frac{1}{a+b}-\frac{2b}{a^2+b^2}+\frac{2b^3}{a^4+b^4}=\frac{2b}{a^2-b^2}-\frac{2b}{a^2+b^2}+\frac{2b^3}{a^4+b^4}$$
$$=\frac{4b^3}{a^4-b^4}+\frac{2b^3}{a^4+b^4}=\frac{6a^4b^3+2b^7}{a^8-b^8}.$$

61. $$\frac{a+x}{a-x}+\frac{a^2+x^2}{a^2-x^2}-\frac{a-x}{a+x}-\frac{a^2-x^2}{a^2+x^2}-\frac{4a^3x+4ax^3}{a^4-x^4}$$
$$=\left(\frac{a+x}{a-x}-\frac{a-x}{a+x}\right)+\left(\frac{a^2+x^2}{a^2-x^2}-\frac{a^2-x^2}{a^2+x^2}\right)-\frac{4ax(a^2+x^2)}{(a^2-x^2)(a^2+x^2)}$$
$$=\frac{4ax}{a^2-x^2}+\frac{4a^2x^2}{a^4-x^4}-\frac{4ax}{a^2-x^2}=\frac{4a^2x^2}{a^4-x^4}.$$

63. $$\frac{1}{(b-c)(a-c)}+\frac{1}{(c-a)(a-b)}+\frac{1}{(b-a)(b-c)}$$
$$=\frac{1}{(b-c)(a-c)}-\frac{1}{(a-c)(a-b)}-\frac{1}{(a-b)(b-c)}$$
$$=\frac{a-b-(b-c)-(a-c)}{(a-b)(a-c)(b-c)}=\frac{a-b-b+c-a+c}{(a-b)(a-c)(b-c)}$$
$$=\frac{-2(b-c)}{(a-b)(a-c)(b-c)}=\frac{2}{(b-a)(a-c)}.$$

64. $$\frac{a+1}{(a-b)(a-c)}+\frac{b+1}{(b-c)(b-a)}+\frac{c+1}{(a-c)(b-c)}$$
$$=\frac{a+1}{(a-b)(a-c)}-\frac{b+1}{(b-c)(a-b)}+\frac{c+1}{(a-c)(b-c)}$$
$$=\frac{(a+1)(b-c)-(b+1)(a-c)+(c+1)(a-b)}{(a-b)(a-c)(b-c)}$$
$$=\frac{ab+b-ac-c-ab-a+bc+c+ac+a-bc-b}{(a-b)(a-c)(b-c)}$$
$$=\frac{0}{(a-b)(a-c)(b-c)}=0.$$

65. $$\frac{c^2ab}{(c-a)(b-c)}-\frac{b^2ca}{(b-a)(b-c)}-\frac{a^2bc}{(a-b)(a-c)}$$
$$=\frac{-c^2ab}{(a-c)(b-c)}+\frac{b^2ca}{(a-b)(b-c)}-\frac{a^2bc}{(a-b)(a-c)}$$
$$=\frac{-c^2ab(a-b)+b^2ca(a-c)-a^2bc(b-c)}{(a-b)(a-c)(b-c)}$$
$$=\frac{-a^2bc^2+ab^2c^2+a^2b^2c-ab^2c^2-a^2b^2c+a^2bc^2}{(a-b)(a-c)(b-c)}$$
$$=\frac{0}{(a-b)(a-c)(b-c)}=0.$$

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66.
$$\frac{b-c}{(b-a)(a-c)}-\frac{c-a}{(b-c)(a-b)}-\frac{a+b}{(a-c)(b-c)}$$
$$=\frac{-(b-c)}{(a-b)(a-c)}+\frac{a-c}{(b-c)(a-b)}-\frac{a+b}{(a-c)(b-c)}$$
$$=\frac{-(b-c)(b-c)+(a-c)(a-c)-(a+b)(a-b)}{(a-b)(a-c)(b-c)}$$
$$=\frac{-b^2+2bc-c^2+a^2-2ac+c^2-a^2+b^2}{(a-b)(a-c)(b-c)}$$
$$=\frac{-2c(a-b)}{(a-b)(a-c)(b-c)}=\frac{2c}{(c-a)(b-c)}.$$

67.
$$\frac{c+a}{(a-b)(b-c)}-\frac{b+c}{(c-a)(b-a)}+\frac{a+b}{(c-b)(a-c)}$$
$$=\frac{a+c}{(a-b)(b-c)}-\frac{b+c}{(a-c)(a-b)}-\frac{a+b}{(b-c)(a-c)}$$
$$=\frac{(a+c)(a-c)-(b+c)(b-c)-(a+b)(a-b)}{(a-b)(a-c)(b-c)}$$
$$=\frac{a^2-c^2-b^2+c^2-a^2+b^2}{(a-b)(a-c)(b-c)}=\frac{0}{(a-b)(a-c)(b-c)}=0.$$

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8. $\frac{a^mb^n}{4x}\times\frac{6x^2}{a^{m-1}b^{2n}}=\frac{a^{m-1}a^1b^n}{4x}\times\frac{6x^2}{a^{m-1}b^nb^n}=\frac{3ax}{2b^n}.$

9. $\frac{a^{m+1}}{b^{m+2}}\times\frac{b^{m+1}}{a^m}=\frac{a^ma^1}{b^{m+1}b^1}\times\frac{b^{m+1}}{a^m}=\frac{a}{b}.$

12. $\frac{1-6x+5x^2}{x^2-3x+2}\times\frac{2-x}{1-x}=\frac{-(x-1)(1-5x)}{-(x-1)(2-x)}\times\frac{2-x}{1-x}=\frac{1-5x}{1-x}.$

13. $\frac{(a-b)^2}{a+b}\times\frac{b}{a^2-ab}\times\frac{(a+b)^2}{a^2-b^2}=\frac{(a-b)^2}{a+b}\times\frac{b}{a(a-b)}\times\frac{(a+b)^2}{(a+b)(a-b)}=\frac{b}{a}.$

14. $\frac{a^4-x^4}{a^3+x^3}\times\frac{a+x}{a^2-x^2}\times\frac{a^2-ax+x^2}{(a+x)^2}$

$$=\frac{(a^2+x^2)(a+x)(a-x)}{(a+x)(a^2-ax+x^2)}\times\frac{a+x}{(a+x)(a-x)}\times\frac{a^2-ax+x^2}{(a+x)^2}=\frac{a^2+x^2}{(a+x)^2}.$$

15. $\frac{4a-b}{2x+y}\times\frac{2a}{4a^2-ab}\times\frac{4x^2-y^2}{4}=\frac{4a-b}{2x+y}\times\frac{2a}{a(4a-b)}\times\frac{(2x+y)(2x-y)}{4}$

$$=\frac{2x-y}{2}.$$

16. $\frac{p+2}{x-3}\times\frac{3x^2-27}{2p^2-8}\times\frac{4}{px+3p}=\frac{p+2}{x-3}\times\frac{3(x+3)(x-3)}{2(p+2)(p-2)}\times\frac{4}{p(x+3)}$

$$=\frac{6}{p(p-2)}.$$

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17. $$\frac{p^4-q^4}{(p-q)^2}\times\frac{p-q}{p^2+pq}\times\frac{p^2}{p^2+q^2}$$
$$=\frac{(p^2+q^2)(p+q)(p-q)}{(p-q)^2}\times\frac{p-q}{p(p+q)}\times\frac{p^2}{p^2+q^2}=p.$$

18. $$\frac{a^3+8}{a^3-8}\times\frac{a^2+2a+4}{a^2-2a+4}=\frac{(a+2)(a^2-2a+4)}{(a-2)(a^2+2a+4)}\times\frac{a^2+2a+4}{a^2-2a+4}=\frac{a+2}{a-2}.$$

19. $$\frac{a^4+a^2x^2+x^4}{a^4-ax^3}\times\frac{x}{a^2-ax+x^2}$$
$$=\frac{(a^2+ax+x^2)(a^2-ax+x^2)}{a(a-x)(a^2+ax+x^2)}\times\frac{x}{a^2-ax+x^2}=\frac{x}{a(a-x)}.$$

20. $$\frac{a^4+4}{a^4+a^2+1}\times\frac{a^2+a+1}{a^2+2a+2}=\frac{(a^2+2a+2)(a^2-2a+2)}{(a^2+a+1)(a^2-a+1)}\times\frac{a^2+a+1}{a^2+2a+2}$$
$$=\frac{a^2-2a+2}{a^2-a+1}.$$

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21. $$\left(1-\frac{x-1}{x^2+6x+5}\right)\left(1-\frac{2}{x^2+7x+12}\right)$$
$$=\frac{x^2+6x+5-x+1}{x^2+6x+5}\times\frac{x^2+7x+12-2}{x^2+7x+12}$$
$$=\frac{x^2+5x+6}{x^2+6x+5}\times\frac{x^2+7x+10}{x^2+7x+12}$$
$$=\frac{(x+3)(x+2)}{(x+5)(x+1)}\times\frac{(x+5)(x+2)}{(x+3)(x+4)}$$
$$=\frac{(x+2)(x+2)}{(x+1)(x+4)}=\frac{x^2+4x+4}{x^2+5x+4}.$$

22. $$\left(1+\frac{7x+11}{x^2-4x-21}\right)\left(1-\frac{17x-11}{x^2+7x+10}\right)$$
$$=\frac{x^2-4x-21+7x+11}{x^2-4x-21}\times\frac{x^2+7x+10-17x+11}{x^2+7x+10}$$
$$=\frac{x^2+3x-10}{x^2-4x-21}\times\frac{x^2-10x+21}{x^2+7x+10}$$
$$=\frac{(x+5)(x-2)}{(x-7)(x+3)}\times\frac{(x-3)(x-7)}{(x+5)(x+2)}=\frac{x^2-5x+6}{x^2+5x+6}.$$

23. $$\frac{a^2+ab+ac+bc}{ax-ay-x^2+xy}\times\frac{a^2-ax+ay-xy}{a^2+ac+ax+cx}\times\frac{x^2-x(y-a)-ay}{a^2-a(y-b)-by}$$
$$=\frac{(a+c)(a+b)}{(a-x)(x-y)}\times\frac{(a+y)(a-x)}{(a+x)(a+c)}\times\frac{(a+x)(x-y)}{(a+b)(a-y)}=\frac{a+y}{a-y}.$$

24. $$\frac{x^3-5x^2+8x-4}{x^3-8x^2+19x-12}\times\frac{x^3-10x^2+33x-36}{x^3-6x^2+11x-6}$$
§ 164, $$=\frac{(x-1)(x-2)(x-2)}{(x-1)(x-3)(x-4)}\times\frac{(x-3)(x-3)(x-4)}{(x-1)(x-2)(x-3)}=\frac{x-2}{x-1}.$$

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25. $\dfrac{x^4 - 3x^3 - 23x^2 + 75x - 50}{x^4 - 5x^3 - 21x^2 + 125x - 100} \times \dfrac{x^3 - 10x^2 + 29x - 20}{x^3 - 12x^2 + 45x - 50}$

§ 164, $= \dfrac{(x-1)(x-2)(x-5)(x+5)}{(x-1)(x-4)(x-5)(x+5)} \times \dfrac{(x-1)(x-4)(x-5)}{(x-2)(x-5)(x-5)} = \dfrac{x-1}{x-5}.$

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15. $\dfrac{my - y^2}{(m+y)^2} \div \dfrac{y^2}{m^2 - y^2} = \dfrac{y(m-y)}{(m+y)^2} \times \dfrac{(m+y)(m-y)}{y^2} = \dfrac{(m-y)^2}{y(m+y)}.$

16. $\dfrac{(a-b)^2}{a+b} \div \dfrac{a^2 - ab}{b} = \dfrac{(a-b)^2}{a+b} \times \dfrac{b}{a(a-b)} = \dfrac{b(a-b)}{a(a+b)}.$

17. $(4a + 2) \div \dfrac{2a+1}{5a} = 2(2a+1) \times \dfrac{5a}{2a+1} = 10a.$

18. $\dfrac{a^4 - b^4}{a^2 - 2ab + b^2} \div \dfrac{a^2 + b^2}{a^2 - ab} = \dfrac{(a^2+b^2)(a+b)(a-b)}{(a-b)^2} \times \dfrac{a(a-b)}{a^2+b^2} = a(a+b).$

19. $\dfrac{x^3 + y^3}{x^2 - y^2} \div \dfrac{x^2 + xy + y^2}{x - y} = \dfrac{(x+y)(x^2 - xy + y^2)}{(x+y)(x-y)} \times \dfrac{x-y}{x^2 + xy + y^2}$

$$= \frac{x^2 - xy + y^2}{x^2 + xy + y^2}.$$

20.

$\dfrac{m^4x + m^5}{m^3x - mx^3} \div \dfrac{m^3x^2 - mx^4}{m^3x^3 - x^6} = \dfrac{m^4(x+m)}{mx(m+x)(m-x)} \times \dfrac{x^3(m-x)(m^2 + mx + x^2)}{mx^2(m+x)(m-x)}$

$$= \frac{m^2(m^2 + mx + x^2)}{m^2 - x^2}.$$

21. $\left(x \div \dfrac{1}{y}\right) \div \left(y^2 \div \dfrac{1}{x^2}\right) = xy \div x^2y^2 = xy \times \dfrac{1}{x^2y^2} = \dfrac{1}{xy}.$

22. $\left(\dfrac{a^3}{b} \div b^2\right) \div \left(\dfrac{a^2}{b^2} \times ab\right) = \left(\dfrac{a^3}{b} \times \dfrac{1}{b^2}\right) \div \dfrac{a^3}{b} = \dfrac{a^3}{b} \times \dfrac{1}{b^2} \times \dfrac{b}{a^3} = \dfrac{1}{b^2}.$

23. $(a + c) \div \left(\dfrac{a^2 - c^2}{1 + x} \div \dfrac{a - c}{1 - x^2}\right) = (a + c) \div \left(\dfrac{a^2 - c^2}{1 + x} \times \dfrac{1 - x^2}{a - c}\right)$

$$= (a + c) \times \frac{1 + x}{(a + c)(a - c)} \times \frac{a - c}{(1 + x)(1 - x)} = \frac{1}{1 - x}.$$

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24. $\dfrac{a^2 + b^2 - c^2 + 2ab}{a^2 - b^2 - c^2 + 2bc} \div \dfrac{a^2 - b^2 + c^2 - 2ac}{a^2 - b^2 + c^2 + 2ac}$

$= \dfrac{a^2 + b^2 - c^2 + 2ab}{a^2 - b^2 - c^2 + 2bc} \times \dfrac{a^2 - b^2 + c^2 + 2ac}{a^2 - b^2 + c^2 - 2ac}$

$= \dfrac{(a+b+c)(a+b-c)}{(a+b-c)(a-b+c)} \times \dfrac{(a+b+c)(a-b+c)}{(a+b-c)(a-b-c)} = \dfrac{(a+b+c)^2}{(a+b-c)(a-b-c)}.$

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25. $\dfrac{x^3-6x^2+11x-6}{x^3+2x^2-19x-20} \div \dfrac{x^3-13x+12}{x^3+10x^2+29x+20}$

$$= \frac{x^3-6x^2+11x-6}{x^3+2x^2-19x-20} \times \frac{x^3+10x^2+29x+20}{x^3-13x+12}$$

§ 164, $$= \frac{(x-1)(x-2)(x-3)}{(x+1)(x-4)(x+5)} \times \frac{(x+1)(x+4)(x+5)}{(x-1)(x-3)(x+4)} = \frac{x-2}{x-4}.$$

26. $\left(y-x+\dfrac{x^2}{y}\right) \div \left(\dfrac{x}{y^2}+\dfrac{y}{x^2}\right) = \dfrac{y^2-xy+x^2}{y} \div \dfrac{x^3+y^3}{x^2y^2}$

$$= \frac{x^2-xy+y^2}{y} \times \frac{x^2y^2}{(x+y)(x^2-xy+y^2)} = \frac{x^2y}{x+y}.$$

27. $\left(x^2-\dfrac{1}{x^2}\right) \div \left(x-\dfrac{1}{x}\right) = \dfrac{x^4-1}{x^2} \div \dfrac{x^2-1}{x}$

$$= \frac{(x^2+1)(x+1)(x-1)}{x^2} \times \frac{x}{(x+1)(x-1)} = \frac{x^2+1}{x}.$$

28. $\left(1-\dfrac{y^2}{x^2}\right) \div \left(1-\dfrac{2x}{y}+\dfrac{x^2}{y^2}\right) = \dfrac{x^2-y^2}{x^2} \div \dfrac{y^2-2xy+x^2}{y^2}$

$$= \frac{(x+y)(x-y)}{x^2} \times \frac{y^2}{(x-y)^2} = \frac{y^2(x+y)}{x^2(x-y)}.$$

29. $\left(1+\dfrac{1}{y^2}+\dfrac{1}{y^4}\right) \div \left(1+\dfrac{1}{y}+\dfrac{1}{y^2}\right) = \dfrac{y^4+y^2+1}{y^4} \div \dfrac{y^2+y+1}{y^2}$

$$= \frac{(y^2+y+1)(y^2-y+1)}{y^4} \times \frac{y^2}{y^2+y+1} = \frac{y^2-y+1}{y^2}.$$

30. $\left(x-4+\dfrac{9}{x+2}\right) \div \left(1-\dfrac{4x-7}{x^2-4}\right) = \dfrac{x^2-2x+1}{x+2} \div \dfrac{x^2-4x+3}{x^2-4}$

$$= \frac{(x-1)^2}{x+2} \times \frac{(x+2)(x-2)}{(x-1)(x-3)} = \frac{(x-1)(x-2)}{x-3}.$$

31. $\left(x+\dfrac{3x+6}{x^2-1}+2\right) \div \left(x+3+\dfrac{1}{x+1}\right)$

$$= \frac{x^3+2x^2+2x+4}{x^2-1} \div \frac{x^2+4x+4}{x+1}$$

$$= \frac{(x^2+2)(x+2)}{(x+1)(x-1)} \times \frac{x+1}{(x+2)^2} = \frac{x^2+2}{(x-1)(x+2)}.$$

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2. $\dfrac{\dfrac{x+y}{ab}}{\dfrac{x^2-y^2}{ab^2}} = \dfrac{x+y}{ab} \div \dfrac{x^2-y^2}{ab^2} = \dfrac{x+y}{ab} \times \dfrac{ab^2}{(x+y)(x-y)} = \dfrac{b}{x-y}.$

3. $\dfrac{a+\dfrac{b}{c}}{b+\dfrac{c}{a}} = \dfrac{ac+b}{c} \div \dfrac{ab+c}{a} = \dfrac{ac+b}{c} \times \dfrac{a}{ab+c} = \dfrac{a(ac+b)}{c(ab+c)}.$

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4. $$\frac{m-\frac{3m}{x}}{x-\frac{x}{m}}=\frac{mx-3m}{x}\div\frac{mx-x}{m}=\frac{m(x-3)}{x}\times\frac{m}{x(m-1)}=\frac{m^2(x-3)}{x^2(m-1)}.$$

5. $$\frac{2+\frac{3a}{4b}}{a+\frac{8b}{3}}=\frac{8b+3a}{4b}\div\frac{3a+8b}{3}=\frac{3a+8b}{4b}\times\frac{3}{3a+8b}=\frac{3}{4b}.$$

6. $$\frac{1-\frac{y^2}{x^2}}{1+\frac{y^2}{x^2}}=\frac{x^2-y^2}{x^2}\div\frac{x^2+y^2}{x^2}=\frac{x^2-y^2}{x^2}\times\frac{x^2}{x^2+y^2}=\frac{x^2-y^2}{x^2+y^2}.$$

7. $$\frac{x-\frac{1}{x}}{1+\frac{1}{x}}=\frac{x^2-1}{x}\div\frac{x+1}{x}=\frac{(x+1)(x-1)}{x}\times\frac{x}{x+1}=x-1.$$

8. $$\frac{b-\frac{c}{2}}{\frac{b}{2}-c}=\frac{2b-c}{2}\div\frac{b-2c}{2}=\frac{2b-c}{2}\times\frac{2}{b-2c}=\frac{2b-c}{b-2c}.$$

9. $$\frac{ax-\frac{x^2}{2}}{\frac{a^2}{2}-ax}=\frac{2ax-x^2}{2}\div\frac{a^2-2ax}{2}=\frac{x(2a-x)}{2}\times\frac{2}{a(a-2x)}=\frac{x(2a-x)}{a(a-2x)}$$

10. $$\frac{\frac{x+y}{y}-\frac{x+y}{x}}{\frac{1}{y}-\frac{1}{x}}=\frac{x^2-y^2}{xy}\div\frac{x-y}{xy}=\frac{(x+y)(x-y)}{xy}\times\frac{xy}{x-y}=x+y.$$

12. $$\frac{\frac{x^2-1}{x}}{\frac{x+1}{x^2}}=\frac{x(x^2-1)}{x+1}=\frac{x(x+1)(x-1)}{x+1}=x(x-1)$$

13. Multiplying the terms by $x(y+z)$, $\dfrac{\frac{1}{x}+\frac{1}{y+z}}{\frac{1}{x}-\frac{1}{y+z}}=\dfrac{y+z+x}{y+z-x}.$

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14. $$\frac{\dfrac{x^3+y^3}{xy}}{\dfrac{x^2-xy+y^2}{xy}}=\frac{x^3+y^3}{x^2-xy+y^2}=\frac{(x+y)(x^2-xy+y^2)}{x^2-xy+y^2}=x+y.$$

15. $$\frac{\dfrac{1}{a+1}}{1-\dfrac{1}{a+1}}=\frac{1}{a+1-1}=\frac{1}{a}.$$

16. $$\frac{\dfrac{x^2+y^2}{2y}-x}{\dfrac{x}{y}-\dfrac{y}{x}}=\frac{x^3+xy^2-2x^2y}{2(x^2-y^2)}=\frac{x(x-y)^2}{2(x+y)(x-y)}=\frac{x(x-y)}{2(x+y)}.$$

17. $$-\frac{\dfrac{1}{1-a}}{\dfrac{a}{a-1}}=\frac{1}{1-a}\div-\frac{a}{a-1}=\frac{1}{1-a}\times\frac{1-a}{a}=\frac{1}{a}.$$

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18. $$\frac{x-2+\dfrac{1}{x+2}}{x+2+\dfrac{1}{x-2}}=\frac{x^2-3}{x+2}\div\frac{x^2-3}{x-2}=\frac{x^2-3}{x+2}\times\frac{x-2}{x^2-3}=\frac{x-2}{x+2}.$$

19. $$\frac{\dfrac{1}{x}+\dfrac{4}{x^2}+\dfrac{4}{x^3}}{1+\dfrac{5}{x}+\dfrac{6}{x^2}}=\frac{x^2+4x+4}{x^3+5x^2+6x}=\frac{(x+2)^2}{x(x+2)(x+3)}=\frac{x+2}{x(x+3)}.$$

20. $$\frac{6a-1-\dfrac{1}{a}}{\dfrac{2a-1}{3a}}=\frac{18a^2-3a-3}{2a-1}=\frac{3(3a+1)(2a-1)}{2a-1}=3(3a+1).$$

21. $$\frac{\dfrac{x-5}{2}-7+\dfrac{24}{x}}{\dfrac{9-3x}{x}}=\frac{x^2-5x-14x+48}{18-6x}=\frac{x^2-19x+48}{18-6x}$$
$$=\frac{(x-3)(x-16)}{-6(x-3)}=\frac{16-x}{6}.$$

22. $$\frac{\dfrac{1}{x+1}}{1-\dfrac{1}{1+x}}+\frac{\dfrac{1}{x+1}}{\dfrac{x}{1-x}}+\frac{\dfrac{1}{1-x}}{\dfrac{x}{1+x}}=\frac{1}{x}+\frac{1-x}{x(1+x)}+\frac{1+x}{x(1-x)}$$
$$=\frac{1-x^2+1-2x+x^2+1+2x+x^2}{x(1-x^2)}=\frac{3+x^2}{x(1-x^2)}.$$

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23. $$\frac{3xyz}{yz+zx+xy}-\frac{\dfrac{x-1}{x}+\dfrac{y-1}{y}+\dfrac{z-1}{z}}{\dfrac{1}{x}+\dfrac{1}{y}+\dfrac{1}{z}}$$

$$=\frac{3xyz}{yz+xz+xy}-\frac{xyz-yz+xyz-xz+xyz-xy}{yz+xz+xy}$$

$$=\frac{3xyz-(3xyz-yz-xz-xy)}{yz+xz+xy}$$

$$=\frac{3xyz-3xyz+yz+xz+xy}{yz+xz+xy}=1.$$

24. $$\frac{\dfrac{1}{x+y}+\dfrac{2}{x-y}-\dfrac{9}{3x-y}}{\dfrac{-8y}{y^2-9x^2}}=\frac{\dfrac{8y^2}{(x+y)(x-y)(3x-y)}}{\dfrac{8y}{9x^2-y^2}}$$

$$=\frac{8y^2}{(x+y)(x-y)(3x-y)}\times\frac{(3x+y)(3x-y)}{8y}=\frac{y(3x+y)}{x^2-y^2}.$$

25. $$\frac{\dfrac{x^2+(a+b)x+ab}{x^2-(a+b)x+ab}}{\dfrac{x^2-b^2}{x^2-a^2}}=\frac{\dfrac{(x+a)(x+b)}{(x-a)(x-b)}}{\dfrac{(x+b)(x-b)}{(x+a)(x-a)}}=\frac{(x+a)(x+b)(x+a)}{(x+b)(x-b)(x-b)}=\frac{(x+a)^2}{(x-b)^2}.$$

26. $$\frac{\dfrac{1}{a}+\dfrac{1}{b+c}}{\dfrac{1}{a}-\dfrac{1}{b+c}}\div\frac{1}{1+\dfrac{b^2+c^2-a^2}{2bc}}=\frac{b+c+a}{b+c-a}\div\frac{2bc}{2bc+b^2+c^2-a^2}$$

$$=\frac{b+c+a}{b+c-a}\times\frac{(b+c+a)(b+c-a)}{2bc}=\frac{(a+b+c)^2}{2bc}.$$

27. $$\frac{x^2-\dfrac{(x^2+y^2-z^2)^2}{4y^2}}{\dfrac{(x+y)^2-z^2}{y^2}\times\dfrac{(x-y+z)^2}{4}}=\frac{4x^2y^2-(x^2+y^2-z^2)^2}{(x+y+z)(x+y-z)(x-y+z)^2}$$

$$=\frac{(x+y+z)(x+y-z)(x-y+z)(z-x+y)}{(x+y+z)(x+y-z)(x-y+z)^2}=\frac{z-x+y}{z+x-y}.$$

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29. $$\frac{1}{x+\dfrac{1}{1+\dfrac{x+1}{3-x}}}=\frac{1}{x+\dfrac{1}{\dfrac{4}{3-x}}}=\frac{1}{x+\dfrac{3-x}{4}}=\frac{1}{\dfrac{3x+3}{4}}=\frac{4}{3x+3}.$$

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30. $$\frac{a}{a+1+\frac{a}{a+1-\frac{1}{a}}}=\frac{a}{a+1+\frac{a}{\frac{a^2+a-1}{a}}}=\frac{a}{a+1+\frac{a^2}{a^2+a-1}}$$
$$=\frac{a}{\frac{a^3+3a^2-1}{a^2+a-1}}=\frac{a(a^2+a-1)}{a^3+3a^2-1}.$$

31. $$\frac{2}{2-\frac{2}{2-\frac{2}{2-x}}}=\frac{2}{2-\frac{2}{\frac{2-2x}{2-x}}}=\frac{2}{2-\frac{2-x}{1-x}}=\frac{2}{\frac{x}{x-1}}=\frac{2(x-1)}{x}.$$

32. $$\frac{x-2}{x-2-\frac{x}{x-\frac{x-1}{x-2}}}=\frac{x-2}{x-2-\frac{x}{\frac{x^2-3x+1}{x-2}}}=\frac{x-2}{x-2-\frac{x^2-2x}{x^2-3x+1}}$$
$$=\frac{x-2}{\frac{x^3-6x^2+9x-2}{x^2-3x+1}}=\frac{(x-2)(x^2-3x+1)}{(x-2)(x^2-4x+1)}=\frac{x^2-3x+1}{x^2-4x+1}.$$

33. $$\frac{1}{a+\frac{1}{a+\frac{1}{a}}}=\frac{1}{a+\frac{1}{\frac{a^2+1}{a}}}=\frac{1}{a+\frac{a}{a^2+1}}=\frac{1}{\frac{a^3+2a}{a^2+1}}=\frac{a^2+1}{a^3+2a}.$$

34. $$1+\frac{c}{1+c+\frac{2c}{1+\frac{1}{c}}}=1+\frac{c}{1+c+\frac{2c}{\frac{c+1}{c}}}=1+\frac{c}{1+c+\frac{2c^2}{c+1}}$$
$$=1+\frac{c}{\frac{3c^2+2c+1}{c+1}}=1+\frac{c^2+c}{3c^2+2c+1}$$
$$=\frac{3c^2+2c+1+c^2+c}{3c^2+2c+1}=\frac{4c^2+3c+1}{3c^2+2c+1}.$$

REVIEW

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18. $$\frac{x^3+x^2+x-3}{x^3+3x^2+5x+3}.$$

Factoring the numerator by the factor theorem, we have,

$$x^3+x^2+x-3=(x^2+2x+3)(x-1).$$

By trial, it is found that x^2+2x+3 is a factor also of the denominator, whose other factor is $x+1$.

$$\therefore \frac{x^3+x^2+x-3}{x^3+3x^2+5x+3}=\frac{(x^2+2x+3)(x-1)}{(x^2+2x+3)(x+1)}=\frac{x-1}{x+1}.$$

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19. $$\frac{x^3-x^2-x-2}{x^3+3x^2+3x+2}.$$

Factoring the numerator by the factor theorem, we have

$$x^3-x^2-x-2=(x^2+x+1)(x-2).$$

By trial, it is found that x^2+x+1 is a factor also of the denominator, whose other factor is $x+2$.

$$\therefore \frac{x^3-x^2-x-2}{x^3+3x^2+3x+2}=\frac{(x^2+x+1)(x-2)}{(x^2+x+1)(x+2)}=\frac{x-2}{x+2}.$$

20. $$\frac{x^3+x^2-22x-40}{x^3-7x^2+2x+40}.$$

Factoring the numerator by the factor theorem, we have

$$x^3+x^2-22x-40=(x^2-3x-10)(x+4).$$

By trial, it is found that $x^2-3x-10$ is a factor also of the denominator, whose other factor is $x-4$.

$$\therefore \frac{x^3+x^2-22x-40}{x^3-7x^2+2x+40}=\frac{(x^2-3x-10)(x+4)}{(x^2-3x-10)(x-4)}=\frac{x+4}{x-4}.$$

21. $$\frac{x^3+10x^2+7x-18}{x^3-8x^2-11x+18}.$$

Factoring the numerator by the factor theorem, we have

$$x^3+10x^2+7x-18=(x^2+x-2)(x+9).$$

By trial, it is found that x^2+x-2 is a factor also of the denominator, whose other factor is $x-9$.

$$\therefore \frac{x^3+10x^2+7x-18}{x^3-8x^2-11x+18}=\frac{(x^2+x-2)(x+9)}{(x^2+x-2)(x-9)}=\frac{x+9}{x-9}.$$

22. $$\frac{x}{2y-1}+\frac{y}{2y+1}-\frac{y-x}{1-4y^2}=\frac{x}{2y-1}+\frac{y}{2y+1}-\frac{x-y}{4y^2-1}$$
$$=\frac{2xy+x+2y^2-y-(x-y)}{4y^2-1}$$
$$=\frac{2xy+2y^2}{4y^2-1}=\frac{2y(x+y)}{4y^2-1}.$$

23. $$\frac{a^2}{4(1-a)^2}-\left(\frac{3}{8(1-a)}+\frac{1}{8(a+1)}-\frac{1-a}{4(a+1)}\right)$$
$$=\frac{a^2}{4(1-a)^2}-\left(\frac{3}{8(1-a)}+\frac{-1+2a}{8(1+a)}\right)$$
$$=\frac{a^2}{4(1-a)^2}-\frac{3+3a-1+3a-2a^2}{8(1-a^2)}=\frac{a^2}{4(1-a)^2}-\frac{1+3a-a^2}{4(1-a^2)}$$
$$=\frac{a^2+a^3-(1+2a-4a^2+a^3)}{4(1-a)^2(1+a)}=\frac{5a^2-2a-1}{4(1-a)^2(1+a)}.$$

24. $$\left(1+\frac{2}{m-1}\right)\left(\frac{m^2+m-2}{m^2+m}\right)=\frac{m+1}{m-1}\times\frac{(m-1)(m+2)}{m(m+1)}=\frac{m+2}{m}.$$

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25. $\left(\frac{a+b}{a-b}-\frac{a-b}{a+b}-\frac{4\,ab}{a^2+b^2}\right)\frac{a^2-b^2}{8\,b^2}=\left(\frac{4\,ab}{a^2-b^2}-\frac{4\,ab}{a^2+b^2}\right)\frac{a^2-b^2}{8\,b^2}$

$$=\frac{8\,ab^3}{(a^2-b^2)(a^2+b^2)}\times\frac{a^2-b^2}{8\,b^2}=\frac{ab}{a^2+b^2}.$$

26. $\left(1+\frac{x}{y}\right)\left(\frac{x}{y^2}-\frac{1}{y}+\frac{1}{x}\right)=\frac{x+y}{y}\times\frac{x^2-xy+y^2}{xy^2}=\frac{x^3+y^3}{xy^3}.$

27. $\left(x+1+\frac{1}{x}+\frac{1}{x^2}\right)\div\left(x+1-\frac{1}{x}-\frac{1}{x^2}\right)$

$$=\frac{x^3+x^2+x+1}{x^2}\div\frac{x^3+x^2-x-1}{x^2}$$

$$=\frac{(x^2+1)(x+1)}{x^2}\times\frac{x^2}{(x^2-1)(x+1)}=\frac{x^2+1}{x^2-1}.$$

28. $\left(\frac{a+b}{a-b}+\frac{a^2+b^2}{a^2-b^2}\right)\left(\frac{a+b}{a-b}-\frac{a^3+b^3}{a^3-b^3}\right)$

$$=\frac{2\,a^2+2\,ab+2\,b^2}{a^2-b^2}\times\frac{2\,a^2b+2\,ab^2}{a^3-b^3}$$

$$=\frac{2(a^2+ab+b^2)}{(a+b)(a-b)}\times\frac{2\,ab(a+b)}{(a-b)(a^2+ab+b^2)}=\frac{4\,ab}{(a-b)^2}.$$

29. $\left(x^2-3\,xy-2\,y^2+\frac{12\,y^3}{x+3\,y}\right)\div\left(3\,x-6\,y-\frac{2\,x^2}{x+3\,y}\right)$

$$=\frac{x^3-11\,xy^2+6\,y^3}{x+3\,y}\div\frac{x^2+3\,xy-18\,y^2}{x+3\,y}$$

$$=\frac{(x-3\,y)(x^2+3\,xy-2\,y^2)}{x+3\,y}\times\frac{x+3\,y}{(x+6\,y)(x-3\,y)}=\frac{x^2+3\,xy-2\,y^2}{x+6\,y}.$$

30. $\left(\frac{m-3\,n}{m+n}\right)\left(1+\frac{4\,n}{m+n}\right)\div\left(\frac{m}{n}+2-\frac{15\,n}{m}\right)$

$$=\left(\frac{m-3\,n}{m+n}\right)\left(\frac{m+5\,n}{m+n}\right)\div\left(\frac{m^2+2\,mn-15\,n^2}{mn}\right)$$

$$=\frac{m-3\,n}{m+n}\times\frac{m+5\,n}{m+n}\times\frac{mn}{(m-3\,n)(m+5\,n)}=\frac{mn}{(m+n)^2}.$$

31. $1-\frac{2\,x+5\,x^2}{2(x+1)^2}-\left\{\frac{x^2+x}{2\,x-\frac{2}{x}}\right\}\left(\frac{3-3\,x}{(x+1)^2}\right)$

$$=1-\frac{2\,x+5\,x^2}{2(x+1)^2}-\frac{x^3+x^2}{2\,x^2-2}\times\frac{3-3\,x}{(x+1)^2}=1+\frac{-2\,x-5\,x^2}{2(x+1)^2}+\frac{3\,x^2}{2(x+1)^2}$$

$$=1-\frac{2\,x^2+2\,x}{2(x+1)^2}=1-\frac{x}{x+1}=\frac{1}{x+1}.$$

32. $\left(1+\frac{x}{a-x}\right)\left(\frac{x}{x+a}-\frac{2\,x^2+2\,ax-a^2}{x^2+3\,ax+2\,a^2}\right)$

$$=\frac{a}{a-x}\times\frac{(a+x)(a-x)}{(x+a)(x+2\,a)}=\frac{a}{x+2\,a}.$$

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33. By § 113, $\left(\frac{x}{y}+\frac{y}{x}\right)\left(\frac{x}{y}-\frac{y}{x}\right)=\frac{x^2}{y^2}-\frac{y^2}{x^2}.$

34. By § 104, $\left(\frac{x}{y}+\frac{y}{x}\right)\left(\frac{x}{y}+\frac{y}{x}\right)=\frac{x^2}{y^2}+2+\frac{y^2}{x^2}.$

35. $\left(\frac{a}{b}-3\right)\left(\frac{a}{b}+8\right)=\frac{a^2+5ab-24b^2}{b^2}=\frac{a^2}{b^2}+\frac{5a}{b}-24.$

36. By § 107, $\left(2x-\frac{1}{2x}\right)\left(2x-\frac{1}{2x}\right)=4x^2-2+\frac{1}{4x^2}.$

37. $$\frac{1+\frac{1}{x^2}+\frac{1}{x^4}}{1+\frac{1}{x}+\frac{1}{x^2}}=\frac{x^4+x^2+1}{x^4+x^3+x^2}=\frac{(x^2+x+1)(x^2-x+1)}{x^2(x^2+x+1)}=\frac{x^2-x+1}{x^2}.$$

38. $$\frac{\left(\frac{m^2+n^2}{n}-m\right)\div\left(\frac{1}{n}-\frac{1}{m}\right)}{\frac{m^3+n^3}{m^2-n^2}}$$

$$=\frac{m^2-mn+n^2}{n}\div\frac{m-n}{mn}\div\frac{(m+n)(m^2-mn+n^2)}{(m+n)(m-n)}$$

$$=\frac{m^2-mn+n^2}{n}\times\frac{mn}{m-n}\times\frac{m-n}{m^2-mn+n^2}=m.$$

39. $$\frac{\frac{c}{(a+1)^2}+\frac{d}{(a+1)^2}}{\frac{a}{(a+1)^4}+\frac{1}{(a+1)^4}}=\frac{c+d}{(a+1)^2}\div\frac{a+1}{(a+1)^4}=\frac{c+d}{(a+1)^2}\times\frac{(a+1)^3}{1}$$

$$=(c+d)(a+1).$$

40. $$\frac{1}{1-\frac{1}{1-\frac{1}{1-x}}}=\frac{1}{1-\frac{1}{\frac{-x}{1-x}}}=\frac{1}{1+\frac{1-x}{x}}=\frac{1}{\frac{1}{x}}=x.$$

41. $$\frac{\frac{6a}{x+y}-\frac{4}{x+y}}{\frac{1}{(x+y)^2}}=\frac{6ax+6ay-(4x+4y)}{1}=2(3a-2)(x+y).$$

42. $$1-\frac{\frac{a^2+3a+2}{a^2+2a+1}}{\frac{a^2+7a+12}{a^2+5a+4}}=1-\frac{\frac{(a+1)(a+2)}{(a+1)(a+1)}}{\frac{(a+3)(a+4)}{(a+1)(a+4)}}=1-\frac{(a+1)^2(a+2)(a+4)}{(a+1)^2(a+3)(a+4)}$$

$$=1-\frac{a+2}{a+3}=\frac{1}{a+3}.$$

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43. $$\frac{\frac{m^3-n^3}{m^3+n^3}\left(1-\frac{2n}{m+n}\right)}{1+\frac{2mn}{m^2-mn+n^2}}=\frac{m^3-n^3}{m^3+n^3}\times\frac{m-n}{m+n}\div\frac{m^2+mn+n^2}{m^2-mn+n^2}$$
$$=\frac{(m-n)(m^2+mn+n^2)}{(m+n)(m^2-mn+n^2)}\times\frac{m-n}{m+n}\times\frac{m^2-mn+n^2}{m^2+mn+n^2}=\frac{(m-n)^2}{(m+n)^2}.$$

44. $$\frac{1}{2-\frac{3}{4-\frac{5}{6-x}}}=\frac{1}{2-\frac{3}{\frac{19-4x}{6-x}}}=\frac{1}{2-\frac{18-3x}{19-4x}}=\frac{1}{\frac{20-5x}{19-4x}}=\frac{19-4x}{20-5x}.$$

45. $$\left(\frac{x+y}{x-y}+\frac{x-y}{x+y}\right)\div\left(\frac{x+y}{2(x-y)}-\frac{x-y}{2x+2y}\right)=\frac{2x^2+2y^2}{x^2-y^2}\div\frac{4xy}{2(x^2-y^2)}$$
$$=\frac{2(x^2+y^2)}{x^2-y^2}\times\frac{2(x^2-y^2)}{4xy}=\frac{x^2+y^2}{xy}.$$

46. $$\frac{\left(\frac{a}{x^2}+\frac{1}{x}+\frac{1}{a}+\frac{x}{a^2}\right)\left(\frac{a^2}{x^3}-\frac{1}{x}+\frac{x}{a^2}\right)}{\frac{a^3}{x^5}\left(1+\frac{x}{a}\right)}=\frac{\frac{a^3+a^2x+ax^2+x^3}{a^2x^2}\times\frac{a^4-a^2x^2+x^4}{a^2x^3}}{\frac{a^3}{x^5}\left(\frac{a+x}{a}\right)}$$
$$=\frac{a^3+a^2x+ax^2+x^3}{a^2x^2}\times\frac{a^4-a^2x^2+x^4}{a^2x^3}\div\frac{a^3}{x^5}\left(\frac{a+x}{a}\right)$$
$$=\frac{(a+x)(a^2+x^2)}{a^2x^2}\times\frac{a^4-a^2x^2+x^4}{a^2x^3}\times\frac{x^5}{a^2(a+x)}$$
$$=\frac{(a^2+x^2)(a^4-a^2x^2+x^4)}{a^6}=\frac{a^6+x^6}{a^6}.$$

47. $$\frac{x^3-\frac{8}{y^3}}{x^3y^3-x^2y^2}\times\frac{\frac{1}{xy}+\frac{1}{x^2y^2}}{1+\frac{2}{xy}+\frac{4}{x^2y^2}}\times\frac{xy-1}{xy+1}$$
$$=\frac{x^3y^3-8}{x^3y^6-x^2y^5}\times\frac{xy+1}{x^2y^2+2xy+4}\times\frac{xy-1}{xy+1}$$
$$=\frac{(xy-2)(x^2y^2+2xy+4)}{x^2y^5(xy-1)}\times\frac{xy+1}{x^2y^2+2xy+4}\times\frac{xy-1}{xy+1}=\frac{xy-2}{x^2y^5}.$$

SIMPLE EQUATIONS

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19. $$\frac{v+1}{3}-\frac{v+4}{5}+\frac{v+3}{4}=16.$$

Clearing, $20(v+1)-12(v+4)+15(v+3)=60\cdot 16.$

$$20v+20-12v-48+15v+45=960.$$
$$\therefore\ v=41.$$

Verifying, $\frac{42}{3}-\frac{45}{5}+\frac{44}{4}=16,$

which reduces to $16=16.$

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20.
$$\frac{7z+2}{6}-\frac{12-z}{4}+\frac{z+2}{2}=6.$$

Clearing, $2(7z+2)-3(12-z)+6(z+2)=12\cdot 6.$
$$14z+4-36+3z+6z+12=72.$$
$$\therefore\ z=4.$$

Verifying, $\frac{30}{6}-\frac{8}{4}+\frac{6}{2}=6,$
which reduces to $6=6.$

21.
$$\frac{u-3}{7}+\frac{u+5}{3}-\frac{u+2}{6}=4.$$

Clearing, $6(u-3)+14(u+5)-7(u+2)=42\cdot 4.$
$$6u-18+14u+70-7u-14=168.$$
$$\therefore\ u=10.$$

Verifying, $\frac{7}{7}+\frac{15}{3}-\frac{12}{6}=4,$
which reduces to $4=4.$

22.
$$\frac{3t-5}{4}-\frac{7t-13}{6}=3-\frac{t+3}{2}.$$

Clearing, $3(3t-5)-2(7t-13)=12\cdot 3-6(t+3).$
$$9t-15-14t+26=36-6t-18.$$
$$\therefore\ t=7.$$

Verifying, $\frac{16}{4}-\frac{36}{6}=3-\frac{10}{2},$
which reduces to $-2=-2.$

23.
$$\frac{5x+2}{3}-\left(x-\frac{3x-1}{2}\right)=\frac{3x+19}{2}-\left(\frac{x+1}{6}+5\right).$$

Clearing, $2(5x+2)-3(2x-3x+1)=3(3x+19)-(x+1+30).$
$$10x+4+3x-3=9x+57-x-31.$$
$$\therefore\ x=5.$$

Verifying, $\frac{27}{3}+\frac{4}{2}=\frac{34}{2}-\frac{36}{6},$
which reduces to $11=11.$

24.
$$1.07x+.32=.15x+10.12+.675x.$$

Clearing of decimal fractions, $1070x+320=150x+10120+675x.$
$$\therefore\ x=40.$$

Verifying, $42.8+.32=6+10.12+27,$
which reduces to $43.12=43.12.$

25.
$$.604x-3.16-.7854x+7.695=0.$$

Clearing of decimal fractions,
$$6040x-31600-7854x+76950=0.$$
$$\therefore\ x=25.$$

Verifying, $15.100-3.16-19.6350+7.695=0,$
which reduces to $0=0.$

26.
$$3.1416x-15.5625+.0216x=.2535.$$

Clearing of decimal fractions,
$$31416x-155625+216x=2535.$$
$$\therefore\ x=5.$$

Verifying, $15.7080-15.5625+.1080=.2535,$
which reduces to $.2535=.2535.$

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27. $$\frac{.2x}{7}-\frac{.1x}{4}-\frac{.1x}{2}+\frac{.4x}{7}=\frac{.3}{14}.$$

Clearing, $$8x-7x-14x+16x=6.$$
$$\therefore\ x=2.$$

Verifying, $$\frac{.4}{7}-\frac{.2}{4}-\frac{.2}{2}+\frac{.8}{7}=\frac{.3}{14},$$

which reduces to $$\frac{.6}{28}=\frac{.6}{28}.$$

28. $$\frac{n+4}{.3}+\frac{2-2n}{.6}=\frac{n+1}{.2}-\frac{10}{.3}.$$

Clearing, $$2n+8+2-2n=3n+3-20.$$
$$\therefore\ n=9.$$

Verifying, $$\frac{13}{.3}+\frac{-16}{.6}=\frac{10}{.2}-\frac{10}{.3},$$

which reduces to $$\frac{10}{.6}=\frac{10}{.6}.$$

29. $$\frac{9x+5}{14}+\frac{8x-7}{6x+2}=\frac{36x+15}{56}+\frac{10\frac{1}{4}}{14}.$$

Dist. Law, $$\frac{9x}{14}+\frac{5}{14}+\frac{8x-7}{6x+2}=\frac{36x}{56}+\frac{15}{56}+\frac{41}{56}.$$

Canceling, etc., $$\frac{8x-7}{6x+2}=\frac{36}{56}=\frac{9}{14}.$$

Clearing, $$112x-98=54x+18.$$
$$\therefore\ x=2.$$

Verifying, $$\tfrac{23}{14}+\tfrac{9}{14}=\tfrac{87}{56}+\tfrac{41}{56},$$
which reduces to $$\tfrac{32}{14}=\tfrac{32}{14}.$$

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30. $$\frac{3x-2}{2x-5}+\frac{3x-21}{5}=\frac{6x-22}{10}.$$

Dist. Law, $$\frac{3x-2}{2x-5}+\frac{3x}{5}-\frac{21}{5}=\frac{3x}{5}-\frac{11}{5}.$$

Canceling, etc., $$\frac{3x-2}{2x-5}=2.$$

Clearing, $$3x-2=4x-10.$$
$$\therefore x=8.$$

Verifying, $$\tfrac{22}{11}+\tfrac{3}{5}=\tfrac{26}{10},$$
which reduces to $$2\tfrac{3}{5}=2\tfrac{3}{5}.$$

31. $$\frac{4x+3}{9}=\frac{8x+19}{18}-\frac{7x-29}{5x-12}.$$

Dist. Law, $$\frac{4x}{9}+\frac{3}{9}=\frac{8x}{18}+\frac{19}{18}-\frac{7x-29}{5x-12}.$$

Canceling, etc., $$\frac{7x-29}{5x-12}=\frac{13}{18}.$$

Clearing, $$126x-522=65x-156.$$
$$\therefore x=6.$$

Verifying, $$\tfrac{27}{9}=\tfrac{67}{18}-\tfrac{13}{18},$$
which reduces to $$3=3.$$

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32. $$\frac{6p+1}{15}-\frac{2p-4}{7p-13}=\frac{2p-1}{5}.$$

Dist. Law, $$\frac{6p}{15}+\frac{1}{15}-\frac{2p-4}{7p-13}=\frac{2p}{5}-\frac{1}{5}.$$

Canceling, etc., $$\frac{4}{15}=\frac{2p-4}{7p-13}.$$

Clearing, $$28p-52=30p-60.$$

$$\therefore p=4.$$

Verifying, $$\tfrac{25}{15}-\tfrac{4}{15}=\tfrac{7}{5},$$
which reduces to $$\tfrac{7}{5}=\tfrac{7}{5}.$$

33. $$\frac{10q+17}{18}-\frac{5q-2}{9}=\frac{12q-1}{11q-8}.$$

Uniting terms in the first member,
$$\frac{7}{6}=\frac{12q-1}{11q-8}.$$

Clearing, $$77q-56=72q-6.$$

$$\therefore q=10.$$

Verifying, $$\tfrac{117}{18}-\tfrac{48}{9}=\tfrac{119}{102},$$
which reduces to $$\tfrac{7}{6}=\tfrac{7}{6}.$$

34. $$\frac{6r+3}{15}-\frac{3r-1}{5r-25}=\frac{2r-9}{5}.$$

$$\frac{6r}{15}+\frac{3}{15}-\frac{3r-1}{5r-25}=\frac{2r}{5}-\frac{9}{5}.$$

Canceling, etc., $$2=\frac{3r-1}{5r-25}.$$

Clearing, $$10r-50=3r-1.$$

$$\therefore r=7.$$

Verifying, $$\tfrac{45}{15}-\tfrac{20}{10}=\tfrac{5}{5},$$
which reduces to $$1=1.$$

36. $$\frac{x-1}{x-2}+\frac{x-7}{x-8}=\frac{x-5}{x-6}+\frac{x-3}{x-4}.$$

Transposing, $$\frac{x-1}{x-2}-\frac{x-3}{x-4}=\frac{x-5}{x-6}-\frac{x-7}{x-8}.$$

Uniting terms in each member,
$$\frac{x^2-5x+4-(x^2-5x+6)}{(x-2)(x-4)}=\frac{x^2-13x+40-(x^2-13x+42)}{(x-6)(x-8)},$$

or $$\frac{-2}{x^2-6x+8}=\frac{-2}{x^2-14x+48}.$$

$$\therefore x^2-6x+8=x^2-14x+48,$$

whence, $$x=5.$$

Verifying, $$\frac{4}{3}+\frac{-2}{-3}=\frac{0}{-1}+\frac{2}{1},\text{ or } 2=2.$$

37. $$\frac{x-3}{x-4}+\frac{x-7}{x-8}=\frac{x-6}{x-7}+\frac{x-4}{x-5}.$$

Transposing, $$\frac{x-7}{x-8}-\frac{x-6}{x-7}=\frac{x-4}{x-5}-\frac{x-3}{x-4}.$$

Uniting terms in each member,

$$\frac{x^2-14x+49-(x^2-14x+48)}{(x-8)(x-7)}=\frac{x^2-8x+16-(x^2-8x+15)}{(x-5)(x-4)},$$

or $$\frac{1}{x^2-15x+56}=\frac{1}{x^2-9x+20}.$$

Clearing, $$x^2-9x+20=x^2-15x+56.$$

$$\therefore x=6.$$

Verifying, $$\frac{3}{2}+\frac{-1}{-2}=\frac{0}{-1}+\frac{2}{1}, \text{ or } 2=2.$$

38. $$\frac{v+2}{v+1}-\frac{v+3}{v+2}=\frac{v+5}{v+4}-\frac{v+6}{v+5}.$$

Uniting terms in each member,

$$\frac{v^2+4v+4-(v^2+4v+3)}{(v+1)(v+2)}=\frac{v^2+10v+25-(v^2+10v+24)}{(v+4)(v+5)},$$

or $$\frac{1}{v^2+3v+2}=\frac{1}{v^2+9v+20}.$$

Clearing, $$v^2+9v+20=v^2+3v+2,$$

$$\therefore v=-3.$$

Verifying, $$\frac{-1}{-2}-\frac{0}{-1}=\frac{2}{1}-\frac{3}{2}, \text{ or } \frac{1}{2}=\frac{1}{2}.$$

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39. $$\frac{s+1}{s+2}+\frac{s+6}{s+7}=\frac{s+2}{s+3}+\frac{s+5}{s+6}.$$

Transposing, $$\frac{s+6}{s+7}-\frac{s+5}{s+6}=\frac{s+2}{s+3}-\frac{s+1}{s+2}.$$

Uniting terms in each member,

$$\frac{s^2+12s+36-(s^2+12s+35)}{(s+7)(s+6)}=\frac{s^2+4s+4-(s^2+4s+3)}{(s+3)(s+2)},$$

or $$\frac{1}{s^2+13s+42}=\frac{1}{s^2+5s+6}.$$

Clearing, $$s^2+5s+6=s^2+13s+42;$$

whence, $$s=-\tfrac{9}{2}.$$

Verifying, $$\frac{-\frac{7}{2}}{-\frac{5}{2}}+\frac{\frac{3}{2}}{\frac{5}{2}}=\frac{-\frac{5}{2}}{-\frac{3}{2}}+\frac{\frac{1}{2}}{\frac{3}{2}}, \text{ or } 2=2.$$

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40. $$\frac{x^3+1}{x-1}-\frac{x^3-1}{x+1}=\frac{8}{x^2-1}+2x.$$

Reducing to mixed numbers,

$$x^2+x+1+\frac{2}{x-1}-\left(x^2-x+1-\frac{2}{x+1}\right)=\frac{8}{x^2-1}+2x.$$

Uniting terms, $$\frac{2}{x-1}+\frac{2}{x+1}=\frac{8}{x^2-1}.$$

Clearing, $$2x+2+2x-2=8.$$

$$\therefore x=2.$$

Verifying, $$\tfrac{9}{1}-\tfrac{7}{3}=\tfrac{8}{3}+4, \text{ or } 6\tfrac{2}{3}=6\tfrac{2}{3}.$$

41. $$\frac{x^3+2}{x+1}-\frac{x^3-2}{x-1}=\frac{10}{x^2-1}-2x.$$

Reducing to mixed numbers,

$$x^2-x+1+\frac{1}{x+1}-\left(x^2+x+1-\frac{1}{x-1}\right)=\frac{10}{x^2-1}-2x.$$

Uniting terms, $$\frac{1}{x+1}+\frac{1}{x-1}=\frac{10}{x^2-1}.$$

Clearing, $$x-1+x+1=10.$$

$$\therefore x=5.$$

Verifying, $$\tfrac{127}{6}-\tfrac{123}{4}=\tfrac{10}{24}-10, \text{ or } -\tfrac{115}{12}=-\tfrac{115}{12}.$$

42. $$\frac{r}{2}(2-r)-\frac{r}{4}(3-2r)=\frac{r+10}{6}.$$

Simplifying each term, $$r-\frac{r^2}{2}-\frac{3r}{4}+\frac{r^2}{2}=\frac{r}{6}+\frac{5}{3}.$$

Clearing, etc., $$12r-9r=2r+20.$$

$$\therefore r=20.$$

Verifying, $$10(2-20)-5(3-40)=5, \text{ or } 5=5.$$

43. $$\frac{3n-4}{4}-\left(\frac{4n}{5}+\frac{n+2}{2}\right)=\frac{9n}{10}-\left(19+\frac{n+4}{2}\right).$$

$$\frac{3n-4}{4}-\frac{4n}{5}-\frac{n+2}{2}=\frac{9n}{10}-\frac{38}{2}-\frac{n+4}{2}.$$

Clearing, $$15n-20-16n-10n-20=18n-380-10n-40.$$

$$\therefore n=20.$$

Verifying, $$\frac{60-4}{4}-\left(\frac{80}{5}+\frac{20+2}{2}\right)=\frac{180}{10}-\left(19+\frac{20+4}{2}\right),$$

which reduces to $$-13=-13.$$

44. $$\frac{(x-3)^2}{7}-\frac{(x+4)^2}{3}=20-\left(8x+\frac{5x+10}{21}\right)-\frac{4x^2}{21},$$

$$\frac{x^2-6x+9}{7}-\frac{x^2+8x+16}{3}=20-8x-\frac{5x+10}{21}-\frac{4x^2}{21}.$$

Clearing, $$3x^2-18x+27-7x^2-56x-112=420-168x-5x-10-4x^2.$$

$$\therefore x=5.$$

Verifying, $$\frac{(5-3)^2}{7}-\frac{(5+4)^2}{3}=20-\left(40+\frac{25+10}{21}\right)-\frac{100}{21},$$

which reduces to $$-\tfrac{555}{21}=-\tfrac{555}{21}.$$

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45.
$$\frac{\frac{2c}{3}+4}{2}=\frac{\frac{1}{2}-c}{3}+\frac{c}{2}\left(\frac{6}{c}-1\right).$$

Simplifying each term, $\frac{c}{3}+2=\frac{5}{2}-\frac{c}{3}+3-\frac{c}{2}.$

Transposing and uniting terms, $\frac{7c}{6}=\frac{7}{2}.$

Multiplying by $\frac{6}{7}$, $c=3.$

Verifying, $\frac{2+4}{2}=\frac{4\frac{1}{2}}{3}+\frac{3}{2}$ of 1, or $3=3.$

46.
$$\frac{\frac{4}{5d}-16}{24}-\frac{\frac{2}{5d}+6}{60}=\frac{4\frac{1}{6}}{5}.$$

Simplifying each term,
$$\frac{1}{30d}-\frac{2}{3}-\frac{1}{150d}-\frac{1}{10}=\frac{5}{6}.$$

Clearing, $5-100d-1-15d=125d.$
$$\therefore d=\tfrac{1}{60}.$$

Verifying, since $\frac{1}{d}=60,\ \frac{4}{5d}=\frac{4}{5}$ of 60, and $\frac{2}{5d}=\frac{2}{5}$ of 60,
$$\frac{48-16}{24}-\frac{24+6}{60}=\frac{5}{6},\text{ or }\frac{5}{6}=\frac{5}{6}.$$

47.
$$\frac{2x\left(1-\frac{5}{x}\right)}{3}+\frac{3x\left(1-\frac{4}{x}\right)}{4}=\frac{x-4}{\frac{4}{5}}$$

Simplifying each term,
$$\frac{2x}{3}-\frac{10}{3}+\frac{3x}{4}-3=\frac{5x}{4}-5.$$

Clearing, $8x-40+9x-36=15x-60.$
$$\therefore x=8.$$

Verifying, $\frac{16\cdot\frac{3}{8}}{3}+\frac{24\cdot\frac{1}{2}}{4}=\frac{4}{\frac{4}{5}};$

that is, $2+3=5$, or $5=5.$

48.
$$\frac{1}{2}x-2\left(\frac{4x}{5}-3\right)=4-\frac{3}{2}\left(\frac{x}{2}+1\right).$$

Expanding, $\frac{1}{2}x-\frac{8x}{5}+6=4-\frac{3x}{4}-\frac{3}{2}.$

Clearing, $10x-32x+120=80-15x-30.$
$$\therefore x=10.$$

Verifying, $5-2(8-3)=4-\frac{3}{2}$ of 6, or $-5=-5.$

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49. $$\frac{(2x+1)^2}{.05}-\frac{(4x-1)^2}{.2}=\frac{15}{.08}+\frac{3(4x+1)}{.4}.$$

Clearing,
$$32x^2+32x+8-32x^2+16x-2=75+12x+3.$$
$$\therefore\ x=2.$$

Verifying, $$\frac{25}{.05}-\frac{49}{.2}=\frac{15}{.08}+\frac{27}{.4},$$

which reduces to $$\frac{102}{.40}=\frac{102}{.40}.$$

50. $$\frac{17+\frac{3}{x}}{3}+\frac{1+\frac{18}{x}}{5}=\frac{\frac{21}{x}-1}{9}+\frac{\frac{100}{x}+\frac{5}{3}}{15}. \qquad (1)$$

$$\frac{17}{3}+\frac{1}{x}+\frac{1}{5}+\frac{18}{5}\cdot\frac{1}{x}=\frac{7}{3}\cdot\frac{1}{x}-\frac{1}{9}+\frac{20}{3}\cdot\frac{1}{x}+\frac{1}{9}. \qquad (2)$$

Canceling and transposing, $$\frac{17}{3}+\frac{1}{5}=\left(\frac{7}{3}+\frac{20}{3}-1-\frac{18}{5}\right)\frac{1}{x}. \qquad (3)$$

Uniting terms, $$\frac{88}{15}=\frac{22}{5}\cdot\frac{1}{x}. \qquad (4)$$

Dividing by $\frac{22}{5}$, $$\frac{4}{3}=\frac{1}{x}. \qquad (5)$$

$$\therefore\ x=\tfrac{3}{4}. \qquad (6)$$

Substituting (5) in (1), $$\frac{17+4}{3}+\frac{1+24}{5}=\frac{28-1}{9}+\frac{\frac{400}{3}+\frac{5}{3}}{15};$$

that is, $$7+5=3+9, \text{ or } 12=12.$$

51. $$\frac{\frac{1}{4}(x-4)}{\frac{3}{2}}-\frac{4x-16}{6}=\frac{3}{5}-\frac{\frac{2x}{5}+5}{\frac{5}{2}}.$$

Simplifying each term, $$\frac{x}{6}-\frac{2}{3}-\frac{2x}{3}+\frac{8}{3}=\frac{3}{5}-\frac{4x}{25}-2.$$

Transposing and uniting, $$-\frac{17x}{50}=-\frac{17}{5}.$$

$$\therefore\ x=10.$$

Verifying, $$\frac{\frac{1}{4}\text{ of }6}{\frac{3}{2}}-\frac{24}{6}=\frac{3}{5}-\frac{9}{\frac{5}{2}};$$

that is, $$1-4=\tfrac{3}{5}-\tfrac{18}{5}, \text{ or } -3=-3.$$

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2. $$\frac{c^2-x}{nx}+\frac{n^2}{cx}=\frac{1}{c}.$$

Clearing, $$c^3-cx+n^3=nx.$$

Transposing, etc., $$cx+nx=c^3+n^3.$$

Dividing by $c+n$, $$x=c^2-cn+n^2.$$

VERIFICATION. — When $c=2$ and $n=1$, $x=4-2+1=3$.

Substituting in given equation, $$\frac{4-3}{3}+\frac{1}{6}=\frac{1}{2}, \text{ or } \frac{1}{2}=\frac{1}{2}.$$

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3.
$$1-\frac{ab}{x}=\frac{7}{ab}-\frac{49}{abx}.$$

Clearing, $abx-a^2b^2=7\,x-49.$

Transposing, $abx-7\,x=a^2b^2-49.$

Dividing by $ab-7$, $x=ab+7.$

VERIFICATION. — When $a=2$ and $b=1$, $x=2+7=9.$

Substituting in given equation, $1-\frac{2}{9}=\frac{7}{2}-\frac{49}{18}$, or $\frac{14}{18}=\frac{14}{18}$.

4.
$$\frac{a^3}{ab^2}-\frac{2\,a^2}{b^2x}=1-\frac{2\,b^2}{a^2x}.$$

Clearing, $a^4x-2\,a^4=a^2b^2x-2\,b^4.$

Transposing, $a^4x-a^2b^2x=2\,a^4-2\,b^4.$

Dividing by a^2-b^2, $a^2x=2\,(a^2+b^2).$

$$\therefore\ x=\frac{2\,(a^2+b^2)}{a^2}.$$

VERIFICATION. — When $a=2$ and $b=1$, $x=\frac{2\,(4+1)}{4}=\frac{10}{4}=\frac{5}{2}.$

Substituting in given equation, $\frac{8}{2}-\frac{8}{\frac{5}{2}}=1-\frac{2}{4\times\frac{5}{2}}$, or $\frac{4}{5}=\frac{4}{5}.$

5.
$$\frac{x}{b}-\frac{x+2\,b}{a}=\frac{a}{b}-3.$$

Clearing, $ax-bx-2\,b^2=a^2-3\,ab.$

Transposing, $ax-bx=a^2-3\,ab+2\,b^2$

Dividing by $a-b$, $x=a-2\,b.$

VERIFICATION. — When $a=2$ and $b=1$, $x=2-2=0.$

Substituting in given equation,
$$\frac{0}{1}-\frac{0+2}{2}=\frac{2}{1}-3,\text{ or }-1=-1.$$

6.
$$\frac{x-2\,ab}{cx}-\frac{1}{x}=\frac{x-3\,c}{abx}.$$

Clearing, $abx-2\,a^2b^2-abc=cx-3\,c^2.$

Transposing, $abx-cx=2\,a^2b^2+abc-3\,c^2.$

Dividing by $ab-c$, $x=2\,ab+3\,c.$

VERIFICATION. — When $a=3$, $b=2$, and $c=1$, $x=12+3=15.$

Substituting in given equation, $\frac{15-12}{15}-\frac{1}{15}=\frac{15-3}{90}$, or $\frac{2}{15}=\frac{2}{15}.$

7.
$$\frac{x-a}{b}+\frac{2\,x}{a}=5+\frac{6\,b}{a}.$$

Clearing, $ax-a^2+2\,bx=5\,ab+6\,b^2.$

Transposing, $ax+2\,bx=a^2+5\,ab+6\,b^2.$

Dividing by $a+2\,b$, $x=a+3\,b.$

VERIFICATION. — When $a=2$ and $b=1$, $x=2+3=5.$

Substituting in given equation, $\frac{5-2}{1}+\frac{10}{2}=5+\frac{6}{2}$, or $8=8.$

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8. $$\frac{a^2}{bx}+\frac{b^2}{ax}=\frac{a+b}{ab}-\frac{3(a+b)}{x}.$$

Clearing, $a^3+b^3=(a+b)x-3\,ab(a+b).$

Dividing by $a+b$, $a^2-ab+b^2=x-3\,ab.$

Transposing, etc., $x=a^2+2\,ab+b^2$, or $(a+b)^2$.

VERIFICATION. — When $a=2$ and $b=1$, $x=(2+1)^2=9.$

Substituting in given equation, $\frac{4}{9}+\frac{1}{18}=\frac{2+1}{2}-\frac{3(2+1)}{9}$, or $\frac{1}{2}=\frac{1}{2}$.

9. $$\frac{a^2+b^2}{2\,bx}-\frac{a-b}{2\,bx^2}=\frac{b}{x}.$$

Clearing, $(a^2+b^2)x-(a-b)=2\,b^2x.$

Transposing, etc., $(a^2-b^2)x=a-b.$

Dividing by a^2-b^2, $x=\frac{a-b}{a^2-b^2}=\frac{1}{a+b}.$

VERIFICATION. — When $a=2$ and $b=1$, $x=\frac{1}{2+1}=\frac{1}{3}.$

Substituting in given equation, $\frac{4+1}{\frac{2}{3}}-\frac{2-1}{\frac{2}{9}}=\frac{1}{\frac{1}{3}}$, or $3=3$.

10. $$\frac{2\,x-a}{x-a}-\frac{x-a}{x+a}=1.$$

Clearing, $2\,x^2+ax-a^2-x^2+2\,ax-a^2=x^2-a^2.$

Transposing, etc., $x=\frac{a}{3}.$

VERIFICATION. — When $a=1$, $x=\frac{1}{3}.$

Substituting in given equation, $\frac{\frac{2}{3}-1}{\frac{1}{3}-1}-\frac{\frac{1}{3}-1}{\frac{1}{3}+1}=1$, or $1=1$.

11. $$\frac{x-2\,a}{a}+\frac{x}{b}=\frac{a^2+b^2}{ab}.$$

Clearing, $bx-2\,ab+ax=a^2+b^2.$

Transposing, $ax+bx=a^2+2\,ab+b^2.$

Dividing by $a+b$, $x=a+b.$

VERIFICATION. — When $a=2$ and $b=1$, $x=2+1=3.$

Substituting in given equation, $\frac{3-4}{2}+\frac{3}{1}=\frac{4+1}{2}$, or $\frac{5}{2}=\frac{5}{2}$.

12. $$6\,x+18\left(1-\frac{a}{2}\right)=a(x-a).$$

Expanding, $6\,x+18-9\,a=ax-a^2.$

Transposing, etc., $ax-6\,x=a^2-9\,a+18.$

Dividing by $a-6$, $x=a-3.$

VERIFICATION. — When $a=1$, $x=1-3=-2.$

Substituting in given equation,

$$-12+18(1-\tfrac{1}{2})=1(-2-1),\text{ or }-3=-3.$$

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13. $$b(2x - 9c - 14b) = c(c - x).$$

Expanding, $$2bx - 9bc - 14b^2 = c^2 - cx.$$

Transposing, $$2bx + cx = 14b^2 + 9bc + c^2.$$

Dividing by $2b + c$, $$x = 7b + c.$$

VERIFICATION. — When $b = 2$ and $c = 1$, $x = 14 + 1 = 15$.

Substituting in given equation,
$$2(30 - 9 - 28) = 1(1 - 15), \text{ or } -14 = -14.$$

14. $$a(x - a - 2b) + b(x - b) + c(x + c) = 0.$$

Expanding, $$ax - a^2 - 2ab + bx - b^2 + cx + c^2 = 0.$$

Transposing, $$ax + bx + cx = a^2 + 2ab + b^2 - c^2.$$

Dividing by $a + b + c$, $$x = a + b - c.$$

VERIFICATION. — When $a = 2$, $b = 1$, and $c = 3$, $x = 2 + 1 - 3 = 0$.

Substituting in given equation,
$$2(0 - 2 - 2) + 1(0 - 1) + 3(0 + 3) = 0, \text{ or } 0 = 0.$$

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15. $$(a - x)(x - b) + (a + x)(x - b) = (a - b)^2.$$

Uniting terms, $$2a(x - b) = (a - b)^2.$$

Expanding, $$2ax - 2ab = a^2 - 2ab + b^2.$$

Canceling $-2ab = -2ab$, and dividing by $2a$,
$$x = \frac{a^2 + b^2}{2a}.$$

VERIFICATION. — When $a = 2$ and $b = 1$, $x = \dfrac{4 + 1}{4} = \dfrac{5}{4}$.

Substituting in given equation,
$$(2 - \tfrac{5}{4})(\tfrac{5}{4} - 1) + (2 + \tfrac{5}{4})(\tfrac{5}{4} - 1) = (2 - 1)^2, \text{ or } 1 = 1.$$

16. $$(a - b)(x - c) - (b - c)(x - a) = (c - a)(x - b).$$

Expanding, etc.,
$$ax - ac - bx + bc - bx + ab + cx - ac = cx - bc - ax + ab.$$

Canceling $ab + cx = cx + ab$, transposing, etc.,
$$2ax - 2bx = 2ac - 2bc.$$

Dividing by $2(a - b)$, $$x = c.$$

VERIFICATION. — When $c = 2$, $$x = 2.$$

Let $$a = 1 \text{ and } b = 3.$$

Substituting in given equation,
$$(1 - 3)(2 - 2) - (3 - 2)(2 - 1) = (2 - 1)(2 - 3),$$
or
$$-1 = -1.$$

17. $$\frac{a - b + c}{x + a} = \frac{b - a + c}{x - a}.$$

Expanding, $ax - bx + cx - a^2 + ab - ac = bx - ax + cx + ab - a^2 + ac.$

Canceling $cx - a^2 + ab = cx + ab - a^2$, transposing and uniting terms,
$$2ax - 2bx = 2ac.$$

Dividing by $2(a - b)$, $$x = \frac{ac}{a - b}.$$

VERIFICATION. — When $a = 2$, $b = 1$, and $c = 3$, $x = \dfrac{6}{2 - 1} = 6$.

Substituting in given equation, $\dfrac{2 - 1 + 3}{6 + 2} = \dfrac{1 - 2 + 3}{6 - 2}$, or $\dfrac{1}{2} = \dfrac{1}{2}$.

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18. $$\frac{1}{a(b-x)}+\frac{1}{b(c-x)}-\frac{1}{a(c-x)}=0.$$

Clearing, $b(c-x)+a(b-x)-b(b-x)=0.$

Expanding, $bc-bx+ab-ax-b^2+bx=0.$

Transposing, etc., $ax=ab-b^2+bc=b(a-b+c).$

Dividing by a, $$x=\frac{b}{a}(a-b+c).$$

VERIFICATION. — When $a=3$, $b=2$, and $c=1$, $x=\frac{2}{3}(3-2+1)=\frac{4}{3}$.
Substituting in given equation,

$$\frac{1}{3(2-\frac{4}{3})}+\frac{1}{2(1-\frac{4}{3})}-\frac{1}{3(1-\frac{4}{3})}=0, \text{ or } 0=0.$$

19. $$\frac{x-1}{a-1}-\frac{a-1}{x-1}=\frac{x^2-a^2}{(a-1)(x-1)}.$$

Clearing, $(x-1)^2-(a-1)^2=x^2-a^2.$

Expanding, etc.,

$$x^2-2x+1-a^2+2a-1=x^2-a^2.$$

Canceling $x^2-a^2=x^2-a^2$ and $1-1=0$, and transposing,

$$-2x=-2a.$$
$$\therefore x=a.$$

VERIFICATION. — When $a=2$, $x=2.$
Substituting in given equation,

$$\frac{2-1}{2-1}-\frac{2-1}{2-1}=\frac{4-4}{(2-1)(2-1)}, \text{ or } 0=0.$$

20. $$\frac{1}{m+n}-\frac{2mn}{(m+n)^3}-\frac{m}{(m+n)^2}=\frac{x-n}{(m+n)^2}.$$

Clearing,

$$m^2+2mn+n^2-2mn-m^2-mn=mx-mn+nx-n^2.$$

Transposing, etc., $mx+nx=2n^2.$

Dividing by $m+n$, $$x=\frac{2n^2}{m+n}.$$

VERIFICATION. — When $m=2$ and $n=1$, $x=\frac{2}{2+1}=\frac{2}{3}.$

Substituting in given equation,

$$\frac{1}{2+1}-\frac{4}{(2+1)^3}-\frac{2}{(2+1)^2}=\frac{(\frac{2}{3}-1)}{(2+1)^2}, \text{ or } -\frac{1}{27}=-\frac{1}{27}.$$

21. $$\frac{x+a}{b}+\frac{x+c}{a}+\frac{x+b}{c}=\frac{a}{b}+\frac{b}{c}+\frac{c}{a}+1.$$

$$\frac{x}{b}+\frac{a}{b}+\frac{x}{a}+\frac{c}{a}+\frac{x}{c}+\frac{b}{c}=\frac{a}{b}+\frac{b}{c}+\frac{c}{a}+1.$$

Canceling, $$\frac{x}{b}+\frac{x}{a}+\frac{x}{c}=1.$$

Clearing, $cax+bcx+abx=abc.$

Dividing by $ab+bc+ca$, $$x=\frac{abc}{ab+bc+ca}.$$

VERIFICATION. — When $a=3$, $b=2$, and $c=1$, $x=\frac{6}{6+2+3}=\frac{6}{11}.$
Substituting in given equation,

$$\frac{\frac{6}{11}+3}{2}+\frac{\frac{6}{11}+1}{3}+\frac{\frac{6}{11}+2}{1}=\frac{3}{2}+\frac{2}{1}+\frac{1}{3}+1, \text{ or } \frac{29}{6}=\frac{29}{6}.$$

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22.
$$\frac{x}{a+b+c}+\frac{x}{a+b-c}=a^2+b^2+c^2+2ab.$$

Clearing,
$$(a+b-c+a+b+c)x=(a^2+2ab+b^2+c^2)(a^2+2ab+b^2-c^2).$$
$$2(a+b)x=(a+b)^4-c^4.$$

Dividing by $2(a+b)$, $\quad x=\dfrac{(a+b)^4-c^4}{2(a+b)}.$

Verification. — When $a=3$, $b=2$, and $c=1$,
$$x=\frac{(3+2)^4-1}{2(3+2)}=\frac{624}{10}=\frac{312}{5}.$$

Substituting in given equation,
$$\frac{\frac{312}{5}}{3+2+1}+\frac{\frac{312}{5}}{3+2-1}=9+4+1+12,\text{ or }26=26.$$

23.
$$\frac{a+x}{a}-\frac{2x}{a+x}+\frac{x^2(x-a)}{a(a^2-x^2)}=\frac{1}{3}.$$
$$\frac{a+x}{a}-\frac{2x}{a+x}-\frac{x^2}{a(a+x)}=\frac{1}{3}.$$

Multiplying by $a(a+x)$,
$$a^2+2ax+x^2-2ax-x^2=\tfrac{1}{3}a(a+x).$$

Canceling and clearing, $\quad 3a^2=a^2+ax.$
$$\therefore x=2a.$$

Verification. — When $a=1$, $\quad x=2.$

Substituting in given equation,
$$\frac{1+2}{1}-\frac{4}{1+2}+\frac{4(2-1)}{1(1-4)}=\frac{1}{3},\text{ or }\frac{1}{3}=\frac{1}{3}.$$

24.
$$\frac{x^2-ax-bx+ab}{x-a}=\frac{x^2-2bx+2b^2}{x-b}-\frac{c^2}{x-c}.$$

Simplifying, $\quad x-b=x-b+\dfrac{b^2}{x-b}-\dfrac{c^2}{x-c}.$

Canceling and clearing, $\quad 0=b^2x-b^2c-c^2x+bc^2.$

Transposing, etc., $\quad b^2x-c^2x=b^2c-bc^2.$

Dividing by b^2-c^2, $\quad x=\dfrac{bc(b-c)}{b^2-c^2}=\dfrac{bc}{b+c}.$

Verification. — When $b=2$ and $c=1$, $x=\dfrac{2\cdot 1}{2+1}=\dfrac{2}{3}.$

Let $\quad a=3.$

Substituting in given equation,
$$\frac{\frac{4}{9}-2-\frac{4}{3}+6}{\frac{2}{3}-3}=\frac{\frac{4}{9}-\frac{8}{3}+8}{\frac{2}{3}-2}-\frac{1}{\frac{2}{3}-1},\text{ or }-\frac{4}{3}=-\frac{4}{3}$$

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1. Let $\quad x=$ number of cents one foot of tubing cost.

Then, $\quad 850-5x=$ number of cents pump cost.
$$850-5x=12x.$$

Solving, $\quad x=50.$
$$\therefore 5x=250,$$

and $\quad 850-5x=600.$

Hence, the pump cost \$6.00 and the tubing cost \$2.50.

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2. Let $x =$ number of bath robes purchased.

Then, $\frac{x}{4} =$ number of bath robes sold at \$6 each,

$\frac{x}{6} =$ number of bath robes sold at \$7 each,

$\frac{x}{3} =$ number of bath robes sold at \$5 each,

and $\frac{x}{4} =$ number of bath robes sold at \$8 each.

$\therefore \frac{6x}{4} + \frac{7x}{6} + \frac{5x}{3} + \frac{8x}{4} =$ number of dollars received for bath robes.

But $480 + 128 =$ number of dollars received for bath robes.

$\therefore \frac{6x}{4} + \frac{7x}{6} + \frac{5x}{3} + \frac{8x}{4} = 480 + 128.$

Solving, $x = 96$;

whence, $\frac{x}{4} = 24$, number of bath robes sold at \$6 each,

$\frac{x}{6} = 16$, number of bath robes sold at \$7 each,

$\frac{x}{3} = 32$, number of bath robes sold at \$5 each,

and $\frac{x}{4} = 24$, number of bath robes sold at \$8 each.

3. Let $x =$ number of tons each barge holds.
Then, $3\frac{1}{2}x =$ number of tons each schooner holds.
$3x + 2(\frac{7}{2})x =$ number of tons in shipment.
But $12{,}000 =$ number of tons in shipment.
$\therefore 3x + 2(\frac{7}{2})x = 12{,}000.$
Solving, $x = 1200,$
and $3\frac{1}{2}x = 4200.$
Hence, the capacity of a barge is 1200 tons, and the capacity of a schooner is 4200 tons.

4. Let $x =$ number of redfish caught.
Then, $960 - x =$ number of king salmon caught.
$2x + 10(960 - x) =$ number of cents received for fish.
But $2480 =$ number of cents received for fish.
$\therefore 2x + 10(960 - x) = 2480.$
Solving, $x = 890,$
and $960 - x = 70.$
Hence, 890 redfish were caught and 70 king salmon.

5. Let $x =$ the weight of the shell in pounds.
Then, $\frac{1}{4}x + 15 =$ the weight of the powder in pounds.
$\therefore x + \frac{1}{4}x + 15 = 1265.$
$\frac{5}{4}x = 1265 - 15$, or $1250.$
Multiplying by 4, $5x = 5000.$
$x = 1000$, the weight of the shell in pounds.
$\frac{1}{4}x + 15 = 265$, the weight of the powder in pounds.

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6. Let $x =$ number of barrels bought at \$ $4\frac{3}{4}$ per barrel.
Then, $62 - x =$ number of barrels bought at \$ $5\frac{1}{2}$ per barrel.
$4\frac{3}{4}x + 5\frac{1}{2}(62 - x) =$ number of dollars paid for flour.
But $320 =$ number of dollars paid for flour.
$\therefore \frac{19}{4}x + \frac{11}{2}(62 - x) = 320.$
Solving, $x = 28,$
and $62 - x = 34.$
Hence, 28 barrels were bought at \$ $4\frac{3}{4}$ per barrel and 34 barrels were bought at \$ $5\frac{1}{2}$ per barrel.

7. Let $x =$ the number of "clear" days.
Then, $x + 6 =$ the number of "cloudy" days,
and $x - 4 =$ the number of "partly cloudy" days.
$\therefore x + x + 6 + x - 4 = 365.$
$3x = 365 - 6 + 4 = 363.$
Solving, $x = 121$, the number of "clear" days.
$x + 6 = 127$, the number of "cloudy" days.
$x - 4 = 117$, the number of "partly cloudy" days.

8. Let $x =$ number of pounds cork bark weighed.
Then, $\frac{x}{5} =$ number of pounds lost by being boiled,
and $\frac{x - \frac{x}{5}}{4} =$ number of pounds lost by being scraped.
$\frac{x}{5} + \frac{x - \frac{x}{5}}{4} =$ number of pounds in entire loss.
But $16 =$ number of pounds in entire loss.
$\therefore \frac{x}{5} + \frac{x - \frac{x}{5}}{4} = 16.$
Solving, $x = 40$, and $x - 16 = 24.$
Hence, cork weighed 40 pounds before and 24 pounds after the two operations.

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9. Let $x =$ number of 60-pound bushels produced.
Then, $60x =$ number of pounds of wheat produced.
$2000 + 60x + 60x =$ number of pounds of straw and grain.
$\therefore 60x = .3(2000 + 120x).$
Solving, $x = 25.$
Hence, 25 60-pound bushels of wheat were produced.

10. Let $x =$ number of dollars cut diamonds are worth.
Then, $\frac{2x}{5} =$ number of dollars uncut diamonds are worth,
and $\frac{x}{5} =$ number of dollars other stones are worth.

(See next page.)

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$$\therefore x + \frac{2x}{5} + \frac{x}{5} = 40{,}000{,}000.$$

Solving, $x = 25{,}000{,}000,$

and $\frac{2x}{5} = 10{,}000{,}000.$

Hence, the cut diamonds are worth $25,000,000 and the uncut diamonds are worth $10,000,000.

11. Let x = number of cents it costs per mile with steam.
Then, $x - 14$ = number of cents it costs per mile with electricity.

$$\therefore x - 14 = \frac{3x}{5}.$$

Solving, $x = 35,$

and $x - 14 = 21.$

Hence, the cost per mile with electricity is 21 cents.

12. Let x = number of cents per pound for first month.
Then, $\frac{x}{2}$ = number of cents per pound for each succeeding month.

$$\therefore 7000\left(x + \frac{3x}{2}\right) = 4375.$$

Solving, $x = \frac{1}{4},$

and $\frac{x}{2} = \frac{1}{8}.$

Hence, the charge per pound for first month was $\frac{1}{4}$ of a cent, and for each succeeding month was $\frac{1}{8}$ of a cent.

13. Let x = number of dollars reporter earns per week.
Then, $\frac{x}{5}$ = number of dollars reporter saves per week.

Condition of problem,

$x + 5$ = number of dollars artist earns per week.

$\frac{x+5}{7}$ = number of dollars artist saves per week.

$$\therefore \frac{x}{5} - 1 = \frac{x+5}{7}.$$

Solving, $x = 30,$

and $x + 5 = 35.$

Hence, reporter earned $30 per week, and artist earned $35 per week.

14. Let x = number of rods in length of field.
Then, $\frac{x}{2}$ = number of rods in width of field

$$\therefore (x + 20)\left(\frac{x}{2} + 30\right) = \frac{x^2}{2} + 2200.$$

Solving, $x = 40,$

and $\frac{x}{2} = 20.$

Hence, the length of field is 40 rods, and the width is 20 rods.

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15. Let $2x =$ number of 5-cent pieces.
Then, $x =$ number of quarters,
and $\frac{4}{3}x =$ number of dimes.
$\therefore 2x \cdot 5 + x \cdot 25 + \frac{4}{3}x \cdot 10 = 145.$
Solving, $x = 3$;
whence, $2x = 6$ and $\frac{4}{3}x = 4$.
Hence, there are 3 quarters, 6 5-cent pieces, and 4 dimes.

16. Let $x =$ numerator.
Then, $x + 15 =$ denominator.
$$\therefore \frac{x}{x+15} = \frac{4}{7}.$$
Solving, $x = 20$;
whence, $x + 15 = 35$.
Hence, the fraction is $\frac{20}{35}$.

17. Let $2x =$ denominator.
Then, $x + 3 =$ numerator.
$$\therefore \frac{x+3}{2x} = \frac{2}{3}.$$
Solving, $x = 9$;
whence, $2x = 18$ and $x + 3 = 12$.
Hence, the fraction is $\frac{12}{18}$.

18. Let $x =$ numerator.
Then, $x + 8 =$ denominator.
$$\therefore \frac{x-5}{x+3} = \frac{1}{3}.$$
Solving, $x = 9$;
whence, $x + 8 = 17$.
Hence, the fraction is $\frac{9}{17}$.

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19. Let $x =$ number reeled by beginner in one day.
Then, $x + 12 =$ number reeled by experienced woman in one day.
$$\therefore 6x + 36 = (x + 12)(6 - \tfrac{3}{4}).$$
Solving, $x = 36$,
and $x + 12 = 48$.
Hence, experienced woman reeled .48 Kg. in one day, and the beginner reeled .36 Kg in one day.

21. Let $x =$ number of days it will take all.
Then, $\frac{1}{x} =$ part all can do in one day.
$$\therefore \frac{1}{x} = \frac{1}{10} + \frac{1}{12} + \frac{1}{8}.$$
Solving, $x = 3\frac{9}{37}$.
Hence, A, B, and C together can do the work in $3\frac{9}{37}$ days.

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22. Let $x =$ number of days in which A can finish the work after both have worked 3 days.

Then, $x + 3 =$ number of days A works,

and $\frac{x+3}{6} =$ part of the work A does.

But, since B does $\frac{3}{8}$ of the work, A must do $\frac{5}{8}$ of it.

$$\therefore \frac{x+3}{6} = \frac{5}{8}.$$

Solving, $x = \frac{3}{4}$.

Hence, after both have worked 3 days, A can finish the work in $\frac{3}{4}$ of a day.

23. Let $\frac{1}{x} =$ part of the wall C can build in 1 day.

Then, $\frac{2}{x} =$ part of the wall B can build in 1 day.

Since A can build $\frac{1}{15}$ of the wall in 1 day, and all can build $\frac{1}{6}$ of it in 1 day, B and C can build $\frac{1}{6} - \frac{1}{15}$ of it in 1 day.

$$\therefore \frac{1}{x} + \frac{2}{x} = \frac{1}{6} - \frac{1}{15}.$$

Solving, $\frac{1}{x} = \frac{1}{30}$;

whence, $\frac{2}{x} = \frac{1}{15}$.

Hence, B can build $\frac{1}{15}$ of the wall in 1 day, or all of it in 15 days, and C can build $\frac{1}{30}$ of it in 1 day, or all of it in 30 days.

24. Let $\frac{1}{x} =$ part of the ditch all can dig in 1 day.

Then, in accordance with the suggestion in the text,

$$\frac{2}{x} = \frac{1}{10} + \frac{1}{6} + \frac{2}{15} = \frac{2}{5}.$$

$\therefore \frac{1}{x} = \frac{1}{5}$, part of the ditch all can dig in 1 day;

whence, $\frac{1}{5} - \frac{1}{6} = \frac{1}{30}$, part of the ditch A can dig in 1 day.

$\frac{1}{5} - \frac{2}{15} = \frac{1}{15}$, part of the ditch B can dig in 1 day,

and $\frac{1}{5} - \frac{1}{10} = \frac{1}{10}$, part of the ditch C can dig in 1 day.

Hence, alone A can dig the ditch in 30 days, B in 15 days, and C in 10 days.

25. Let $\frac{1}{x} =$ part of car all can load in 1 hour.

Then, from conditions of problem,

$$\frac{2}{x} = \frac{1}{3} + \frac{2}{5} + \frac{7}{15}.$$

$\therefore \frac{1}{x} = \frac{9}{15}$, part all can load in 1 hour;

whence, $\frac{9}{15} - \frac{2}{5} = \frac{1}{5}$, part A can load in 1 hour,

$\frac{9}{15} - \frac{7}{15} = \frac{2}{15}$, part B can load in 1 hour,

and $\frac{9}{15} - \frac{1}{3} = \frac{4}{15}$, part C can load in 1 hour.

Hence, alone A can load the car in 5 hours, B in $7\frac{1}{2}$ hours, and C in $3\frac{3}{4}$ hours.

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27. Let $x =$ digit in units' place.
Then, $3x =$ digit in tens' place.
$30x + x$, or $31x =$ the number,
and $2x =$ difference of digits.

$$\therefore \frac{31x - 33}{2x} = 10.$$

Solving, $x = 3$, digit in units' place;
whence, $3x = 9$, digit in tens' place.
Hence, the number is 93.

28. Let $x =$ digit in tens' place.
Then, $2x =$ digit in units' place.
$10x + 2x$, or $12x =$ the number,
and $3x =$ sum of the digits.

$$\therefore \frac{12x + 27}{3x} = 6\tfrac{1}{4}.$$

Solving, $x = 4$, digit in tens' place;
whence, $2x = 8$, digit in units' place.
Hence, the number is 48.

29. Let $x =$ number of cents she had.

Then, $\frac{x-5}{18} =$ number of cents 1 apple cost,

and $\frac{x-11}{12} =$ number of cents 1 orange cost.

$$\therefore 8\left(\frac{x-5}{18}\right) + 6\left(\frac{x-11}{12}\right) = x - 10.$$

Solving, $x = 41$.
Hence, she had 41 cents.

30. Let $x =$ number of men on a side.
Then, $x^2 =$ number of men in the square.
From conditions of problem, $x^2 + 34 = (x+1)^2 - 35$.
Solving, $x = 34$,
and $x^2 + 34 = 1190$.
Hence, there were 1190 men.

31. Let $x =$ number of men on a side at first.
Then, $x^2 =$ number of men at first,
and $x^2 + 240 =$ number of men after arrival of reënforcements.
$\therefore x^2 + 240 = (x+4)^2$.
Solving, $x = 28$;
whence, $x^2 = 784$.
Hence, there were 784 men in the regiment at first.

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33. Let $x =$ number of dollars property is worth.
Then, $\frac{4}{100} \cdot \frac{2}{3}x + \frac{3}{100} \cdot \frac{1}{4}x + \frac{2}{100} \cdot (x - \frac{2}{3}x - \frac{1}{4}x) = 860$,
or $\frac{2}{75}x + \frac{3}{400}x + \frac{1}{600}x = 860$.
Clearing, $32x + 9x + 2x = 860 \cdot 1200$.
$\therefore x = 20 \cdot 1200 = 24{,}000$.

Hence, his property is worth \$24,000.

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34. Let $x =$ number of dollars in first investment.
Then, $4330 - x =$ number of dollars in second investment.

$$\therefore .12x - .05(4330 - x) = 251.$$

Expanding, $.12x - 216.5 + .05x = 251.$

$$\therefore x = 2750;$$

whence, $4330 - x = 1580.$

Hence, he invested \$2750 at a gain of 12% and \$1580 at a loss of 5%.

35. Let $x =$ number of dollars Mr. Bailey loaned.

Then, $\frac{4x}{5} =$ number of dollars Mr. Day loaned.

$$\therefore \frac{4x}{100} + 48 = \frac{4x}{5}\left(\frac{6}{100}\right).$$

Solving, $x = 6000,$

and $\frac{4x}{5} = 4800.$

Hence, Mr. Bailey loaned \$6000 and Mr. Day loaned \$4800.

36. Let $x =$ number of cents per dollar for insuring first house.

Then, $x + \frac{1}{800} =$ number of cents per dollar for insuring second house.

$$\therefore 6000x + 4000(x + \tfrac{1}{800}) = 80.$$

Solving, $x = \frac{75}{100000} = \frac{3}{4000},$

and $x + \frac{1}{800} = \frac{7}{8000}.$

Hence, the rate for first house was $\frac{3}{4}$% and for second house $\frac{7}{8}$%.

37. Let $x =$ number of dollars first invested.
Then, $2x =$ number of dollars borrowed.

$$\frac{7x}{100} + \frac{7}{100}(2x) = \text{number of dollars in income.}$$

$$\tfrac{5}{100}(2x) = \text{number of dollars interest paid.}$$

$$\therefore \frac{7x}{100} + \frac{14x}{100} - \frac{10x}{100} = 385.$$

Solving, $x = 3500,$

and $2x = 7000.$

Hence, the sum first invested was \$3500 and sum borrowed was \$7000.

38. Let $x =$ number of 50-dollar shares.

Then, $\frac{2x}{3} =$ number of 100-dollar shares.

$$\therefore \frac{2}{100} \cdot 50x + \frac{1\frac{1}{2}}{100} \cdot 100 \cdot \frac{2}{3}x = 120.$$

Solving, $x = 60,$

and $\frac{2x}{3} = 40.$

Hence, \$3000 was invested in 50-dollar shares, and \$4000 was invested in 100-dollar shares.

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39. Let $6x =$ number of dollars in savings bank.

Then, $x =$ number of dollars with trust company.

$$\therefore \tfrac{3}{400}(x + 400) + \tfrac{4}{400}(6x - 400) = \tfrac{1}{2} + \tfrac{4}{400}(6x).$$

Solving, $x = 200$,

and $6x = 1200$.

Hence, \$1200 is deposited in savings bank and \$200 with trust company.

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41. Let $x =$ number of minute spaces the minute hand travels after 1 o'clock before they come together.

Then, $\frac{x}{12} =$ number of minute spaces the hour hand travels in the same time.

Since they are five minute spaces apart at 1 o'clock,

$$x - \frac{x}{12} = 5.$$

Solving, $x = 5\tfrac{5}{11}$, the number of minutes after 1 o'clock.

Hence, at $1:05\tfrac{5}{11}$ o'clock the hands will be together.

42. Let $x =$ number of minute spaces the minute hand travels after 6 o'clock before they come together.

Then, $\frac{x}{12} =$ number of minute spaces the hour hand travels in the same time.

Since the hour hand is 30 minute spaces in advance at 6 o'clock,

$$x - \frac{x}{12} = 30.$$

Solving, $x = 32\tfrac{8}{11}$, the number of minutes after 6 o'clock.

Hence, at $6:32\tfrac{8}{11}$ o'clock the hands will be together.

43. Let $x =$ number of minute spaces the minute hand travels after 10 o'clock before the hands are opposite.

Then, $\frac{x}{12} =$ number of minute spaces the hour hand travels in the same time.

Since the hour hand is 50 minutes in advance at 10 o'clock, and 30 minutes in advance when the hands are opposite,

$$x - \frac{x}{12} = 50 - 30.$$

Solving, $x = 21\tfrac{9}{11}$, the number of minutes after 10 o'clock.

Hence, at $10:21\tfrac{9}{11}$ o'clock the hands will extend in opposite directions.

44. Let $x =$ number of minute spaces the minute hand travels after 4 o'clock before it is 15 minute spaces *behind* the hour hand.

Then, $\frac{x}{12} =$ number of minute spaces the hour hand travels in the same time.

Since the minute hand gains 20 — 15, or 5, minute spaces,

$$x - \frac{x}{12} = 5.$$

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Solving, $x = 5\frac{5}{11}$, the number of minutes after 4 o'clock.

Again, let $x =$ number of minute spaces the minute hand travels before it is 15 minute spaces *ahead* of the hour hand.

Then, $\frac{x}{12} =$ number of minute spaces the hour hand travels in the same time.

Since the minute hand gains 20 + 15, or 35, minute spaces,

$$x - \frac{x}{12} = 35.$$

Solving, $x = 38\frac{2}{11}$, the number of minutes after 4 o'clock.

Hence, the required times are $4:05\frac{5}{11}$ o'clock and $4:38\frac{2}{11}$ o'clock.

45. Let $x =$ number of miles downstream.

Then, $\frac{x}{6} =$ number of hours downstream,

and $\frac{x}{3} =$ number of hours upstream.

$$\therefore \frac{x}{6} + \frac{x}{3} = 9.$$

Solving, $x = 18.$

Hence, he can row 18 miles downstream and return in 9 hours.

46. Let $x =$ number of miles up the river.

Then, $\frac{x}{17\frac{1}{2}} =$ number of hours up the river,

and $\frac{x}{21} =$ number of hours down the river.

$$\therefore \frac{x}{17\frac{1}{2}} + \frac{x}{21} = 2\frac{56}{60}.$$

Solving, $x = 28.$

Hence, the boat went 28 miles up the river.

47. Let $x =$ number of miles an hour freight train runs.

Then, $\frac{8x}{3} =$ number of miles freight train runs,

and $\frac{96}{60}$ of 40, or $64 =$ number of miles express train runs.

Since the trains run the same distance,

$$\frac{8x}{3} = 64.$$

$$\therefore x = 24.$$

Hence, the freight train runs 24 miles per hour.

48. Let $5x =$ number of miles an hour downstream.

Then, $3x =$ number of miles an hour upstream.

By the 2d condition,

$$5x - 4 = 2(3x - 4).$$

Solving, $x = 4$;

whence, $5x = 20$, number of miles an hour downstream,

and $3x = 12$, number of miles an hour upstream.

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49. Let $x =$ number of miles from Albany.

Then, $\dfrac{x}{\frac{3}{2}} =$ number of hours it takes boat from Albany,

and $\dfrac{166 - x}{\frac{5}{4}} =$ number of hours it takes boat from Syracuse.

$$\therefore \frac{2x}{3} = \frac{4(166 - x)}{5}.$$

Solving, $x = 90\frac{6}{11}$.

Hence, the canal boats meet $90\frac{6}{11}$ miles from Albany.

50. Let $x =$ number of knots from Southampton.

Then, $\dfrac{x}{20} =$ number of hours from Southampton,

and $\dfrac{3046 - x}{18} =$ number of hours from New York.

$$\therefore \frac{x}{20} - \frac{31}{2} = \frac{3046 - x}{18}.$$

Solving, $x = 1750$.

Hence, the boats meet 1750 knots from Southampton.

51. Let $x =$ number of miles by rail.

Then, $x + 60 =$ number of miles by boat.

$\dfrac{x}{44} =$ number of hours by rail,

and $\dfrac{x + 60}{13} =$ number of hours by boat.

$$\therefore \frac{x}{44} + \frac{x + 60}{13} = 73\tfrac{1}{2} - 36.$$

Solving, $x = 330$, and $x + 60 = 390$.

Then, $\dfrac{x}{44} = 7\frac{1}{2}$,

and $\dfrac{x + 60}{13} = 30$.

Hence, by boat it is 390 miles, and takes 30 hours. By train it is 330 miles, and takes $7\frac{1}{2}$ hours.

52. Let $x =$ number of miles freight train went per hour.

Then, $3x =$ number of miles passenger train went per hour.

$\dfrac{5280x}{3600} =$ number of feet freight train went per second,

and $\dfrac{5280(3x)}{3600} =$ number of feet passenger train went per second.

$$\therefore \frac{5280x}{3600} + \frac{5280(3x)}{3600} = \frac{660}{7\frac{1}{2}}.$$

Solving, $x = 15$, and $3x = 45$.

Hence, the rate of the freight train was 15 miles per hour and of the passenger train 45 miles per hour.

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53. Let $x =$ number of pounds of zinc used.
Then, $2x =$ number of pounds of tin used,
and $17x =$ number of pounds of copper used.
$\therefore x + 2x + 17x = 5000.$
Solving, $x = 250,$
$2x = 500,$
and $17x = 4250.$
Hence, there were used 250 pounds of zinc, 500 pounds of tin, and 4250 pounds of copper.

54. Let $x =$ number of pounds of gunpowder.
Then, $\frac{3}{4}x + 10 =$ number of pounds of niter,
$\frac{1}{12}x + 3 =$ number of pounds of sulphur,
and $\frac{3}{40}x + 1 - 3 =$ number of pounds of charcoal.
$\therefore x = \frac{3}{4}x + 10 + \frac{1}{12}x + 3 + \frac{3}{40}x + 1 - 3.$
Solving, $x = 120$, number of pounds of gunpowder.

55. Let $x =$ number of ounces of pure silver to be added.
Then, $\frac{4}{5}(250) + x =$ number of ounces of pure silver after the addition.
$\frac{9}{10}(250 + x) =$ number of ounces of pure silver after the addition
$\therefore \frac{4}{5}(250) + x = \frac{9}{10}(250 + x).$
Solving, $x = 250$, number of ounces of pure silver to be added.

56. Let $x =$ number of ounces of copper to be added.
Then, $90 + x =$ number of ounces new alloy weighs.
Since the weight of the alloy must contain 10 ounces the same number of times that the total weight of silver in the new alloy, or 6 ounces, contains the weight of silver in each 10 ounces of the new alloy, or $\frac{2}{5}$ of an ounce, $\frac{90 + x}{10} = \frac{6}{\frac{2}{5}} = 15.$
Solving, $x = 60.$
Hence, 60 ounces of copper must be added.

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57. Let $x =$ number of pounds of fresh water to be added.
Then, $80 + x =$ number of pounds of water after addition of fresh water.
Since the number of times the total weight of water contains 49 pounds is the same as the number of times the total weight of salt contains $1\frac{3}{4}$ pounds, $\frac{80 + x}{49} = \frac{4}{1\frac{3}{4}}.$
Solving, $x = 32.$
Hence, 32 pounds of fresh water must be added.

58. Let $x =$ number of pounds of copper to be added.
Then, $75 + x =$ number of pounds new alloy weighs.
$\frac{12\frac{1}{2}}{100}(75 + x) =$ number of pounds of tin in new alloy.
$$\therefore \frac{12\frac{1}{2}}{100}(75 + x) = 12.$$
Solving, $x = 21.$
Hence, 21 pounds of copper must be added.

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59. Let x = number of pounds of fresh water to be evaporated.

Then, $60 - x$ = number of pounds of solution remaining.

Since the number of times the total weight of new solution contains 25 pounds is same as number of times the total weight of salt contains $2\frac{1}{2}$ pounds, $\frac{60-x}{25}=\frac{3}{2\frac{1}{2}}$.

Solving, $x = 30$.

Hence, 30 pounds of water must be evaporated.

60. Let x = number of pounds of pure water to be added.

Then, $24 + x$ = number of pounds new solution weighs,

and $\frac{4}{100}(24 + x)$ = number of pounds of salt in new solution.

$\therefore \frac{4}{100}(24 + x) = \frac{12}{100} \cdot 24.$

Solving, $x = 48$.

Hence, 48 pounds of pure water must be added.

61. Let x = number of gallons of water required.

Then, $6 + x$ = number of gallons in new mixture.

$\therefore \frac{75}{100}(6 + x) = \frac{95}{100} \cdot 6.$

Solving, $x = 1\frac{3}{5} = 1.6.$

Hence, 1.6 gallons of water must be added.

62. Let x = number of gallons of water to be added.

Then, $4 + x$ = number of gallons after water is added.

$\therefore \frac{50}{100}(4 + x) = \frac{90}{100}(4).$

Solving, $x = 3.2$.

Hence, 3.2 gallons of water must be added.

63. Let x = number of grams alcohol weighs per cubic centimeter.

Then, $94 - 51.6$ = number of grams the alcohol weighs.

$$\therefore \frac{94 - 51.6}{x} = 50.$$

Solving, $x = .848.$

Hence, the alcohol weighs .848 gram per cubic centimeter.

64. In accordance with the suggestion in the text,

let x = the number of pounds of brass.

Then, $57 - x$ = the number of pounds of iron,

$x \div 8\frac{2}{5}$ = number of pounds of water brass displaces,

and $(57 - x) \div 7\frac{1}{2}$ = number of pounds of water iron displaces.

$\therefore x \div 8\frac{2}{5} + (57 - x) \div 7\frac{1}{2} = 7.$

Solving, $x = 42$;

whence, $57 - x = 15.$

Hence, the mass contains 42 pounds of brass and 15 pounds of iron.

65. Let x = number of pounds of lead.

Then, $159 - x$ = number of pounds of iron.

Since, when weighed in water, the lead loses $\frac{2}{23}x$ pounds, the iron $\frac{2}{15}(159 - x)$ pounds, and both together 16 pounds,

$$\frac{2x}{23} + \frac{2(159 - x)}{15} = 16.$$

Solving, $x = 112\frac{1}{8}$,

and $159 - x = 46\frac{7}{8}$.

Hence, there are $112\frac{1}{8}$ pounds of lead and $46\frac{7}{8}$ pounds of iron.

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66. Let $x =$ number of pounds the tin weighs.
Then, $60 - x =$ number of pounds the lead weighs.

$$\frac{5x}{37} = \text{number of pounds of tin lost,}$$

and $$\frac{2(60-x)}{23} = \text{number of pounds of lead lost.}$$

$$\therefore \frac{5x}{37} + \frac{2(60-x)}{23} = 7.$$

Solving, $x = 37.$
Hence, there are 37 pounds of tin in the mass.

67. Let $x =$ number of ounces of gold.
Then, $320 - x =$ number of ounces of silver.
Since in water 1 ounce of gold loses $\frac{5}{97}$ ounces in weight, and 1 ounce of silver $\frac{2}{21}$ ounces,

$$\tfrac{5}{97}x + \tfrac{2}{21}(320 - x) = 320 - 298.$$

Solving, $x = 194;$
whence, $320 - x = 126.$
Hence, there are 194 ounces of gold and 126 ounces of silver.

68. Let $x =$ number per cent by volume of zinc in the alloy.
Then, $100 - x =$ number per cent by volume of copper in the alloy.

$$\therefore \frac{x}{100}(437.5) + \frac{100-x}{100}(550) = 532.$$

Solving, $x = 16,$ and $100 - x = 84.$
Hence, there is 16 % of zinc, and 84 % of copper in the alloy.

---

2. From $A = \frac{1}{2}bh,$ $\frac{1}{2}bh = A.$

$$\therefore b = \frac{2A}{h}.$$

Substituting given values, $b = \dfrac{2 \times 600}{40} = 30.$

Hence, the base of the triangle is 30 feet.

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3. From $A = \dfrac{b+b'}{2}\cdot h,$ $\dfrac{b+b'}{2}\cdot h = A.$

$$\therefore h = \frac{2A}{b+b'}.$$

Substituting the given values, $h = \dfrac{192}{14+10} = 8$

Hence, the altitude of the trapezoid is 8 inches.

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4. From $V = \frac{1}{3} Bh$, $\frac{1}{3} Bh = V.$

$$\therefore B = \frac{3V}{h}.$$

Substituting the given values, $B = \frac{756}{9} = 84.$
Hence, the area of the base of the pyramid is 84 square feet.

5. $c = 40 + 3(n - 10).$

Substituting 16 for n, $c = 40 + 3(16 - 10) = 58.$
Hence, the cost of a 16-word message is 58 cents.
From $c = 40 + 3(n - 10)$, $3(n - 10) = c - 40.$

Solving for n, $n = \dfrac{c - 10}{3}.$

Substituting given value for c, $n = \dfrac{100 - 10}{3} = 30.$

Hence, 30 words may be sent for \$1.

6. From $i = p \cdot \dfrac{r}{100} \cdot t$, $p \cdot r \cdot t = 100\,i.$ (1)

Solving for t, $t = \dfrac{100\,i}{p \cdot r}.$

Substituting given values, $t = \dfrac{100 \times 60}{300 \times 5} = 4.$

Hence, \$300 must be on interest 4 years to yield \$60 interest at 5%.

Solving for r in (1), $r = \dfrac{100\,i}{p \cdot t}.$

Substituting given values, $r = \dfrac{100 \times 900}{4500 \times 5} = 4.$

Hence, \$4500 must be on interest at 4% to yield \$900 interest in 5 years.

Solving for p in (1), $p = \dfrac{100\,i}{r \cdot t}.$

Substituting given values, $p = \dfrac{100 \times 210}{3\frac{1}{2} \times 1} = 6000.$

Hence, \$6000 at $3\frac{1}{2}$% will yield \$210 annually.

7. From $s = vt$, $v = \dfrac{s}{t}.$

Substituting given values, $v = \frac{8640}{8} = 1080.$
Hence, the velocity of sound is 1080 feet.

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8. From $C = \frac{5}{9}(F - 32)$, $9C = 5F - 5 \cdot 32.$
Solving for F, $F = \frac{9}{5}C + 32.$
Substituting 25 for C, $F = \frac{9}{5} \cdot 25 + 32 = 77.$
Hence, mean annual temperature in Havana is 77° Fahrenheit.

9. From $L = L_0 + L_0 \cdot at$, $L_0at = L - L_0.$

Solving for a, $a = \dfrac{L - L_0}{L_0t}.$

Substituting given values, $a = \dfrac{30.001632 - 30}{30 \times 50} = .000001088.$

Hence, the value of a is .000001088 feet.

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20. From $pd = WD$, $W = \frac{pd}{D}$.

Substituting given values, $W = \frac{150 \times 7}{3} = 350$.

Hence, a power of 150 pounds will support a weight of 350 pounds.

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21. Substituting given values in $pd = WD$, $6p = 144 \times 2$.
Solving, $p = 48$.

22. Substituting given values in $pd = WD$, $300d = 100(8 - d)$.
Solving, $d = 2$.

23. From $pd = WD$, $W = \frac{pd}{D}$.

Substituting given values, $W = \frac{600 \times 3}{5} = 360$.

24. Substituting in $pd = WD$, $8p = 700 \times 5$.
Solving, $p = 437\frac{1}{2}$.

25. From $pd = WD$, $d = \frac{WD}{p}$.

Substituting given values, $d = \frac{102(9 - d)}{114}$.

Solving, $d = 4\frac{1}{4}$.
Hence, Philip is $4\frac{1}{4}$ feet from the fulcrum.

26. From $pd = WD$, $W = \frac{pd}{D}$.

Substituting given values, $W = \frac{16(100) \cdot 5}{50} = 160$.

Hence, a weight of 160 pounds must be hung on end of lever.

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35. Substituting given values in $P = wAh$, $P = 62\frac{1}{2} \cdot 1 \cdot 30$.
Solving, $P = 1875$.
Hence, the pressure is 1875 pounds.
Substituting second set of values in $P = wAh$, $P = 64 \cdot 1 \cdot 3000$.
Solving. $P = 192,000$.
Hence, the pressure is 192,000 pounds.

36. From $P = wAh$, $h = \frac{P}{wA}$.

Substituting given values, $h = \frac{5000}{62\frac{1}{2} \times 8} = 10$.

37. From $P = wAh$, $h = \frac{P}{wA}$.

Substituting given values, $h = \frac{2000}{62\frac{1}{2} \cdot 4} = 8$.

Hence, the depth is 8 feet.

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38. From $P = wAh$, $h = \dfrac{P}{wA}.$

Substituting given values, $h = \dfrac{36{,}000}{62\frac{1}{2} \times 6^2} = 16.$

Hence, the depth is 16 feet.

39. From $P = wAh$, $h = \dfrac{P}{wA}.$

Substituting given values, $h = \dfrac{20{,}160}{64} = 315.$

Hence, the diver may go 315 feet deep without danger.

40. From $P = wAh$, $w = \dfrac{P}{Ah}.$

Substituting the given values, $w = \dfrac{990}{18} = 55.$

Hence, the petroleum weighs 55 pounds per cubic foot.

41. From $P = wAh$, $w = \dfrac{P}{Ah}.$

Substituting the given values, $w = \dfrac{8000}{5 \times 25} = 64.$

Hence, the chest was submerged in salt water.

42. From $P = wAh$, $h = \dfrac{P}{wA}.$

Substituting the given values, $h = \dfrac{(60-40)144}{62\frac{1}{2}} = 46.08.$

Hence, the faucet in the top story is 46.08 feet higher.

SIMULTANEOUS SIMPLE EQUATIONS

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2. $\begin{cases} 7x - 5y = 52, & (1) \\ 2x + 5y = 47. & (2) \end{cases}$

(1) + (2), $9x = 99.$

$x = 11.$ (3)

Substituting (3) in (2), $y = 5.$

3. $\begin{cases} 3x + 2y = 23, & (1) \\ x + y = 8. & (2) \end{cases}$

(2) × 2, $2x + 2y = 16.$ (3)

(1) − (3), $x = 7.$ (4)

Substituting (4) in (2), $y = 1.$

4. $\begin{cases} 3x - 4y = 7, & (1) \\ x + 10y = 25. & (2) \end{cases}$

(2) × 3, $3x + 30y = 75.$ (3)

(3) − (1), $34y = 68.$

$y = 2.$ (4)

Substituting (4) in (1), $x = 5.$

5. $\begin{cases} 2x - 10y = 15, & (1) \\ 2x - 4y = 18. & (2) \end{cases}$

(2) − (1), $6y = 3.$

$y = \frac{1}{2}.$ (3)

Substituting (3) in (1), $x = 10.$

6. $\begin{cases} 3u - v = 4, & (1) \\ u + 3v = -2. & (2) \end{cases}$

(1) × 3, $9u - 3v = 12.$ (3)

(2) + (3), $10u = 10.$

$u = 1.$ (4)

Substituting (4) in (2), $v = -1.$

7. $\begin{cases} 4x - y = 19, & (1) \\ x + 3y = 21. & (2) \end{cases}$

(1) × 3, $12x - 3y = 57.$ (3)

(2) + (3), $13x = 78.$

$x = 6.$ (4)

Substituting (4) in (2), $y = 5.$

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8. $\begin{cases} l + 2r = 5, & (1) \\ 2l + r = 1. & (2) \end{cases}$

$[(1) + (2)] \div 3, \quad l + r = 2. \quad (3)$

$(2) - (3), \quad l = -1.$

$(1) - (3), \quad r = 3.$

9. $\begin{cases} 2a + 3b = 17, & (1) \\ 3a + 2b = 18. & (2) \end{cases}$

$[(1) + (2)] \div 5, \quad a + b = 7. \quad (3)$

$(3) \times 2, \quad 2a + 2b = 14. \quad (4)$

$(2) - (4), \quad a = 4.$

$(1) - (4), \quad b = 3.$

10. $\begin{cases} 7s - 9v = 6, & (1) \\ s + 2v = 14. & (2) \end{cases}$

$(2) \times 7, \quad 7s + 14v = 98. \quad (3)$

$[(3) - (1)] \div 23 \quad v = 4. \quad (4)$

Substituting (4) in (2), $s = 6$.

11. $\begin{cases} 13t - u = 20, & (1) \\ 4t + 2u = 20. & (2) \end{cases}$

$(2) \div 2, \quad 2t + u = 10. \quad (3)$

$(1) + (3), \quad 15t = 30.$

$t = 2. \quad (4)$

Substituting (4) in (3), $u = 6$.

12. $\begin{cases} 3d + 4y = 25, & (1) \\ 4d + 3y = 31. & (2) \end{cases}$

$(1) + (2), \quad 7d + 7y = 56. \quad (3)$

$(3) \times \frac{3}{7}, \quad 3d + 3y = 24. \quad (4)$

$(2) - (4), \quad d = 7.$

$(1) - (4), \quad y = 1.$

13. $\begin{cases} 5p + 6q = 32, & (1) \\ 7p - 3q = 22. & (2) \end{cases}$

$(2) \times 2, \quad 14p - 6q = 44. \quad (3)$

$(1) + (3), \quad 19p = 76.$

$p = 4. \quad (4)$

Substituting (4) in (1), $q = 2$.

14. $\begin{cases} 3a + 6z = 39, & (1) \\ 9a - 4z = 51. & (2) \end{cases}$

$(1) \times \frac{2}{3}, \quad 2a + 4z = 26. \quad (3)$

$(2) + (3), \quad 11a = 77.$

$a = 7. \quad (4)$

Substituting (4) in (1), $z = 3$.

15. $\begin{cases} 8x - 3y = 44, & (1) \\ 7x - 5y = 29. & (2) \end{cases}$

$(1) \times 5, \quad 40x - 15y = 220. \quad (3)$

$(2) \times 3, \quad 21x - 15y = 87. \quad (4)$

$(3) - (4), \quad 19x = 133.$

$x = 7. \quad (5)$

Substituting (5) in (1), $y = 4$.

16. $\begin{cases} 6x - 5y = 33, & (1) \\ 4x + 4y = 44. & (2) \end{cases}$

$(2) \div 4, \quad x + y = 11. \quad (3)$

$(3) \times 5, \quad 5x + 5y = 55. \quad (4)$

$(1) + (4), \quad 11x = 88.$

$x = 8. \quad (5)$

Substituting (5) in (3), $y = 3$.

17. $\begin{cases} 3m + 11n = 67, & (1) \\ 5m - 3n = 5. & (2) \end{cases}$

$(1) \times 3, \quad 9m + 33n = 201. \quad (3)$

$(2) \times 11, \quad 55m - 33n = 55. \quad (4)$

$(3) + (4), \quad 64m = 256.$

$m = 4. \quad (5)$

Substituting (5) in (1), $n = 5$.

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2. $\begin{cases} 3x - 2y = 10, & (1) \\ x + y = 70. & (2) \end{cases}$

From (1), $\quad x = \dfrac{2y + 10}{3}. \quad (3)$

From (2), $\quad x = 70 - y. \quad (4)$

Equating the values of x,

$$\frac{2y + 10}{3} = 70 - y.$$

$y = 40. \quad (5)$

Substituting (5) in (4),

$x = 30.$

3. $\begin{cases} 5x + y = 22, & (1) \\ x + 5y = 14. & (2) \end{cases}$

From (1), $\quad y = 22 - 5x. \quad (3)$

From (2) $\quad y = \dfrac{14 - x}{5}. \quad (4)$

Equating the values of y,

$$22 - 5x = \frac{14 - x}{5}.$$

$x = 4. \quad (5)$

Substituting (5) in (3),

$y = 2.$

4. $\begin{cases} 2x + 3y = 24, & (1) \\ 5x - 3y = 18. & (2) \end{cases}$

From (1), $\quad 3y = 24 - 2x. \quad (3)$

From (2), $\quad 3y = 5x - 18. \quad (4)$

Equating the values of $3y$,

$$24 - 2x = 5x - 18.$$

$x = 6. \quad (5)$

Substituting (5) in (3),

$y = 4.$

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5. $\begin{cases} 3x + 5y = 14, & (1) \\ 2x - 3y = 3. & (2) \end{cases}$

From (1), $x = \dfrac{14 - 5y}{3}$. (3)

From (2), $x = \dfrac{3y + 3}{2}$. (4)

Equating the values of x,

$$\frac{14 - 5y}{3} = \frac{3y + 3}{2}.$$

$y = 1$. (5)

Substituting (5) in (4),

$x = 3$.

6. $\begin{cases} 3v + 2y = 36, & (1) \\ 5v - 9y = 23. & (2) \end{cases}$

From (1) $v = \dfrac{36 - 2y}{3}$. (3)

From (2), $v = \dfrac{9y + 23}{5}$. (4)

Equating the values of v,

$$\frac{36 - 2y}{3} = \frac{9y + 23}{5}.$$

$y = 3$, (5)

Substituting (5) in (3),

$v = 10$.

7. $\begin{cases} 2s + 7t = 8, & (1) \\ 3s + 9t = 9. & (2) \end{cases}$

From (1), $s = \dfrac{8 - 7t}{2}$. (3)

From (2), $s = 3 - 3t$. (4)

Equating the values of s,

$$\frac{8 - 7t}{2} = 3 - 3t.$$

$t = 2$. (5)

Substituting (5) in (4),

$s = -3$.

8. $\begin{cases} 4u + 6v = 19, & (1) \\ 3u - 2v = \frac{9}{2}. & (2) \end{cases}$

From (1), $v = \dfrac{19 - 4u}{6}$. (3)

From (2), $v = \dfrac{6u - 9}{4}$. (4)

Equating the values of v,

$$\frac{19 - 4u}{6} = \frac{6u - 9}{4}.$$

$u = \frac{5}{2}$. (5)

Substituting (5) in (3),

$v = \frac{3}{2}$.

9. $\begin{cases} 4v + 3w = 34, & (1) \\ 11v + 5w = 87. & (2) \end{cases}$

From (1), $w = \dfrac{34 - 4v}{3}$. (3)

From (2), $w = \dfrac{87 - 11v}{5}$. (4)

Equating the values of w,

$$\frac{34 - 4v}{3} = \frac{87 - 11v}{5}.$$

$v = 7$. (5)

Substituting (5) in (3),

$w = 2$.

10. $\begin{cases} 4x - 13y = 5, & (1) \\ 3x + 11y = -17. & (2) \end{cases}$

From (1), $x = \dfrac{13y + 5}{4}$. (3)

From (2), $x = \dfrac{-11y - 17}{3}$. (4)

Equating the values of x,

$$\frac{13y + 5}{4} = \frac{-11y - 17}{3}.$$

$y = -1$. (5)

Substituting (5) in (3),

$x = -2$.

11. $\begin{cases} 18x - 3y = 4y, & (1) \\ 1 - 4x + 3y = 27. & (2) \end{cases}$

From (1), $y = \dfrac{18x}{7}$. (3)

From (2), $y = \dfrac{4x + 26}{3}$. (4)

Equating the values of y,

$$\frac{18x}{7} = \frac{4x + 26}{3}.$$

$x = 7$. (5)

Substituting (5) in (3),

$y = 18$.

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1. $\begin{cases} x - y = 4, & (1) \\ 4y - x = 14. & (2) \end{cases}$

From (1), $x = y + 4.$ (3)

Substituting (3) in (2),

$4y - (y + 4) = 14.$

$y = 6.$ (4)

Substituting (4) in (3),

$x = 10.$

2. $\begin{cases} x + y = 10, & (1) \\ 6x - 7y = 34. & (2) \end{cases}$

From (1), $x = 10 - y.$ (3)

Substituting (3) in (2),

$6(10 - y) - 7y = 34.$

$y = 2.$ (4)

Substituting (4) in (3),

$x = 8.$

3. $\begin{cases} 3x - 4y = 26, & (1) \\ x - 8y = 22. & (2) \end{cases}$

From (2), $x = 8y + 22.$ (3)

Substituting (3) in (1),

$3(8y + 22) - 4y = 26.$

$y = -2.$ (4)

Substituting (4) in (3).

$x = 6.$

4. $\begin{cases} 6y - 10x = 14, & (1) \\ y - x = 3. & (2) \end{cases}$

From (2), $y = x + 3.$ (3)

Substituting (3) in (1),

$6(x + 3) - 10x = 14.$

$x = 1.$ (4)

Substituting (4) in (3),

$y = 4.$

5. $\begin{cases} y + 1 = 3x, & (1) \\ 5x + 9 = 3y. & (2) \end{cases}$

From (1), $y = 3x - 1.$ (3)

Substituting (3) in (2).

$5x + 9 = 9x - 3.$

$x = 3.$ (4)

Substituting (4) in (3),

$y = 8.$

6. $\begin{cases} 17 = 3x + z, & (1) \\ 7 = 3z - 2x. & (2) \end{cases}$

From (1), $z = 17 - 3x.$ (3)

Substituting (3) in (2).

$7 = 3(17 - 3x) - 2x.$

$x = 4.$ (4)

Substituting (4) in (3).

$z = 5.$

7. $\begin{cases} 4y = 10 - x; & (1) \\ y - x = 5. & (2) \end{cases}$

From (2), $y = x + 5.$ (3)

Substituting (3) in (1),

$4x + 20 = 10 - x.$

$x = -2.$ (4)

Substituting (4) in (3).

$y = 3.$

8. $\begin{cases} 7z - 3x = 18, & (1) \\ 2z - 5x = 1. & (2) \end{cases}$

From (2), $z = \dfrac{5x + 1}{2}.$ (3)

Substituting (3) in (1),

$7\left(\dfrac{5x + 1}{2}\right) - 3x = 18.$

$35x + 7 - 6x = 36.$

$x = 1.$ (4)

Substituting (4) in (3),

$z = 3.$

9. $\begin{cases} 3 - 15y = -x, & (1) \\ 3 + 15y = 4x. & (2) \end{cases}$

From (1), $15y = x + 3.$ (3)

Substituting (3) in (2),

$3 + x + 3 = 4x.$

$x = 2.$ (4)

Substituting (4) in (3),

$y = \frac{1}{3}.$

10. $\begin{cases} 1 - x = 3y, & (1) \\ 3(1 - x) = 40 - y. & (2) \end{cases}$

Substituting $3y$ for $1 - x$ in (2),

$9y = 40 - y.$

$y = 4.$ (3)

Substituting (3) in (1),

$1 - x = 12.$

$x = -11.$

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1. $\begin{cases} x + z = 13, & (1) \\ x - z = 5. & (2) \end{cases}$

$(1)+(2),\quad 2x = 18.$

$x = 9. \quad (3)$

Substituting (3) in (1),

$z = 4.$

2. $\begin{cases} 3x + y = 10, & (1) \\ x + 3y = 6. & (2) \end{cases}$

$(2)\times 3,\quad 3x + 9y = 18. \quad (3)$

$(3)-(1),\quad 8y = 8.$

$y = 1. \quad (4)$

Substituting (4) in (2),

$x = 3.$

3. $\begin{cases} 4x + 5y = -2, & (1) \\ 5x + 4y = 2. & (2) \end{cases}$

$(1)\times 4,\quad 16x + 20y = -8. \quad (3)$

$(2)\times 5,\quad 25x + 20y = 10. \quad (4)$

$(4)-(3),\quad 9x = 18.$

$x = 2. \quad (5)$

Substituting (5) in (1),

$y = -2.$

4. $\begin{cases} 5x - y = 28, & (1) \\ 3x + 5y = 28. & (2) \end{cases}$

$(1)\times 5,\quad 25x - 5y = 140. \quad (3)$

$(2)+(3),\quad 28x = 168.$

$x = 6. \quad (4)$

Substituting (4) in (1),

$y = 2.$

5. $\begin{cases} x + 3 = y - 3, & (1) \\ 2(x+3) = 6 - y. & (2) \end{cases}$

Substituting $(y-3)$ for $(x+3)$ in (2),

$2(y-3) = 6 - y.$

$y = 4. \quad (3)$

Substituting (3) in (1),

$x = -2.$

6. $\begin{cases} 5x - y = 12, & (1) \\ x + 3y = 12. & (2) \end{cases}$

$(1)\times 3,\quad 15x - 3y = 36. \quad (3)$

$(2)+(3),\quad 16x = 48.$

$x = 3. \quad (4)$

Substituting (4) in (2),

$y = 3.$

7. $\begin{cases} 4(2-x) = 3y, & (1) \\ 2(2-x) = 2(y-2). & (2) \end{cases}$

From (1), $\quad 2 - x = \frac{3}{4}y. \quad (3)$

From (2), $\quad 2 - x = y - 2. \quad (4)$

Equating values of $(2-x)$,

$\frac{3}{4}y = y - 2.$

$y = 8. \quad (5)$

Substituting (5) in (1),

$x = -4.$

8. $\begin{cases} (x+1)+(y-2) = 7, & (1) \\ (x+1)-(y-2) = 5. & (2) \end{cases}$

$(1)+(2),\quad 2(x+1) = 12.$

$x = 5. \quad (3)$

Substituting (3) in (1),

$y = 3.$

9. $\begin{cases} x + \dfrac{y}{3} = 11, & (1) \\ \dfrac{x}{3} + 3y = 21. & (2) \end{cases}$

From (2), $\quad \frac{x}{3} = 21 - 3y. \quad (3)$

Substituting $(3)\times 3$ in (1),

$63 - 9y + \frac{y}{3} = 11.$

$-26y = -156.$

$y = 6. \quad (4)$

Substituting (4) in (1),

$x = 9.$

10. $\begin{cases} \dfrac{3x}{4} + \dfrac{4y}{5} = 21, & (1) \\ \dfrac{2x}{3} + \dfrac{3y}{5} = 17. & (2) \end{cases}$

$(1)\times 3,\quad \frac{9x}{4} + \frac{12y}{5} = 63. \quad (3)$

$(2)\times 4,\quad \frac{8x}{3} + \frac{12y}{5} = 68. \quad (4)$

$(4)-(3),\quad \frac{5x}{12} = 5.$

$x = 12. \quad (5)$

Substituting (5) in (2),

$\frac{3y}{5} = 9.$

$y = 15.$

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11. $\begin{cases} \frac{x}{3} = 11 - \frac{y}{2}, & (1) \\ \frac{x}{3} + \frac{2y}{7} = 8. & (2) \end{cases}$

Substituting (1) in (2),

$$11 - \frac{y}{2} + \frac{2y}{7} = 8.$$

$$\frac{-3y}{14} = -3.$$

$$y = 14. \quad (3)$$

Substituting (3) in (1),

$$x = 12.$$

12. $\begin{cases} \frac{x}{2} - \frac{2y}{3} = -2, & (1) \\ \frac{5x}{2} + \frac{y}{3} = 12. & (2) \end{cases}$

From (1), $\frac{x}{2} = \frac{2y}{3} - 2.$ (3)

Substituting (3) in (2),

$$\frac{10y}{3} - 10 + \frac{y}{3} = 12.$$

$$y = 6. \quad (4)$$

Substituting (4) in (1),

$$x = 4.$$

13. $\begin{cases} \frac{x+y}{2} - \frac{x-y}{3} = 8, & (1) \\ \frac{x+y}{3} + \frac{x-y}{4} = 11. & (2) \end{cases}$

Multiplying (1) by 2, $x + y - \frac{2(x-y)}{3} = 16.$ (3)

Multiplying (2) by 3, $x + y + \frac{3(x-y)}{4} = 33.$ (4)

Subtracting (3) from (4), $\frac{17(x-y)}{12} = 17.$

$$x - y = 12. \quad (5)$$

Substituting (5) in (1), $\frac{x+y}{2} - 4 = 8.$

$$x + y = 24. \quad (6)$$

Adding (5) and (6), $2x = 36.$

$$x = 18.$$

Subtracting (5) from (6), $2y = 12.$

$$y = 6.$$

14. $\begin{cases} \frac{x}{2} - \frac{y}{3} - 1 = 0, & (1) \\ \frac{2x-1}{2} - \frac{3y-1}{3} = \frac{5}{6}. & (2) \end{cases}$

Clearing (1) of fractions, etc., $3x - 2y = 6.$ (3)

Simplifying (2), $x - \frac{1}{2} - y + \frac{1}{3} = \frac{5}{6}.$

$$x - y = 1. \quad (4)$$

Multiplying (4) by 2, $2x - 2y = 2.$ (5)

Subtracting (5) from (3), $x = 4.$ (6)

Substituting (6) in (4), $y = 3.$

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15. $\begin{cases} \dfrac{x}{3} = \dfrac{y}{2}, & (1) \\ \dfrac{x}{3} - \dfrac{y}{3} = 1. & (2) \end{cases}$

Substituting (1) in (2),

$$\frac{y}{2} - \frac{y}{3} = 1.$$

$$\frac{y}{6} = 1.$$

$$y = 6. \qquad (3)$$

Substituting (3) in (1),

$$\frac{x}{3} = 3.$$

$$x = 9.$$

16. $\begin{cases} \dfrac{3x}{4} + \dfrac{2y}{3} = 20, & (1) \\ \dfrac{x}{2} + \dfrac{3y}{4} = 17. & (2) \end{cases}$

(1) × 2, $\dfrac{3x}{2} + \dfrac{4y}{3} = 40.$ (3)

(2) × 3, $\dfrac{3x}{2} + \dfrac{9y}{4} = 51.$ (4)

(4) − (3), $\dfrac{11y}{12} = 11.$

$y = 12.$ (5)

Substituting (5) in (2),

$$\frac{x}{2} + 9 = 17.$$

$$x = 16.$$

17. $\begin{cases} \dfrac{x-1}{4} + y = 3, & (1) \\ \dfrac{x-1}{4} + 4y = 9. & (2) \end{cases}$

(2) − (1), $3y = 6.$

$y = 2.$ (3)

Substituting (3) in (1),

$$\frac{x-1}{4} + 2 = 3.$$

$$x = 5.$$

18. $\begin{cases} \dfrac{x}{8} + 4y = 15, & (1) \\ \dfrac{x}{6} + \dfrac{2y}{3} = 6. & (2) \end{cases}$

(2) × 6, $x + 4y = 36.$ (3)

(3) − (1), $\dfrac{7x}{8} = 21.$

$x = 24.$ (4)

Substituting (4) in (3),

$$y = 3.$$

19. $\begin{cases} \dfrac{1}{x-1} - \dfrac{3}{x+y} = 0, & (1) \\ \dfrac{3}{x-y} + 3 = 0. & (2) \end{cases}$

Solving (1) for y, $y = 2x - 3.$ (3)

Solving (2) for y, $y = x + 1.$ (4)

Equating the values of y, $2x - 3 = x + 1.$

$x = 4.$ (5)

Substituting (5) in (4), $y = 5.$

20. $\begin{cases} \dfrac{x}{2} - 12 = \dfrac{y+32}{4}, & (1) \\ \dfrac{y}{8} + \dfrac{3x-2y}{5} = 25. & (2) \end{cases}$

Reducing (1), $2x - y = 80.$ (3)

Reducing (2), $24x - 11y = 1000.$ (4)

Multiplying (3) by 12, $24x - 12y = 960.$ (5)

Subtracting (5) from (4), $y = 40.$ (6)

Substituting (6) in (3), $x = 60.$

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21.
$$\begin{cases} \dfrac{7+x}{5} - \dfrac{2x-y}{4} = 3y - 5, & (1) \\ \dfrac{5y-7}{2} + \dfrac{4x-3}{6} = 18 - 5x. & (2) \end{cases}$$

Reducing (1), $6x + 55y = 128.$ (3)
Reducing (2), $34x + 15y = 132.$ (4)
Subtracting (3) from (4), $28x - 40y = 4.$ (5)
Multiplying (5) by $\frac{3}{4}$, $21x - 30y = 3.$ (6)
Multiplying (4) by 2, $68x + 30y = 264.$ (7)
Adding (6) and (7), $89x = 267.$
$x = 3.$ (8)
Substituting (8) in (4), $y = 2.$

22.
$$\begin{cases} \dfrac{.2y + .5}{1.5} = \dfrac{.49x - .7}{4.2}, & (1) \\ \dfrac{.5x - .2}{1.6} = \dfrac{41}{16} - \dfrac{1.5y - 11}{8}. & (2) \end{cases}$$

Reducing the second member of (1) by dividing both terms by .7,
$$\frac{.2y + .5}{1.5} = \frac{.7x - 1}{6}. \quad (3)$$
Multiplying (3) by 6, $.8y + 2 = .7x - 1.$ (4)
$7x - 8y = 30.$ (5)
Multiplying (2) by 16, $5x - 2 = 41 - 3y + 22.$
$5x + 3y = 65.$ (6)
Multiplying (5) by 3, $21x - 24y = 90.$ (7)
Multiplying (6) by 8, $40x + 24y = 520.$ (8)
Adding (7) and (8), $61x = 610.$
$x = 10.$ (9)
Substituting (9) in (6), $y = 5.$

23.
$$\begin{cases} \dfrac{x+y}{5} + \dfrac{x-y}{5} = \dfrac{2x-y}{8} + \dfrac{4y-x}{5} + 1, & (1) \\ x = 2y. & (2) \end{cases}$$

Reducing (1), $14x - 27y = 40.$ (3)
Multiplying (2) by 14 and transposing,
$14x - 28y = 0.$ (4)
Subtracting (4) from (3), $y = 40.$ (5)
Substituting (5) in (2), $x = 80.$

24.
$$\begin{cases} x + \frac{1}{2}(3x - y - 1) = \frac{1}{4} + \frac{3}{4}(y - 1), & (1) \\ \frac{1}{5}(4x + 3y) = \frac{1}{10}(7y + 24). & (2) \end{cases}$$

Reducing (1), $10x - 5y = 0.$
$y = 2x.$ (3)
Reducing (2), $8x - y = 24.$ (4)
Substituting (3) in (4), $x = 4.$ (5)
Substituting (5) in (3), $y = 8.$

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25.
$$\begin{cases} \dfrac{6x+9}{4} + \dfrac{3x+5y}{4x-6} = 3\tfrac{1}{4} + \dfrac{3x+4}{2}, & (1) \\[2ex] \dfrac{8y+7}{10} + \dfrac{6x-3y}{2(y-4)} = 4 + \dfrac{4y-9}{5}. & (2) \end{cases}$$

From (1), $\dfrac{6x}{4} + 2\tfrac{1}{4} + \dfrac{3x+5y}{4x-6} = 3\tfrac{1}{4} + \dfrac{3x}{2} + 2.$

Canceling, etc., $\dfrac{3x+5y}{4x-6} = 3.$ (3)

Reducing (3), $9x - 5y = 18.$ (4)

From (2), $\dfrac{8y}{10} + \dfrac{7}{10} + \dfrac{6x-3y}{2(y-4)} = 4 + \dfrac{4y}{5} - \dfrac{9}{5}.$

Canceling, etc., $\dfrac{6x-3y}{2(y-4)} = \dfrac{3}{2}.$ (5)

Reducing (5), $x = y - 2.$ (6)

Substituting (6) in (4), $9y - 18 - 5y = 18.$

$y = 9.$ (7)

Substituting (7) in (6), $x = 7.$

26.
$$\begin{cases} \dfrac{3x-5y}{3} - \dfrac{2x-8y-9}{12} = \dfrac{31}{12}, & (1) \\[2ex] \left(\dfrac{x}{7} + \dfrac{y}{4} + 1\tfrac{1}{3}\right) - \left(4x - \dfrac{y}{8} - 25\right) = \dfrac{5}{6}. & (2) \end{cases}$$

Reducing (1), $5x - 6y = 11.$ (3)

Clearing (2) of fractions,

$24x + 42y + 224 - 672x + 21y + 4200 = 140.$

$-648x + 63y = -4284.$ (4)

Multiplying (4) by 2, $-1296x + 126y = -8568.$ (5)

Multiplying (3) by 21, $105x - 126y = 231.$ (6)

Adding (5) and (6), $-1191x = -8337.$

$x = 7.$ (7)

Substituting (7) in (3), $y = 4.$

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27.
$$\begin{cases} \tfrac{1}{2}R - \tfrac{1}{3}(r+1) = 1\tfrac{1}{2}, & (1) \\ \tfrac{1}{3}(R-1) - \tfrac{1}{2}r = 4\tfrac{1}{2}. & (2) \end{cases}$$

Reducing (1), $3R - 2r = 11.$ (3)

Reducing (2), $2R - 3r = 29.$ (4)

Multiplying (3) by 3, $9R - 6r = 33.$ (5)

Multiplying (4) by 2, $4R - 6r = 58.$ (6)

Subtracting (6) from (5), $5R = -25.$

$R = -5.$ (7)

Substituting (7) in (3), $r = -13.$

28.
$$\begin{cases} 2.4d - .04u = .62, & (1) \\ .7u + .15d = 1.795. & (2) \end{cases}$$

Clearing (1) of decimals, $240d - 4u = 62.$ (3)

Clearing (2) of decimals, $700u + 150d = 1795.$ (4)

Multiplying (1) by 5, $1200d - 20u = 310.$ (5)

Multiplying (3) by 8, $1200d + 5600u = 14{,}360.$ (6)

Subtracting (4) from (6), $5620u = 14{,}050.$

$u = 2.5.$ (7)

Substituting (7) in (1), $d = .3$

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29. $$\begin{cases} (u+.3)(v+.5)=(u-.3)(v+2), & (1)\\ (2u+.1)(3v+.5)=6v(u+.3). & (2)\end{cases}$$

Reducing (1) and clearing of decimals,

$$60v-150u=-75. \qquad (3)$$

Reducing (2) and clearing of decimals,

$$-150v+100u=-5. \qquad (4)$$

Multiplying (3) by $\frac{2}{5}$, $\quad 24v-60u=-30. \qquad (5)$

Multiplying (4) by $\frac{3}{5}$, $\quad -90v+60u=-3. \qquad (6)$

Adding (5) and (6), $\quad -66v=-33.$

$$v=.5. \qquad (7)$$

Substituting (7) in (1), $\quad u=.7.$

30. $$\begin{cases} x-20-\dfrac{2y-x}{23-x}=\dfrac{2x-59}{2}, & (1)\\ y-\dfrac{3-y}{x-18}-30=\dfrac{3y-73}{3}. & (2)\end{cases}$$

From (1), canceling $x-20=\dfrac{2x-40}{2}$,

$$-\frac{2y-x}{23-x}=-\frac{19}{2},$$

or $\quad 17x+4y=437. \qquad (3)$

From (2), canceling $y-30=\dfrac{3y-90}{3}$,

$$-\frac{3-y}{x-18}=\frac{17}{3},$$

or $\quad 17x-3y=297. \qquad (4)$

Subtracting (4) from (3), $\quad 7y=140.$

$$y=20. \qquad (5)$$

Substituting (5) in (4), $\quad x=21.$

31. $$\begin{cases} \dfrac{x}{2}+\dfrac{16-x}{2}=30+\dfrac{5y+2x}{40-x}, & (1)\\ \dfrac{4(x-6)}{y+8}+\dfrac{83-8y}{8}=10-y. & (2)\end{cases}$$

From (1), $$\frac{x}{2}+8-\frac{x}{2}=30+\frac{5y+2x}{40-x},$$

or $$\frac{5y+2x}{40-x}=-22,$$

or $\quad 4x-y=176. \qquad (3)$

From (2), $$\frac{4(x-6)}{y+8}+10\tfrac{3}{8}-y=10-y,$$

or $$\frac{4(x-6)}{y+8}=-\frac{3}{8},$$

or $\quad 32x+3y=168. \qquad (4)$

Multiplying (3) by 3, $\quad 12x-3y=528. \qquad (5)$

Adding (4) and (5), $\quad 44x=696.$

$$x=15\tfrac{9}{11}. \qquad (6)$$

Substituting (6) in (3), $\quad y=-112\tfrac{8}{11}.$

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33. $\begin{cases} \dfrac{5}{x}-\dfrac{3}{y}=-2, & (1) \\ \dfrac{25}{x}+\dfrac{1}{y}=6. & (2) \end{cases}$

(2) × 3, $\dfrac{75}{x}+\dfrac{3}{y}=18.$ (3)

(1) + (3), $\dfrac{80}{x}=16.$

$x=5.$ (4)

Substituting (4) in (2),

$y=1.$

34. $\begin{cases} \dfrac{2}{x}-\dfrac{3}{y}=5, & (1) \\ \dfrac{5}{x}-\dfrac{2}{y}=7. & (2) \end{cases}$

(1) × 2, $\dfrac{4}{x}-\dfrac{6}{y}=10.$ (3)

(2) × 3, $\dfrac{15}{x}-\dfrac{6}{y}=21.$ (4)

(4) − (3), $\dfrac{11}{x}=11.$

$x=1.$ (5)

Substituting (5) in (1),

$y=-1.$

35. $\begin{cases} \dfrac{4}{x}+\dfrac{3}{y}=\dfrac{9}{8}, & (1) \\ \dfrac{3}{x}+\dfrac{4}{y}=\dfrac{11}{12}. & (2) \end{cases}$

(1) + (2), $\dfrac{7}{x}+\dfrac{7}{y}=\dfrac{49}{24}.$ (3)

(3) × $\frac{3}{7}$, $\dfrac{3}{x}+\dfrac{3}{y}=\dfrac{7}{8}.$ (4)

(1) − (4), $\dfrac{1}{x}=\dfrac{1}{4}.$

$x=4.$

(2) − (4), $\dfrac{1}{y}=\dfrac{1}{24}.$

$y=24.$

36. $\begin{cases} \dfrac{7}{x}+\dfrac{8}{y}=30, & (1) \\ \dfrac{7}{y}+\dfrac{8}{x}=30. & (2) \end{cases}$

(2) − (1), $\dfrac{1}{x}-\dfrac{1}{y}=0.$

$y=x.$ (3)

Substituting (3) in (1),

$\dfrac{15}{x}=30.$

$x=\frac{1}{2}.$ (4)

Substituting (4) in (3),

$y=\frac{1}{2}.$

37. $\begin{cases} \dfrac{5}{x}+\dfrac{6}{y}=64, & (1) \\ \dfrac{6}{x}+\dfrac{5}{y}=73\frac{1}{2}. & (2) \end{cases}$

(1) + (2), $\dfrac{11}{x}+\dfrac{11}{y}=\dfrac{275}{2}.$ (3)

(3) × $\frac{5}{11}$, $\dfrac{5}{x}+\dfrac{5}{y}=\dfrac{125}{2}.$ (4)

(2) − (4), $\dfrac{1}{x}=11.$

$x=\frac{1}{11}.$

(1) − (4), $\dfrac{1}{y}=\dfrac{3}{2}.$

$y=\frac{2}{3}.$

38. $\begin{cases} \dfrac{3}{2x}-\dfrac{1}{y}=-3, & (1) \\ \dfrac{5}{2x}+\dfrac{3}{y}=23. & (2) \end{cases}$

(1) × 3, $\dfrac{9}{2x}-\dfrac{3}{y}=-9.$ (3)

(2) + (3), $\dfrac{7}{x}=14.$

$x=\frac{1}{2}.$ (4)

Substituting (4) in (1),

$y=\frac{1}{6}.$

39. $\begin{cases} \dfrac{10}{x}+\dfrac{5}{y}=20, & (1) \\ \dfrac{5}{x}+\dfrac{10}{y}=57\frac{1}{2}. & (2) \end{cases}$

(1) + (2), $\dfrac{15}{x}+\dfrac{15}{y}=\dfrac{155}{2}.$ (3)

(3) ÷ 3, $\dfrac{5}{x}+\dfrac{5}{y}=\dfrac{155}{6}.$ (4)

(1) − (4), $\dfrac{5}{x}=-\dfrac{35}{6}.$

$x=-\frac{6}{7}.$

(2) − (4), $\dfrac{5}{y}=\dfrac{95}{3}.$

$y=\frac{3}{19}.$

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40. $\begin{cases} \dfrac{7}{8x} - \dfrac{2}{3y} = 10, & (1) \\ \dfrac{5}{6x} - \dfrac{2}{11y} = 17. & (2) \end{cases}$

(1) × 3, $\dfrac{21}{8x} - \dfrac{2}{y} = 30.$ (3)

(2) × 11, $\dfrac{55}{6x} - \dfrac{2}{y} = 187.$ (4)

(4) − (3), $\dfrac{55}{6x} - \dfrac{21}{8x} = 157.$

$\dfrac{157}{24x} = 157.$

$\dfrac{1}{x} = 24.$ (5)

$x = \frac{1}{24}.$

Substituting (5) in (3),

$-\dfrac{2}{y} = -33.$

$y = \frac{2}{33}.$

41. $\begin{cases} \dfrac{1}{x-1} + \dfrac{1}{y+1} = 5, & (1) \\ \dfrac{2}{x-1} + \dfrac{3}{y+1} = 12. & (2) \end{cases}$

(1) × 3, $\dfrac{3}{x-1} + \dfrac{3}{y+1} = 15.$ (3)

(3) − (2), $\dfrac{1}{x-1} = 3.$ (4)

$\therefore x = \frac{4}{3}.$

Substituting (4) in (1),

$\dfrac{1}{y+1} = 2.$

$\therefore y = -\frac{1}{2}.$

42. $\begin{cases} \dfrac{5}{x-1} - \dfrac{3}{y-1} = 14, & (1) \\ \dfrac{2}{x-1} - \dfrac{1}{y-1} = 6. & (2) \end{cases}$

(2) × 3, $\dfrac{6}{x-1} - \dfrac{3}{y-1} = 18.$ (3)

(3) − (1), $\dfrac{1}{x-1} = 4.$ (4)

$\therefore x = \frac{5}{4}.$

Substituting (4) in (2),

$\dfrac{1}{y-1} = 2.$

$\therefore y = \frac{3}{2}.$

43. $\begin{cases} \dfrac{1}{y} = \dfrac{3}{2-x}, & (1) \\ \dfrac{5}{y} = \dfrac{6}{2-x} + 9. & (2) \end{cases}$

Substituting (1) in (2),

$\dfrac{15}{2-x} = \dfrac{6}{2-x} + 9.$

Solving, $x = 1.$ (3)

Substituting (3) in (1),

$y = \frac{1}{3}.$

44. $\begin{cases} \dfrac{4}{x} = \dfrac{1}{y+3}, & (1) \\ \dfrac{7}{x} = \dfrac{3}{y+3} - 10. & (2) \end{cases}$

Substituting (1) in (2),

$\dfrac{7}{x} = \dfrac{12}{x} - 10.$

$\therefore x = \frac{1}{2}.$ (3)

Substituting (3) in (1),

$y = -\frac{23}{8}.$

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2. $\begin{cases} ax + by = m, & (1) \\ bx - ay = c. & (2) \end{cases}$

(1) × a, $a^2x + aby = am.$ (3)

(2) × b, $b^2x - aby = bc.$ (4)

(3) + (4), $(a^2 + b^2)x = am + bc.$

$\therefore x = \dfrac{am + bc}{a^2 + b^2}.$

(1) × b, $abx + b^2y = bm.$ (5)

(2) × a, $abx - a^2y = ac.$ (6)

(5) − (6), $(a^2 + b^2)y = bm - ac.$

$\therefore y = \dfrac{bm - ac}{a^2 + b^2}.$

3. $\begin{cases} ax - by = m, & (1) \\ cx - dy = r. & (2) \end{cases}$

(1) × d, $adx - bdy = dm.$ (3)

(2) × b, $bcx - bdy = br.$ (4)

(3) − (4), $(ad - bc)x = dm - br.$

$\therefore x = \dfrac{dm - br}{ad - bc}.$

(1) × c, $acx - bcy = cm.$ (5)

(2) × a, $acx - ady = ar.$ (6)

(5) − (6), $(ad - bc)y = cm - ar.$

$\therefore y = \dfrac{cm - ar}{ad - bc}.$

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4. $\begin{cases} ax = by, & (1) \\ x + y = ab. & (2) \end{cases}$

(2) × a, $ax + ay = a^2b.$ (3)

Substituting (1) in (3),

$$by + ay = a^2b.$$

$$\therefore\ y = \frac{a^2b}{a+b}. \quad (4)$$

(4) × b, $by = \dfrac{a^2b^2}{a+b}.$ (5)

Substituting (5) in (1),

$$ax = \frac{a^2b^2}{a+b}.$$

$$\therefore\ x = \frac{ab^2}{a+b}.$$

5. $\begin{cases} m(x+y) = a, & (1) \\ n(x-y) = 2a. & (2) \end{cases}$

(1) × n, $mnx + mny = an.$ (3)

(2) × m, $mnx - mny = 2am.$ (4)

(3) + (4), $2mnx = an + 2am.$

$$\therefore\ x = \frac{an + 2am}{2mn}. \quad (5)$$

Substituting (5) in (1),

$$y = \frac{an - 2am}{2mn}.$$

6. $\begin{cases} a(x-y) = 5, & (1) \\ bx - cy = n. & (2) \end{cases}$

(1) × c, $acx - acy = 5c.$ (3)

(2) × a, $abx - acy = an.$ (4)

(4) − (3), $(ab - ac)x = an - 5c.$

$$\therefore\ x = \frac{an - 5c}{ab - ac}.$$

(1) × b, $abx - aby = 5b.$ (5)

(2) × a, $abx - acy = an.$ (6)

(6) − (5), $(ab - ac)y = an - 5b.$

$$\therefore\ y = \frac{an - 5b}{ab - ac}.$$

7. $\begin{cases} a(a-x) = b(y-b), & (1) \\ ax = by. & (2) \end{cases}$

(2) − (1), $-a^2 + 2ax = b^2.$

$$\therefore\ x = \frac{a^2 + b^2}{2a}.$$

(1) + (2), $a^2 = 2by - b^2.$

$$\therefore\ y = \frac{a^2 + b^2}{2b}.$$

8. $\begin{cases} x + y = b - a, & (1) \\ bx - ay + 2ab = 0. & (2) \end{cases}$

From (1), $y = b - a - x.$ (3)

Substituting (3) in (2),

$$bx - ab + a^2 + ax + 2ab = 0.$$

$$\therefore\ x = \frac{-a^2 - ab}{a+b} = \frac{-a(a+b)}{a+b};$$

that is, $x = -a.$ (4)

Substituting (4) in (1),

$$y = b.$$

9. $\begin{cases} x - y = a - b, & (1) \\ ax + by = a^2 - b^2. & (2) \end{cases}$

From (1), $x = a - b + y.$ (3)

Substituting (3) in (2),

$$y = \frac{b(a-b)}{a+b}. \quad (4)$$

Substituting (4) in (3),

$$x = \frac{a^2 + ab - 2b^2}{a+b}.$$

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10. $\begin{cases} \dfrac{x}{a} + \dfrac{y}{b} - 2 = 0, & (1) \\ bx - ay = 0. & (2) \end{cases}$

From (1), $bx + ay = 2ab.$ (3)

(3) + (2), $2bx = 2ab.$

$$\therefore\ x = a.$$

(3) − (2), $2ay = 2ab.$

$$\therefore\ y = b.$$

11. $\begin{cases} \dfrac{1}{x} + \dfrac{1}{y} = \dfrac{1}{a}, & (1) \\ \dfrac{1}{x} - \dfrac{1}{y} = \dfrac{1}{b}. & (2) \end{cases}$

(1) + (2), $\dfrac{2}{x} = \dfrac{1}{a} + \dfrac{1}{b}$

$$= \frac{a+b}{ab}.$$

$$\therefore\ x = \frac{2ab}{a+b}.$$

(1) − (2), $\dfrac{2}{y} = \dfrac{1}{a} - \dfrac{1}{b}$

$$= \frac{b-a}{ab}.$$

$$\therefore\ y = \frac{2ab}{b-a}.$$

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12. $\begin{cases} \dfrac{a}{x}-\dfrac{b}{y}=-1, & (1)\\ \dfrac{b}{x}-\dfrac{a}{y}=-1. & (2)\end{cases}$

(1)−(2), $\dfrac{a-b}{x}+\dfrac{a-b}{y}=0.$

$$\frac{a-b}{y}=-\frac{a-b}{x}.$$

$$y=-x. \quad (3)$$

Substituting (3) in (1),

$$\frac{a}{x}+\frac{b}{x}=-1.$$

$$\therefore x=-(a+b). \quad (4)$$

Substituting (4) in (3),

$$y=a+b.$$

13. $\begin{cases} \dfrac{x}{a}+\dfrac{y}{b}=2\,ab, & (1)\\ \dfrac{x}{ab}+\dfrac{y}{ab}=a+b. & (2)\end{cases}$

(1) ÷ a, $\dfrac{x}{a^2}+\dfrac{y}{ab}=2\,b.$ (3)

(2)−(3), $\left(\dfrac{1}{ab}-\dfrac{1}{a^2}\right)x=a-b.$

$$\left(\frac{a-b}{a^2b}\right)x=a-b.$$

$$\therefore x=a^2b. \quad (4)$$

Substituting (4) in (1),

$$ab+\frac{y}{b}=2\,ab.$$

$$\therefore y=ab^2.$$

14. $\begin{cases} \dfrac{x}{a}+\dfrac{y}{b}=1, & (1)\\ \dfrac{x}{b}-\dfrac{y}{a}=\dfrac{1}{2}. & (2)\end{cases}$

(1) ÷ a, $\dfrac{x}{a^2}+\dfrac{y}{ab}=\dfrac{1}{a}.$ (3)

(2) ÷ b, $\dfrac{x}{b^2}-\dfrac{y}{ab}=\dfrac{1}{2\,b}.$ (4)

(3)+(4), $\left(\dfrac{1}{b^2}+\dfrac{1}{a^2}\right)x=\dfrac{1}{2\,b}+\dfrac{1}{a}.$

$$\frac{a^2+b^2}{a^2b^2}x=\frac{a+2\,b}{2\,ab}.$$

$$\therefore x=\frac{ab\,(a+2\,b)}{2\,(a^2+b^2)}. \quad (5)$$

Substituting (5) in (1),

$$\frac{b\,(a+2\,b)}{2(a^2+b^2)}+\frac{y}{b}=1.$$

$$ab^2+2\,b^3+2\,(a^2+b^2)\,y=2\,a^2b+2\,b^3.$$

$$\therefore y=\frac{2\,a^2b-ab^2}{2\,(a^2+b^2)}=\frac{ab(2\,a-b)}{2\,(a^2+b^2)}.$$

15. $\begin{cases} \dfrac{x+1}{y+1}=\dfrac{a+b+1}{a-b+1}, & (1)\\ x-y=2\,b. & (2)\end{cases}$

From (2), $x=y+2\,b.$ (3)

Substituting (3) in (1),

$$\frac{y+2\,b+1}{y+1}=\frac{a+b+1}{a-b+1}.$$

Reducing to mixed numbers and canceling 1 from each member,

$$\frac{2\,b}{y+1}=\frac{2\,b}{a-b+1}.$$

$$y+1=a-b+1.$$

$$\therefore y=a-b. \quad (4)$$

Substituting (4) in (3),

$$x=a+b.$$

16. $\begin{cases} \dfrac{1}{x-a}=\dfrac{1}{a-y}, & (1)\\ \dfrac{x+y}{x-y}=a. & (2)\end{cases}$

From (1), $x=2\,a-y.$ (3)

(3) in (2), $\dfrac{2\,a}{2\,a-2\,y}=a.$

$$\frac{1}{a-y}=1.$$

$$\therefore y=a-1. \quad (4)$$

Substituting (4) in (3),

$$x=a+1.$$

17. $\begin{cases} \dfrac{x}{a}+\dfrac{y}{b}=c, & (1)\\ \dfrac{x}{b}+\dfrac{y}{c}=d. & (2)\end{cases}$

(1) × b, $\dfrac{bx}{a}+y=bc.$ (3)

(2) × c, $\dfrac{cx}{b}+y=cd.$ (4)

(3) − (4), $\dfrac{bx}{a}-\dfrac{cx}{b}=bc-cd.$

$$(b^2-ac)\,x=ab\,(bc-cd).$$

$$\therefore x=\frac{abc\,(b-d)}{b^2-ac}.$$

(1) × a, $x+\dfrac{ay}{b}=ac.$ (5)

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(2) × b, $\quad x+\dfrac{by}{c}=bd.$ (6)

(6) − (5), $\quad \dfrac{by}{c}-\dfrac{ay}{b}=bd-ac.$

$$(b^2-ac)y=bc(bd-ac).$$

$$\therefore y=\frac{bc(bd-ac)}{b^2-ac}.$$

18. $\begin{cases}\dfrac{a}{x}+\dfrac{b}{y}=c, & (1)\\[2mm] \dfrac{m}{x}+\dfrac{n}{y}=e. & (2)\end{cases}$

(1) × n, $\quad \dfrac{an}{x}+\dfrac{bn}{y}=cn.$ (3)

(2) × b, $\quad \dfrac{bm}{x}+\dfrac{bn}{y}=be.$ (4)

(3) − (4), $(an-bm)\dfrac{1}{x}=cn-be.$

$$\therefore x=\frac{an-bm}{cn-be}.$$

(1) × m, $\quad \dfrac{am}{x}+\dfrac{bm}{y}=cm.$ (5)

(2) × a, $\quad \dfrac{am}{x}+\dfrac{an}{y}=ae.$ (6)

(6) − (5), $(an-bm)\dfrac{1}{y}=ae-cm.$

$$\therefore y=\frac{an-bm}{ae-cm}.$$

19. $\begin{cases}\dfrac{1}{ax}+\dfrac{1}{by}=c, & (1)\\[2mm] \dfrac{1}{bx}-\dfrac{1}{ay}=d. & (2)\end{cases}$

(1) × b, $\quad \dfrac{b}{ax}+\dfrac{1}{y}=bc.$ (3)

(2) × a, $\quad \dfrac{a}{bx}-\dfrac{1}{y}=ad.$ (4)

(3) + (4), $\quad \dfrac{a}{bx}+\dfrac{b}{ax}=ad+bc.$ (5)

From (5), $a^2+b^2=abx(ad+bc).$

$$\therefore x=\frac{a^2+b^2}{ab(ad+bc)}.$$

(1) × a, $\quad \dfrac{1}{x}+\dfrac{a}{by}=ac.$ (6)

(2) × b, $\quad \dfrac{1}{x}-\dfrac{b}{ay}=bd.$ (7)

(6) − (7), $\quad \dfrac{a}{by}+\dfrac{b}{ay}=ac-bd.$ (8)

From (8), $a^2+b^2=aby(ac-bd).$

$$\therefore y=\frac{a^2+b^2}{ab(ac-bd)}.$$

20. Substituting given values,

$\begin{cases}F=15\,a, & (1)\\ 72=\frac{1}{2}\cdot a\cdot 36. & (2)\end{cases}$

From (2), $\quad a=4.$ (3)

Substituting (3) in (1),

$$F=60.$$

21. Substituting first values,

$\begin{cases}l=a+49\cdot 2, & (1)\\ 2500=25(a+l). & (2)\end{cases}$

Substituting (1) in (2),

$a=1.$ (3)

Substituting (3) in (1),

$$l=99.$$

Substituting second values,

$\begin{cases}50=a+24\,d, & (1)\\ 650=\frac{25}{2}(a+50). & (2)\end{cases}$

From (2), $\quad a=2.$ (3)

Substituting (3) in (1),

$$d=2.$$

22. $\begin{cases}l=a\cdot 2^{10}, & (1)\\[2mm] 2047=\dfrac{2\,l-a}{1}. & (2)\end{cases}$

Substituting (1) in (2),

$$2047=2048\,a-a.$$

$\therefore a=1.$ (3)

Substituting (3) in (1),

$$l=1024.$$

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1. Let $x=$ the larger number,

and $y=$ the smaller number.

Then, $x+y=14,$ (1)

and $x-y=8.$ (2)

Adding, $2x=22$; $\therefore x=11$, the larger number.

Substituting 11 for x in (1), $11+y=14$; $\therefore y=3$, the smaller number.

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2. Let $x =$ the first number,
and $y =$ the second number.

Then, $2x + 3y = 34$, (1)
and $2x + 5y = 50$. (2)

Subtracting (1) from (2), $2y = 16$; $\therefore y = 8$, the second number.
Substituting 8 for y in (1), $2x + 24 = 34$; $\therefore x = 5$, the first number.

3. Let $x =$ the first number,
and $y =$ the second number.

Then, $x + y = 18$, (1)
and $x + 2y = 20$. (2)

Subtracting (1) from (2), $y = 2$, the second number.
Substituting 2 for y in (1), $x + 2 = 18$; $\therefore x = 16$, the first number.

4. Let $x =$ the number of cents per box of raspberries,
and $y =$ the number of cents per box of cherries.

Then, $2x + 3y = 54$, (1)
and $3x + 2y = 56$. (2)

Multiplying (1) by 3, $6x + 9y = 162$. (3)
Multiplying (2) by 2, $6x + 4y = 112$. (4)
Subtracting (4) from (3), $5y = 50$; $\therefore y = 10$.
Substituting 10 for y in (1), $2x + 30 = 54$; $\therefore x = 12$.
Hence, raspberries were 12¢ per box, cherries 10¢ per box.

5. Let $x =$ the number of 3-gr. capsules,
and $y =$ the number of 2-gr. capsules.

Then, $x + y = 220$, (1)
and $3x + 2y = 500$. (2)

Multiplying (1) by 2, $2x + 2y = 440$. (3)
Subtracting (3) from (2), $x = 60$.
Substituting 60 for x in (1), $60 + y = 220$; $\therefore y = 160$.
Hence, there were 60 3-gr. capsules and 160 2-gr. capsules.

6. Let $x =$ the number of glasses @ 5¢,
and $y =$ the number of glasses @ 10¢.

Then, $x + y = 850$, (1)
and $5x + 10y = 5500$. (2)

Multiplying (1) by 5, $5x + 5y = 4250$. (3)
Subtracting (3) from (2), $5y = 1250$; $\therefore y = 250$, the number @ 10¢.
Substituting 250 for y in (1), $x + 250 = 850$; $\therefore x = 600$, the number @ 5¢.

7. Let $x =$ the number @ 12¢,
and $y =$ the number @ 10¢.

Then, $x + y = 36$, (1)
and $12x + 10y = 250 + 150$, or 400. (2)

Multiplying (1) by 10, $10x + 10y = 360$. (3)
Subtracting (3) from (2), $2x = 40$; $\therefore x = 20$, the number @ 12¢.
Substituting 20 for x in (1), $20 + y = 36$; $\therefore y = 16$, the number @ 10¢.

8. Let $x =$ the number for adults,
and $y =$ the number for children.

Then, $x + y = 300$, (1)
and $\frac{1}{2}x + \frac{1}{4}y = 125$. (2)

Multiplying (2) by 4, $2x + y = 500$. (3)
(3) – (1), $x = 200$, the number for adults. (4)
Substituting (4) in (1), $y = 100$, the number for children.

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9. Let $x =$ the number of 1-dollar bills,
and $y =$ the number of 2-dollar bills.
Then, $x + y = 38,$ (1)
and $x + 2y = 50.$ (2)
Subtracting (1) from (2), $y = 12$, the number of 2-dollar bills.
Substituting 12 for y in (1), $x + 12 = 38$; $\therefore x = 26$, the number of 1-dollar bills.

10. Let $x =$ number of pounds one Brazil tree yields,
and $y =$ number of pounds one Ceylon tree yields.
Then, $2x = 8y + 4,$ (1)
and $3x = 10y + 10.$ (2)
Multiplying (1) by 3, $6x = 24y + 12.$ (3)
Multiplying (2) by 2, $6x = 20y + 20.$ (4)
Subtracting (4) from (3), $4y = 8$; $\therefore y = 2.$ (5)
Substituting (5) in (1), $x = 10.$
Hence, one Brazil tree yields 10 pounds, and one Ceylon tree yields 2 pounds.

11. Let $x =$ the number of cents for the first ten words,
and $y =$ the number of cents for each additional word.
Then, $x + 5y = 40,$ (1)
and $x + 12y = 54.$ (2)
Subtracting (1) from (2), $7y = 14$; $\therefore y = 2.$
Substituting 2 for y in (1), $x + 10 = 40$; $\therefore x = 30.$
Hence, 10 words cost 30¢, and each additional word 2¢, or the rate is 30 – 2.

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12. Let $x =$ the number of men,
and $y =$ the number of women.
Then, $x + y = 1000,$ (1)
and $2.50x + 1.50y = 2340.$ (2)
Multiplying (1) by 1.50, $1.50x + 1.50y = 1500.$ (3)
Subtracting (3) from (2), $x = 840.$
Substituting 840 for x in (1), $840 + y = 1000$; $\therefore y = 160.$
Hence, there were 840 men and 160 women.

13. Let $x =$ the number of inches in length,
and $y =$ the number of inches in width.
Then, $x - y = 6,$ (1)
and $2x + 2y = 60.$ (2)
Dividing (2) by 2, $x + y = 30.$ (3)
Adding (1) and (3), $2x = 36$; $\therefore x = 18$, the length in inches.
Substituting 18 for x in (3), $18 + y = 30$; $\therefore y = 12$, the width in inches.

14. Let $x =$ the number of months sea duty,
and $y =$ the number of months shore duty.
Then, $x + y = 12,$ (1)
and $150x + 127\frac{1}{2}y = 1620.$ (2)
Multiplying (2) by 2, $300x + 255y = 3240.$ (3)
Multiplying (1) by 300, $300x + 300y = 3600.$ (4)
Subtracting (3) from (4), $45y = 360$; $\therefore y = 8.$
Substituting 8 for y in (1), $x + 8 = 12$; $\therefore x = 4.$
Hence, he was 4 months on sea duty and 8 months on shore duty.

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15. Let $x =$ the weight of a shaft in tons,
and $y =$ the weight of a capital in tons.

Then, $x + y = 94,$ (1)

and $x - y = 74.$ (2)

Adding (1) and (2), $2x = 168$; $\therefore x = 84.$

Substituting 84 for x in (1), $84 + y = 94$; $\therefore y = 10.$

Hence, a shaft weighed 84 T., a capital 10 T.

16. Let $x =$ the number of tickets to the grounds,
and $y =$ the number of tickets to the grand stand.

Then, $\frac{1}{2}x + \frac{1}{4}y = 700,$ (1)

and $\frac{1}{2}x + 2(\frac{1}{4}y) = 800,$

or $\frac{1}{2}x + \frac{1}{2}y = 800.$ (2)

Subtracting (1) from (2), $\frac{1}{4}y = 100$; $\therefore y = 400.$

Substituting 400 for y in (2), $\frac{1}{2}x + 200 = 800$; $\therefore x = 1200.$

Hence, there were 1200 tickets to grounds and 400 to grand stand.

17. Let $x =$ the number of cents duty per M on shingles,
and $y =$ the number of cents duty per M on laths.

Then, $40x + 160y = 5200,$ (1)

and $80x + 70y = 4150.$ (2)

Multiplying (1) by 2, $80x + 320y = 10,400.$ (3)

Subtracting (2) from (3), $250y = 6250$; $\therefore y = 25.$

Substituting 25 for y in (1), $40x + 4000 = 5200$; $\therefore x = 30.$

Therefore, the rate per M on shingles was 30¢; on laths, 25¢.

18. Let $x =$ the numerator of the fraction,
and $y =$ the denominator.

Then, $\frac{x+1}{y} = \frac{3}{4},$

and $\frac{x}{y+2} = \frac{1}{2}.$

Solving, $x = 5$, and $y = 8.$

Hence, the fraction is $\frac{5}{8}$.

19. Let $\frac{x}{y} =$ the fraction.

Then, by 1st condition, $\frac{x-2}{y} = \frac{x}{y} - \frac{1}{2},$ (1)

and by 2d condition, $\frac{x}{y+4} = \frac{x}{y} - \frac{5}{8}.$ (2)

Reducing (1), $y = 4.$ (3)

Substituting (3) in (2), $\frac{x}{8} = \frac{x}{4} - \frac{5}{8};$ (4)

whence, $x = 5. \quad \therefore \frac{x}{y} = \frac{5}{4}.$

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20. Let $x =$ the digit in tens' place,
and $y =$ the digit in units' place.
Then, $10x + y =$ the number,
and $10y + x =$ the number with its digits reversed;
$\therefore 10x + y = 7(x + y),$
and $10x + y - 27 = 10y + x.$
Solving, $x = 6$, and $y = 3$.
Hence, $10x + y = 60 + 3$, or 63, the number.

21. Let $x =$ digit in tens' place,
and $y =$ digit in units' place,
whence, $10x + y =$ the number.
Then, $x + y = 12,$
and $10x + y = 6(x + y) + 3.$
Substituting 12 for $x + y$ in the 2d equation,

$$9x + 12 = 72 + 3;$$

$$\therefore x = 7,$$

whence, from the 1st equation, $y = 5$.
Hence, the number is $70 + 5$, or 75.

22. Let $x =$ digit in tens' place,
and $y =$ digit in units' place,
whence, $10x + y =$ the number.
Then, $\dfrac{10x + y}{x + y} = 8,$
and $x - 3y = 1.$
Solving, $x = 7$, and $y = 2$.
Hence, the number is $70 + 2$, or 72.

23. Let $x =$ number of feet in length,
and $y =$ number of feet in width.
Then, $(x + 5)(y + 2) = xy + 140,$ (1)
and $(x + 10)(y + 7) = xy + 390.$ (2)
Reducing (1), $2x + 5y = 130.$ (3)
Reducing (2), $7x + 10y = 320.$ (4)
Solving (3) and (4), $x = 20$, and $y = 18$.
Hence, the floor is 20 feet long and 18 feet wide.

24. Let $x =$ number of miles an hour the crew can row in still water,
and $y =$ number of miles an hour the stream runs.

Then, since the rate of the crew in still water is increased by that of the current when they row downstream, and decreased by that of the current when they row upstream, and since the time is equal to the distance divided by the rate,

$$\frac{8}{x + y} + \frac{8}{x - y} = \frac{3}{2}, \quad (1)$$

and

$$\frac{12}{x + y} + \frac{6}{x - y} = \frac{3}{2}. \quad (2)$$

Subtracting (1) from (2),

$$\frac{4}{x + y} - \frac{2}{x - y} = 0. \quad (3)$$

(See next page.)

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Clearing of fractions, $4x - 4y - 2x - 2y = 0.$

$$x = 3y. \tag{4}$$

Substituting (4) in (1), $\frac{6}{y} = \frac{3}{2};$

$$\therefore y = 4. \tag{5}$$

Substituting (5) in (4), $x = 12.$

Hence, the rate of the crew in still water is 12 miles an hour, and the velocity of the stream is 4 miles an hour.

25. Let $x =$ number of miles he can row in still water,
and $y =$ number of miles an hour the stream runs.

Then, since his rate of rowing is increased when he rows downstream, and diminished when he rows upstream, by the velocity of the current, and since the time is equal to the distance divided by the rate,

$$\frac{12}{x+y} + \frac{12}{x-y} = 11, \tag{1}$$

and
$$\frac{8}{x+y} = \frac{3}{x-y}. \tag{2}$$

From (2), $y = \frac{5}{11}x.$ (3)

Substituting (3) in (1), $x = \frac{11}{4},$ (4)

whence, by (3), $y = \frac{5}{4}.$

Hence, the man's rate of rowing in still water is $2\frac{3}{4}$ miles an hour, and the velocity of the stream is $1\frac{1}{4}$ miles an hour.

26. Let $x =$ number of tons necessary to fill a car,
and $y =$ number of tons necessary to fill a pocket.

From problem, $25x = 7y + \frac{1}{2}y,$ (1)

and $\frac{1}{2}y = 2x - 16.$ (2)

From (2), $y = 4x - 32.$ (3)

Substituting (3) in (1), $x = 48.$ (4)

Substituting (4) in (3), $y = 160.$

Hence, the capacity of a car is 48 tons, and the capacity of a pocket is 160 tons.

27. Let $x =$ number of pounds of water 1 pound of coal can evaporate.
and $y =$ number of pounds of water 1 gallon of oil can evaporate.

From problem, $100x - 6y = 50,$ (1)

and $60x - 4y = -10.$ (2)

Multiplying (1) by 2, $200x - 12y = 100.$ (3)

Multiplying (2) by 3, $180x - 12y = -30.$ (4)

Subtracting (4) from (3), $20x = 130.$

$$\therefore x = 6\tfrac{1}{2}. \tag{5}$$

Substituting (5) in (1), $y = 100.$

Hence, 1 pound of coal can evaporate $6\frac{1}{2}$ pounds of water, and 1 gallon of oil can evaporate 100 pounds of water.

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28. Let $x =$ number of pineapples in first size crate,
and $y =$ number of pineapples in second size crate.

Then, $450x + 375y = 24{,}300,$ (1)
and $675x + 550y = 36{,}000.$ (2)
Reducing (1), $6x + 5y = 324.$ (3)
Reducing (2), $27x + 22y = 1440.$ (4)
Multiplying (3) by 9, $54x + 45y = 2916.$ (5)
Multiplying (4) by 2, $54x + 44y = 2880.$ (6)
Subtracting (6) from (5), $y = 36.$ (7)
Substituting (7) in (3), $x = 24.$

Hence, crate of first size held 24 pineapples, and crate of second size held 36 pineapples.

29. Let $x =$ number of gallons of naphtha,
and $y =$ number of gallons of petroleum.

From problem, $\frac{23}{4}x + \frac{15}{2}y = 12{,}400,$ (1)
and $\frac{55}{8}x + \frac{15}{2}y = 14{,}500.$ (2)
Multiplying (1) by 15, $\frac{345}{4}x + \frac{195}{2}y = 186{,}000.$ (3)
Multiplying (2) by 13, $\frac{715}{8}x + \frac{195}{2}y = 188{,}500.$ (4)
Subtracting (3) from (4), $\frac{25}{8}x = 2500.$
$\therefore x = 800.$ (5)
Substituting (5) in (1), $y = 1200.$

Hence, there were 800 gallons of naphtha and 1200 gallons of petroleum.

30. Let $x =$ number of cubic meters removed per hour by contract,
and $y =$ number of cubic meters actually removed per hour.

From problem, $y = 1400 + x,$ (1)
and $\frac{9}{5}y = \frac{5}{2}x.$ (2)
Reducing (2), $18y = 25x.$ (3)
Substituting (1) in (3), $18(1400 + x) = 25x.$ (4)
Solving (4), $x = 3600.$ (5)
Substituting (5) in (1), $y = 5000.$

Hence, the contract rate is 3600 cubic meters per hour, and the actual rate is 5000 cubic meters per hour.

31. Let $x =$ number of days it will take A,
and $y =$ number of days it will take B.

Then, $$\frac{1}{x} + \frac{1}{y} = \frac{1}{12},\qquad (1)$$
and $$\frac{5}{x} + \frac{26}{y} = 1.\qquad (2)$$

Multiplying (1) by 5 and subtracting the result from (2),
$$\frac{21}{y} = \frac{7}{12}.$$
$$\therefore y = 36;$$
whence, substituting in (1), $x = 18.$

Hence, A can do the work in 18 days, and B in 36 days.

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32. Let x = number of days it would take larger machine,
and y = number of days it would take smaller machine.

Then, $\frac{1}{x}$ = amount larger machine does in 1 day,

and $\frac{1}{y}$ = amount smaller machine does in 1 day.

$$\therefore \frac{1}{x}+\frac{1}{y}=\frac{1}{6}, \quad (1)$$

and $$\frac{8}{x}+\frac{3}{y}=1. \quad (2)$$

Multiplying (1) by 3, $$\frac{3}{x}+\frac{3}{y}=\frac{1}{2}. \quad (3)$$

(2) − (3), $$\frac{5}{x}=\frac{1}{2}.$$

$$\therefore x=10. \quad (4)$$

Substituting (4) in (1), $$y=15.$$

Hence, the larger machine can do it in 10 days, and the smaller in 15 days.

33. Let x = number of days it would take 1 man,
and y = number of days it would take 1 boy.

Then, $\frac{1}{x}$ = the amount one man can do in 1 day,

and $\frac{1}{y}$ = the amount 1 boy can do in 1 day.

$$\frac{6}{x}+\frac{4}{y}=\frac{1}{30}, \quad (1)$$

and $$\frac{5}{x}+\frac{5}{y}=\frac{1}{32}. \quad (2)$$

Multiplying (1) by 5, $$\frac{30}{x}+\frac{20}{y}=\frac{1}{6}. \quad (3)$$

Multiplying (2) by 4, $$\frac{20}{x}+\frac{20}{y}=\frac{1}{8}. \quad (4)$$

Subtracting (4) from (3), $$\frac{10}{x}=\frac{1}{24}.$$

$$\therefore x=240,$$

and $$\frac{x}{12}=20.$$

Hence, 12 men can do the work in 20 days.

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34. Let $x =$ number of days it takes A,
and $y =$ number of days it takes B.

Then, $$\frac{1}{x}+\frac{1}{y}=\frac{1}{a}, \quad (1)$$

and $$\frac{m}{x}+\frac{n}{y}=1. \quad (2)$$

Multiplying (1) by n, $$\frac{n}{x}+\frac{n}{y}=\frac{n}{a}. \quad (3)$$

Subtracting (3) from (2), $$\frac{m-n}{x}=\frac{a-n}{a};$$

whence, $$x=\frac{a(m-n)}{a-n}.$$

Similarly, $(1) \times m - (2)$, $$y=\frac{a(m-n)}{m-a}.$$

Hence, A can do the work in $\frac{a(m-n)}{a-n}$ days, and B in $\frac{a(m-n)}{m-a}$ days.

35. Let $x =$ number of days A must work,
and $y =$ number of days B must work.

Then, $$\frac{x}{c}+\frac{y}{d}=1, \quad (1)$$

and $$x+y=a. \quad (2)$$

Clearing (1) of fractions, $dx + cy = cd$. (3)

$(2) \times c - (3)$, $$(c-d)x = ac - cd.$$
$$\therefore x=\frac{c(a-d)}{c-d}.$$

$(3)-(2) \times d$, $$(c-d)y = cd - ad,$$
$$\therefore y=\frac{d(c-a)}{c-d}.$$

Hence, A must work $\frac{c(a-d)}{c-d}$ days, and B $\frac{d(c-a)}{c-d}$ days.

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36. Let $x =$ number of dollars in principal,
and $y =$ number of hundredths indicated by rate.

Then, since the interest for one year is equal to the difference between the amount and the principal, divided by the number of years,

$$\frac{xy}{100}=\frac{2472-x}{\frac{3}{4}}, \quad (1)$$

and $$\frac{xy}{100}=\frac{2528-x}{\frac{4}{3}}. \quad (2)$$

Eliminating xy by comparison,

$$\frac{4(2472-x)}{3}=\frac{3(2528-x)}{4}. \quad (3)$$

Solving (3), $x = 2400$. (4)

Substituting (4) in (1), $y = 4$.

Hence, the principal was \$2400, and the rate 4%.

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37. Let $x =$ number of dollars invested at 5 %,
and $y =$ number of dollars invested at 4 %.

Then, $x + y = 4000,$ (1

and $.05\,x + .04\,y = 175.$ (2

Multiplying (2) by 25, $1.25\,x + y = 4375.$ (3

Subtracting (1) from (3), $.25\,x = 375.$

$\therefore\ x = 1500.$ (4

Substituting (4) in (1), $y = 2500.$

Hence, he invested \$ 1500 at 5 % and \$ 2500 at 4 %.

38. Let $x =$ number of dollars invested at r %,
and $y =$ number of dollars invested at s %.

Then, $x + y = a,$ (1)

and
$$\frac{rx}{100} + \frac{sy}{100} = b. \qquad (2)$$

Clearing (2) of fractions, $rx + sy = 100\,b.$ (3)

Multiplying (1) by s, $sx + sy = sa.$ (4)

Subtracting (4) from (3),
$$(r - s)\,x = 100\,b - sa.$$
$$\therefore\ x = \frac{100\,b - sa}{r - s}.$$

Multiplying (1) by r, $rx + ry = ra.$ (5)

Subtracting (3) from (5),
$$(r - s)\,y = ra - 100\,b.$$
$$\therefore\ y = \frac{ra - 100\,b}{r - s}.$$

Hence, he invested
$$\frac{100\,b - sa}{r - s} \text{ dollars at } r\,\%, \text{ and } \frac{ra - 100\,b}{r - s} \text{ dollars at } s\,\%.$$

39. Let $x =$ number of dollars in principal,
and $y =$ number of hundredths indicated by rate.

Then, since the interest for one year is equal to the difference between the amount and the principal, divided by the number of years,
$$\frac{xy}{100} = \frac{b - x}{t}, \qquad (1)$$
$$\frac{xy}{100} = \frac{a - x}{s}. \qquad (2)$$

Eliminating xy by comparison, $x = \dfrac{bs - at}{s - t}.$ (3)

Substituting (3) in (1) or in (2),
$$y = \frac{100\,(a - b)}{bs - at}.$$

Hence, the principal was $\dfrac{bs - at}{s - t}$ dollars, and the rate $\dfrac{100\,(a - b)}{bs - at}$ %.

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40. Let $x =$ number of persons,
and $y =$ number of cents each should pay.

Then, $xy =$ total expense.

By the first condition, $xy = ax + b$. (1)

By the second condition, $xy = cx - d$. (2)

Eliminating xy by comparison, $ax + b = cx - d$; (3)

whence, $$x = \frac{b+d}{c-a}. \quad (4)$$

Substituting (4) in (1) or in (2), $$y = \frac{ad+bc}{b+d}.$$

Hence, there are $\frac{b+d}{c-a}$ persons, and each should pay $\frac{ad+bc}{b+d}$ cents.

41. Let $x =$ number of gallons per hour discharged by larger pump,
and $y =$ number of gallons per hour discharged by smaller pump.

Then, $x + y = m$, (1)

and $$x = \frac{bm}{a}. \quad (2)$$

Substituting (2) in (1), $$y = \frac{m(a-b)}{a}. \quad (3)$$

Substituting 5 for a, 4 for b, and 1250 for m,
(2) becomes $x = 1000$,
and (3) becomes $y = 250$.

Hence, the larger pump discharges $\frac{bm}{a}$, or 1000, gallons per hour; and the smaller discharges $\frac{m(a-b)}{a}$, or 250, gallons per hour.

42. Let $x =$ number of miles per hour 1st train runs,
and $y =$ number of miles per hour 2d train runs.

Since they are m miles apart, and approach each other at the rate of $(x + y)$ miles per hour, meeting in b hours,

$$\frac{m}{x+y} = b. \quad (1)$$

Also, since the first train has traveled ax miles when the second starts by the 2d condition,

$$\frac{m-ax}{x+y} = c. \quad (2)$$

Reducing (1), $bx + by = m$. (3)

Reducing (2), $(a + c)x + cy = m$. (4)

Subtracting (3) from (4), and transposing the terms containing y,

$$(a - b + c)x = (b - c)y.$$

$$\therefore x = \frac{b-c}{a-b+c}y. \quad (5)$$

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Substituting (5) in (3), $\frac{b^2-bc}{a-b+c}y+\frac{ab-b^2+bc}{a-b+c}y=m.$

$$\therefore y=\frac{m(a-b+c)}{ab}. \quad (6)$$

Substituting (6) in (5), $$x=\frac{m(b-c)}{ab}. \quad (7)$$

Substituting 800 for m, 9 for c, $1\frac{3}{5}$ for a, and 10 for b, in (7) and (6),

$$x=50, \text{ and } y=30.$$

Hence, the rate of the train from A is $\frac{m(b-c)}{ab}$, or 50, miles per hour; and the rate of the train from B is $\frac{m(a-b+c)}{ab}$, or 30, miles per hour.

43. Let x = number of barrels alone needed,

and y = number of bags alone needed.

Then, $\frac{1}{x}$ = the amount one barrel holds,

and $\frac{1}{y}$ = the amount one bag holds.

From problem, $$\frac{c}{x}+\frac{d}{y}=1, \quad (1)$$

and $$\frac{a}{x}+\frac{b}{y}=\frac{m}{n}. \quad (2)$$

Multiplying (1) by b, $$\frac{bc}{x}+\frac{bd}{y}=b. \quad (3)$$

Multiplying (2) by d, $$\frac{ad}{x}+\frac{bd}{y}=\frac{dm}{n}. \quad (4)$$

Subtracting (4) from (3), $$\frac{bc-ad}{x}=b-\frac{dm}{n}.$$

$$\therefore x=\frac{bcn-adn}{bn-dm}. \quad (5)$$

Substituting (5) in (1), $$y=\frac{adn-bcn}{an-cm}. \quad (6)$$

Substituting the given values in (5), $x=\frac{(240-90)2}{30-15}=20.$

Substituting the given values in (6), $y=\frac{2(90-240)}{12-16}=75.$

Hence, $\frac{bcn-adn}{bn-dm}$ barrels alone were needed and $\frac{adn-bcn}{an-cm}$ bags alone. Also 20 barrels alone were needed and 75 bags alone.

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2. $\begin{cases} x + 3y - z = 10, & (1) \\ 2x + 5y + 4z = 57, & (2) \\ 3x - y + 2z = 15. & (3) \end{cases}$

(3) × 3, $9x - 3y + 6z = 45.$ (4)
(1) + (4), $10x + 5z = 55.$
$2x + z = 11.$ (5)
(3) × 5, $15x - 5y + 10z = 75.$ (6)
(2) + (6), $17x + 14z = 132.$ (7)
(5) × 14, $28x + 14z = 154.$ (8)
(8) − (7), $11x = 22.$
$x = 2.$ (9)
Substituting (9) in (5), $z = 7.$ (10)
Substituting (9) and (10) in (1),
$y = 5.$

3. $\begin{cases} x + y + z = 53, & (1) \\ x + 2y + 3z = 105, & (2) \\ x + 3y + 4z = 134. & (3) \end{cases}$

(2) − (1), $y + 2z = 52.$ (4)
(3) − (2), $y + z = 29.$ (5)
(4) − (5), $z = 23.$ (6)
Substituting (6) in (5), $y = 6.$ (7)
Substituting (7) and (6) in (1),
$x = 24.$

4. $\begin{cases} x - y + z = 30, & (1) \\ 3y - x - z = 12, & (2) \\ 7z - y + 2x = 141. & (3) \end{cases}$

(1) + (2), $2y = 42.$
$y = 21.$ (4)
Substituting (4) in (1),
$x + z = 51.$ (5)
(3) − (1), $x + 6z = 111.$ (6)
(6) − (5), $5z = 60.$
$z = 12.$ (7)
(5) − (7), $x = 39.$

5. $\begin{cases} 8x - 5y + 2z = 53, & (1) \\ x + y - z = 9, & (2) \\ 13x - 9y + 3z = 71. & (3) \end{cases}$

(2) × 2, $2x + 2y - 2z = 18.$ (4)
(1) + (4), $10x - 3y = 71.$ (5)
(2) × 3, $3x + 3y - 3z = 27.$ (6)
(3) + (6), $16x - 6y = 98.$
$8x - 3y = 49.$ (7)
(5) − (7), $2x = 22.$
$x = 11.$ (8)
Substituting (8) in (5), $y = 13.$ (9)
Substituting (8) and (9) in (2),
$z = 15.$

6. $\begin{cases} x + 3y + 4z = 83, & (1) \\ x + y + z = 29, & (2) \\ 6x + 8y + 3z = 156. & (3) \end{cases}$

(1) − (2), $2y + 3z = 54.$ (4)
(2) × 6, $6x + 6y + 6z = 174.$ (5)
(3) − (5), $2y - 3z = -18.$ (6)
(4) + (6), $4y = 36.$
$y = 9.$ (7)
(4) − (6), $6z = 72.$
$z = 12.$ (8)
(2) − (7) − (8), $x = 8.$

7. $\begin{cases} 2x + 3y + 4z = 29, & (1) \\ 3x + 2y + 5z = 32, & (2) \\ 4x + 3y + 2z = 25. & (3) \end{cases}$

(3) − (2), $x + y - 3z = -7.$ (4)
(1) + (3), $6x + 6y + 6z = 54.$
$x + y + z = 9.$ (5)
(5) − (4), $4z = 16.$
$z = 4.$ (6)
(2) − (1), $x - y + z = 3.$ (7)
(5) − (7), $2y = 6.$
$y = 3.$ (8)
(5) − (6) − (8), $x = 2.$

8. $\begin{cases} 3x - 2y + z = 2, & (1) \\ 2x + 5y + 2z = 27, & (2) \\ x + 3y + 3z = 25. & (3) \end{cases}$

(1) + (2), $5x + 3y + 3z = 29.$ (4)
(4) − (3), $4x = 4.$
$x = 1.$ (5)
Substituting (5) in (1),
$-2y + z = -1.$ (6)
Substituting (5) in (3),
$3y + 3z = 24.$
$y + z = 8.$ (7)
(7) − (6), $3y = 9.$
$y = 3.$ (8)
(7) − (8), $z = 5.$

9. $\begin{cases} 2x - 3y + 4z - v = 4, & (1) \\ 4x + 2y - z + 2v = 13, & (2) \\ x - y + 2z + 3v = 17, & (3) \\ 3x + 2y - z + 4v = 20. & (4) \end{cases}$

(1) × 2, $4x - 6y + 8z - 2v = 8.$ (5)
(5) + (2), $8x - 4y + 7z = 21.$ (6)
(2) × 2, $8x + 4y - 2z + 4v = 26.$ (7)
(7) − (4), $5x + 2y - z = 6.$ (8)
(1) × 3, $6x - 9y + 12z - 3v = 12.$ (9)
(3) + (9), $7x - 10y + 14z = 29.$ (10)
(6) × 2, $16x - 8y + 14z = 42.$ (11)
(11) − (10), $9x + 2y = 13.$ (12)

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(8) × 7, $35x + 14y - 7z = 42$. (13)
(6) + (13), $43x + 10y = 63$. (14)
(12) × 5, $45x + 10y = 65$. (15)
(15) − (14), $2x = 2$.
$x = 1$. (16)
Substituting (16) in (12),
$y = 2$. (17)
Substituting (16) and (17) in (8),
$z = 3$. (18)
Substituting (16), (17), and (18) in (1), $v = 4$.

10. $\begin{cases} 4x - 5y + 3z = 14, & (1) \\ x + 7y - z = 13, & (2) \\ 2x + 5y + 5z = 36. & (3) \end{cases}$

(2) × 2, $2x + 14y - 2z = 26$. (4)
(4) − (3), $9y - 7z = -10$. (5)
(3) × 2, $4x + 10y + 10z = 72$. (6)
(6) − (1), $15y + 7z = 58$. (7)
(5) + (7), $24y = 48$.
$y = 2$. (8)
Substituting (8) in (7),
$7z = 28$.
$z = 4$. (9)
Substituting (8) and (9) in (2),
$x = 3$.

11. $\begin{cases} 2x + y - 3z + 4w = 44, & (1) \\ 3x - 2y + z - w = -1, & (2) \\ 4x - y + 2z + w = 55, & (3) \\ 5x - 3y + 4z - w = 39. & (4) \end{cases}$

To eliminate w. (2) × 4,
$12x - 8y + 4z - 4w = -4$; (5)
(1) + (5), $14x - 7y + z = 40$; (6)
(2) + (3), $7x - 3y + 3z = 54$; (7)
(4) − (2), $2x - y + 3z = 40$. (8)
To eliminate x from (6), (7), & (8).
(7) × 2, $14x - 6y + 6z = 108$; (9)
(9) − (6), $y + 5z = 68$; (10)
(8) × 7, $14x - 7y + 21z = 280$; (11)
(11) − (6), $20z = 240$;
$z = 12$. (12)
To eliminate z from (10) & (12).
Substituting (12) in (10),
$y = 8$. (13)
Substituting (12) and (13) in (8),
$x = 6$. (14)
Substituting (14), (13), and (12) in (3), $w = 15$.

12. $\begin{cases} 7x - 1 = 3y, & (1) \\ 11z - 1 = 7v, & (2) \\ 4z - 1 = 7y, & (3) \\ 19x - 1 = 3v. & (4) \end{cases}$

(2) − (3), $7z = 7(v - y)$.
$z = v - y$. (5)
(4) − (1), $12x = 3v - 3y$.
$4x = v - y$. (6)
From (5) and (6), $z = 4x$. (7)
(1) × 7, $49x - 7 = 21y$. (8)
(2) × 3, $33z - 3 = 21v$. (9)
Substituting (7) in (9),
$132x - 3 = 21v$. (10)
(10) − (8), $83x + 4 = 21(v - y)$. (11)
Substituting (6) in (11),
$83x + 4 = 21(4x)$.
$x = 4$. (12)
Substituting (12) in (7),
$z = 16$.
Substituting (12) in (1),
$y = 9$.
Substituting (12) in (4),
$v = 25$.

13. $\begin{cases} x + \frac{1}{2}y + \frac{1}{3}z = 32, & (1) \\ \frac{1}{3}x + \frac{1}{4}y + \frac{1}{5}z = 15, & (2) \\ \frac{1}{4}x + \frac{1}{5}y + \frac{1}{6}z = 12. & (3) \end{cases}$

Clearing of fractions,
$6x + 3y + 2z = 192$, (4)
$20x + 15y + 12z = 900$, (5)
$15x + 12y + 10z = 720$. (6)
(5) − (4), $14x + 12y + 10z = 708$. (7)
(6) − (7), $x = 12$. (8)
Substituting (8) in (4) and (6),
$3y + 2z = 120$, (9)
and $12y + 10z = 540$. (10)
(9) × 5, $15y + 10z = 600$. (11)
(11) − (10), $3y = 60$.
$y = 20$. (12)
Substituting (12) in (9),
$z = 30$.

14. $\begin{cases} \frac{1}{6}x - \frac{1}{5}y + \frac{1}{4}z = 3, & (1) \\ \frac{1}{5}x - \frac{1}{4}y + \frac{1}{5}z = 1, & (2) \\ \frac{1}{4}x - \frac{1}{3}y + \frac{1}{2}z = 5. & (3) \end{cases}$

(1) × 2, $\frac{1}{3}x - \frac{2}{5}y + \frac{1}{2}z = 6$. (4)
(4) − (3), $\frac{1}{12}x - \frac{1}{15}y = 1$. (5)
(2) × $\frac{5}{2}$, $\frac{1}{2}x - \frac{5}{8}y + \frac{1}{2}z = \frac{5}{2}$. (6)
(6) − (3), $\frac{1}{4}x - \frac{7}{24}y = -\frac{5}{2}$. (7)
(5) × 3, $\frac{1}{4}x - \frac{1}{5}y = 3$. (8)
(8) − (7), $\frac{11}{120}y = \frac{11}{2}$.
$y = 60$. (9)
Substituting (9) in (8),
$x = 60$. (10)
Substituting (9) and (10) in (1),
$z = 20$.

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15. $\begin{cases} \dfrac{x+y}{3} + 3z = 29, & (1) \\ \dfrac{2x-y}{2} + 2z = 22, & (2) \\ 3x - y = 3(z-1). & (3) \end{cases}$

Reducing the given equations,

$$x + y + 9z = 87, \quad (4)$$
$$2x - y + 4z = 44, \quad (5)$$
$$3x - y - 3z = -3. \quad (6)$$

(4) + (5), $3x + 13z = 131.$ (7)

(6) − (5), $x - 7z = -47.$ (8)

(8) × 3, $3x - 21z = -141.$ (9)

(7) − (9), $34z = 272.$

$$z = 8. \quad (10)$$

Substituting (10) in (8),

$$x = 9. \quad (11)$$

Substituting (10) and (11) in (3),

$$y = 6.$$

16. $\begin{cases} 3x + y - z + 2v = 0, & (1) \\ 3y - 2x + z - 4v = 21, & (2) \\ x - y + 2z - 3v = 6, & (3) \\ 4x + 2y - 3z + v = 12. & (4) \end{cases}$

To eliminate z.

(1) + (2), $x + 4y - 2v = 21;$ (5)

adding (2), (3), and (4),

$$3x + 4y - 6v = 39; \quad (6)$$

adding (1) × 2 to (3),

$$7x + y + v = 6. \quad (7)$$

To eliminate y from (5), (6), & (7).

(6) − (5), $2x - 4v = 18,$

or $x - 2v = 9.$ (8)

Subtracting (6) from (7) × 4,

$$25x + 10v = -15,$$

or $5x + 2v = -3,$ (9)

(8) + (9), $6x = 6.$

$$x = 1. \quad (10)$$

Substituting (10) in (9),

$$v = -4. \quad (11)$$

Substituting (10) and (11) in (7),

$$y = 3. \quad (12)$$

Substituting (10), (11), and (12) in (2), $z = -2.$

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18. $\begin{cases} x + y = 9, & (1) \\ y + z = 7, & (2) \\ z + x = 5. & (3) \end{cases}$

Adding the given equations,

$$2x + 2y + 2z = 21,$$
$$x + y + z = \tfrac{21}{2}. \quad (4)$$

Subtracting (2), (3), and (1), successively, from (4),

$$x = 3\tfrac{1}{2},\ y = 5\tfrac{1}{2},\ \text{and } z = 1\tfrac{1}{2}.$$

19. $\begin{cases} v + x + y = 15, & (1) \\ x + y + z = 18, & (2) \\ y + z + v = 17, & (3) \\ z + v + x = 16. & (4) \end{cases}$

Adding the given equations,

$$3v + 3x + 3y + 3z = 66.$$
$$v + x + y + z = 22. \quad (5)$$

Subtracting (2), (3), (4), and (1), successively, from (5),

$$v = 4,\ x = 5,\ y = 6,\ \text{and } z = 7.$$

20. $\begin{cases} \dfrac{1}{x} + \dfrac{1}{y} = 6, & (1) \\ \dfrac{1}{y} + \dfrac{1}{z} = 10, & (2) \\ \dfrac{1}{z} + \dfrac{1}{x} = 8. & (3) \end{cases}$

Adding the given equations,

$$\frac{2}{x} + \frac{2}{y} + \frac{2}{z} = 24.$$
$$\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 12. \quad (4)$$

Subtracting (2), (3), and (1) successively, from (4),

$$\frac{1}{x} = 2,\ \frac{1}{y} = 4,\ \text{and } \frac{1}{z} = 6.$$
$$\therefore x = \tfrac{1}{2},\ y = \tfrac{1}{4},\ \text{and } z = \tfrac{1}{6}.$$

21. $\begin{cases} \dfrac{xy}{x+y} = \dfrac{1}{5}, & (1) \\ \dfrac{yz}{y+z} = \dfrac{1}{6}, & (2) \\ \dfrac{zx}{z+x} = \dfrac{1}{7}. & (3) \end{cases}$

Taking reciprocals Ax. 5,

From (1), $\dfrac{1}{y} + \dfrac{1}{x} = 5,$ (4)

From (2), $\dfrac{1}{z} + \dfrac{1}{y} = 6,$ (5)

From (3), $\dfrac{1}{x} + \dfrac{1}{z} = 7.$ (6)

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Adding (4), (5), and (6),

$$\frac{2}{x}+\frac{2}{y}+\frac{2}{z}=18.$$

$$\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=9. \quad (7)$$

Subtracting (5), (6), and (4), successively from (7),

$$\frac{1}{x}=3,\ \frac{1}{y}=2,\text{ and }\frac{1}{z}=4.$$

$$\therefore x=\tfrac{1}{3},\ y=\tfrac{1}{2},\text{ and }z=\tfrac{1}{4}.$$

22.
$$\begin{cases} x+3y+z=14, & (1)\\ x+y+3z=16, & (2)\\ 3x+y+z=20. & (3)\end{cases}$$

Adding the given equations,

$$5x+5y+5z=50.$$

$$x+y+z=10. \quad (4)$$

Subtracting (4) from (3), (1), and (2) in succession,

$$2x=10,\ 2y=4,\text{ and }2z=6.$$

$$\therefore x=5,\ y=2,\text{ and }z=3.$$

23.
$$\begin{cases} y+z+v-x=22, & (1)\\ z+v+x-y=18, & (2)\\ v+x+y-z=14, & (3)\\ x+y+z-v=10. & (4)\end{cases}$$

Adding the given equations,

$$2v+2x+2y+2z=64.$$

$$v+x+y+z=32. \quad (5)$$

Subtracting (4), (1), (2), and (3), in succession, from (5),

$$2v=22,\ 2x=10,\ 2y=14,\text{ and }2z=18.$$

$$\therefore v=11,\ x=5,\ y=7,\text{ and }z=9.$$

24.
$$\begin{cases} \frac{1}{x}+\frac{1}{y}-1=0, & (1)\\ \frac{1}{y}+\frac{1}{z}+3=0, & (2)\\ \frac{1}{z}+\frac{1}{x}-2=0. & (3)\end{cases}$$

Adding the given equations,

$$\frac{2}{x}+\frac{2}{y}+\frac{2}{z}=0,$$

$$\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=0. \quad (4)$$

Subtracting (2), (3), and (1), in succession, from (4),

$$\frac{1}{x}-3=0, \quad (5)$$

$$\frac{1}{y}+2=0, \quad (6)$$

and $$\frac{1}{z}+1=0. \quad (7)$$

From (5), (6), and (7),

$$x=\tfrac{1}{3},\ y=-\tfrac{1}{2},\text{ and }z=-1.$$

25.
$$\begin{cases} \frac{xy}{x+y}=\frac{1}{8}, & (1)\\ \frac{yz}{y+z}=\frac{1}{4}, & (2)\\ \frac{zx}{z+x}=\frac{1}{2}. & (3)\end{cases}$$

Taking the reciprocals of both members of each equation, Ax. 5,

$$\frac{1}{y}+\frac{1}{x}=8, \quad (4)$$

$$\frac{1}{z}+\frac{1}{y}=4, \quad (5)$$

$$\frac{1}{x}+\frac{1}{z}=2. \quad (6)$$

Adding (4), (5), and (6),

$$\frac{2}{x}+\frac{2}{y}+\frac{2}{z}=14. \quad (7)$$

Subtracting (5), (6), and (4), each multiplied by 2, in succession, from (7),

$$\frac{2}{x}=6,$$

whence, $x=\frac{1}{3}$;

$$\frac{2}{y}=10,$$

whence, $y=\frac{1}{5}$;

and $\frac{2}{z}=-2,$

whence $z=-1.$

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27.
$$\begin{cases} axy - x - y = 0, & (1) \\ bzx - z - x = 0, & (2) \\ cyz - y - z = 0. & (3) \end{cases}$$

Dividing (1) by xy, (2) by zx, and (3) by yz,

$$a - \frac{1}{y} - \frac{1}{x} = 0, \quad (4)$$

$$b - \frac{1}{x} - \frac{1}{z} = 0. \quad (5)$$

$$c - \frac{1}{z} - \frac{1}{y} = 0. \quad (6)$$

Adding (4), (5), and (6),

$$\frac{2}{x} + \frac{2}{y} + \frac{2}{z} = a + b + c. \quad (7)$$

Adding (6), (5), and (4), each multiplied by 2, in succession, to (7),

$$\frac{2}{x} = a + b - c,$$

whence, $$x = \frac{2}{a + b - c};$$

$$\frac{2}{y} = a - b + c,$$

whence, $$y = \frac{2}{a - b + c};$$

$$\frac{2}{z} = b + c - a,$$

whence, $$z = \frac{2}{b + c - a}.$$

28.
$$\begin{cases} x + y - z = 0, & (1) \\ x - y = 2b, & (2) \\ x + z = 3a + b. & (3) \end{cases}$$

Adding the given equations, $$3x = 3a + 3b.$$

$$x = a + b. \quad (4)$$

Subtracting (4) from (3), $$z = 2a.$$

Subtracting (2) from (4), $$y = a - b.$$

29.
$$\begin{cases} v + x = 2a, & (1) \\ x + y = 2a - z, & (2) \\ y + z = a + b, & (3) \\ v - z = a + c. & (4) \end{cases}$$

Subtracting (3) from (2), $$x - z = a - b - z.$$

$$x = a - b. \quad (5)$$

Subtracting (5) from (1), $$v = a + b. \quad (6)$$

Subtracting (4) from (6), $$z = b - c. \quad (7)$$

Subtracting (7) from (3), $$y = a + c.$$

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30.
$$\begin{cases} y+z-3x=2a, & (1)\\ z+x-3y=2b, & (2)\\ x+y-3z=2c, & (3)\\ 2x+2y+v=0. & (4)\end{cases}$$

Adding (1), (2), and (3), $-x-y-z=2a+2b+2c.$ (5)

Adding (1) and (5), $-4x=4a+2b+2c.$

$\therefore x=-\frac{1}{2}(2a+b+c).$ (6)

Adding (2) and (5), $-4y=2a+4b+2c.$

$\therefore y=-\frac{1}{2}(2b+c+a).$ (7)

Adding (3) and (5), $-4z=2a+2b+4c.$

$\therefore z=-\frac{1}{2}(2c+a+b).$

Substituting (6) and (7) in (4), $v=3a+3b+2c.$

31.
$$\begin{cases} abxyz+cxy-ayz-bzx=0, & (1)\\ bcxyz+ayz-bzx-cxy=0, & (2)\\ caxyz+bzx-cxy-ayz=0. & (3)\end{cases}$$

Dividing each equation by xyz, transposing, etc.,

$$\frac{a}{x}+\frac{b}{y}-\frac{c}{z}=ab, \quad (4)$$

$$-\frac{a}{x}+\frac{b}{y}+\frac{c}{z}=bc, \quad (5)$$

and
$$\frac{a}{x}-\frac{b}{y}+\frac{c}{z}=ca. \quad (6)$$

Adding (4), (5), and (6), $\frac{a}{x}+\frac{b}{y}+\frac{c}{z}=ab+bc+ca.$ (7)

Subtracting (5) from (7), $\frac{2a}{x}=ab+ca.$

$$x=\frac{2}{b+c}.$$

Subtracting (6) from (7), $\frac{2b}{y}=ab+bc.$

$$y=\frac{2}{a+c}.$$

Subtracting (4) from (7), $\frac{2c}{z}=bc+ca.$ $z=\frac{2}{a+b}.$

32.
$$\begin{cases} x+y+z=a+b+c, & (1)\\ x+2y+3z=b+2c, & (2)\\ x+3y+4z=b+3c. & (3)\end{cases}$$

Subtracting (1) from (2), $y+2z=c-a.$ (4)

Subtracting (2) from (3), $y+z=c.$ (5)

Subtracting (5) from (4), $z=-a.$ (6)

Subtracting (6) from (5), $y=a+c.$

Subtracting (5) from (1), $x=a+b.$

33.
$$\begin{cases} v+x+y=a+2b+c, & (1)\\ x+y+z=3b, & (2)\\ y+z+v=a+b, & (3)\\ z+v+x=a+3b-c. & (4)\end{cases}$$

Adding given equations, $3v+3x+3y+3z=3a+9b.$

$v+x+y+z=a+3b.$ (5)

Subtracting (2), (3), (4), and (1) successively from (5),

$v=a,\ x=2b,\ y=c,$ and $z=b-c.$

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34.
$$\begin{cases} ax + by + cz = 3, & (1) \\ x + y = \dfrac{a+b}{ab}, & (2) \\ y + z = \dfrac{b+c}{bc}. & (3) \end{cases}$$

Multiplying (3) by c, $$cy + cz = \frac{b+c}{b} = 1 + \frac{c}{b}. \quad (4)$$

Subtracting (4) from (1), $$ax + (b-c)y = 2 - \frac{c}{b}. \quad (5)$$

Multiplying (2) by a, $$ax + ay = 1 + \frac{a}{b}. \quad (6)$$

Subtracting (6) from (5), $$(b-c-a)\,y = 1 - \frac{c}{b} - \frac{a}{b} = \frac{b-c-a}{b}.$$

$$y = \frac{1}{b}. \quad (7)$$

Substituting (7) in (2) and in (3), $x = \dfrac{1}{a}$ and $z = \dfrac{1}{c}$.

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1. Let $x =$ first number,
$y =$ second number,
and $z =$ third number.

Then, $$\tfrac{1}{2}x + \tfrac{1}{3}y + \tfrac{1}{4}z = 12, \quad (1)$$
$$\tfrac{1}{3}x + \tfrac{1}{4}y + \tfrac{1}{5}z = 9, \quad (2)$$
and $$x + y + z = 38. \quad (3)$$

Subtracting (3) successively from (1) × 4 and (2) × 5,
$$x + \tfrac{1}{3}y = 10, \quad (4)$$
and $$\tfrac{2}{3}x + \tfrac{1}{4}y = 7. \quad (5)$$

Subtracting (4) from (5) × $\frac{3}{2}$, $\frac{1}{24}y = \frac{1}{2}$.
$\therefore y = 12$, second number,
whence, from (4), $x = 6$, first number,
and from (3), $z = 20$, third number.

2. Let $x =$ first part,
$y =$ second part,
and $z =$ third part.

Then, $$x + y + z = 800, \quad (1)$$
$$x + \tfrac{1}{2}y + \tfrac{2}{5}z = 400, \quad (2)$$
and $$\tfrac{3}{4}x + y + \tfrac{1}{4}z = 400. \quad (3)$$

Subtracting (1) from (2) × 2, and (3) from (1),
$$x - \tfrac{1}{5}z = 0, \quad (4)$$
and $$\tfrac{1}{4}x + \tfrac{3}{4}z = 400. \quad (5)$$

From (4), $$z = 5x. \quad (6)$$

Substituting (6) in (5), $\frac{1}{4}x + \frac{15}{4}x = 400$.
$\therefore x = 100$, first part,
whence, from (6), $z = 500$, third part,
and from (1), $y = 200$, second part.

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3. Let $x =$ number of days it will take A,
$y =$ number of days it will take B,
and $z =$ number of days it will take C.

Then,
$$\frac{1}{x} + \frac{1}{y} = \frac{1}{10}, \tag{1}$$
$$\frac{1}{x} + \frac{1}{z} = \frac{1}{8}, \tag{2}$$
and
$$\frac{1}{y} + \frac{1}{z} = \frac{1}{12}. \tag{3}$$

Adding the given equations and dividing the result by 2,
$$\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = \frac{37}{240}. \tag{4}$$

Subtracting (3), (2), and (1) successively from (4), and solving,
$$x = 14\tfrac{2}{17},$$
$$y = 34\tfrac{2}{7},$$
$$z = 18\tfrac{6}{13}.$$

Hence, it will take A $14\frac{2}{17}$ days, B $34\frac{2}{7}$ days, and C $18\frac{6}{13}$ days.

4. Let $x =$ number of miles from A to B,
$y =$ number of miles from B to C,
and $z =$ number of miles from C to A.

Then,
$$z + y = 130, \tag{1}$$
$$x + z = 110, \tag{2}$$
and
$$y + x = 140. \tag{3}$$

Adding the given equations and dividing by 2,
$$x + y + z = 190. \tag{4}$$

Subtracting (1), (2), and (3) successively from (4),
$$x = 60, \quad y = 80, \quad z = 50.$$

Hence, B is 60 miles from A, C 80 miles from B, and A 50 miles from C.

5. Let $x =$ digit in hundreds' place,
$y =$ digit in tens' place,
and $z =$ digit in units' place.

Then,
$$x + y + z = 14, \tag{1}$$
$$100\,x + 10\,y + z + 693 = 100\,z + 10\,y + x, \tag{2}$$
and
$$z = y + 6. \tag{3}$$

Reducing (2), $x - z = -7.$ (4)

Subtracting (4) from (1), $y + 2\,z = 21.$ (5)

Adding (3) and (5), $3\,z = 27.$

$\therefore z = 9,$ (6)

whence, in (3), $y = 3.$ (7)

Substituting (6) and (7) in (1), $x = 2.$

Hence, the number is 239.

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6. Let $x =$ number of dollars A has,
$y =$ number of dollars B has,
and $z =$ number of dollars C has.

Then,
$$x - 100 = y + 100, \quad (1)$$
$$2(x - 100) = z + 100, \quad (2)$$
and
$$4(y - 100) = z + 100. \quad (3)$$

Eliminating z between (2) and (3) by comparison,
$$x - 100 = 2(y - 100). \quad (4)$$

Eliminating x between (4) and (1) by comparison,
$$2(y - 100) = y + 100.$$
$$\therefore y = 300,$$
whence, by substitution, $x = 500$, and $z = 700$.

Hence, A has \$500, B \$300, and C \$700.

7. Let $x =$ number of gallons first jar holds,
$y =$ number of gallons second jar holds,
and $z =$ number of gallons third jar holds.

Then,
$$x + y + z = 4z, \quad (1)$$
$$x + y + z = 2x + 4, \quad (2)$$
$$x + y + z = 3y + 2. \quad (3)$$

From (1), $x = 3z - y$. (4)

From (2), $x = y + z - 4$. (5)

Eliminating x between (4) and (5), transposing, etc.,
$$y - z = 2. \quad (6)$$

From (3), $x = 2y - z + 2$. (7)

Eliminating x between (5) and (7), and transposing,
$$y - 2z = -6. \quad (8)$$

Subtracting (8) from (6), $z = 8$,
whence, by substitution, $y = 10$, and $x = 14$.

Hence, the capacity of the largest jar is 14 gallons, of the second, 10 gallons, and of the third, 8 gallons.

8. From problem, $3a + 3b + 2c + 2d = 4230$, (1)
$$2a + 2b + 3c + 3d = 4320, \quad (2)$$
$$3a + 2b + 2c + 2d = 3870, \quad (3)$$
and
$$a + 2b + 2c + d = 2470. \quad (4)$$

Multiplying (1) by 3, $9a + 9b + 6c + 6d = 12690$. (5)

Multiplying (2) by 2, $4a + 4b + 6c + 6d = 8640$. (6)

Subtracting (6) from (5), $5a + 5b = 4050$. (7)

Subtracting (3) from (1), $b = 360$. (8)

Substituting (8) in (7), $a = 450$. (9)

Subtracting (4) from (3), $2a + d = 1400$. (10)

Substituting (9) in (10), $d = 500$. (11)

Substituting (8), (9), and (11) in (1), $c = 400$.

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GRAPHIC SOLUTIONS

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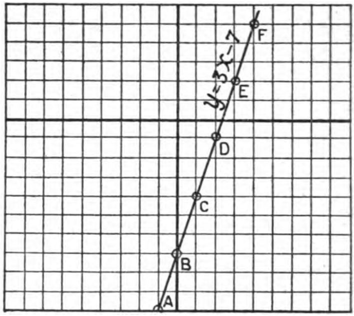
16. $y = 3x - 7$.

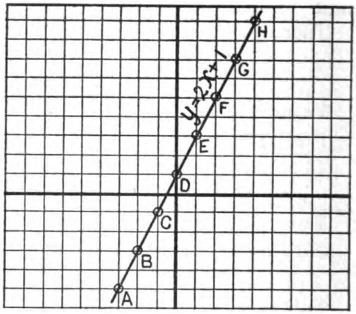
x	y	POINT
-1	-10	A
0	-7	B
1	-4	C
2	-1	D
3	2	E
4	5	F



A line drawn through A, B, C, D, etc., is the graph of $y = 3x - 7$.

17. $y = 2x + 1$.

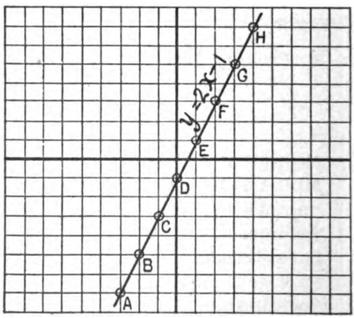
x	y	POINT
-3	-5	A
-2	-3	B
-1	-1	C
0	1	D
1	3	E
2	5	F
3	7	G
4	9	H



A line drawn through A, B, C, D, etc., is the graph of $y = 2x + 1$.

18. $y = 2x - 1$.

x	y	POINT
-3	-7	A
-2	-5	B
-1	-3	C
0	-1	D
1	1	E
2	3	F
3	5	G
4	7	H



A line drawn through A, B, C, D, etc.. is the graph of $y = 2x - 1$.

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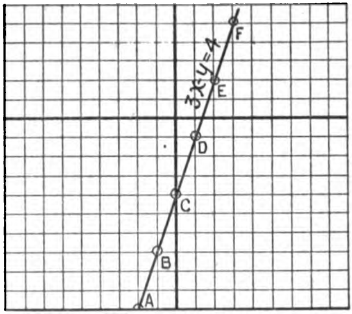
19. Solving for y,

$$y = 3x - 4.$$

x	y	POINT
-2	-10	A
-1	-7	B
0	-4	C
1	-1	D
2	2	E
3	5	F

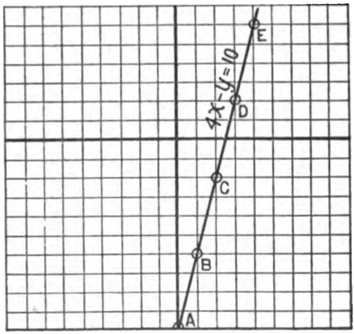


A line drawn through A, B, C, D, etc., is the graph of $3x - y = 4$.

20. Solving for y,

$$y = 4x - 10.$$

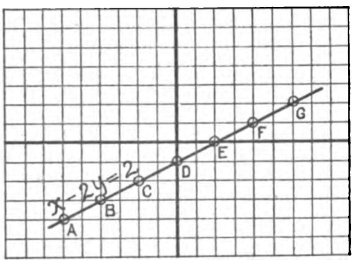
x	y	POINT
0	-10	A
1	-6	B
2	-2	C
3	2	D
4	6	E



A line drawn through A, B, C, D, etc., is the graph of $4x - y = 10$.

21. Solving for y, $y = \frac{1}{2}(x - 2)$.

x	y	POINT
-6	-4	A
-4	-3	B
-2	-2	C
0	-1	D
2	0	E
4	1	F
6	2	G



A line drawn through A, B, C, D, etc., is the graph of $x - 2y = 2$.

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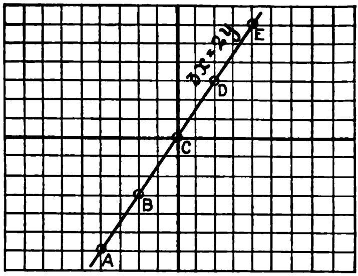
22. Solving for y,

$$y = \tfrac{3}{2}x.$$

x	y	Point
-4	-6	A
-2	-3	B
0	0	C
2	3	D
4	6	E

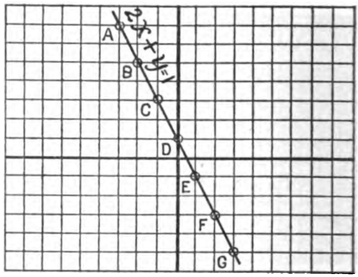


A line drawn through A, B, C, D, etc., is the graph of $3x = 2y$.

23. Solving for y,

$$y = 1 - 2x.$$

x	y	Point
-3	7	A
-2	5	B
-1	3	C
0	1	D
1	-1	E
2	-3	F
3	-5	G

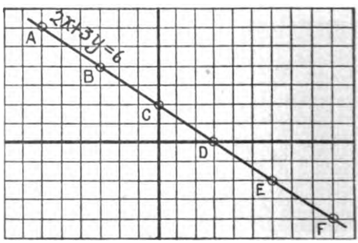


A line drawn through A, B, C, D, etc., is the graph of $2x + y = 1$.

24. Solving for y,

$$y = 2 - \tfrac{2}{3}x.$$

x	y	Point
-6	6	A
-3	4	B
0	2	C
3	0	D
6	-2	E
9	-4	F



A line drawn through A, B, C, D, etc., is the graph of $2x + 3y = 6$.

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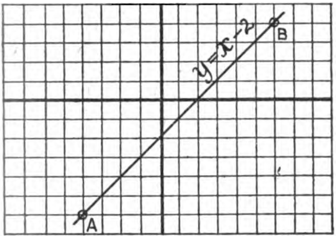
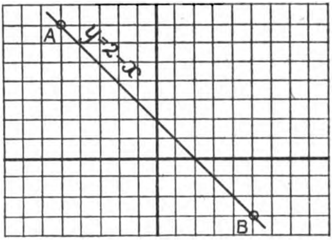
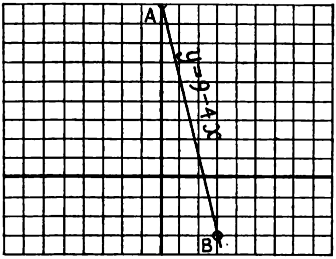
1. $y = x - 2.$
When $x = -4, y = -6;$
when $x = 6, y = 4.$
Locate $A = (-4, -6), B = (6, 4).$
A straight line drawn through A and B is the graph of $y = x - 2.$



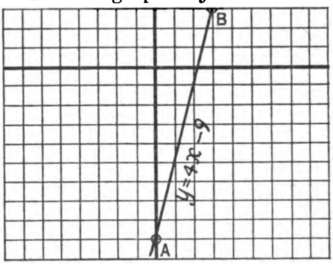
2. $y = 2 - x.$
When $x = -5, y = 7;$
when $x = 5, y = -3.$
Locate $A = (-5, 7), B = (5, -3).$
A straight line drawn through A and B is the graph of $y = 2 - x.$



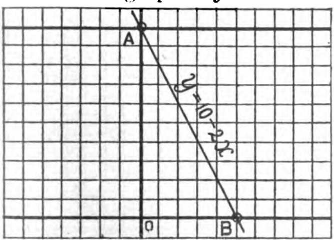
3. $y = 9 - 4x.$
When $x = 0, y = 9;$
when $x = 3, y = -3.$
Locate $A = (0, 9), B = (3, -3).$
A straight line drawn through A and B is the graph of $y = 9 - 4x.$



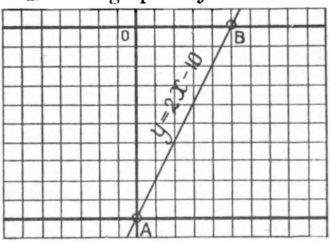
4. $y = 4x - 9.$
When $x = 0, y = -9;$
when $x = 3, y = 3.$
Locate $A = (0, -9), B = (3, 3).$
A straight line drawn through A and B is the graph of $y = 4x - 9.$



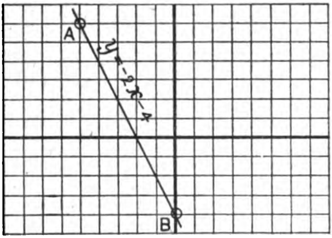
5. $y = 10 - 2x.$
When $x = 0, y = 10;$
when $y = 0, x = 5.$
Locate $A = (0, 10), B = (5, 0).$
A straight line drawn through A and B is the graph of $y = 10 - 2x.$



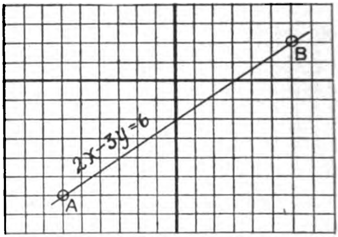
6. $y = 2x - 10.$
When $x = 0, y = -10;$
when $y = 0, x = 5.$
Locate $A = (0, -10), B = (5, 0).$
A straight line drawn through A and B is the graph of $y = 2x - 10.$



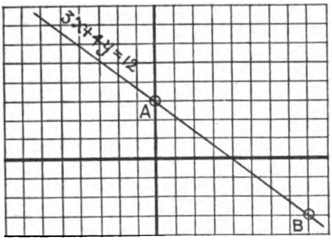
7. $y = -2x - 4.$
When $x = -5,\ y = 6;$
when $x = 0,\ y = -4.$
Locate $A = (-5, 6),\ B = (0, -4).$
A straight line drawn through A and B is the graph of $y = -2x - 4.$



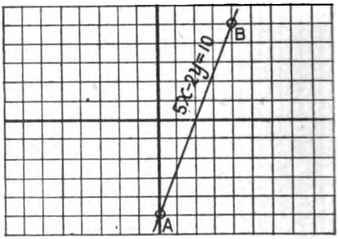
8. Solving for y, $y = \frac{2}{3}x - 2.$
When $x = -6,\ y = -6;$
when $x = 6,\ y = 2.$
Locate $A = (-6, -6),\ B = (6, 2).$
A straight line drawn through A and B is the graph of $2x - 3y = 6.$



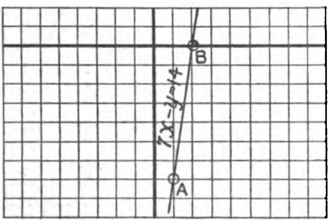
9. Solving for y, $y = 3 - \frac{3}{4}x.$
When $x = 0,\ y = 3;$
when $x = 8,\ y = -3.$
Locate $A = (0, 3),\ B = (8, -3).$
A straight line drawn through A and B is the graph of $3x + 4y = 12.$



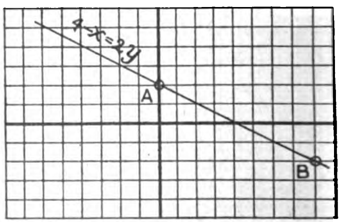
10. Solving for y, $y = \frac{5}{2}x - 5.$
When $x = 0,\ y = -5;$
when $x = 4,\ y = 5.$
Locate $A = (0, -5),\ B = (4, 5).$
A straight line drawn through A and B is the graph of $5x - 2y = 10.$



11. Solving for y, $y = 7x - 14.$
When $x = 1,\ y = -7;$
when $x = 2,\ y = 0.$
Locate $A = (1, -7),\ B = (2, 0).$
A straight line drawn through A and B is the graph of $7x - y = 14.$



12. Solving for y, $y = 2 - \frac{1}{2}x.$
When $x = 0,\ y = 2;$
when $x = 8,\ y = -2.$
Locate $A = (0, 2),\ B = (8, -2).$
A straight line drawn through A and B is the graph of $4 - x = 2y.$



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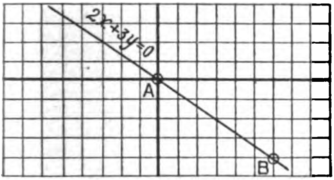
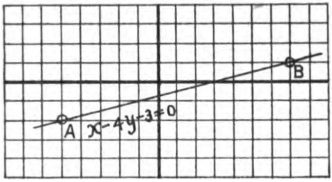
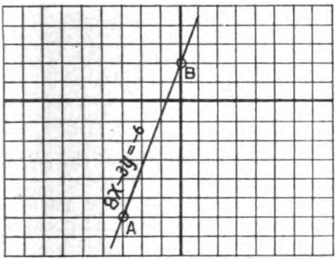
13. Solving for y, $y = -\frac{2}{3}x$.
When $x = 0$, $y = 0$;
when $x = 6$, $y = -4$.
Locate $A = (0, 0)$, $B = (6, -4)$.
A straight line drawn through A and B is the graph of $2x + 3y = 0$



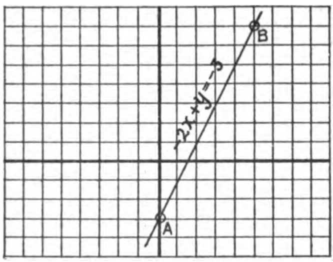
14. Solving for y, $y = \frac{1}{4}(x - 3)$.
When $x = -5$, $y = -2$;
when $x = 7$, $y = 1$.
Locate $A = (-5, -2)$, $B = (7, 1)$.
A straight line drawn through A and B is the graph of $x - 4y - 3 = 0$.



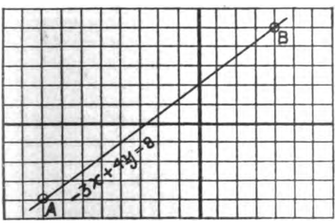
15. Solving for y, $y = \frac{8}{3}x + 2$.
When $x = -3$, $y = -6$;
when $x = 0$, $y = 2$.
Locate $A = (-3, -6)$, $B = (0, 2)$.
A straight line drawn through A and B is the graph of $8x - 3y = -6$.



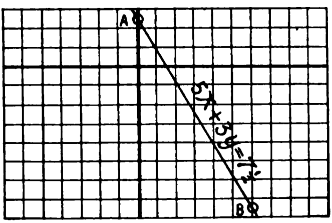
16. Solving for y, $y = 2x - 3$.
When $x = 0$, $y = -3$;
when $x = 5$, $y = 7$.
Locate $A = (0, -3)$, $B = (5, 7)$.
A straight line drawn through A and B is the graph of $-2x + y = -3$.



17. Solving for y, $y = \frac{3}{4}x + 2$.
When $x = -8$, $y = -4$;
when $x = 4$, $y = 5$.
Locate $A = (-8, -4)$, $B = (4, 5)$.
A straight line drawn through A and B is the graph of $-3x + 4y = 8$.



18. Solving for y, $y = \frac{5}{2} - \frac{5}{3}x$.
When $x = 0$, $y = 2\frac{1}{2}$;
when $x = 6$, $y = -7\frac{1}{2}$.
Locate $A = (0, 2\frac{1}{2})$, $B = (6, -7\frac{1}{2})$.
A straight line drawn through A and B is the graph of $5x + 3y = 7\frac{1}{2}$.



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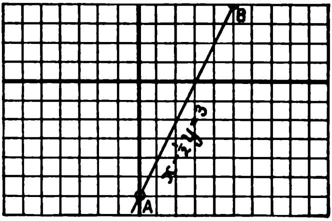
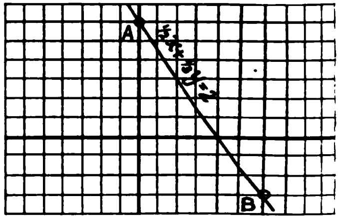
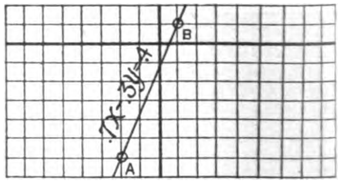
19. Solving for y, $y = 2x - 6$.
When $x = 0$, $y = -6$;
when $x = 5$, $y = 4$.
Locate $A = (0, -6)$, $B = (5, 4)$.
A straight line drawn through A and B is the graph of $x - \frac{1}{2}y = 3$.



20. Solving for y, $y = 6 - \frac{3}{2}x$.
When $x = 0$, $y = 6$;
when $x = 6$, $y = -3$.
Locate $A = (0, 6)$, $B = (6, -3)$.
A straight line drawn through A and B is the graph of $\frac{1}{2}x + \frac{1}{3}y = 2$.



21. Solving for y, $y = \frac{1}{3}(7x - 4)$.
When $x = -2$, $y = -6$;
when $x = 1$, $y = 1$.
Locate $A = (-2, -6)$, $B = (1, 1)$.
A straight line drawn through A and B is the graph of $.7x - .3y = .4$.



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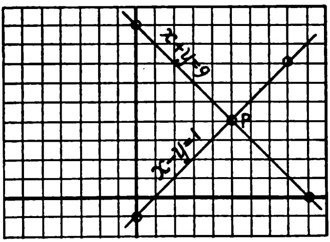
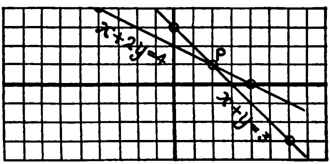
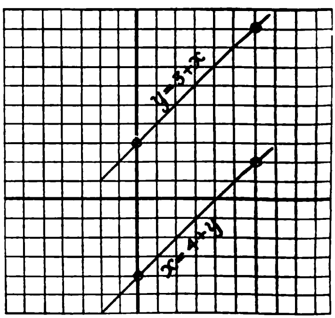
2. The graphs of the equations intersect at $P = (5, 4)$.
Hence, $x = 5$ and $y = 4$.



3. The graphs of the equations intersect at $P = (2, 1)$.
Hence, $x = 2$ and $y = 1$.



4. Since the graphs of the equations are everywhere seven units apart vertically, they are parallel straight lines and have no point in common.
Hence, the equations are *inconsistent.*



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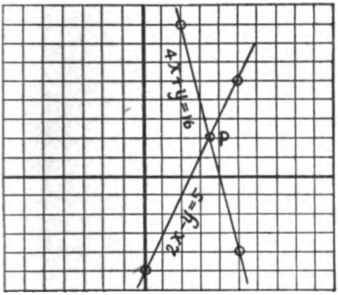
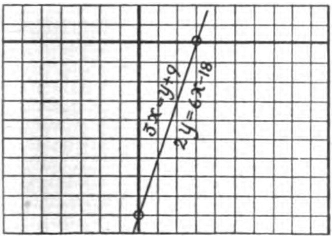
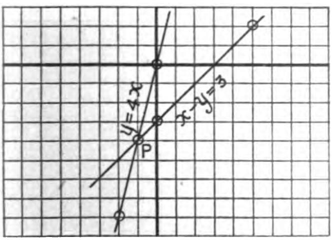
5. The graphs of the equations intersect at $P = (3.5, 2)$.
Hence, $x = 3.5$ and $y = 2$.



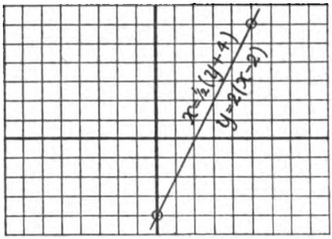
6. The graphs of the equations coincide. Hence, the equations are *indeterminate.*



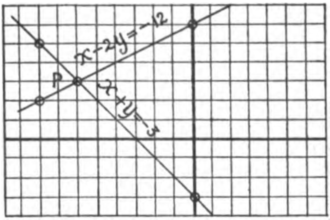
7. The graphs of the equations intersect at $P = (-1, -4)$.
Hence, $x = -1$ and $y = -4$.



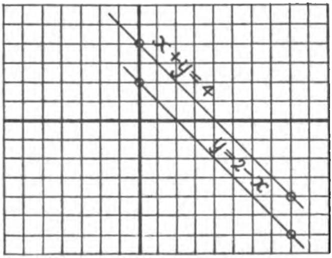
8. The graphs of the equations coincide. Hence, the equations are *indeterminate.*



9. The graphs of the equations intersect at $P = (-6, 3)$.
Hence, $x = -6$ and $y = 3$.



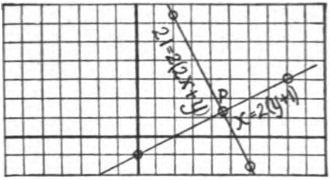
10. Since the graphs of the equations are everywhere two units apart vertically, they are parallel straight lines and have no point in common. Hence, the equations are *inconsistent.*



11. The graphs of the equations intersect approximately at

$P = (4.6, 1.3)$.

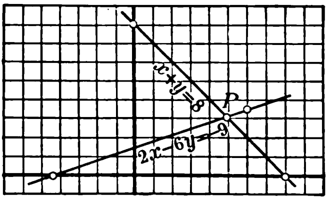
Hence, $x = 4.6$ and $y = 1.3$.



12. The graphs of the equations intersect approximately at

$P = (4.9, 3.1)$.

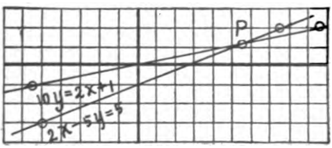
Hence, $x = 4.9$ and $y = 3.1$.



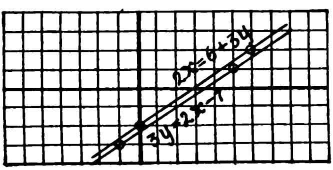
13. The graphs of the equations intersect approximately at

$P = (5.5, 1.2)$.

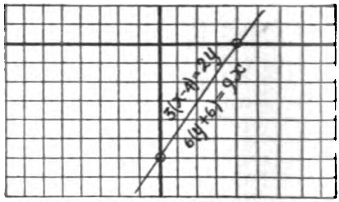
Hence, $x = 5.5$ and $y = 1.2$.



14. The graphs of the equations are parallel straight lines and have no point in common. Hence, the equations are *inconsistent*.



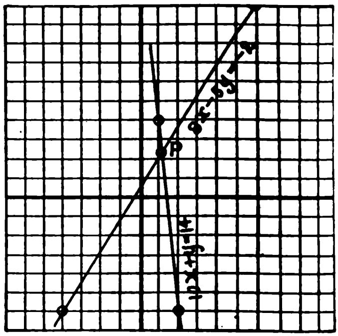
15. The graphs of the equations coincide. Hence, the equations are ***indeterminate.***



16. The graphs of the equations intersect approximately at

$P = (1.2, 2.3)$.

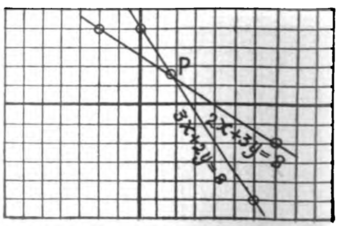
Hence, $x = 1.2$ and $y = 2.3$.



17. The graphs of the equations intersect approximately at

$P = (1.6, 1.6)$.

Hence, $x = 1.6$ and $y = 1.6$.



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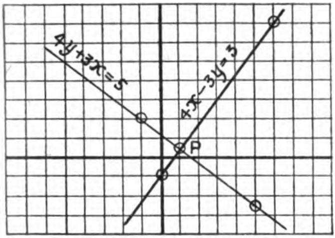
18. The graphs of the equations intersect approximately at

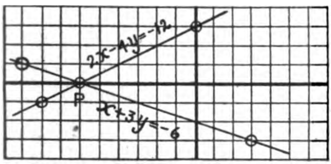
$P = (1.1, .4).$

Hence, $x = 1.1$ and $y = .4.$



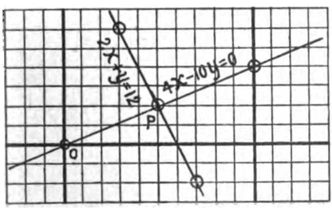
19. The graphs of the equations intersect at $P = (-6, 0)$.

Hence, $x = -6$ and $y = 0$.



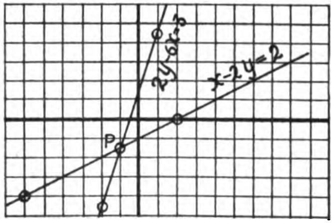
20. The graphs of the equations intersect at $P = (5, 2)$.

Hence, $x = 5$ and $y = 2$.

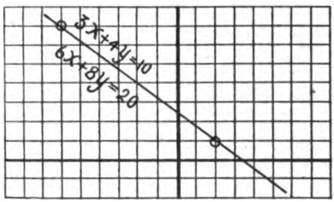


21. The graphs of the equations intersect at $P = (-1, -1.5)$.

Hence, $x = -1$, and $y = -1.5$.

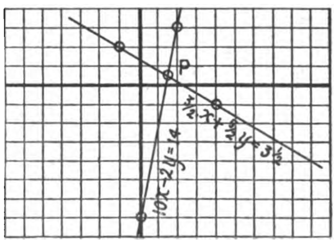


22. The graphs of the equations coincide. Hence, the equations are *indeterminate.*



23. The graphs of the equations intersect at $P = (1.5, .5)$.

Hence, $x = 1.5$ and $y = .5$.



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INVOLUTION

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16. $(x+2)^3 = x^3 + 3x^2(2) + 3x(2)^2 + (2)^3 = x^3 + 6x^2 + 12x + 8.$

17. $(a+3)^3 = a^3 + 3a^2(3) + 3a(3)^2 + (3)^3 = a^3 + 9a^2 + 27a + 27.$

18. $(x+4)^3 = x^3 + 3x^2(4) + 3x(4)^2 + (4)^3 = x^3 + 12x^2 + 48x + 64.$

19. $(x+5)^3 = x^3 + 3x^2(5) + 3x(5)^2 + (5)^3 = x^3 + 15x^2 + 75x + 125.$

20. $(x-2)^3 = x^3 - 3x^2(2) + 3x(2)^2 - (2)^3 = x^3 - 6x^2 + 12x - 8.$

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28. $(x+2y)^4 = x^4 + 4x^3(2y) + 6x^2(2y)^2 + 4x(2y)^3 + (2y)^4$
$= x^4 + 8x^3y + 24x^2y^2 + 32xy^3 + 16y^4.$

29. $(2x-y)^3 = (2x)^3 - 3(2x)^2y + 3(2x)y^2 - y^3$
$= 8x^3 - 12x^2y + 6xy^2 - y^3.$

30. $(2x-5)^3 = (2x)^3 - 3(2x)^2(5) + 3(2x)(5)^2 - (5)^3$
$= 8x^3 - 60x^2 + 150x - 125.$

31. $(x^2-10)^4 = (x^2)^4 - 4(x^2)^3(10) + 6(x^2)^2(10)^2 - 4(x^2)(10)^3 + (10)^4$
$= x^8 - 40x^6 + 600x^4 - 4000x^2 + 10000.$

32. $(1-3x^2)^4 = (1)^4 - 4(1)^3(3x^2) + 6(1)^2(3x^2)^2 - 4(1)(3x^2)^3 + (3x^2)^4$
$= 1 - 12x^2 + 54x^4 - 108x^6 + 81x^8.$

33. $(5x^2-ab)^3 = (5x^2)^3 - 3(5x^2)^2(ab) + 3(5x^2)(ab)^2 - (ab)^3$
$= 125x^6 - 75abx^4 + 15a^2b^2x^2 - a^3b^3.$

34. $(1+a^2b^2)^4 = (1)^4 + 4(1)^3(a^2b^2) + 6(1)^2(a^2b^2)^2 + 4(1)(a^2b^2)^3 + (a^2b^2)^4$
$= 1 + 4a^2b^2 + 6a^4b^4 + 4a^6b^6 + a^8b^8.$

35. $(2ax-b)^5 = (2ax)^5 - 5(2ax)^4b + 10(2ax)^3b^2 - 10(2ax)^2b^3$
$+ 5(2ax)b^4 - b^5$
$= 32a^5x^5 - 80a^4bx^4 + 80a^3b^2x^3 - 40a^2b^3x^2 + 10ab^4x - b^5.$

36. $(1-x)^7 = (1)^7 - 7(1)^6x + 21(1)^5x^2 - 35(1)^4x^3 + 35(1)^3x^4$
$- 21(1)^2x^5 + 7(1)x^6 - x^7$
$= 1 - 7x + 21x^2 - 35x^3 + 35x^4 - 21x^5 + 7x^6 - x^7.$

37. $(1-2x)^6 = (1)^6 - 6(1)^5(2x) + 15(1)^4(2x)^2 - 20(1)^3(2x)^3$
$+ 15(1)^2(2x)^4 - 6(1)(2x)^5 + (2x)^6$
$= 1 - 12x + 60x^2 - 160x^3 + 240x^4 - 192x^5 + 64x^6.$

38. $(x-\frac{1}{2})^6 = x^6 - 6x^5(\frac{1}{2}) + 15x^4(\frac{1}{2})^2 - 20x^3(\frac{1}{2})^3 + 15x^2(\frac{1}{2})^4 - 6x(\frac{1}{2})^5 + (\frac{1}{2})^6$
$= x^6 - 3x^5 + \frac{15}{4}x^4 - \frac{5}{2}x^3 + \frac{15}{16}x^2 - \frac{3}{16}x + \frac{1}{64}.$

39. $(\frac{1}{2}x - \frac{1}{3}y)^4 = (\frac{1}{2}x)^4 - 4(\frac{1}{2}x)^3(\frac{1}{3}y) + 6(\frac{1}{2}x)^2(\frac{1}{3}y)^2 - 4(\frac{1}{2}x)(\frac{1}{3}y)^3 + (\frac{1}{3}y)^4$
$= \frac{1}{16}x^4 - \frac{1}{6}x^3y + \frac{1}{6}x^2y^2 - \frac{2}{27}xy^3 + \frac{1}{81}y^4.$

40. $(2a+\frac{1}{2})^5 = (2a)^5 + 5(2a)^4(\frac{1}{2}) + 10(2a)^3(\frac{1}{2})^2 + 10(2a)^2(\frac{1}{2})^3$
$+ 5(2a)(\frac{1}{2})^4 + (\frac{1}{2})^5$
$= 32a^5 + 40a^4 + 20a^3 + 5a^2 + \frac{5}{8}a + \frac{1}{32}.$

41. $\left(\frac{x}{y} - \frac{y}{x}\right)^4 = \left(\frac{x}{y}\right)^4 - 4\left(\frac{x}{y}\right)^3\left(\frac{y}{x}\right) + 6\left(\frac{x}{y}\right)^2\left(\frac{y}{x}\right)^2 - 4\left(\frac{x}{y}\right)\left(\frac{y}{x}\right)^3 + \left(\frac{y}{x}\right)^4$
$= \frac{x^4}{y^4} - 4\frac{x^2}{y^2} + 6 - 4\frac{y^2}{x^2} + \frac{y^4}{x^4}.$

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42. $\left(\frac{x}{y}-\frac{y}{x}\right)^6=\left(\frac{x}{y}\right)^6-6\left(\frac{x}{y}\right)^5\left(\frac{y}{x}\right)+15\left(\frac{x}{y}\right)^4\left(\frac{y}{x}\right)^2-20\left(\frac{x}{y}\right)^3\left(\frac{y}{x}\right)^3$
$+15\left(\frac{x}{y}\right)^2\left(\frac{y}{x}\right)^4-6\left(\frac{x}{y}\right)\left(\frac{y}{x}\right)^5+\left(\frac{y}{x}\right)^6$
$=\frac{x^6}{y^6}-6\frac{x^4}{y^4}+15\frac{x^2}{y^2}-20+15\frac{y^2}{x^2}-6\frac{y^4}{x^4}+\frac{y^6}{x^6}.$

43. $\left(3a^2+\frac{b}{6}\right)^3=(3a^2)^3+3(3a^2)^2\left(\frac{b}{6}\right)+3(3a^2)\left(\frac{b}{6}\right)^2+\left(\frac{b}{6}\right)^3$
$=27a^6+\frac{9}{2}a^4b+\frac{1}{4}a^2b^2+\frac{1}{216}b^3.$

44. $\left(1+\frac{3x}{2}\right)^5$
$=(1)^5+5(1)^4\left(\frac{3x}{2}\right)+10(1)^3\left(\frac{3x}{2}\right)^2+10(1)^2\left(\frac{3x}{2}\right)^3+5(1)\left(\frac{3x}{2}\right)^4+\left(\frac{3x}{2}\right)^5$
$=1+\frac{15}{2}x+\frac{45}{2}x^2+\frac{135}{4}x^3+\frac{405}{16}x^4+\frac{243}{32}x^5.$

45. $\left(\frac{3}{5}+\frac{5x}{3}\right)^4=\left(\frac{3}{5}\right)^4+4\left(\frac{3}{5}\right)^3\left(\frac{5x}{3}\right)+6\left(\frac{3}{5}\right)^2\left(\frac{5x}{3}\right)^2+4\left(\frac{3}{5}\right)\left(\frac{5x}{3}\right)^3+\left(\frac{5x}{3}\right)^4$
$=\frac{81}{625}+\frac{36}{25}x+6x^2+\frac{100}{9}x^3+\frac{625}{81}x^4.$

46. $\left(\frac{1}{2x}-2x\right)^5$
$=\left(\frac{1}{2x}\right)^5-5\left(\frac{1}{2x}\right)^4(2x)+10\left(\frac{1}{2x}\right)^3(2x)^2-10\left(\frac{1}{2x}\right)^2(2x)^3$
$+5\left(\frac{1}{2x}\right)(2x)^4-(2x)^5$
$=\frac{1}{32x^5}-\frac{5}{8x^3}+\frac{5}{x}-20x+40x^3-32x^5.$

47. $\left(\frac{1}{a}-a\right)^6$
$=\left(\frac{1}{a}\right)^6-6\left(\frac{1}{a}\right)^5a+15\left(\frac{1}{a}\right)^4a^2-20\left(\frac{1}{a}\right)^3a^3+15\left(\frac{1}{a}\right)^2a^4-6\left(\frac{1}{a}\right)a^5+a^6$
$=\frac{1}{a^6}-\frac{6}{a^4}+\frac{15}{a^2}-20+15a^2-6a^4+a^6.$

48. $\left(x+\frac{1}{x}\right)^7=x^7+7x^6\left(\frac{1}{x}\right)+21x^5\left(\frac{1}{x}\right)^2+35x^4\left(\frac{1}{x}\right)^3+35x^3\left(\frac{1}{x}\right)^4$
$+21x^2\left(\frac{1}{x}\right)^5+7x\left(\frac{1}{x}\right)^6+\left(\frac{1}{x}\right)^7$
$=x^7+7x^5+21x^3+35x+\frac{35}{x}+\frac{21}{x^3}+\frac{7}{x^5}+\frac{1}{x^7}.$

50. $(a+b-c-d)^3=(\overline{a+b}-\overline{c+d})^3$
$=(a+b)^3-3(a+b)^2(c+d)+3(a+b)(c+d)^2-(c+d)^3$
$=a^3+3a^2b+3ab^2+b^3-3(a^2c+2abc+b^2c+a^2d+2abd+b^2d)$
$+3(ac^2+2acd+ad^2+bc^2+2bcd+bd^2)-(c^3+3c^2d+3cd^2+d^3)$
$=a^3+3a^2b+3ab^2+b^3-3a^2c-6abc-3b^2c-3a^2d-6abd-3b^2d$
$+3ac^2+6acd+3ad^2+3bc^2+6bcd+3bd^2-c^3-3c^2d-3cd^2-d^3.$

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51. $(a+x-y)^3 = (\overline{a+x}-y)^3$
$= (a+x)^3 - 3(a+x)^2y + 3(a+x)y^2 - y^3$
$= a^3+3a^2x+3ax^2+x^3-3y(a^2+2ax+x^2)+3ay^2+3xy^2-y^3$
$= a^3+3a^2x+3ax^2+x^3-3a^2y-6axy-3x^2y+3ay^2+3xy^2-y^3.$

52. $(a-m-n)^3=(\overline{a-m}-n)^3$
$=(a-m)^3-3(a-m)^2n+3(a-m)n^2-n^3$
$=a^3-3a^2m+3am^2-m^3-3n(a^2-2am+m^2)+3an^2-3mn^2-n^3$
$=a^3-3a^2m+3am^2-m^3-3a^2n+6amn-3m^2n+3an^2-3mn^2-n^3.$

53. $(a-x+y)^3=(\overline{a-x}+y)^3$
$=(a-x)^3+3(a-x)^2y+3(a-x)y^2+y^3$
$=a^3-3a^2x+3ax^2-x^3+3y(a^2-2ax+x^2)+3ay^2-3xy^2+y^3$
$=a^3-3a^2x+3ax^2-x^3+3a^2y-6axy+3x^2y+3ay^2-3xy^2+y^3.$

54. $(a-x-y)^3=(\overline{a-x}-y)^3$
$=(a-x)^3-3(a-x)^2y+3(a-x)y^2-y^3$
$=a^3-3a^2x+3ax^2-x^3-3y(a^2-2ax+x^2)+3ay^2-3xy^2-y^3$
$=a^3-3a^2x+3ax^2-x^3-3a^2y+6axy-3x^2y+3ay^2-3xy^2-y^3.$

55. $(a+x+2)^3=(\overline{a+x}+2)^3$
$=(a+x)^3+3(a+x)^2(2)+3(a+x)(2)^2+(2)^3$
$=a^3+3a^2x+3ax^2+x^3+6(a^2+2ax+x^2)+12a+12x+8$
$=a^3+3a^2x+3ax^2+x^3+6a^2+12ax+6x^2+12a+12x+8.$

56. $(a-x-2)^3=(\overline{a-x}-2)^3$
$=(a-x)^3-3(a-x)^2(2)+3(a-x)(2)^2-(2)^3$
$=a^3-3a^2x+3ax^2-x^3-6(a^2-2ax+x^2)+12a-12x-8$
$=a^3-3a^2x+3ax^2-x^3-6a^2+12ax-6x^2+12a-12x-8.$

57. $(a+2b-3c)^3=(\overline{a+2b}-3c)^3$
$=(a+2b)^3-3(a+2b)^2(3c)+3(a+2b)(3c)^2-(3c)^3$
$=a^3+3a^2(2b)+3a(2b)^2+(2b)^3-9c(a^2+4ab+4b^2)+27ac^2+54bc^2-27c^3$
$=a^3+6a^2b+12ab^2+8b^3-9a^2c-36abc-36b^2c+27ac^2+54bc^2-27c^3.$

58. $(a+b+x+y)^3=(\overline{a+b}+\overline{x+y})^3$
$=(a+b)^3+3(a+b)^2(x+y)+3(a+b)(x+y)^2+(x+y)^3$
$=a^3+3a^2b+3ab^2+b^3+3(a^2x+2abx+b^2x+a^2y+2aby+b^2y)$
$+3(ax^2+2axy+ay^2+bx^2+2bxy+by^2)+x^3+3x^2y+3xy^2+y^3$
$=a^3+3a^2b+3ab^2+b^3+3a^2x+6abx+3b^2x+3a^2y+6aby+3b^2y$
$+3ax^2+6axy+3ay^2+3bx^2+6bxy+3by^2+x^3+3x^2y+3xy^2+y^3.$

59. $(a+b-x-y)^3=(\overline{a+b}-\overline{x+y})^3$
$=(a+b)^3-3(a+b)^2(x+y)+3(a+b)(x+y)^2-(x+y)^3$
$=a^3+3a^2b+3ab^2+b^3-3(a^2x+2abx+b^2x+a^2y+2aby+b^2y)$
$+3(ax^2+2axy+ay^2+bx^2+2bxy+by^2)-(x^3+3x^2y+3xy^2+y^3)$
$=a^3+3a^2b+3ab^2+b^3-3a^2x-6abx-3b^2x-3a^2y-6aby-3b^2y$
$+3ax^2+6axy+3ay^2+3bx^2+6bxy+3by^2-x^3-3x^2y-3xy^2-y^3.$

60. $(a-b+x-y)^3=(\overline{a-b}+\overline{x-y})^3$
$=(a-b)^3+3(a-b)^2(x-y)+3(a-b)(x-y)^2+(x-y)^3$
$=a^3-3a^2b+3ab^2-b^3+3(a^2x-2abx+b^2x-a^2y+2aby-b^2y)$
$+3(ax^2-2axy+ay^2-bx^2+2bxy-by^2)+x^3-3x^2y+3xy^2-y^3$
$=a^3-3a^2b+3ab^2-b^3+3a^2x-6abx+3b^2x-3a^2y+6aby-3b^2y$
$+3ax^2-6axy+3ay^2-3bx^2+6bxy-3by^2+x^3-3x^2y+3xy^2-y^3.$

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61. $(a-b-x+y)^3 = (\overline{a-b} - \overline{x-y})^3$
$= (a-b)^3 - 3(a-b)^2(x-y) + 3(a-b)(x-y)^2 - (x-y)^3$
$= a^3 - 3a^2b + 3ab^2 - b^3 - 3(a^2x - 2abx + b^2x - a^2y + 2aby - b^2y)$
$\quad + 3(ax^2 - 2axy + ay^2 - bx^2 + 2bxy - by^2) - (x^3 - 3x^2y + 3xy^2 - y^3)$
$= a^3 - 3a^2b + 3ab^2 - b^3 - 3a^2x + 6abx - 3b^2x + 3a^2y - 6aby + 3b^2y$
$\quad + 3ax^2 - 6axy + 3ay^2 - 3bx^2 + 6bxy - 3by^2 - x^3 + 3x^2y - 3xy^2 + y^3.$

62. $(a-b-x-y)^3 = (\overline{a-b} - \overline{x+y})^3$
$= (a-b)^3 - 3(a-b)^2(x+y) + 3(a-b)(x+y)^2 - (x+y)^3$
$= a^3 - 3a^2b + 3ab^2 - b^3 - 3(a^2x - 2abx + b^2x + a^2y - 2aby + b^2y)$
$\quad + 3(ax^2 + 2axy + ay^2 - bx^2 - 2bxy - by^2) - (x^3 + 3x^2y + 3xy^2 + y^3)$
$= a^3 - 3a^2b + 3ab^2 - b^3 - 3a^2x + 6abx - 3b^2x - 3a^2y + 6aby - 3b^2y$
$\quad + 3ax^2 + 6axy + 3ay^2 - 3bx^2 - 6bxy - 3by^2 - x^3 - 3x^2y - 3xy^2 - y^3.$

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10.
$$\begin{array}{l|l|l}
 & 25a^2 - 40a + 16 & 5a - 4 \\
 & 25a^2 & \\
\hline
10a & -40a + 16 & \\
10a - 4 & -40a + 16 & \\
\hline
\end{array}$$

11.
$$\begin{array}{l|l|l}
 & 900x^2 + 60x + 1 & 30x + 1 \\
 & 900x^2 & \\
\hline
60x & 60x + 1 & \\
60x + 1 & 60x + 1 & \\
\hline
\end{array}$$

12.
$$\begin{array}{l|l|l}
 & x^2 + xy + \frac{1}{4}y^2 & x + \frac{1}{2}y \\
 & x^2 & \\
\hline
2x & xy + \frac{1}{4}y^2 & \\
2x + \frac{1}{2}y & xy + \frac{1}{4}y^2 & \\
\hline
\end{array}$$

13.
$$\begin{array}{l|l|l}
 & 4x^4 - 52x^2 + 169 & 2x^2 - 13 \\
 & 4x^4 & \\
\hline
4x^2 & -52x^2 + 169 & \\
4x^2 - 13 & -52x^2 + 169 & \\
\hline
\end{array}$$

14.
$$\begin{array}{l|l|l}
 & \frac{4}{9}d^6 - \frac{2}{3}d^3n^2 + \frac{1}{4}n^4 & \frac{2}{3}d^3 - \frac{1}{2}n^2 \\
 & \frac{4}{9}d^6 & \\
\hline
\frac{4}{3}d^3 & -\frac{2}{3}d^3n^2 + \frac{1}{4}n^4 & \\
\frac{4}{3}d^3 - \frac{1}{2}n^2 & -\frac{2}{3}d^3n^2 + \frac{1}{4}n^4 & \\
\hline
\end{array}$$

15.
$$\begin{array}{l|l|l}
 & (a+b)^2 - 4(a+b) + 4 & (a+b) - 2 = a + b - 2 \\
 & (a+b)^2 & \\
\hline
2(a+b) & -4(a+b) + 4 & \\
2(a+b) - 2 & -4(a+b) + 4 & \\
\hline
\end{array}$$

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16.

$$\begin{array}{l|l}
 & 9x^4 - 12x^3 + 10x^2 - 4x + 1 \;\big|\; \underline{3x^2 - 2x + 1} \\
 & 9x^4 \\ \hline
6x^2 & -12x^3 + 10x^2 \\
6x^2 - 2x & -12x^3 + 4x^2 \\ \hline
6x^2 - 4x & 6x^2 - 4x + 1 \\
6x^2 - 4x + 1 & 6x^2 - 4x + 1 \\ \hline
\end{array}$$

17.

$$\begin{array}{l|l}
 & x^4 - 6x^3y + 13x^2y^2 - 12xy^3 + 4y^4 \;\big|\; \underline{x^2 - 3xy + 2y^2} \\
 & x^4 \\ \hline
2x^2 & -6x^3y + 13x^2y^2 \\
2x^2 - 3xy & -6x^3y + 9x^2y^2 \\ \hline
2x^2 - 6xy & 4x^2y^2 - 12xy^3 + 4y^4 \\
2x^2 - 6xy + 2y^2 & 4x^2y^2 - 12xy^3 + 4y^4 \\ \hline
\end{array}$$

18.

$$\begin{array}{l|l}
 & x^8 - 2a^2x^6 - a^4x^4 + 2a^6x^2 + a^8 \;\big|\; \underline{x^4 - a^2x^2 - a^4} \\
 & x^8 \\ \hline
2x^4 & -2a^2x^6 - a^4x^4 \\
2x^4 - a^2x^2 & -2a^2x^6 + a^4x^4 \\ \hline
2x^4 - 2a^2x^2 & -2a^4x^4 + 2a^6x^2 + a^8 \\
2x^4 - 2a^2x^2 - a^4 & -2a^4x^4 + 2a^6x^2 + a^8 \\ \hline
\end{array}$$

19.

$$\begin{array}{l|l}
 & 25x^4 - 30x^3 + 29x^2 - 12x + 4 \;\big|\; \underline{5x^2 - 3x + 2} \\
 & 25x^4 \\ \hline
10x^2 & -30x^3 + 29x^2 \\
10x^2 - 3x & -30x^3 + 9x^2 \\ \hline
10x^2 - 6x & 20x^2 - 12x + 4 \\
10x^2 - 6x + 2 & 20x^2 - 12x + 4 \\ \hline
\end{array}$$

20.

$$\begin{array}{l|l}
 & 1 - 2x + 3x^2 - 4x^3 + 3x^4 - 2x^5 + x^6 \;\big|\; \underline{1 - x + x^2 - x^3} \\
 & 1 \\ \hline
2 & -2x + 3x^2 \\
2 - x & -2x + x^2 \\ \hline
2 - 2x & 2x^2 - 4x^3 + 3x^4 \\
2 - 2x + x^2 & 2x^2 - 2x^3 + x^4 \\ \hline
2 - 2x + 2x^2 & -2x^3 + 2x^4 - 2x^5 + x^6 \\
2 - 2x + 2x^2 - x^3 & -2x^3 + 2x^4 - 2x^5 + x^6 \\ \hline
\end{array}$$

21.

$$\begin{array}{l|l}
 & a^4 - 2a^2b + 2a^2c^2 - 2bc^2 + b^2 + c^4 \;\big|\; \underline{a^2 - b + c^2} \\
 & a^4 \\ \hline
2a^2 & -2a^2b + 2a^2c^2 - 2bc^2 + b^2 + c^4 \\
2a^2 - b & -2a^2b + b^2 \\ \hline
2a^2 - 2b & 2a^2c^2 - 2bc^2 + c^4 \\
2a^2 - 2b + c^2 & 2a^2c^2 - 2bc^2 + c^4 \\ \hline
\end{array}$$

22.

$$\begin{array}{l|l}
 & 4a^2 - 12ab + 9b^2 + 16ac - 24bc + 16c^2 \;\big|\; \underline{2a - 3b + 4c} \\
 & 4a^2 \\ \hline
4a & -12ab + 9b^2 \\
4a - 3b & -12ab + 9b^2 \\ \hline
4a - 6b & 16ac - 24bc + 16c^2 \\
4a - 6b + 4c & 16ac - 24bc + 16c^2 \\ \hline
\end{array}$$

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23.

$$\begin{array}{l|l}
 & 9x^2 - 30xy + 25y^2 + 18xz - 30yz + 9z^2 \;\underline{|\,3x - 5y + 3z} \\
 & \underline{9x^2} \\
6x & -30xy + 25y^2 \\
6x - 5y & \underline{-30xy + 25y^2} \\
6x - 10y & 18xz - 30yz + 9z^2 \\
6x - 10y + 3z & \underline{18xz - 30yz + 9z^2}
\end{array}$$

24.

$$\begin{array}{l|l}
 & \dfrac{a^2}{9} + \dfrac{10a}{3} + 25 \;\underline{\Big|\,\dfrac{a}{3} + 5} \\
 & \underline{\dfrac{a^2}{9}} \\
\dfrac{2a}{3} & \dfrac{10a}{3} + 25 \\
\dfrac{2a}{3} + 5 & \underline{\dfrac{10a}{3} + 25}
\end{array}$$

25.

$$\begin{array}{l|l}
 & \dfrac{25}{4n^2} + 15 + 9n^2 \;\underline{\Big|\,\dfrac{5}{2n} + 3n} \\
 & \underline{\dfrac{25}{4n^2}} \\
\dfrac{5}{n} & 15 + 9n^2 \\
\dfrac{5}{n} + 3n & \underline{15 + 9n^2}
\end{array}$$

26.

$$\begin{array}{l|l}
 & \dfrac{25d^2}{16r^2} - \dfrac{5d}{r} + 4 \;\underline{\Big|\,\dfrac{5d}{4r} - 2} \\
 & \underline{\dfrac{25d^2}{16r^2}} \\
\dfrac{5d}{2r} & -\dfrac{5d}{r} + 4 \\
\dfrac{5d}{2r} - 2 & \underline{-\dfrac{5d}{r} + 4}
\end{array}$$

27.

$$\begin{array}{l|l}
 & \dfrac{x^4}{64} + \dfrac{x^3}{8} - x + 1 \;\underline{\Big|\,\dfrac{x^2}{8} + \dfrac{x}{2} - 1} \\
 & \underline{\dfrac{x^4}{64}} \\
\dfrac{x^2}{4} & \dfrac{x^3}{8} \\
\dfrac{x^2}{4} + \dfrac{x}{2} & \underline{\dfrac{x^3}{8} + \dfrac{x^2}{4}} \\
\dfrac{x^2}{4} + x & -\dfrac{x^2}{4} - x + 1 \\
\dfrac{x^2}{4} + x - 1 & \underline{-\dfrac{x^2}{4} - x + 1}
\end{array}$$

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28.

$$x^2+2x-1-\frac{2}{x}+\frac{1}{x^2}\;\Big|\;\underline{x+1-\frac{1}{x}}$$

$$\begin{array}{l|l} & x^2 \\ \hline 2x & 2x-1 \\ 2x+1 & 2x+1 \\ \hline 2x+2 & -2-\frac{2}{x}+\frac{1}{x^2} \\ 2x+2-\frac{1}{x} & -2-\frac{2}{x}+\frac{1}{x^2} \\ \hline \end{array}$$

29.

$$x^4+x^3+\frac{13x^2}{20}+\frac{x}{5}+\frac{1}{25}\;\Big|\;\underline{x^2+\frac{x}{2}+\frac{1}{5}}$$

$$\begin{array}{l|l} & x^4 \\ \hline 2x^2 & x^3+\frac{13x^2}{20} \\ 2x^2+\frac{x}{2} & x^3+\frac{x^2}{4} \\ \hline 2x^2+x & \frac{2x^2}{5}+\frac{x}{5}+\frac{1}{25} \\ 2x^2+x+\frac{1}{5} & \frac{2x^2}{5}+\frac{x}{5}+\frac{1}{25} \\ \hline \end{array}$$

30.

$$x^8+4x^7-2x^6-20x^5-3x^4+32x^3+4x^2-16x+4\;\Big|\;\underline{x^4+2x^3-3x^2-4x}\ [+2$$

$$\begin{array}{l|l} & x^8 \\ \hline 2x^4 & 4x^7-2x^6 \\ 2x^4+2x^3 & 4x^7+4x^6 \\ \hline 2x^4+4x^3 & -6x^6-20x^5-3x^4 \\ 2x^4+4x^3-3x^2 & -6x^6-12x^5+9x^4 \\ \hline 2x^4+4x^3-6x^2 & -8x^5-12x^4+32x^3+4x^2 \\ 2x^4+4x^3-6x^2-4x & -8x^5-16x^4+24x^3+16x^2 \\ \hline 2x^4+4x^3-6x^2-8x & 4x^4+8x^3-12x^2-16x+4 \\ 2x^4+4x^3-6x^2-8x+2 & 4x^4+8x^3-12x^2-16x+4 \\ \hline \end{array}$$

31.

$$\frac{4x^4}{y^4}-\frac{4x^3}{y^3}-\frac{3x^2}{y^2}+\frac{2x}{y}+1\;\Big|\;\underline{\frac{2x^2}{y^2}-\frac{x}{y}-1}$$

$$\begin{array}{l|l} & \frac{4x^4}{y^4} \\ \hline \frac{4x^2}{y^2} & -\frac{4x^3}{y^3}-\frac{3x^2}{y^2} \\ \frac{4x^2}{y^2}-\frac{x}{y} & -\frac{4x^3}{y^3}+\frac{x^2}{y^2} \\ \hline \frac{4x^2}{y^2}-\frac{2x}{y} & -\frac{4x^2}{y^2}+\frac{2x}{y}+1 \\ \frac{4x^2}{y^2}-\frac{2x}{y}-1 & -\frac{4x^2}{y^2}+\frac{2x}{y}+1 \\ \hline \end{array}$$

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32.

$$\frac{a^4}{4} + a^3x + \frac{4\,a^2x^2}{3} + \frac{2\,ax^3}{3} + \frac{x^4}{9} \;\Big|\; \frac{a^2}{2} + ax + \frac{x^2}{3}$$

$$\begin{array}{l|l}
 & \frac{a^4}{4} \\
\hline
a^2 & a^3x + \frac{4\,a^2x^2}{3} \\
a^2 + ax & a^3x + a^2x^2 \\
\hline
a^2 + 2\,ax & \frac{a^2x^2}{3} + \frac{2\,ax^3}{3} + \frac{x^4}{9} \\
a^2 + 2\,ax + \frac{x^2}{3} & \frac{a^2x^2}{3} + \frac{2\,ax^3}{3} + \frac{x^4}{9} \\
\hline
\end{array}$$

33.

$$\frac{a^4}{9} - \frac{4\,a^3}{3} + \frac{a^2b}{3} + 4\,a^2 - 2\,ab + \frac{b^2}{4} \;\Big|\; \frac{a^2}{3} - 2\,a + \frac{b}{2}$$

$$\begin{array}{l|l}
 & \frac{a^4}{9} \\
\hline
\frac{2\,a^2}{3} & -\frac{4\,a^3}{3} + 4\,a^2 \\
\frac{2\,a^2}{3} - 2\,a & -\frac{4\,a^3}{3} + 4\,a^2 \\
\hline
\frac{2\,a^2}{3} - 4\,a & \frac{a^2b}{3} - 2\,ab + \frac{b^2}{4} \\
\frac{2\,a^2}{3} - 4\,a + \frac{b}{2} & \frac{a^2b}{3} - 2\,ab + \frac{b^2}{4} \\
\hline
\end{array}$$

34.

$$\frac{4\,m^6}{9} - \frac{4\,m^5}{3} + \frac{19\,m^4}{15} + \frac{3\,m^3}{5} - \frac{73\,m^2}{50} + \frac{3\,m}{10} + \frac{9}{16} \;\Big|\; \frac{2\,m^3}{3} - m^2 + \frac{m}{5} + \frac{3}{4}$$

$$\begin{array}{l|l}
 & \frac{4\,m^6}{9} \\
\hline
\frac{4\,m^3}{3} & -\frac{4\,m^5}{3} + \frac{19\,m^4}{15} \\
\frac{4\,m^3}{3} - m^2 & -\frac{4\,m^5}{3} + m^4 \\
\hline
\frac{4\,m^3}{3} - 2\,m^2 & \frac{4\,m^4}{15} + \frac{3\,m^3}{5} - \frac{73\,m^2}{50} \\
\frac{4\,m^3}{3} - 2\,m^2 + \frac{m}{5} & \frac{4\,m^4}{15} - \frac{2\,m^3}{5} + \frac{m^2}{25} \\
\hline
\frac{4\,m^3}{3} - 2\,m^2 + \frac{2\,m}{5} & m^3 - \frac{3\,m^2}{2} + \frac{3\,m}{10} + \frac{9}{16} \\
\frac{4\,m^3}{3} - 2\,m^2 + \frac{2\,m}{5} + \frac{3}{4} & m^3 - \frac{3\,m^2}{2} + \frac{3\,m}{10} + \frac{9}{16} \\
\hline
\end{array}$$

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35.

$$
\begin{array}{l|l}
 & r^8 - \frac{4}{5}r^7 + \frac{4}{25}r^6 + \frac{5}{3}r^5 - \frac{20}{3}r^4 + \frac{12}{5}r^3 + \frac{25}{36}r^2 - 5r + 9 \;\big|\; \underline{r^4 - \frac{2}{5}r^3 + \frac{5}{6}r - 3} \\
 & r^8 \\
\hline
2r^4 & -\frac{4}{5}r^7 + \frac{4}{25}r^6 \\
2r^4 - \frac{2}{5}r^3 & -\frac{4}{5}r^7 + \frac{4}{25}r^6 \\
\hline
2r^4 - \frac{4}{5}r^3 & \frac{5}{3}r^5 - \frac{20}{3}r^4 + \frac{12}{5}r^3 + \frac{25}{36}r^2 \\
2r^4 - \frac{4}{5}r^3 + \frac{5}{6}r & \frac{5}{3}r^5 - \frac{2}{3}r^4 \qquad\qquad + \frac{25}{36}r^2 \\
\hline
2r^4 - \frac{4}{5}r^3 + \frac{5}{3}r & -6r^4 + \frac{12}{5}r^3 \qquad -5r + 9 \\
2r^4 - \frac{4}{5}r^3 + \frac{5}{3}r - 3 & -6r^4 + \frac{12}{5}r^3 \qquad -5r + 9 \\
\hline
\end{array}
$$

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37.

$$
\begin{array}{l|l}
 & 1 - a \;\big|\; \underline{1 - \frac{1}{2}a - \frac{1}{8}a^2 - \frac{1}{16}a^3} \\
 & 1 \\
\hline
2 & -a \\
2 - \frac{1}{2}a & -a + \frac{1}{4}a^2 \\
\hline
2 - a & -\frac{1}{4}a^2 \\
2 - a - \frac{1}{8}a^2 & -\frac{1}{4}a^2 + \frac{1}{8}a^3 + \frac{1}{64}a^4 \\
\hline
2 - a - \frac{1}{4}a^2 & -\frac{1}{8}a^3 - \frac{1}{64}a^4
\end{array}
$$

38.

$$
\begin{array}{l|l}
 & a^2 + 1 \;\Big|\; \underline{a + \frac{1}{2a} - \frac{1}{8a^3} + \frac{1}{16a^5}} \\
 & a^2 \\
\hline
2a & 1 \\
2a + \frac{1}{2a} & 1 + \frac{1}{4a^2} \\
\hline
2a + \frac{1}{a} & -\frac{1}{4a^2} \\
2a + \frac{1}{a} - \frac{1}{8a^3} & -\frac{1}{4a^2} - \frac{1}{8a^4} + \frac{1}{64a^6} \\
\hline
2a + \frac{1}{a} - \frac{1}{4a^3} & \frac{1}{8a^4} - \frac{1}{64a^6}
\end{array}
$$

39.

$$
\begin{array}{l|l}
 & x^2 - 1 \;\Big|\; \underline{x - \frac{1}{2x} - \frac{1}{8x^3} - \frac{1}{16x^5}} \\
 & x^2 \\
\hline
2x & -1 \\
2x - \frac{1}{2x} & -1 + \frac{1}{4x^2} \\
\hline
2x - \frac{1}{x} & -\frac{1}{4x^2} \\
2x - \frac{1}{x} - \frac{1}{8x^3} & -\frac{1}{4x^2} + \frac{1}{8x^4} + \frac{1}{64x^6} \\
\hline
2x - \frac{1}{x} - \frac{1}{4x^3} & -\frac{1}{8x^4} - \frac{1}{64x^6}
\end{array}
$$

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40.

$$
\begin{array}{l|l}
 & 4-a \;\big|\; 2-\frac{a}{4}-\frac{a^2}{64}-\frac{a^3}{512} \\
 & 4 \\
\hline
4 & -a \\
4-\frac{a}{4} & -a+\frac{a^2}{16} \\
\hline
4-\frac{a}{2} & -\frac{a^2}{16} \\
4-\frac{a}{2}-\frac{a^2}{64} & -\frac{a^2}{16}+\frac{a^3}{128}+\frac{a^4}{4096} \\
\hline
4-\frac{a}{2}-\frac{a^2}{32} & -\frac{a^3}{128}-\frac{a^4}{4096}
\end{array}
$$

41.

$$
\begin{array}{l|l}
 & y^2+3 \;\big|\; y+\frac{3}{2y}-\frac{9}{8y^3}+\frac{27}{16y^5} \\
 & y^2 \\
\hline
2y & 3 \\
2y+\frac{3}{2y} & 3+\frac{9}{4y^2} \\
\hline
2y+\frac{3}{y} & -\frac{9}{4y^2} \\
2y+\frac{3}{y}-\frac{9}{8y^3} & -\frac{9}{4y^2}-\frac{27}{8y^4}+\frac{81}{64y^6} \\
\hline
2y+\frac{3}{y}-\frac{9}{4y^3} & \frac{27}{8y^4}-\frac{81}{64y^6}
\end{array}
$$

42.

$$
\begin{array}{l|l}
 & a^2+2b \;\big|\; a+\frac{b}{a}-\frac{b^2}{2a^3}+\frac{b^3}{2a^5} \\
 & a^2 \\
\hline
2a & 2b \\
2a+\frac{b}{a} & 2b+\frac{b^2}{a^2} \\
\hline
2a+\frac{2b}{a} & -\frac{b^2}{a^2} \\
2a+\frac{2b}{a}-\frac{b^2}{2a^3} & -\frac{b^2}{a^2}-\frac{b^3}{a^4}+\frac{b^4}{4a^6} \\
\hline
2a+\frac{2b}{a}-\frac{b^2}{a^3} & \frac{b^3}{a^4}-\frac{b^4}{4a^6}
\end{array}
$$

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6.

$$
\begin{array}{l|l}
 & 5'71'21 \;\big|\; 239 \\
 & 4 \\
\hline
20\times 2=40 & 1\ 71 \\
40+3=43 & 1\ 29 \\
\hline
230\times 2=460 & 42\ 21 \\
460+9=469 & 42\ 21 \\
\hline
\end{array}
$$

7.

$$
\begin{array}{l|l}
 & 4'20'25 \;\big|\; 205 \\
 & 4 \\
\hline
200\times 2=400 & 20\ 25 \\
400+5=405 & 20\ 25 \\
\hline
\end{array}
$$

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8.

$$\begin{array}{r|l} & 9'54'81 \quad \underline{|309} \\ & 9 \\ \hline 300 \times 2 = 600 & 54\ 81 \\ 600 + 9 = 609 & 54\ 81 \\ \hline \end{array}$$

9.

$$\begin{array}{r|l} & 24'80.04 \quad \underline{|49.8} \\ & 16 \\ \hline 40 \times 2 = 80 & 8\ 80 \\ 80 + 9 = 89 & 8\ 01 \\ \hline 490 \times 2 = 980 & 79\ 04 \\ 980 + 8 = 988 & 79\ 04 \\ \hline \end{array}$$

10.

$$\begin{array}{r|l} & 10.95'61 \quad \underline{|3.31} \\ & 9 \\ \hline 30 \times 2 = 60 & 1\ 95 \\ 60 + 3 = 63 & 1\ 89 \\ \hline 330 \times 2 = 660 & 6\ 61 \\ 660 + 1 = 661 & 6\ 61 \\ \hline \end{array}$$

11.

$$\begin{array}{r|l} & .00'12'25 \quad \underline{|.035} \\ & 9 \\ \hline 30 \times 2 = 60 & 3\ 25 \\ 60 + 5 = 65 & 3\ 25 \\ \hline \end{array}$$

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12.

$$\begin{array}{r|l} & 18'66'24 \quad \underline{|432} \\ & 16 \\ \hline 40 \times 2 = 80 & 2\ 66 \\ 80 + 3 = 83 & 2\ 49 \\ \hline 430 \times 2 = 860 & 17\ 24 \\ 860 + 2 = 862 & 17\ 24 \\ \hline \end{array}$$

13.

$$\begin{array}{r|l} & 13'32.25 \quad \underline{|36.5} \\ & 9 \\ \hline 30 \times 2 = 60 & 4\ 32 \\ 60 + 6 = 66 & 3\ 96 \\ \hline 360 \times 2 = 720 & 36\ 25 \\ 720 + 5 = 725 & 36\ 25 \\ \hline \end{array}$$

14.

$$\begin{array}{r|l} & 1'11.09'16 \quad \underline{|10.54} \\ & 1 \\ \hline 100 \times 2 = 200 & 11\ 09 \\ 200 + 5 = 205 & 10\ 25 \\ \hline 1050 \times 2 = 2100 & 84\ 16 \\ 2100 + 4 = 2104 & 84\ 16 \\ \hline \end{array}$$

23.

$$\begin{array}{r|l} & 3.00'00'00'00 \quad \underline{|1.7320+} \\ & 1 \\ \hline 10 \times 2 = 20 & 2\ 00 \\ 20 + 7 = 27 & 1\ 89 \\ \hline 170 \times 2 = 340 & 11\ 00 \\ 340 + 3 = 343 & 10\ 29 \\ \hline 1730 \times 2 = 3460 & 71\ 00 \\ 3460 + 2 = 3462 & 69\ 24 \\ \hline 17320 \times 2 = 34640 & 1\ 76\ 00 \end{array}$$

Since the square root of 3 is 1.7320+ and the square root of 4 is 2, the square root of $\frac{3}{4}$ to four decimal places is $\frac{1.7320}{2}$, or .8660.

24. $\frac{4}{5} = .8.$

$$\begin{array}{r|l} & .80'00'00'00 \quad \underline{|.8944} \\ & 64 \\ \hline 80 \times 2 = 160 & 16\ 00 \\ 160 + 9 = 169 & 15\ 21 \\ \hline 890 \times 2 = 1780 & 79\ 00 \\ 1780 + 4 = 1784 & 71\ 36 \\ \hline 8940 \times 2 = 17880 & 7\ 64\ 00 \\ 17880 + 4 = 17884 & 7\ 15\ 36 \\ \hline \end{array}$$

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25. $\frac{5}{8} = .625.$

$$\begin{array}{r|l}
 & .62'50'00'00 \quad \underline{|.7905} \\
 & 49 \\
70 \times 2 = 140 & 13\ 50 \\
140 + 9 = 149 & 13\ 41 \\
\hline
7900 \times 2 = 15800 & 9\ 00\ 00 \\
15800 + 5 = 15805 & 7\ 90\ 25 \\
\hline
\end{array}$$

26

$$\begin{array}{r|l}
 & .60'00'00'00 \quad \underline{|.7745} \\
 & 49 \\
70 \times 2 = 140 & 11\ 00 \\
140 + 7 = 147 & 10\ 29 \\
\hline
770 \times 2 = 1540 & 71\ 00 \\
1540 + 4 = 1544 & 61\ 76 \\
\hline
7740 \times 2 = 15480 & 9\ 24\ 00 \\
15480 + 5 = 15485 & 7\ 74\ 25 \\
\hline
\end{array}$$

27. $\frac{5}{6} = .83333333+.$

$$\begin{array}{r|l}
 & .83'33'33'33+ \quad \underline{|.9128} \\
 & 81 \\
90 \times 2 = 180 & 2\ 33 \\
180 + 1 = 181 & 1\ 81 \\
\hline
910 \times 2 = 1820 & 52\ 33 \\
1820 + 2 = 1822 & 36\ 44 \\
\hline
9120 \times 2 = 18240 & 15\ 89\ 33 \\
18240 + 8 = 18248 & 14\ 59\ 84 \\
\hline
\end{array}$$

28. $\frac{2}{9} = .22222222+.$

$$\begin{array}{r|l}
 & .22'22'22'22+ \quad \underline{|.4714} \\
 & 16 \\
40 \times 2 = 80 & 6\ 22 \\
80 + 7 = 87 & 6\ 09 \\
\hline
470 \times 2 = 940 & 13\ 22 \\
940 + 1 = 941 & 9\ 41 \\
\hline
4710 \times 2 = 9420 & 3\ 81\ 22 \\
9420 + 4 = 9424 & 3\ 76\ 96 \\
\hline
\end{array}$$

29. $\frac{7}{8} = .875.$

$$\begin{array}{r|l}
 & .87'50'00'00 \quad \underline{|.9354} \\
 & 81 \\
90 \times 2 = 180 & 6\ 50 \\
180 + 3 = 183 & 5\ 49 \\
\hline
930 \times 2 = 1860 & 1\ 01\ 00 \\
1860 + 5 = 1865 & 93\ 25 \\
\hline
9350 \times 2 = 18700 & 7\ 75\ 00 \\
18700 + 4 = 18704 & 7\ 48\ 16 \\
\hline
\end{array}$$

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30.

$$\begin{array}{r|l}
 & 5.00'00'00'00 \;\underline{|\,2.2360+} \\
 & 4 \\
\hline
20 \times 2 = 40 & 1\ 00 \\
40 + 2 = 42 & \ \ 84 \\
\hline
220 \times 2 = 440 & 16\ 00 \\
440 + 3 = 443 & 13\ 29 \\
\hline
2230 \times 2 = 4460 & 2\ 71\ 00 \\
4460 + 6 = 4466 & 2\ 67\ 96 \\
\hline
22360 \times 2 = 44720 & 3\ 04\ 00
\end{array}$$

Since the square root of 5 is 2.2360+ and the square root of 16 is 4, the square root of $\frac{5}{16}$ to four decimal places is $\frac{2.2360}{4}$, or .5590.

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3.

$$\begin{array}{r|l}
 & x^3 - 3x^2y + 3xy^2 - y^3 \;\underline{|\,x - y} \\
 & x^3 \\
\hline
3x^2 & -3x^2y + 3xy^2 - y^3 \\
3x^2 - 3xy + y^2 & -3x^2y + 3xy^2 - y^3 \\
\hline
\end{array}$$

4.

$$\begin{array}{r|l}
 & m^3 - 9m^2 + 27m - 27 \;\underline{|\,m - 3} \\
 & m^3 \\
\hline
3m^2 & -9m^2 + 27m - 27 \\
3m^2 - 9m + 9 & -9m^2 + 27m - 27 \\
\hline
\end{array}$$

5.

$$\begin{array}{r|l}
 & 8m^3 - 60m^2n + 150mn^2 - 125n^3 \;\underline{|\,2m - 5n} \\
 & 8m^3 \\
\hline
12m^2 & -60m^2n + 150mn^2 - 125n^3 \\
12m^2 - 30mn + 25n^2 & -60m^2n + 150mn^2 - 125n^3 \\
\hline
\end{array}$$

6.

$$\begin{array}{r|l}
 & 27x^3 - 189x^2y + 441xy^2 - 343y^3 \;\underline{|\,3x - 7y} \\
 & 27x^3 \\
\hline
27x^2 & -189x^2y + 441xy^2 - 343y^3 \\
27x^2 - 63xy + 49y^2 & -189x^2y + 441xy^2 - 343y^3 \\
\hline
\end{array}$$

7.

$$\begin{array}{r|l}
 & 125a^3 + 675a^2x + 1215ax^2 + 729x^3 \;\underline{|\,5a + 9x} \\
 & 125a^3 \\
\hline
75a^2 & 675a^2x + 1215ax^2 + 729x^3 \\
75a^2 + 135ax + 81x^2 & 675a^2x + 1215ax^2 + 729x^3 \\
\hline
\end{array}$$

8.

$$\begin{array}{r|l}
 & 1000p^6 - 300p^4q + 30p^2q^2 - q^3 \;\underline{|\,10p^2 - q} \\
 & 1000p^6 \\
\hline
300p^4 & -300p^4q + 30p^2q^2 - q^3 \\
300p^4 - 30p^2q + q^2 & -300p^4q + 30p^2q^2 - q^3 \\
\hline
\end{array}$$

9.

$$\begin{array}{r|l}
 & m^6 + 6m^5 + 15m^4 + 20m^3 + 15m^2 + 6m + 1 \;\underline{|\,m^2 + 2m + 1} \\
 & m^6 \\
\hline
3m^4 & 6m^5 + 15m^4 + 20m^3 \\
3m^4 + 6m^3 + 4m^2 & 6m^5 + 12m^4 + 8m^3 \\
\hline
3m^4 + 12m^3 + 12m^2 & 3m^4 + 12m^3 + 15m^2 + 6m + 1 \\
3m^4 + 12m^3 + 15m^2 + 6m + 1 & 3m^4 + 12m^3 + 15m^2 + 6m + 1 \\
\hline
\end{array}$$

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10. $$x^6 - 6x^5 + 15x^4 - 20x^3 + 15x^2 - 6x + 1 \;\big|\; \underline{x^2 - 2x + 1}$$

$$\begin{array}{l|l}
 & x^6 \\ \hline
3x^4 & -6x^5 + 15x^4 - 20x^3 \\
3x^4 - 6x^3 + 4x^2 & -6x^5 + 12x^4 - 8x^3 \\ \hline
3x^4 - 12x^3 + 12x^2 & 3x^4 - 12x^3 + 15x^2 - 6x + 1 \\
3x^4 - 12x^3 + 15x^2 - 6x + 1 & 3x^4 - 12x^3 + 15x^2 - 6x + 1 \\ \hline
\end{array}$$

11. $$x^6 + 3x^5 + 9x^4 + 13x^3 + 18x^2 + 12x + 8 \;\big|\; \underline{x^2 + x + 2}$$

$$\begin{array}{l|l}
 & x^6 \\ \hline
3x^4 & 3x^5 + 9x^4 + 13x^3 \\
3x^4 + 3x^3 + x^2 & 3x^5 + 3x^4 + x^3 \\ \hline
3x^4 + 6x^3 + 3x^2 & 6x^4 + 12x^3 + 18x^2 + 12x + 8 \\
3x^4 + 6x^3 + 9x^2 + 6x + 4 & 6x^4 + 12x^3 + 18x^2 + 12x + 8 \\ \hline
\end{array}$$

12. $$x^6 + 12x^5 + 63x^4 + 184x^3 + 315x^2 + 300x + 125 \;\big|\; \underline{x^2 + 4x + 5}$$

$$\begin{array}{l|l}
 & x^6 \\ \hline
3x^4 & 12x^5 + 63x^4 + 184x^3 \\
3x^4 + 12x^3 + 16x^2 & 12x^5 + 48x^4 + 64x^3 \\ \hline
3x^4 + 24x^3 + 48x^2 & 15x^4 + 120x^3 + 315x^2 + 300x + 125 \\
3x^4 + 24x^3 + 63x^2 + 60x + 25 & 15x^4 + 120x^3 + 315x^2 + 300x + 125 \\ \hline
\end{array}$$

13. $$x^6 + 6x^5 - 18x^4 - 112x^3 + 180x^2 + 600x - 1000 \;\big|\; \underline{x^2 + 2x - 10}$$

$$\begin{array}{l|l}
 & x^6 \\ \hline
3x^4 & 6x^5 - 18x^4 - 112x^3 \\
3x^4 + 6x^3 + 4x^2 & 6x^5 + 12x^4 + 8x^3 \\ \hline
3x^4 + 12x^3 + 12x^2 & -30x^4 - 120x^3 + 180x^2 + 600x - 1000 \\
3x^4 + 12x^3 - 18x^2 - 60x + 100 & -30x^4 - 120x^3 + 180x^2 + 600x - 1000 \\ \hline
\end{array}$$

14. $$1 - 6a + 21a^2 - 44a^3 + 63a^4 - 54a^5 + 27a^6 \;\big|\; \underline{1 - 2a + 3a^2}$$

$$\begin{array}{l|l}
 & 1 \\ \hline
3 & -6a + 21a^2 - 44a^3 \\
3 - 6a + 4a^2 & -6a + 12a^2 - 8a^3 \\ \hline
3 - 12a + 12a^2 & 9a^2 - 36a^3 + 63a^4 - 54a^5 + 27a^6 \\
3 - 12a + 21a^2 - 18a^3 + 9a^4 & 9a^2 - 36a^3 + 63a^4 - 54a^5 + 27a^6 \\ \hline
\end{array}$$

15. $$8n^3 + 36n^4 + 42n^5 - 9n^6 - 21n^7 + 9n^8 - n^9 \;\big|\; \underline{2n + 3n^2 - n^3}$$

$$\begin{array}{l|l}
 & 8n^3 \\ \hline
12n^2 & 36n^4 + 42n^5 - 9n^6 \\
12n^2 + 18n^3 + 9n^4 & 36n^4 + 54n^5 + 27n^6 \\ \hline
12n^2 + 36n^3 + 27n^4 & -12n^5 - 36n^6 - 21n^7 + 9n^8 - n^9 \\
12n^2 + 36n^3 + 21n^4 - 9n^5 + n^6 & -12n^5 - 36n^6 - 21n^7 + 9n^8 - n^9 \\ \hline
\end{array}$$

16. $$x^3 - 12x^2 + 54x - 112 + \frac{108}{x} - \frac{48}{x^2} + \frac{8}{x^3} \;\Big|\; \underline{x - 4 + \frac{2}{x}}$$

$$\begin{array}{l|l}
 & x^3 \\ \hline
3x^2 & -12x^2 + 54x - 112 \\
3x^2 - 12x + 16 & -12x^2 + 48x - 64 \\ \hline
3x^2 - 24x + 48 & 6x - 48 + \frac{108}{x} - \frac{48}{x^2} + \frac{8}{x^3} \\
3x^2 - 24x + 54 - \frac{24}{x} + \frac{4}{x^2} & 6x - 48 + \frac{108}{x} - \frac{48}{x^2} + \frac{8}{x^3} \\ \hline
\end{array}$$

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17.

$$\begin{array}{l|l}
 & \dfrac{a^3b^3x^9}{c^3} - \dfrac{3a^2bx^8}{c} + \dfrac{3acx^7}{b} - \dfrac{c^3x^6}{b^3} \;\Big|\; \dfrac{abx^3}{c} - \dfrac{cx^2}{b} \\
 & \dfrac{a^3b^3x^9}{c^3} \\
\hline
\dfrac{3a^2b^2x^6}{c^2} & -\dfrac{3a^2bx^8}{c} + \dfrac{3acx^7}{b} - \dfrac{c^3x^6}{b^3} \\
\dfrac{3a^2b^2x^6}{c^2} - 3ax^5 + \dfrac{c^2x^4}{b^2} & -\dfrac{3a^2bx^8}{c} + \dfrac{3acx^7}{b} - \dfrac{c^3x^6}{b^3} \\
\hline
\end{array}$$

18.

$$\begin{array}{l|l}
 & x^6 + 6x^4 + 15x^2 + 20 + \dfrac{15}{x^2} + \dfrac{6}{x^4} + \dfrac{1}{x^6} \;\Big|\; x^2 + 2 + \dfrac{1}{x^2} \\
 & x^6 \\
\hline
3x^4 & 6x^4 + 15x^2 + 20 \\
3x^4 + 6x^2 + 4 & 6x^4 + 12x^2 + 8 \\
\hline
3x^4 + 12x^2 + 12 & 3x^2 + 12 + \dfrac{15}{x^2} + \dfrac{6}{x^4} + \dfrac{1}{x^6} \\
3x^4 + 12x^2 + 15 + \dfrac{6}{x^2} + \dfrac{1}{x^4} & 3x^2 + 12 + \dfrac{15}{x^2} + \dfrac{6}{x^4} + \dfrac{1}{x^6} \\
\hline
\end{array}$$

19.

$$\begin{array}{l|l}
 & \dfrac{1}{x^3} - \dfrac{3}{2x^2} + \dfrac{27}{4x} - \dfrac{49}{8} + \dfrac{27x}{2} - 6x^2 + 8x^3 \;\Big|\; \dfrac{1}{x} - \dfrac{1}{2} + 2x \\
 & \dfrac{1}{x^3} \\
\hline
\dfrac{3}{x^2} & -\dfrac{3}{2x^2} + \dfrac{27}{4x} - \dfrac{49}{8} \\
\dfrac{3}{x^2} - \dfrac{3}{2x} + \dfrac{1}{4} & -\dfrac{3}{2x^2} + \dfrac{3}{4x} - \dfrac{1}{8} \\
\hline
\dfrac{3}{x^2} - \dfrac{3}{x} + \dfrac{3}{4} & \dfrac{6}{x} - 6 + \dfrac{27x}{2} - 6x^2 + 8x^3 \\
\dfrac{3}{x^2} - \dfrac{3}{x} + \dfrac{27}{4} - 3x + 4x^2 & \dfrac{6}{x} - 6 + \dfrac{27x}{2} - 6x^2 + 8x^3 \\
\hline
\end{array}$$

20.

$$\begin{array}{l|l}
 & \dfrac{n^3}{8} - 3n^2 + \dfrac{51n}{2} - 88 + \dfrac{102}{n} - \dfrac{48}{n^2} + \dfrac{8}{n^3} \;\Big|\; \dfrac{n}{2} - 4 + \dfrac{2}{n} \\
 & \dfrac{n^3}{8} \\
\hline
\dfrac{3n^2}{4} & -3n^2 + \dfrac{51n}{2} - 88 \\
\dfrac{3n^2}{4} - 6n + 16 & -3n^2 + 24n - 64 \\
\hline
\dfrac{3n^2}{4} - 12n + 48 & \dfrac{3n}{2} - 24 + \dfrac{102}{n} - \dfrac{48}{n^2} + \dfrac{8}{n^3} \\
\dfrac{3n^2}{4} - 12n + 51 - \dfrac{24}{n} + \dfrac{4}{n^2} & \dfrac{3n}{2} - 24 + \dfrac{102}{n} - \dfrac{48}{n^2} + \dfrac{8}{n^3} \\
\hline
\end{array}$$

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21.

$$c^6 - 3c^5d - 3c^4d^2 + 11c^3d^3 + 6c^2d^4 - 12cd^5 - 8d^6 \;\Big|\; c^2 - cd - 2d^2$$

$$\begin{array}{l|l}
 & c^6 \\
\hline
3c^4 & -3c^5d - 3c^4d^2 + 11c^3d^3 \\
3c^4 - 3c^3d + c^2d^2 & -3c^5d + 3c^4d^2 - c^3d^3 \\
\hline
3c^4 - 6c^3d + 3c^2d^2 & -6c^4d^2 + 12c^3d^3 + 6c^2d^4 - 12cd^5 - 8d^6 \\
3c^4 - 6c^3d - 3c^2d^2 + 6cd^3 + 4d^4 & -6c^4d^2 + 12c^3d^3 + 6c^2d^4 - 12cd^5 - 8d^6 \\
\hline
\end{array}$$

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22.

$$\frac{r^3}{s^3} - \frac{9r^2}{s^2} + \frac{30r}{s} - 45 + \frac{30s}{r} - \frac{9s^2}{r^2} + \frac{s^3}{r^3} \;\Big|\; \frac{r}{s} - 3 + \frac{s}{r}$$

$$\begin{array}{l|l}
 & \frac{r^3}{s^3} \\
\hline
\frac{3r^2}{s^2} & -\frac{9r^2}{s^2} + \frac{30r}{s} - 45 \\
\frac{3r^2}{s^2} - \frac{9r}{s} + 9 & -\frac{9r^2}{s^2} + \frac{27r}{s} - 27 \\
\hline
\frac{3r^2}{s^2} - \frac{18r}{s} + 27 & \frac{3r}{s} - 18 + \frac{30s}{r} - \frac{9s^2}{r^2} + \frac{s^3}{r^3} \\
\frac{3r^2}{s^2} - \frac{18r}{s} + 30 - \frac{9s}{r} + \frac{s^2}{r^2} & \frac{3r}{s} - 18 + \frac{30s}{r} - \frac{9s^2}{r^2} + \frac{s^3}{r^3} \\
\hline
\end{array}$$

23.

$$27x^6 - 9x^4 + 55x^2 - \frac{325}{27} + \frac{110}{3x^2} - \frac{4}{x^4} + \frac{8}{x^6} \;\Big|\; 3x^2 - \frac{1}{3} + \frac{2}{x^2}$$

$$\begin{array}{l|l}
 & 27x^6 \\
\hline
27x^4 & -9x^4 + 55x^2 - \frac{325}{27} \\
27x^4 - 3x^2 + \frac{1}{9} & -9x^4 + x^2 - \frac{1}{27} \\
\hline
27x^4 - 6x^2 + \frac{1}{3} & 54x^2 - 12 + \frac{110}{3x^2} - \frac{4}{x^4} + \frac{8}{x^6} \\
27x^4 - 6x^2 + \frac{55}{3} - \frac{2}{x^2} + \frac{4}{x^4} & 54x^2 - 12 + \frac{110}{3x^2} - \frac{4}{x^4} + \frac{8}{x^6} \\
\hline
\end{array}$$

24.

$$\frac{8a^9}{27} + 2a^7 - \frac{4a^6}{3} + \frac{9a^5}{2} - 6a^4 + \frac{43a^3}{8} - \frac{27a^2}{4} + \frac{9a}{2} - 1 \;\Big|\; \frac{2a^3}{3} + \frac{3a}{2} - 1$$

$$\begin{array}{l|l}
 & \frac{8a^9}{27} \\
\hline
\frac{4a^6}{3} & 2a^7 - \frac{4a^6}{3} + \frac{9a^5}{2} - 6a^4 + \frac{43a^3}{8} \\
\frac{4a^6}{3} + 3a^4 + \frac{9a^2}{4} & 2a^7 + \frac{9a^5}{2} + \frac{27a^3}{8} \\
\hline
\frac{4a^6}{3} + 6a^4 + \frac{27a^2}{4} & -\frac{4a^6}{3} - 6a^4 + 2a^3 - \frac{27a^2}{4} + \frac{9a}{2} - 1 \\
\frac{4a^6}{3} + 6a^4 - 2a^3 + \frac{27a^2}{4} - \frac{9a}{2} + 1 & -\frac{4a^6}{3} - 6a^4 + 2a^3 - \frac{27a^2}{4} + \frac{9a}{2} - 1 \\
\hline
\end{array}$$

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25.

$$\frac{b^9}{a^6}+\frac{3b^8}{a^7}+\frac{6b^7}{a^8}+\frac{7b^6}{a^9}+\frac{6b^5}{a^{10}}+\frac{3b^4}{a^{11}}+\frac{b^3}{a^{12}}\ \Big|\ \frac{b^3}{a^2}+\frac{b^2}{a^3}+\frac{b}{a^4}$$

$$\frac{b^9}{a^6}$$

$$\begin{array}{l|l}
\frac{3b^6}{a^4} & \frac{3b^8}{a^7}+\frac{6b^7}{a^8}+\frac{7b^6}{a^9} \\
\frac{3b^6}{a^4}+\frac{3b^5}{a^5}+\frac{b^4}{a^6} & \frac{3b^8}{a^7}+\frac{3b^7}{a^8}+\frac{b^6}{a^9} \\
\hline
\frac{3b^6}{a^4}+\frac{6b^5}{a^5}+\frac{3b^4}{a^6} & \frac{3b^7}{a^8}+\frac{6b^6}{a^9}+\frac{6b^5}{a^{10}}+\frac{3b^4}{a^{11}}+\frac{b^3}{a^{12}} \\
\frac{3b^6}{a^4}+\frac{6b^5}{a^5}+\frac{6b^4}{a^6}+\frac{3b^3}{a^7}+\frac{b^2}{a^8} & \frac{3b^7}{a^8}+\frac{6b^6}{a^9}+\frac{6b^5}{a^{10}}+\frac{3b^4}{a^{11}}+\frac{b^3}{a^{12}} \\
\hline
\end{array}$$

26.

$$n^6-\tfrac{3}{2}n^5+\tfrac{9}{4}n^4-\tfrac{13}{8}n^3+\tfrac{9}{8}n^2-\tfrac{3}{8}n+\tfrac{1}{8}\ \Big|\ n^2-\tfrac{1}{2}n+\tfrac{1}{2}$$

$$n^6$$

$$\begin{array}{l|l}
3n^4 & -\tfrac{3}{2}n^5+\tfrac{9}{4}n^4-\tfrac{13}{8}n^3 \\
3n^4-\tfrac{3}{2}n^3+\tfrac{1}{4}n^2 & -\tfrac{3}{2}n^5+\tfrac{3}{4}n^4-\tfrac{1}{8}n^3 \\
\hline
3n^4-3n^3+\tfrac{3}{4}n^2 & \tfrac{3}{2}n^4-\tfrac{3}{2}n^3+\tfrac{9}{8}n^2-\tfrac{3}{8}n+\tfrac{1}{8} \\
3n^4-3n^3+\tfrac{9}{4}n^2-\tfrac{3}{4}n+\tfrac{1}{4} & \tfrac{3}{2}n^4-\tfrac{3}{2}n^3+\tfrac{9}{8}n^2-\tfrac{3}{8}n+\tfrac{1}{8} \\
\hline
\end{array}$$

27.

$$\tfrac{1}{27}r^6-\tfrac{1}{6}r^5-\tfrac{3}{4}r^4+\tfrac{23}{8}r^3+\tfrac{27}{4}r^2-\tfrac{27}{2}r-27\ \Big|\ \tfrac{1}{3}r^2-\tfrac{1}{2}r-3$$

$$\tfrac{1}{27}r^6$$

$$\begin{array}{l|l}
\tfrac{1}{3}r^4 & -\tfrac{1}{6}r^5-\tfrac{3}{4}r^4+\tfrac{23}{8}r^3 \\
\tfrac{1}{3}r^4-\tfrac{1}{2}r^3+\tfrac{1}{4}r^2 & -\tfrac{1}{6}r^5+\tfrac{1}{4}r^4-\tfrac{1}{8}r^3 \\
\hline
\tfrac{1}{3}r^4-r^3+\tfrac{3}{4}r^2 & -r^4+3r^3+\tfrac{27}{4}r^2-\tfrac{27}{2}r-27 \\
\tfrac{1}{3}r^4-r^3-\tfrac{9}{4}r^2+\tfrac{9}{2}r+9 & -r^4+3r^3+\tfrac{27}{4}r^2-\tfrac{27}{2}r-27 \\
\hline
\end{array}$$

28.

$$\Big|\ \tfrac{1}{2}x^2+xy-\tfrac{2}{3}y^2$$

$$\tfrac{1}{8}x^6+\tfrac{3}{4}x^5y+x^4y^2-x^3y^3-\tfrac{4}{3}x^2y^4+\tfrac{4}{3}xy^5-\tfrac{8}{27}y^6$$

$$\tfrac{1}{8}x^6$$

$$\begin{array}{l|l}
\tfrac{3}{4}x^4 & \tfrac{3}{4}x^5y+x^4y^2-x^3y^3 \\
\tfrac{3}{4}x^4+\tfrac{3}{2}x^3y+x^2y^2 & \tfrac{3}{4}x^5y+\tfrac{3}{2}x^4y^2+x^3y^3 \\
\hline
\tfrac{3}{4}x^4+3x^3y+3x^2y^2 & -\tfrac{1}{2}x^4y^2-2x^3y^3-\tfrac{4}{3}x^2y^4+\tfrac{4}{3}xy^5-\tfrac{8}{27}y^6 \\
\tfrac{3}{4}x^4+3x^3y+2x^2y^2-2xy^3+\tfrac{4}{9}y^4 & -\tfrac{1}{2}x^4y^2-2x^3y^3-\tfrac{4}{3}x^2y^4+\tfrac{4}{3}xy^5-\tfrac{8}{27}y^6 \\
\hline
\end{array}$$

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3.

$$\begin{array}{lr|l}
 & 29'791 & \underline{31} \\
 & 27 & \\
\hline
 & & 2\,791 \\
3(30)^2 = & 2700 & \\
3(30\times 1) = & 90 & \\
1^2 = & 1 & \\
\hline
 & 2791 & 2\,791 \\
\hline
\end{array}$$

4.

$$\begin{array}{lr|l}
 & 54'872 & \underline{38} \\
 & 27 & \\
\hline
3(30)^2 = & 2700 & 27\,872 \\
3(30\times 8) = & 720 & \\
8^2 = & 64 & \\
\hline
 & 3484 & 27\,872 \\
\hline
\end{array}$$

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5.

$$\begin{array}{lr|l} & 110'592 & \underline{48} \\ & 64 & \\ \hline 3(40)^2 = & 4800 & 46\ 592 \\ 3(40 \times 8) = & 960 & \\ 8^2 = & 64 & \\ \hline & 5824 & 46\ 592 \end{array}$$

7.

$$\begin{array}{lr|l} & 681'472 & \underline{88} \\ & 512 & \\ \hline 3(80)^2 = & 19200 & 169\ 472 \\ 3(80 \times 8) = & 1920 & \\ 8^2 = & 64 & \\ \hline & 21184 & 169\ 472 \end{array}$$

6.

$$\begin{array}{lr|l} & 300'763 & \underline{67} \\ & 216 & \\ \hline 3(60)^2 = & 10800 & 84\ 763 \\ 3(60 \times 7) = & 1260 & \\ 7^2 = & 49 & \\ \hline & 12109 & 84\ 763 \end{array}$$

8.

$$\begin{array}{lr|l} & 941'192 & \underline{98} \\ & 729 & \\ \hline 3(90)^2 = & 24300 & 212\ 192 \\ 3(90 \times 8) = & 2160 & \\ 8^2 = & 64 & \\ \hline & 26524 & 212\ 192 \end{array}$$

9.

$$\begin{array}{lr|l} & 2'406'104 & \underline{134} \\ & 1 & \\ \hline 3(10)^2 = & 300 & 1\ 406 \\ 3(10 \times 3) = & 90 & \\ 3^2 = & 9 & \\ \hline & 399 & 1\ 197 \\ \hline 3(130)^2 = & 50700 & 209\ 104 \\ 3(130 \times 4) = & 1560 & \\ 4^2 = & 16 & \\ \hline & 52276 & 209\ 104 \end{array}$$

11.

$$\begin{array}{lr|l} & 28'372'625 & \underline{305} \\ & 27 & \\ \hline 3(30)^2 = & 2700 & 1\ 372 \\ \hline 3(300)^2 = & 270000 & 1\ 372\ 625 \\ 3(300 \times 5) = & 4500 & \\ 5^2 = & 25 & \\ \hline & 274525 & 1\ 372\ 625 \end{array}$$

12.

$$\begin{array}{lr|l} & 48.228'544 & \underline{3.64} \\ & 27 & \\ \hline 3(30)^2 = & 2700 & 21\ 228 \\ 3(30 \times 6) = & 540 & \\ 6^2 = & 36 & \\ \hline & 3276 & 19\ 656 \\ \hline 3(360)^2 = & 388800 & 1\ 572\ 544 \\ 3(360 \times 4) = & 4320 & \\ 4^2 = & 16 & \\ \hline & 393136 & 1\ 572\ 544 \end{array}$$

10.

$$\begin{array}{lr|l} & 69'426'531 & \underline{411} \\ & 64 & \\ \hline 3(40)^2 = & 4800 & 5\ 426 \\ 3(40 \times 1) = & 120 & \\ 1^2 = & 1 & \\ \hline & 4921 & 4\ 921 \\ \hline 3(410)^2 = & 504300 & 505\ 531 \\ 3(410 \times 1) = & 1230 & \\ 1^2 = & 1 & \\ \hline & 505531 & 505\ 531 \end{array}$$

13.

$$\begin{array}{lr|l} & 17'173.512 & \underline{25.8} \\ & 8 & \\ \hline 3(20)^2 = & 1200 & 9\ 173 \\ 3(20 \times 5) = & 300 & \\ 5^2 = & 25 & \\ \hline & 1525 & 7\ 625 \\ \hline 3(250)^2 = & 187500 & 1\ 548\ 512 \\ 3(250 \times 8) = & 6000 & \\ 8^2 = & 64 & \\ \hline & 193564 & 1\ 548\ 512 \end{array}$$

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14.

$95.443'993 \,|\, \underline{4.57}$

64

$3(40)^2$	=	4800	31 443
$3(40 \times 5)$	=	600	
5^2	=	25	
		5425	27 125
$3(450)^2$	=	607500	4 318 993
$3(450 \times 7)$	=	9450	
7^2	=	49	
		616999	4 318 993

15.

$.000'024'389 \,|\, \underline{.029}$

8

$3(20)^2$	=	1200	16 389
$3(20 \times 9)$	=	540	
9^2	=	81	
		1821	16 389

16.

$.001'906'624 \,|\, \underline{.124}$

1

$3(10)^2$	=	300	906
$3(10 \times 2)$	=	60	
2^2	=	4	
		364	728
$3(120)^2$	=	43200	178 624
$3(120 \times 4)$	=	1440	
4^2	=	16	
		44656	178 624

17.

$.000'912'673 \,|\, \underline{.097}$

729

$3(90)^2$	=	24300	183 673
$3(90 \times 7)$	=	1890	
7^2	=	49	
		26239	183 673

18.

$.259'694'072 \,|\, \underline{.638}$

216

$3(60)^2$	=	10800	43 694
$3(60 \times 3)$	=	540	
3^2	=	9	
		11349	34 047
$3(630)^2$	=	1190700	9 647 072
$3(630 \times 8)$	=	15120	
8^2	=	64	
		1205884	9 647 072

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19.

		926.859′375	9.75
		729	
$3(90)^2 =$	24300	197 859	
$3(90 \times 7) =$	1890		
$7^2 =$	49		
	26239	183 673	
$3(970)^2 =$	2822700	14 186 375	
$3(970 \times 5) =$	14550		
$5^2 =$	25		
	2837275	14 186 375	

20.

		514′500.058′197	80.13
		512	
$3(80)^2 =$	19200	2 500	
$3(800)^2 =$	1920000	2 500 058	
$3(800 \times 1) =$	2400		
$1^2 =$	1		
	1922401	1 922 401	
$3(8010)^2 =$	192480300	577 657 197	
$3(8010 \times 3) =$	72090		
$3^2 =$	9		
	192552399	577 657 197	

21.

		2.000′000′000	1.259
		1	
$3(10)^2 =$	300	1 000	
$3(10 \times 2) =$	60		
$2^2 =$	4		
	364	728	
$3(120)^2 =$	43200	272 000	
$3(120 \times 5) =$	1800		
$5^2 =$	25		
	45025	225 125	
$3(1250)^2 =$	4687500	46 875 000	
$3(1250 \times 9) =$	33750		
$9^2 =$	81		
	4721331	42 491 979	

22.

		5.000′000′000	1.709
		1	
$3(10)^2 =$	300	4 000	
$3(10 \times 7) =$	210		
$7^2 =$	49		
	559	3 913	
$3(170)^2 =$	86700	87 000	
$3(1700)^2 =$	8670000	87 000 000	
$3(1700 \times 9) =$	45900		
$9^2 =$	81		
	8715981	78 443 829	

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23.

$$
\begin{array}{lcr|l}
 & & & .800'000'000 \;\underline{|.928} \\
 & & & 729 \\
\hline
3(90)^2 & = & 24300 & 71\,000 \\
3(90 \times 2) & = & 540 & \\
2^2 & = & 4 & \\
\hline
 & & 24844 & 49\,688 \\
\hline
3(920)^2 & = & 2539200 & 21\,312\,000 \\
3(920 \times 8) & = & 22080 & \\
8^2 & = & 64 & \\
\hline
 & & 2561344 & 20\,490\,752 \\
\end{array}
$$

24.

$$
\begin{array}{lcr|l}
 & & & .160'000'000 \;\underline{|.542} \\
 & & & 125 \\
\hline
3(50)^2 & = & 7500 & 35\,000 \\
3(50 \times 4) & = & 600 & \\
4^2 & = & 16 & \\
\hline
 & & 8116 & 32\,464 \\
\hline
3(540)^2 & = & 874800 & 2\,536\,000 \\
3(540 \times 2) & = & 3240 & \\
2^2 & = & 4 & \\
\hline
 & & 878044 & 1\,756\,088 \\
\end{array}
$$

25. Since, Ex. 22, the cube root of 5 is 1.709+, and since the cube root of 64 is 4, the cube root of $\frac{5}{64}$ to three decimal places is $\frac{1.709}{4}$, or .427

26. $\frac{2}{3} = .666666666+.$

$$
\begin{array}{lcr|l}
 & & & .666'666'666+ \;\underline{|.873} \\
 & & & 512 \\
\hline
3(80)^2 & = & 19200 & 154\,666 \\
3(80 \times 7) & = & 1680 & \\
7^2 & = & 49 & \\
\hline
 & & 20929 & 146\,503 \\
\hline
3(870)^2 & = & 2270700 & 8\,163\,666 \\
3(870 \times 3) & = & 7830 & \\
3^2 & = & 9 & \\
\hline
 & & 2278539 & 6\,835\,617 \\
\end{array}
$$

27. $\frac{7}{8} = .875.$

$$
\begin{array}{lcr|l}
 & & & .875'000'000 \;\underline{|.956} \\
 & & & 729 \\
\hline
3(90)^2 & = & 24300 & 146\,000 \\
3(90 \times 5) & = & 1350 & \\
5^2 & = & 25 & \\
\hline
 & & 25675 & 128\,375 \\
\hline
3(950)^2 & = & 2707500 & 17\,625\,000 \\
3(950 \times 6) & = & 17100 & \\
6^2 & = & 36 & \\
\hline
 & & 2724636 & 16\,347\,816 \\
\end{array}
$$

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28. $\frac{3}{16} = .1875.$

$$
\begin{array}{lr|l}
 & & .187'500'000\ \underline{|.572} \\
 & & 125 \\
\hline
3(50)^2 = & 7500 & 62\ 500 \\
3(50 \times 7) = & 1050 & \\
7^2 = & 49 & \\
\hline
 & 8599 & 60\ 193 \\
\hline
3(570)^2 = & 974700 & 2\ 307\ 000 \\
3(570 \times 2) = & 3420 & \\
2^2 = & 4 & \\
\hline
 & 978124 & 1\ 956\ 248 \\
\hline
\end{array}
$$

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1. $\sqrt{a^6 - 12a^3 + 36} = \sqrt{(a^3-6)(a^3-6)} = a^3 - 6.$

2. $\sqrt[3]{125 - 75x + 15x^2 - x^3} = \sqrt[3]{(5-x)(5-x)(5-x)} = 5 - x.$

3.

$$
\begin{array}{l|l}
 & 16 - 32x + 24x^2 - 8x^3 + x^4\ \underline{|4 - 4x + x^2} \\
 & 16 \\
\hline
8 & -32x + 24x^2 \\
8 - 4x & -32x + 16x^2 \\
\hline
8 - 8x & 8x^2 - 8x^3 + x^4 \\
8 - 8x + x^2 & 8x^2 - 8x^3 + x^4 \\
\hline
\end{array}
\qquad
\begin{array}{l|l}
 & 4 - 4x + x^2\ \underline{|2 - x} \\
 & 4 \\
\hline
4 & -4x + x^2 \\
4 - x & -4x + x^2 \\
\hline
\end{array}
$$

Hence, the fourth root of $16 - 32x + 24x^2 - 8x^3 + x^4$ is $2 - x$.

4.

$$
\begin{array}{l|l}
 & x^4 + 12x^3y + 54x^2y^2 + 108xy^3 + 81y^4\ \underline{|x^2 + 6xy + 9y^2} \\
 & x^4 \\
\hline
2x^2 & 12x^3y + 54x^2y^2 \\
2x^2 + 6xy & 12x^3y + 36x^2y^2 \\
\hline
2x^2 + 12xy & 18x^2y^2 + 108xy^3 + 81y^4 \\
2x^2 + 12xy + 9y^2 & 18x^2y^2 + 108xy^3 + 81y^4 \\
\hline
\end{array}
\qquad
\begin{array}{l|l}
 & x^2 + 6xy + 9y^2\ \underline{|x + 3y} \\
 & x^2 \\
\hline
2x & 6xy + 9y^2 \\
2x + 3y & 6xy + 9y^2 \\
\hline
\end{array}
$$

Hence, the fourth root of $x^4 + 12x^3y + 54x^2y^2 + 108xy^3 + 81y^4$ is $x + 3y$.

5.

$$
\begin{array}{l|l}
 & 16m^4 - 32m^3 + 24m^2 - 8m + 1\ \underline{|4m^2 - 4m + 1} \\
 & 16m^4 \\
\hline
8m^2 & -32m^3 + 24m^2 \\
8m^2 - 4m & -32m^3 + 16m^2 \\
\hline
8m^2 - 8m & 8m^2 - 8m + 1 \\
8m^2 - 8m + 1 & 8m^2 - 8m + 1 \\
\hline
\end{array}
\qquad
\begin{array}{l|l}
 & 4m^2 - 4m + 1\ \underline{|2m - 1} \\
 & 4m^2 \\
\hline
4m & -4m + 1 \\
4m - 1 & -4m + 1 \\
\hline
\end{array}
$$

Hence, the fourth root of $16m^4 - 32m^3 + 24m^2 - 8m + 1$ is $2m - 1$.

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6.

$$32x^5 + 80x^4 + 80x^3 + 40x^2 + 10x + 1 \,|\, \underline{2x + 1}$$
$$32x^5$$

$5(2x)^4 = 80x^4$ | $80x^4 + 80x^3 + 40x^2 + 10x + 1$

$5(2x)^4 + 10(2x)^3(1) + 10(2x)^2(1)^2 + 5(2x)(1)^3 + (1)^4$
$= 80x^4 + 80x^3 + 40x^2 + 10x + 1$ | $80x^4 + 80x^3 + 40x^2 + 10x + 1$

7.

$$a^{10} + 15a^8 + 90a^6 + 270a^4 + 405a^2 + 243 \,|\, \underline{a^2 + 3}$$
$$a^{10}$$

$5(a^2)^4 = 5a^8$ | $15a^8 + 90a^6 + 270a^4 + 405a^2 + 243$

$5(a^2)^4 + 10(a^2)^3(3) + 10(a^2)^2(3)^2 + 5(a^2)(3)^3 + (3)^4$
$= 5a^8 + 30a^6 + 90a^4 + 135a^2 + 81$ | $15a^8 + 90a^6 + 270a^4 + 405a^2 + 243$

8. See next page.

9.

$$64x^6 - 576x^5 + 2160x^4 - 4320x^3 + 4860x^2 - 2916x + 729 \,|\, \underline{4x^2 - 12x + 9}$$
$$64x^6$$

$48x^4$ | $-576x^5 + 2160x^4 - 4320x^3$
$48x^4 - 144x^3 + 144x^2$ | $-576x^5 + 1728x^4 - 1728x^3$

$48x^4 - 288x^3 + 432x^2$ | $432x^4 - 2592x^3 + 4860x^2 - 2916x + 729$
$48x^4 - 288x^3 + 540x^2 - 324x + 81$ | $432x^4 - 2592x^3 + 4860x^2 - 2916x + 729$

The square root of $4x^2 - 12x + 9$ is found to be $2x - 3$.
Hence, the sixth root of $64x^6 - 576x^5 + 2160x^4 - 4320x^3 + 4860x^2 - 2916x + 729$ is $2x - 3$.

10.

$$x^6 + 6acx^5 + 15a^2c^2x^4 + 20a^3c^3x^3 + 15a^4c^4x^2 + 6a^5c^5x + a^6c^6 \,|\, \underline{x^2 + 2acx + a^2c^2}$$
$$x^6$$

$3x^4$ | $6acx^5 + 15a^2c^2x^4 + 20a^3c^3x^3$
$3x^4 + 6acx^3 + 4a^2c^2x^2$ | $6acx^5 + 12a^2c^2x^4 + 8a^3c^3x^3$

$3x^4 + 12acx^3 + 12a^2c^2x^2$ | $3a^2c^2x^4 + 12a^3c^3x^3 + 15a^4c^4x^2 + 6a^5c^5x + a^6c^6$
$3x^4 + 12acx^3 + 15a^2c^2x^2 + 6a^3c^3x + a^4c^4$ | $3a^2c^2x^4 + 12a^3c^3x^3 + 15a^4c^4x^2 + 6a^5c^5x + a^6c^6$

The square root of $x^2 + 2acx + a^2c^2$ is found to be $x + ac$.
Hence, the sixth root of $x^6 + 6acx^5 + 15a^2c^2x^4 + 20a^3c^3x^3 + 15a^4c^4x^2 + 6a^5c^5x + a^6c^6$ is $x + ac$.

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8.

$$
\begin{array}{l|l}
 & x^6-12x^5+60x^4-160x^3+240x^2-192x+64 \;\underline{|x^2-4x+4} \\
 & x^6 \\
\hline
3x^4 & -12x^5+60x^4-160x^3 \\
3x^4-12x^3+16x^2 & -12x^5+48x^4-\;\;64x^3 \\
\hline
3x^4-24x^3+48x^2 & 12x^4-\;\;96x^3+240x^2-192x+64 \\
3x^4-24x^3+60x^2-48x+16 & 12x^4-\;\;96x^3+240x^2-192x+64 \\
\hline
\end{array}
$$

The square root of x^2-4x+4 is found to be $x-2$.

Hence, the sixth root of $x^6-12x^5+60x^4-160x^3+240x^2-192x+64$ is $x-2$.

11. $3375 = 3^3 \times 5^3$;

$\therefore \sqrt[3]{3375} = 3 \times 5 = 15.$

12. $1296 = 2^4 \times 3^4$;

$\therefore \sqrt[4]{1296} = 2 \times 3 = 6.$

13.

$$
\begin{array}{r|l}
 & 5'06'25 \;\underline{|225} \\
 & 4 \\
\hline
20 \times 2 = 40 & 1\;06 \\
40 + 2 = 42 & \;\;\,84 \\
\hline
220 \times 2 = 440 & \;22\;25 \\
440 + 5 = 445 & \;22\;25 \\
\hline
\end{array}
$$

$$
\begin{array}{r|l}
 & 2'25 \;\underline{|15} \\
 & 1 \\
\hline
10 \times 2 = 20 & 1\;25 \\
20 + 5 = 25 & 1\;25 \\
\hline
\end{array}
$$

Hence, $\sqrt[4]{50625} = 15.$

14.

$$
\begin{array}{r|l}
 & 4'66'56 \;\underline{|216} \\
 & 4 \\
\hline
20 \times 2 = 40 & 66 \\
40 + 1 = 41 & 41 \\
\hline
210 \times 2 = 420 & 25\;56 \\
420 + 6 = 426 & 25\;56 \\
\hline
\end{array}
$$

$\sqrt[3]{216} = 6.$

Hence, $\sqrt[6]{46656} = 6.$

15. $262144 = 2^6 \times 2^6 \times 2^6$;

$\therefore \sqrt[6]{262144} = 2 \times 2 \times 2 = 8.$

16. $759375 = 3^5 \times 5^5$;

$\therefore \sqrt[5]{759375} = 3 \times 5 = 15.$

17. $531441 = 3^6 \times 3^6$;

$\therefore \sqrt[6]{531441} = 3 \times 3 = 9.$

18.

$$
\begin{array}{r|l}
 & 5'76'48'01 \;\underline{|2401} \\
 & 4 \\
\hline
20 \times 2 = 40 & 1\;76 \\
40 + 4 = 44 & 1\;76 \\
\hline
240 \times 2 = 480 & \;\;48 \\
\hline
2400 \times 2 = 4800 & \;\;48\;01 \\
4800 + 1 = 4801 & \;\;48\;01 \\
\hline
\end{array}
$$

$$
\begin{array}{r|l}
 & 24'01 \;\underline{|49} \\
 & 16 \\
\hline
40 \times 2 = 80 & 8\;01 \\
80 + 9 = 89 & 8\;01 \\
\hline
\end{array}
$$

$\sqrt{49} = 7.$

Hence, $\sqrt[8]{5764801} = 7.$

19. $4084101 = 3^5 \times 7^5$;

$\therefore \sqrt[5]{4084101} = 3 \times 7 = 21.$

20.

$$
\begin{array}{r|l}
 & 16'77'72'16 \;\underline{|4096} \\
 & 16 \\
\hline
40 \times 2 = 80 & 77 \\
\hline
400 \times 2 = 800 & 77\;72 \\
800 + 9 = 809 & 72\;81 \\
\hline
4090 \times 2 = 8180 & \;4\;91\;16 \\
8180 + 6 = 8186 & \;4\;91\;16 \\
\hline
\end{array}
$$

$$
\begin{array}{r|l}
 & 40'96 \;\underline{|64} \\
 & 36 \\
\hline
60 \times 2 = 120 & 4\;96 \\
120 + 4 = 124 & 4\;96 \\
\hline
\end{array}
$$

$\sqrt{64} = 8.$

Hence, $\sqrt[8]{16777216} = 8.$

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21.

$$
\begin{array}{r|l}
 & 24'13'75'69 \;\underline{|4913} \\
 & 16 \\
\hline
40 \times 2 = 80 & 8\ 13 \\
80 + 9 = 89 & 8\ 01 \\
\hline
490 \times 2 = 980 & \quad 12\ 75 \\
980 + 1 = 981 & \quad 9\ 81 \\
\hline
4910 \times 2 = 9820 & \quad 2\ 94\ 69 \\
9820 + 3 = 9823 & \quad 2\ 94\ 69 \\
\hline
\end{array}
\qquad
\begin{array}{r|l}
 & 4'913 \;\underline{|17} \\
 & 1 \\
\hline
3(10)^2 \quad = 300 & 3\ 913 \\
3(10 \times 7) = 210 & \\
7^2 \quad = \;\; 49 & \\
\hline
559 & 3\ 913 \\
\hline
\end{array}
$$

Hence, $\sqrt[6]{24137569} = 17.$

22.

$$
\begin{array}{llr|l}
 & & & 10'604'499'373 \;\underline{|2197} \\
 & & & 8 \\
\hline
3(20)^2 & = & 1200 & 2\ 604 \\
3(20 \times 1) & = & 60 & \\
1^2 & = & 1 & \\
\hline
 & & 1261 & 1\ 261 \\
\hline
3(210)^2 & = & 132300 & 1\ 343\ 499 \\
3(210 \times 9) & = & 5670 & \\
9^2 & = & 81 & \\
\hline
 & & 138051 & 1\ 242\ 459 \\
\hline
3(2190)^2 & = & 14388300 & \quad 101\ 040\ 373 \\
3(2190 \times 7) & = & 45990 & \\
7^2 & = & 49 & \\
\hline
 & & 14434339 & \quad 101\ 040\ 373 \\
\hline
\end{array}
$$

$$
\begin{array}{r|l}
 & 2'197 \;\underline{|13} \\
 & 1 \\
\hline
3(10)^2 \quad = 300 & 1\ 197 \\
3(10 \times 3) = \;\; 90 & \\
3^2 \quad = \;\;\; 9 & \\
\hline
399 & 1\ 197 \\
\hline
\end{array}
$$

Hence, $\sqrt[9]{10604499373} = 13.$

THEORY OF EXPONENTS

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2. $8^{\frac{1}{3}} = 2.$

3. $8^{\frac{2}{3}} = (8^{\frac{1}{3}})^2 = 2^2 = 4.$

4. $8^{-\frac{1}{3}} = \dfrac{1}{8^{\frac{1}{3}}} = \dfrac{1}{2}.$

5. $(-8)^{\frac{1}{3}} = -2.$

6. $(64)^{\frac{2}{3}} = (64^{\frac{1}{3}})^2 = 4^2 = 16.$

7. $32^{\frac{3}{5}} = (32^{\frac{1}{5}})^3 = 2^3 = 8.$

8. $25^{\frac{3}{2}} = (25^{\frac{1}{2}})^3 = (\pm 5)^3 = \pm 125.$

9. $81^{\frac{3}{4}} = (81^{\frac{1}{4}})^3 = (\pm 3)^3 = \pm 27.$

10. $64^{-\frac{2}{3}} = \dfrac{1}{(64^{\frac{1}{3}})^2} = \dfrac{1}{4^2} = \dfrac{1}{16}.$

11. $(-8)^{-\frac{4}{3}} = \dfrac{1}{(-8)^{\frac{4}{3}}} = \dfrac{1}{(-2)^4} = \dfrac{1}{16}.$

12. $(-32)^{-\frac{3}{5}} = \dfrac{1}{(-32)^{\frac{3}{5}}} = \dfrac{1}{(-2)^3} = -\dfrac{1}{8}.$

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13. $16^{-\frac{3}{4}} = \dfrac{1}{16^{\frac{3}{4}}} = \dfrac{1}{(\pm 2)^3} = \pm\dfrac{1}{8}.$

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36.
$$\begin{aligned}\sqrt[3]{x^2} + x^{\frac{1}{3}} + 8^{\frac{2}{3}} + 3x^{\frac{2}{3}} - 5\sqrt[3]{x} - \sqrt[3]{27^2} &= x^{\frac{2}{3}} + x^{\frac{1}{3}} + 4 + 3x^{\frac{2}{3}} - 5x^{\frac{1}{3}} - 3^2 \\ &= 4x^{\frac{2}{3}} - 4x^{\frac{1}{3}} - 5.\end{aligned}$$

37.
$$\begin{aligned}&4\sqrt[5]{x} + 5x^0 - 3x^{-\frac{1}{5}} + 2\sqrt[5]{x^{-1}} - 8^{\frac{2}{3}} - 2x^{\frac{1}{5}} \\ &\quad= 4x^{\frac{1}{5}} + 5 - 3x^{-\frac{1}{5}} + 2x^{-\frac{1}{5}} - 4 - 2x^{\frac{1}{5}} = 2x^{\frac{1}{5}} + 1 - \frac{1}{x^{\frac{1}{5}}}\end{aligned}$$

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16. $(a^{\frac{1}{2}} + b^{\frac{1}{2}})(a^{\frac{1}{2}} - b^{\frac{1}{2}}) = (a^{\frac{1}{2}})^2 - (b^{\frac{1}{2}})^2 = a - b.$

17. $(x^{\frac{2}{3}} + y^{\frac{2}{3}})(x^{\frac{2}{3}} - y^{\frac{2}{3}}) = (x^{\frac{2}{3}})^2 - (y^{\frac{2}{3}})^2 = x^{\frac{4}{3}} - y^{\frac{4}{3}}.$

18. $(x^{-\frac{1}{2}} + 10)(x^{-\frac{1}{2}} - 1) = (x^{-\frac{1}{2}})^2 + 9x^{-\frac{1}{2}} - 10 = x^{-1} + 9x^{-\frac{1}{2}} - 10.$

19. $(x^{\frac{2}{5}} - 4)(x^{\frac{2}{5}} + 5) = (x^{\frac{2}{5}})^2 + x^{\frac{2}{5}} - 20 = x^{\frac{4}{5}} + x^{\frac{2}{5}} - 20.$

20.
$$\begin{array}{l}x^{\frac{2}{5}} - x^{\frac{1}{5}}y^{-\frac{1}{5}} + y^{-\frac{2}{5}} \\ \underline{x^{\frac{1}{5}} + y^{-\frac{1}{5}}} \\ x^{\frac{3}{5}} - x^{\frac{2}{5}}y^{-\frac{1}{5}} + x^{\frac{1}{5}}y^{-\frac{2}{5}} \\ \underline{\quad + x^{\frac{2}{5}}y^{-\frac{1}{5}} - x^{\frac{1}{5}}y^{-\frac{2}{5}} + y^{-\frac{3}{5}}} \\ x^{\frac{3}{5}} \qquad\qquad\qquad\qquad + y^{-\frac{3}{5}}\end{array}$$

21.
$$\begin{array}{l}a^{\frac{2}{3}} + b^{-\frac{2}{3}} + a^{\frac{1}{3}}b^{-\frac{1}{3}} + 1 \\ \underline{a^{\frac{1}{3}} - b^{-\frac{1}{3}}} \\ a + a^{\frac{1}{3}}b^{-\frac{2}{3}} + a^{\frac{2}{3}}b^{-\frac{1}{3}} + a^{\frac{1}{3}} \\ \underline{\quad - a^{\frac{1}{3}}b^{-\frac{2}{3}} - a^{\frac{2}{3}}b^{-\frac{1}{3}} \qquad - b^{-1} - b^{-\frac{1}{3}}} \\ a \qquad\qquad\qquad\qquad + a^{\frac{1}{3}} - b^{-1} - b^{-\frac{1}{3}}\end{array}$$

22.
$$\begin{array}{l}1 - x + x^2 \\ \underline{x^{-3} + x^{-2} + x^{-1}} \\ x^{-3} - x^{-2} + x^{-1} \\ \quad + x^{-2} - x^{-1} + x^0 \\ \underline{\qquad\quad + x^{-1} - x^0 + x} \\ x^{-3} \qquad + x^{-1} \qquad + x\end{array}$$

23.
$$\begin{array}{l}a^{-1} + b^{-\frac{1}{2}} + c^{\frac{1}{2}} \\ \underline{a^{-1} + b^{-\frac{1}{2}} + 2c^{\frac{1}{2}}} \\ a^{-2} + a^{-1}b^{-\frac{1}{2}} + a^{-1}c^{\frac{1}{2}} \\ \quad + a^{-1}b^{-\frac{1}{2}} \qquad\qquad + b^{-1} + b^{-\frac{1}{2}}c^{\frac{1}{2}} \\ \underline{\qquad\qquad\quad + 2a^{-1}c^{\frac{1}{2}} \qquad + 2b^{-\frac{1}{2}}c^{\frac{1}{2}} + 2c} \\ a^{-2} + 2a^{-1}b^{-\frac{1}{2}} + 3a^{-1}c^{\frac{1}{2}} + b^{-1} + 3b^{-\frac{1}{2}}c^{\frac{1}{2}} + 2c\end{array}$$

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34. $(a - b) \div (a^{\frac{1}{2}} + b^{\frac{1}{2}}) = [(a^{\frac{1}{2}})^2 - (b^{\frac{1}{2}})^2] \div [(a^{\frac{1}{2}}) + (b^{\frac{1}{2}})] = a^{\frac{1}{2}} - b^{\frac{1}{2}}.$

35.
$$\begin{aligned}(a - b) \div (a^{\frac{1}{3}} - b^{\frac{1}{3}}) &= [(a^{\frac{1}{3}})^3 - (b^{\frac{1}{3}})^3] \div [(a^{\frac{1}{3}}) - (b^{\frac{1}{3}})] \\ &= (a^{\frac{1}{3}})^2 + a^{\frac{1}{3}}b^{\frac{1}{3}} + (b^{\frac{1}{3}})^2 = a^{\frac{2}{3}} + a^{\frac{1}{3}}b^{\frac{1}{3}} + b^{\frac{2}{3}}.\end{aligned}$$

36.
$$\begin{aligned}(a + b) \div (a^{\frac{1}{5}} + b^{\frac{1}{5}}) &= [(a^{\frac{1}{5}})^5 + (b^{\frac{1}{5}})^5] \div [(a^{\frac{1}{5}}) + (b^{\frac{1}{5}})] \\ &= (a^{\frac{1}{5}})^4 - (a^{\frac{1}{5}})^3(b^{\frac{1}{5}}) + (a^{\frac{1}{5}})^2(b^{\frac{1}{5}})^2 - (a^{\frac{1}{5}})(b^{\frac{1}{5}})^3 + (b^{\frac{1}{5}})^4 \\ &= a^{\frac{4}{5}} - a^{\frac{3}{5}}b^{\frac{1}{5}} + a^{\frac{2}{5}}b^{\frac{2}{5}} - a^{\frac{1}{5}}b^{\frac{3}{5}} + b^{\frac{4}{5}}.\end{aligned}$$

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37.
$$\begin{array}{l|l}
a^2+b^2 & a^{\frac23}+b^{\frac23} \\ \cline{2-2}
\underline{a^2+a^{\frac43}b^{\frac23}} & a^{\frac43}-a^{\frac23}b^{\frac23}+b^{\frac43} \\
-a^{\frac43}b^{\frac23}+b^2 & \\
\underline{-a^{\frac43}b^{\frac23}-a^{\frac23}b^{\frac43}} & \\
a^{\frac23}b^{\frac43}+b^2 & \\
\underline{a^{\frac23}b^{\frac43}+b^2} &
\end{array}$$

39.
$$\begin{array}{l|l}
x^{\frac47}-2+x^{-\frac47} & x^{\frac27}-x^{-\frac27} \\ \cline{2-2}
\underline{x^{\frac47}-x^0} & x^{\frac27}-x^{-\frac27} \\
-1+x^{-\frac47} & \\
\underline{-x^0+x^{-\frac47}} &
\end{array}$$

38.
$$\begin{array}{l|l}
x-1 & x^{\frac23}+x^{\frac13}+1 \\ \cline{2-2}
\underline{x+x^{\frac23}+x^{\frac13}} & x^{\frac13}-1 \\
-x^{\frac23}-x^{\frac13}-1 & \\
\underline{-x^{\frac23}-x^{\frac13}-1} &
\end{array}$$

40.
$$\begin{array}{l|l}
x^{-2}-4x^{-1}+3 & x^{-1}-3 \\ \cline{2-2}
\underline{x^{-2}-3x^{-1}} & x^{-1}-1 \\
-\ x^{-1}+3 & \\
\underline{-\ x^{-1}+3} &
\end{array}$$

41.
$$\begin{array}{l|l}
a^2-b^3 & a^{\frac13}+b^{\frac12} \\ \cline{2-2}
\underline{a^2+a^{\frac53}b^{\frac12}} & a^{\frac53}-a^{\frac43}b^{\frac12}+ab-a^{\frac23}b^{\frac32}+a^{\frac13}b^2-b^{\frac52} \\
-a^{\frac53}b^{\frac12}-b^3 & \\
\underline{-a^{\frac53}b^{\frac12}-a^{\frac43}b} & \\
a^{\frac43}b-b^3 & \\
\underline{a^{\frac43}b+ab^{\frac32}} & \\
-ab^{\frac32}-b^3 & \\
\underline{-ab^{\frac32}-a^{\frac23}b^2} & \\
a^{\frac23}b^2-b^3 & \\
\underline{a^{\frac23}b^2+a^{\frac13}b^{\frac52}} & \\
-a^{\frac13}b^{\frac52}-b^3 & \\
\underline{-a^{\frac13}b^{\frac52}-b^3} &
\end{array}$$

45. $\sqrt[4]{x^{\frac23}}=(x^{\frac23})^{\frac14}=x^{\frac16}.$

46. $\sqrt[4]{a^{-\frac12}}=(a^{-\frac12})^{\frac14}=a^{-\frac18}.$

47. $\sqrt[4]{a^{-\frac83}}=(a^{-\frac83})^{\frac14}=a^{-\frac23}.$

51. $\sqrt[3]{a^{-\frac12}b^{-3}}=(a^{-\frac12}b^{-3})^{\frac13}=a^{-\frac16}b^{-1}$

52. $\sqrt{x^{\frac43}y^{-3}}=(x^{\frac43}y^{-3})^{\frac12}=x^{\frac23}y^{-\frac32}.$

53. $\sqrt[n]{a^xb^y}=(a^xb^y)^{\frac1n}=a^{\frac xn}b^{\frac yn}.$

57. $(\frac1{36}a^{-\frac23}b^{\frac13})^{-\frac32}=(\frac1{36})^{-\frac32}(a^{-\frac23})^{-\frac32}(b^{\frac13})^{-\frac32}=216\,ab^{-\frac12}.$

58. $(\frac19 m^{-1}n^{-\frac12})^{\frac12}=(\frac19)^{\frac12}(m^{-1})^{\frac12}(n^{-\frac12})^{\frac12}=\frac13 m^{-\frac12}n^{-\frac14}.$

59. $(4\,x^{2n}y^{-3}z^4)^{\frac52}=4^{\frac52}(x^{2n})^{\frac52}(y^{-3})^{\frac52}(z^4)^{\frac52}=32\,x^{5n}y^{-\frac{15}2}z^{10}.$

60. $(a^{\frac12}-b^{\frac12})^2=(a^{\frac12})^2-2(a^{\frac12})(b^{\frac12})+(b^{\frac12})^2=a-2\,a^{\frac12}b^{\frac12}+b.$

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61. $(a^{\frac{1}{2}}+b^{\frac{1}{3}})^3=(a^{\frac{1}{2}})^3+3(a^{\frac{1}{2}})^2(b^{\frac{1}{3}})+3(a^{\frac{1}{2}})(b^{\frac{1}{3}})^2+(b^{\frac{1}{3}})^3$
$=a^{\frac{3}{2}}+3\,ab^{\frac{1}{3}}+3\,a^{\frac{1}{2}}b^{\frac{2}{3}}+b.$

62. $(a^{-1}-b^{\frac{2}{3}})^3=(a^{-1})^3-3(a^{-1})^2(b^{\frac{2}{3}})+3(a^{-1})(b^{\frac{2}{3}})^2-(b^{\frac{2}{3}})^3$
$=a^{-3}-3\,a^{-2}b^{\frac{2}{3}}+3\,a^{-1}b^{\frac{4}{3}}-b^2.$

63. $(x^{-\frac{1}{2}}-y^{\frac{1}{2}})^4=(x^{-\frac{1}{2}})^4-4(x^{-\frac{1}{2}})^3(y^{\frac{1}{2}})+6(x^{-\frac{1}{2}})^2(y^{\frac{1}{2}})^2-4(x^{-\frac{1}{2}})(y^{\frac{1}{2}})^3$
$+(y^{\frac{1}{2}})^4$
$=x^{-2}-4\,x^{-\frac{3}{2}}y^{\frac{1}{2}}+6\,x^{-1}y-4\,x^{-\frac{1}{2}}y^{\frac{3}{2}}+y^2.$

64. $(a^{-\frac{1}{2}}+\frac{1}{2})^3=(a^{-\frac{1}{2}})^3+3(a^{-\frac{1}{2}})^2(\frac{1}{2})+3(a^{-\frac{1}{2}})(\frac{1}{2})^2+(\frac{1}{2})^3$
$=a^{-\frac{3}{2}}+\frac{3}{2}\,a^{-1}+\frac{3}{4}\,a^{-\frac{1}{2}}+\frac{1}{8}.$

65. $(1-x^{\frac{3}{2}})^4=(1)^4-4(1)^3(x^{\frac{3}{2}})+6(1)^2(x^{\frac{3}{2}})^2-4(1)(x^{\frac{3}{2}})^3+(x^{\frac{3}{2}})^4$
$=1-4\,x^{\frac{3}{2}}+6\,x^3-4\,x^{\frac{9}{2}}+x^6.$

66.
$$\begin{array}{l|l}
 & x^2+2\,x^{\frac{3}{2}}+3\,x+4\,x^{\frac{1}{2}}+3+2\,x^{-\frac{1}{2}}+x^{-1}\ \underline{|x+x^{\frac{1}{2}}+1+x^{-\frac{1}{2}}} \\
 & x^2 \\ \hline
2\,x & 2\,x^{\frac{3}{2}}+3\,x \\
2\,x+x^{\frac{1}{2}} & 2\,x^{\frac{3}{2}}+\ \ x \\ \hline
2\,x+2\,x^{\frac{1}{2}} & 2\,x+4\,x^{\frac{1}{2}}+3 \\
2\,x+2\,x^{\frac{1}{2}}+1 & 2\,x+2\,x^{\frac{1}{2}}+1 \\ \hline
2\,x+2\,x^{\frac{1}{2}}+2 & 2\,x^{\frac{1}{2}}+2+2\,x^{-\frac{1}{2}}+x^{-1} \\
2\,x+2\,x^{\frac{1}{2}}+2+x^{-\frac{1}{2}} & 2\,x^{\frac{1}{2}}+2+2\,x^{-\frac{1}{2}}+x^{-1} \\ \hline
\end{array}$$

67.
$$\begin{array}{l|l}
 & x^2-2\,xy^{\frac{1}{2}}+y+4\,xz^{-1}-4\,y^{\frac{1}{2}}z^{-1}+4\,z^{-2}\ \underline{|x-y^{\frac{1}{2}}+2\,z^{-1}} \\
 & x^2 \\ \hline
2\,x & -2\,xy^{\frac{1}{2}}+y \\
2\,x-y^{\frac{1}{2}} & -2\,xy^{\frac{1}{2}}+y \\ \hline
2\,x-2\,y^{\frac{1}{2}} & 4\,xz^{-1}-4\,y^{\frac{1}{2}}z^{-1}+4\,z^{-2} \\
2\,x-2\,y^{\frac{1}{2}}+2\,z^{-1} & 4\,xz^{-1}-4\,y^{\frac{1}{2}}z^{-1}+4\,z^{-2} \\ \hline
\end{array}$$

68.
$$\begin{array}{l|l}
 & a-4\,a^{\frac{1}{2}}b^{\frac{1}{3}}+4\,b^{\frac{2}{3}}+6\,a^{\frac{1}{2}}c^{\frac{1}{4}}-12\,b^{\frac{1}{3}}c^{\frac{1}{4}}+9\,c^{\frac{1}{2}}\ \underline{|a^{\frac{1}{2}}-2\,b^{\frac{1}{3}}+3\,c^{\frac{1}{4}}} \\
 & a \\ \hline
2\,a^{\frac{1}{2}} & -4\,a^{\frac{1}{2}}b^{\frac{1}{3}}+4\,b^{\frac{2}{3}} \\
2\,a^{\frac{1}{2}}-2\,b^{\frac{1}{3}} & -4\,a^{\frac{1}{2}}b^{\frac{1}{3}}+4\,b^{\frac{2}{3}} \\ \hline
2\,a^{\frac{1}{2}}-4\,b^{\frac{1}{3}} & 6\,a^{\frac{1}{2}}c^{\frac{1}{4}}-12\,b^{\frac{1}{3}}c^{\frac{1}{4}}+9\,c^{\frac{1}{2}} \\
2\,a^{\frac{1}{2}}-4\,b^{\frac{1}{3}}+3\,c^{\frac{1}{4}} & 6\,a^{\frac{1}{2}}c^{\frac{1}{4}}-12\,b^{\frac{1}{3}}c^{\frac{1}{4}}+9\,c^{\frac{1}{2}} \\ \hline
\end{array}$$

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69.

$$\begin{array}{r|l}
 & a^2+6a^{\frac{4}{3}}+12a^{\frac{2}{3}}+8 \;\underline{|\,a^{\frac{2}{3}}+2} \\
 & \underline{a^2\phantom{+6a^{\frac{4}{3}}+12a^{\frac{2}{3}}+8}} \\
3a^{\frac{4}{3}} & 6a^{\frac{4}{3}}+12a^{\frac{2}{3}}+8 \\
\underline{3a^{\frac{4}{3}}+6a^{\frac{2}{3}}+4} & \underline{6a^{\frac{4}{3}}+12a^{\frac{2}{3}}+8}
\end{array}$$

70.

$$\begin{array}{r|l}
 & a-3a^{\frac{2}{3}}b^{\frac{2}{3}}+3a^{\frac{1}{3}}b^{\frac{4}{3}}-b^2 \;\underline{|\,a^{\frac{1}{3}}-b^{\frac{2}{3}}} \\
 & \underline{a\phantom{-3a^{\frac{2}{3}}b^{\frac{2}{3}}+3a^{\frac{1}{3}}b^{\frac{4}{3}}-b^2}} \\
3a^{\frac{2}{3}} & -3a^{\frac{2}{3}}b^{\frac{2}{3}}+3a^{\frac{1}{3}}b^{\frac{4}{3}}-b^2 \\
\underline{3a^{\frac{2}{3}}-3a^{\frac{1}{3}}b^{\frac{2}{3}}+b^{\frac{4}{3}}} & \underline{-3a^{\frac{2}{3}}b^{\frac{2}{3}}+3a^{\frac{1}{3}}b^{\frac{4}{3}}-b^2}
\end{array}$$

71.

$$\begin{array}{r|l}
 & 8x^{-1}-12x^{-\frac{2}{3}}y+6x^{-\frac{1}{3}}y^2-y^3 \;\underline{|\,2x^{-\frac{1}{3}}-y} \\
 & \underline{8x^{-1}\phantom{-12x^{-\frac{2}{3}}y+6x^{-\frac{1}{3}}y^2-y^3}} \\
12x^{-\frac{2}{3}} & -12x^{-\frac{2}{3}}y+6x^{-\frac{1}{3}}y^2-y^3 \\
\underline{12x^{-\frac{2}{3}}-6x^{-\frac{1}{3}}y+y^2} & \underline{-12x^{-\frac{2}{3}}y+6x^{-\frac{1}{3}}y^2-y^3}
\end{array}$$

72.

$$\begin{array}{l|l}
 & x^{\frac{3}{2}}-6x+15x^{\frac{1}{2}}-20+15x^{-\frac{1}{2}}-6x^{-1}+x^{-\frac{3}{2}} \;\underline{|\,x^{\frac{1}{2}}-2+x^{-\frac{1}{2}}} \\
 & \underline{x^{\frac{3}{2}}} \\
3x & -6x+15x^{\frac{1}{2}}-20 \\
\underline{3x-6x^{\frac{1}{2}}+4} & \underline{-6x+12x^{\frac{1}{2}}-8} \\
3x-12x^{\frac{1}{2}}+12 & 3x^{\frac{1}{2}}-12+15x^{-\frac{1}{2}}-6x^{-1}+x^{-\frac{3}{2}} \\
\underline{3x-12x^{\frac{1}{2}}+15-6x^{-\frac{1}{2}}+x^{-1}} & \underline{3x^{\frac{1}{2}}-12+15x^{-\frac{1}{2}}-6x^{-1}+x^{-\frac{3}{2}}}
\end{array}$$

74. By § 152, $a^{-2}-b^{-2}=(a^{-1}+b^{-1})(a^{-1}-b^{-1})=\left(\frac{1}{a}+\frac{1}{b}\right)\left(\frac{1}{a}-\frac{1}{b}\right).$

75. By § 152, $9-x^{-2}=(3+x^{-1})(3-x^{-1})=\left(3+\frac{1}{x}\right)\left(3-\frac{1}{x}\right).$

76. By § 152, $16-a^{-4}=(4+a^{-2})(2+a^{-1})(2-a^{-1})$

$$=\left(4+\frac{1}{a^2}\right)\left(2+\frac{1}{a}\right)\left(2-\frac{1}{a}\right).$$

77. By § 159, $27-b^{-3}=(3-b^{-1})(9+3b^{-1}+b^{-2})$

$$=\left(3-\frac{1}{b}\right)\left(9+\frac{3}{b}+\frac{1}{b^2}\right).$$

78. By § 159, $b^{-3}+y^{-3}=(b^{-1}+y^{-1})(b^{-2}-b^{-1}y^{-1}+y^{-2})$

$$=\left(\frac{1}{b}+\frac{1}{y}\right)\left(\frac{1}{b^2}-\frac{1}{by}+\frac{1}{y^2}\right).$$

79. By § 159, $x^3-x^{-3}=(x-x^{-1})(x^2+x^0+x^{-2})$

$$=\left(x-\frac{1}{x}\right)\left(x^2+1+\frac{1}{x^2}\right)$$

§ 167, $$=\left(x-\frac{1}{x}\right)\left(x+1+\frac{1}{x}\right)\left(x-1+\frac{1}{x}\right).$$

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80. By § 148, $a^2+2+a^{-2}=(a+a^{-1})(a+a^{-1})=\left(a+\dfrac{1}{a}\right)\left(a+\dfrac{1}{a}\right)$.

81. By § 148, $b^4-8+16\,b^{-4}=(b^2-4\,b^{-2})(b^2-4\,b^{-2})$
§ 152, $=(b+2\,b^{-1})(b-2\,b^{-1})(b+2\,b^{-1})(b-2\,b^{-1})$
$=\left(b+\dfrac{2}{b}\right)\left(b-\dfrac{2}{b}\right)\left(b+\dfrac{2}{b}\right)\left(b-\dfrac{2}{b}\right)$.

82. § 156, $12-x^{-1}-x^{-2}=(4+x^{-1})(3-x^{-1})=\left(4+\dfrac{1}{x}\right)\left(3-\dfrac{1}{x}\right)$.

83. § 156, $2-3\,x^{-1}-2\,x^{-2}=(2+x^{-1})(1-2x^{-1})=\left(2+\dfrac{1}{x}\right)\left(1-\dfrac{2}{x}\right)$.

84. $x^{\frac12}=7$.
$(x^{\frac12})^2=7^2$.
$x=49$.

85. $x^{\frac34}=8$.
$(x^{\frac34})^{\frac43}=8^{\frac43}$.
$x=16$.

86. $x^{\frac25}=9$.
$(x^{\frac25})^{\frac52}=9^{\frac52}$.
$x=243$.

87. $x^{\frac43}=81$.
$(x^{\frac43})^{\frac34}=81^{\frac34}$.
$x=27$.

88. $\frac13 x^{\frac32}=72$.
$x^{\frac32}=216$.
$(x^{\frac32})^{\frac23}=216^{\frac23}$.
$x=36$.

89. $x^{-\frac12}=12$.
$(x^{-\frac12})^{-2}=12^{-2}$.
$x=\frac{1}{144}$.

90. $\frac14 x^{\frac23}=25$.
$x^{\frac23}=100$.
$(x^{\frac23})^{\frac32}=100^{\frac32}$.
$x=1000$.

91. $2\,x^{-\frac32}=\frac{1}{32}$.
$x^{-\frac32}=\frac{1}{64}$.
$(x^{-\frac32})^{-\frac23}=(\frac{1}{64})^{-\frac23}$.
$x=64^{\frac23}=16$.

92. $x^{-\frac12}=6$.
$(x^{-\frac12})^{-2}=6^{-2}$.
$x=\dfrac{1}{6^2}=\dfrac{1}{36}$.

93. $x^{-\frac23}=144$.
$(x^{-\frac23})^{-\frac32}=144^{-\frac32}$.
$x=\frac{1}{1728}$.

94. $25\,x^{-\frac23}=1$.
$x^{-\frac23}=\frac{1}{25}$.
$(x^{-\frac23})^{-\frac32}=(\frac{1}{25})^{-\frac32}$.
$x=25^{\frac32}=125$.

95. $x^{\frac52}=243$.
$(x^{\frac52})^{\frac25}=243^{\frac25}$.
$x=9$.

96. $x^{\frac53}+32=0$.
$x^{\frac53}=-32$.
$(x^{\frac53})^{\frac35}=(-32)^{\frac35}$.
$x=-8$.

97. $x^{\frac34}-a^6=0$.
$x^{\frac34}=a^6$.
$(x^{\frac34})^{\frac43}=(a^6)^{\frac43}$.
$x=a^8$.

98. $x^{\frac65}-64=0$.
$x^{\frac65}=64$.
$(x^{\frac65})^{\frac56}=64^{\frac56}$.
$x=32$.

99. $x^{-\frac32}-27=0$.
$x^{-\frac32}=27$.
$(x^{-\frac32})^{-\frac23}=(27)^{-\frac23}$.
$x=(\frac{1}{27})^{\frac23}=\frac19$.

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100. $\left(\dfrac{\sqrt{3^{\frac13}}}{\sqrt[3]{2^{-2}}}\right)^{-6}=\left(\dfrac{3^{\frac16}}{2^{-\frac23}}\right)^{-6}=(3^{\frac16}\cdot 2^{\frac23})^{-6}=\left(\dfrac{1}{3^{\frac16}\cdot 2^{\frac23}}\right)^{6}=\dfrac{1}{3\cdot 2^4}=\dfrac{1}{48}$.

101. $\left(\dfrac{9^{-3}}{x^{-4}y^{-2}}\right)^{-\frac12}=\left[\dfrac{\dfrac{1}{9^3}}{\dfrac{1}{x^4y^2}}\right]^{-\frac12}=\left(\dfrac{x^4y^2}{9^3}\right)^{-\frac12}=\left(\dfrac{9^3}{x^4y^2}\right)^{\frac12}=\dfrac{9^{\frac32}}{x^2y}=\dfrac{27}{x^2y}$.

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102. $\left(\dfrac{16\,m^{-\frac{2}{3}}}{r^{-4}}\right)^{-\frac{3}{4}}=\left[\dfrac{\dfrac{16}{m^{\frac{2}{3}}}}{\dfrac{1}{r^4}}\right]^{-\frac{3}{4}}=\left(\dfrac{16\,r^4}{m^{\frac{2}{3}}}\right)^{-\frac{3}{4}}=\left(\dfrac{m^{\frac{2}{3}}}{16\,r^4}\right)^{\frac{3}{4}}=\dfrac{m^{\frac{1}{2}}}{8\,r^3}.$

103. $\left(\dfrac{m^{-2}n^{-\frac{2}{3}}}{x^{\frac{1}{6}}}\right)^{-3}=\left[\dfrac{\dfrac{1}{m^2n^{\frac{2}{3}}}}{x^{\frac{1}{6}}}\right]^{-3}=\left(\dfrac{1}{m^2n^{\frac{2}{3}}x^{\frac{1}{6}}}\right)^{-3}=(m^2n^{\frac{2}{3}}x^{\frac{1}{6}})^3=m^6n^2x^{\frac{1}{2}}.$

104. $\sqrt[3]{\dfrac{9^r\times 3^{2+r}}{27^r}}=\left(\dfrac{9^r\times 3^{2+r}}{27^r}\right)^{\frac{1}{3}}=\left(\dfrac{3^{2r}\times 3^{2+r}}{3^{3r}}\right)^{\frac{1}{3}}=\left(\dfrac{3^{2+3r}}{3^{3r}}\right)^{\frac{1}{3}}=(3^2)^{\frac{1}{3}}=9^{\frac{1}{3}}.$

105. $\dfrac{3\,a^{\frac{1}{4}}\times 4\,a^{-1}}{6\,\sqrt[4]{a^5}}=\dfrac{3\,a^{\frac{1}{4}}\times 4\,a^{-1}}{6\,a^{\frac{5}{4}}}=\dfrac{4\,a^{-1}}{2\,a}=\dfrac{2}{a^2}.$

106. $\dfrac{\sqrt[3]{a^2}\times\sqrt{b^3}}{\sqrt[4]{b^6}\times\sqrt[6]{a^{-2}}}=\dfrac{a^{\frac{2}{3}}\times b^{\frac{3}{2}}}{b^{\frac{3}{2}}\times a^{-\frac{1}{3}}}=\dfrac{a^{\frac{2}{3}}}{a^{-\frac{1}{3}}}=a.$

107. $\dfrac{(9^n\times 3^n\times 9)-27^{n+1}}{3^2\times 3^{3n}}=\dfrac{(3^{2n}\times 3^n\times 3^2)-27^{n+1}}{3^{2+3n}}=\dfrac{3^{2+3n}-3^{3n+3}}{3^{2+3n}}=-2.$

108. $\dfrac{x^2y^{\frac{3}{5}}\times\sqrt[3]{x^{-5}}\times x^{\frac{2}{3}}}{\sqrt[5]{y^{-2}}\times xy}=\dfrac{x^2y^{\frac{3}{5}}\times x^{-\frac{5}{3}}\times x^{\frac{2}{3}}}{y^{-\frac{2}{5}}\times xy}=\dfrac{x^2y^{\frac{3}{5}}\times x^{\frac{2}{3}}\times y^{\frac{2}{5}}}{x^{\frac{5}{3}}\times xy}=\dfrac{x^{\frac{8}{3}}y}{x^{\frac{8}{3}}y}=1.$

109. $\dfrac{9^{r+1}}{(3^{r-1})^{r+1}}\div\dfrac{3^{r+1}}{(3^r)^{r-1}}=\dfrac{3^{2r+2}}{3^{r^2-1}}\times\dfrac{3^{r^2-r}}{3^{r+1}}=\dfrac{3^{r^2+r+2}}{3^{r^2+r}}=3^2=9.$

110. $\dfrac{m^{\frac{1}{2}}-n^{\frac{1}{2}}}{\sqrt{m^3}-\sqrt{n^3}}\div\dfrac{\sqrt{m^3}+\sqrt{n^3}}{m^{\frac{1}{2}}+n^{\frac{1}{2}}}=\dfrac{m^{\frac{1}{2}}-n^{\frac{1}{2}}}{m^{\frac{3}{2}}-n^{\frac{3}{2}}}\times\dfrac{m^{\frac{1}{2}}+n^{\frac{1}{2}}}{m^{\frac{3}{2}}+n^{\frac{3}{2}}}=\dfrac{m-n}{m^3-n^3}$

$$=\frac{1}{m^2+mn+n^2}.$$

111. $\left(2+\dfrac{6}{\sqrt{x}}\right)^{-1}\div\dfrac{x-6\sqrt{x}}{x-3\,x^{\frac{1}{2}}-18}=\dfrac{x^{\frac{1}{2}}}{2\,x^{\frac{1}{2}}+6}\times\dfrac{(x^{\frac{1}{2}}-6)(x^{\frac{1}{2}}+3)}{x-6\,x^{\frac{1}{2}}}=\dfrac{1}{2}.$

112. $\left(\dfrac{2\,a^{\frac{1}{3}}-b^{\frac{1}{3}}}{8\,a-b}\right)^{-1}-\left(\dfrac{1}{4\,a^{\frac{2}{3}}b^{\frac{2}{3}}}\right)^{-\frac{1}{2}}=\dfrac{8\,a-b}{2\,a^{\frac{1}{3}}-b^{\frac{1}{3}}}-(4\,a^{\frac{2}{3}}b^{\frac{2}{3}})^{\frac{1}{2}}$

$$=4\,a^{\frac{2}{3}}+2\,a^{\frac{1}{3}}b^{\frac{1}{3}}+b^{\frac{2}{3}}-2\,a^{\frac{1}{3}}b^{\frac{1}{3}}=4\,a^{\frac{2}{3}}+b^{\frac{2}{3}}.$$

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113. $$\left(\frac{c^{-3}x}{a^{-\frac{2}{3}}}\right)^{-\frac{3}{2}} \div \left(\frac{\sqrt[6]{a^3}\times\sqrt{c^{-\frac{1}{2}}}}{c^2x^{-1}}\right)^{-2} = \left(\frac{a^{\frac{2}{3}}x}{c^3}\right)^{-\frac{3}{2}} \div \left(\frac{a^{\frac{1}{2}}x}{c^2\times c^{\frac{1}{4}}}\right)^{-2}$$
$$= \left(\frac{c^3}{a^{\frac{2}{3}}x}\right)^{\frac{3}{2}} \times \frac{ax^2}{c^{\frac{9}{2}}}$$
$$= \frac{c^{\frac{9}{2}}}{ax^{\frac{3}{2}}} \times \frac{ax^2}{c^{\frac{9}{2}}} = x^{\frac{1}{2}}.$$

114. $$\sqrt{\left\{\left(\frac{r^{\frac{1}{3}}}{s^{\frac{1}{8}}}\right)^2 \div \frac{s^{-\frac{1}{2}}}{r^{-\frac{1}{3}}} \times \frac{\sqrt[4]{s^{-1}}}{\sqrt[3]{r}}\right\}^6} = \sqrt{\left\{\frac{r^{\frac{2}{3}}}{s^{\frac{1}{4}}} \div \frac{r^{\frac{1}{3}}}{s^{\frac{1}{2}}} \times \frac{1}{r^{\frac{1}{3}}s^{\frac{1}{4}}}\right\}^6}$$
$$= \sqrt{\left\{\frac{r^{\frac{2}{3}}}{s^{\frac{1}{4}}} \times \frac{s^{\frac{1}{2}}}{r^{\frac{1}{3}}} \times \frac{1}{r^{\frac{1}{3}}s^{\frac{1}{4}}}\right\}^6} = 1.$$

115. $$\left[\sqrt[4]{\frac{r^2n^{-\frac{3}{2}}}{l^{-1}\sqrt{m^5}}} \times \sqrt[3]{\frac{m^{-\frac{1}{3}}\sqrt{nr^3}}{m^{-\frac{13}{8}}l^{\frac{3}{4}}\sqrt[8]{n^7}}}\right]^{-4} = \left[\frac{r^{\frac{1}{2}}n^{-\frac{3}{8}}}{l^{-\frac{1}{4}}m^{\frac{5}{8}}} \times \frac{m^{-\frac{1}{9}}n^{\frac{1}{6}}r^{\frac{1}{2}}}{m^{-\frac{13}{24}}l^{\frac{1}{4}}n^{\frac{7}{24}}}\right]^{-4}$$
$$= \left[\frac{r^{\frac{1}{2}}l^{\frac{1}{4}}}{n^{\frac{3}{8}}m^{\frac{5}{8}}} \times \frac{m^{\frac{13}{24}}n^{\frac{1}{6}}r^{\frac{1}{2}}}{m^{\frac{1}{9}}l^{\frac{1}{4}}n^{\frac{7}{24}}}\right]^{-4}$$
$$= \left[\frac{r}{m^{\frac{7}{36}}n^{\frac{1}{2}}}\right]^{-4} = \left[\frac{m^{\frac{7}{36}}n^{\frac{1}{2}}}{r}\right]^{4} = \frac{m^{\frac{7}{9}}n^2}{r^4}.$$

116. $$\left[\sqrt{\left(\frac{nr^{-3}}{\sqrt[3]{m^{-2}}}\right)^{-3}} \div \left(\sqrt[4]{\frac{r^{-1}}{n^7}} \times \frac{\sqrt[6]{m^3}}{r^2}\right)^{-2}\right]^{-\frac{3}{5}} = \left[\sqrt{\frac{n^{-3}r^9}{m^2}} \div \left(\frac{r^{-\frac{1}{4}}}{n^{\frac{7}{4}}} \times \frac{m^{\frac{1}{2}}}{r^2}\right)^{-2}\right]^{-\frac{3}{5}}$$
$$= \left[\frac{r^{\frac{9}{2}}}{mn^{\frac{3}{2}}} \div \left(\frac{m^{\frac{1}{2}}}{n^{\frac{7}{4}}r^{\frac{9}{4}}}\right)^{-2}\right]^{-\frac{3}{5}}$$
$$= \left[\frac{r^{\frac{9}{2}}}{mn^{\frac{3}{2}}} \times \frac{m}{n^{\frac{7}{2}}r^{\frac{9}{2}}}\right]^{-\frac{3}{5}}$$
$$= \left[\frac{1}{n^{\frac{10}{2}}}\right]^{-\frac{3}{5}} = (n^5)^{\frac{3}{5}} = n^3.$$

117. $$\{-2^{-\frac{1}{4}} \div 32^{-\frac{1}{4}}\}^{-3} \div \{16^{\frac{5}{8}} \times 8^{\frac{5}{6}}\}^{-1}$$
$$= \left\{-\frac{1}{2^{\frac{1}{4}}} \div \frac{1}{2^{\frac{5}{4}}}\right\}^{-3} \div \{2^{\frac{5}{2}} \times 2^{\frac{5}{2}}\}^{-1}$$
$$= \left\{-\frac{1}{2^{\frac{1}{4}}} \times 2^{\frac{5}{4}}\right\}^{-3} \div \frac{1}{2^5} = (-2)^{-3} \times 2^5$$
$$= \frac{2^5}{(-2)^3} = \frac{32}{-8} = -4.$$

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118.

$$\begin{array}{l|l}
 & x^3 - \frac{4x^{\frac52}}{y^{\frac12}} + \frac{4x^2}{y} + x^{\frac32}y^{\frac12} - 2x + \frac14 y \;\Big|\; x^{\frac32} - \frac{2x}{y^{\frac12}} + \frac{y^{\frac12}}{2} \\
 & x^3 \\
2x^{\frac32} & -\frac{4x^{\frac52}}{y^{\frac12}} + \frac{4x^2}{y} \\
2x^{\frac32} - \frac{2x}{y^{\frac12}} & -\frac{4x^{\frac52}}{y^{\frac12}} + \frac{4x^2}{y} \\
\hline
2x^{\frac32} - \frac{4x}{y^{\frac12}} & x^{\frac32}y^{\frac12} - 2x + \frac14 y \\
2x^{\frac32} - \frac{4x}{y^{\frac12}} + \frac{y^{\frac12}}{2} & x^{\frac32}y^{\frac12} - 2x + \frac14 y \\
\hline
\end{array}$$

RADICALS

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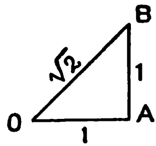
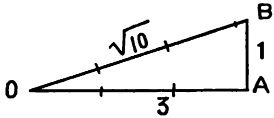


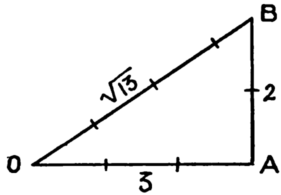
1. Proceeding as in § 324, since $2 = 1^2 + 1^2$, make the sides of the right angle each 1 unit in length.

Then, the length OB represents $\sqrt{2}$ in its relation to the unit length.



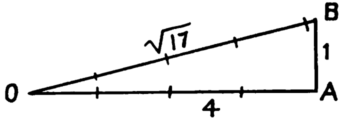
2. Proceeding as in § 324, since $10 = 3^2 + 1^2$, make the sides of the right angle respectively 3 units and 1 unit in length.

Then, the length OB represents $\sqrt{10}$ in its relation to the unit length.



3. Proceeding as in § 324, since $13 = 3^2 + 2^2$, make the sides of the right angle respectively 3 units and 2 units in length.

Then, the length OB represents $\sqrt{13}$ in its relation to the unit length.



4. Proceeding as in § 324, since $17 = 4^2 + 1^2$, make the sides of the right angle respectively 4 units and 1 unit in length.

Then, the length OB represents $\sqrt{17}$ in its relation to the unit length.

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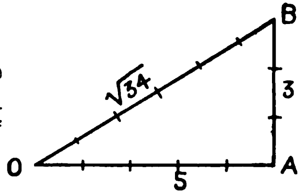
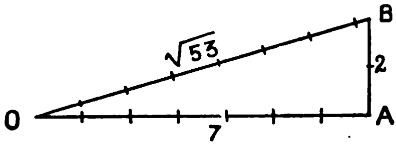


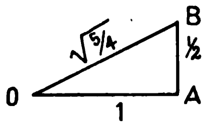
5. Proceeding as in § 324, since $34 = 5^2 + 3^2$, make the sides of the right angle respectively 5 units and 3 units in length.

Then, the length of OB represents $\sqrt{34}$ in its relation to the unit length.



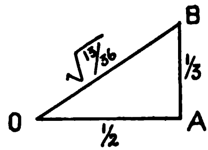
6. Proceeding as in § 324, since $53 = 7^2 + 2^2$, make the sides of the right angle respectively 7 units and 2 units in length.

Then, the length OB represents $\sqrt{53}$ in its relation to the unit length.



7. Proceeding as in § 324, since $\frac{5}{4} = \frac{4}{4} + \frac{1}{4} = 1^2 + (\frac{1}{2})^2$, make the sides of the right angle respectively 1 unit and $\frac{1}{2}$ unit in length.

Then, the length of OB represents $\sqrt{\frac{5}{4}}$ in its relation to the unit length.



8. Proceeding as in § 324, since $\frac{13}{36} = \frac{9}{36} + \frac{4}{36} = (\frac{3}{6})^2 + (\frac{2}{6})^2 = (\frac{1}{2})^2 + (\frac{1}{3})^2$, make the sides of the right angle respectively $\frac{1}{2}$ unit and $\frac{1}{3}$ unit in length.

Then, the length of OB represents $\sqrt{\frac{13}{36}}$ in its relation to the unit length.

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3. $\sqrt{12} = \sqrt{4 \times 3} = \sqrt{4} \times \sqrt{3} = 2\sqrt{3}.$

4. $\sqrt{75} = \sqrt{25 \times 3} = \sqrt{25} \times \sqrt{3} = 5\sqrt{3}.$

5. $\sqrt[3]{16} = \sqrt[3]{8 \times 2} = \sqrt[3]{8} \times \sqrt[3]{2} = 2\sqrt[3]{2}.$

6. $\sqrt{128} = \sqrt{64 \times 2} = \sqrt{64} \times \sqrt{2} = 8\sqrt{2}.$

7. $\sqrt[3]{250} = \sqrt[3]{125 \times 2} = \sqrt[3]{125} \times \sqrt[3]{2} = 5\sqrt[3]{2}.$

8. $\sqrt[4]{32} = \sqrt[4]{16 \times 2} = \sqrt[4]{16} \times \sqrt[4]{2} = 2\sqrt[4]{2}.$

9. $\sqrt{162} = \sqrt{81 \times 2} = \sqrt{81} \times \sqrt{2} = 9\sqrt{2}.$

10. $\sqrt{18a^2} = \sqrt{9a^2 \times 2} = \sqrt{9a^2} \times \sqrt{2} = 3a\sqrt{2}.$

11. $\sqrt{25b} = \sqrt{25 \times b} = \sqrt{25} \times \sqrt{b} = 5\sqrt{b}.$

12. $\sqrt{98c^3} = \sqrt{49c^2 \times 2c} = \sqrt{49c^2} \times \sqrt{2c} = 7c\sqrt{2c}.$

13. $\sqrt{50a} = \sqrt{25 \times 2a} = \sqrt{25} \times \sqrt{2a} = 5\sqrt{2a}.$

14. $\sqrt[5]{640} = \sqrt[5]{32 \times 20} = \sqrt[5]{32} \times \sqrt[5]{20} = 2\sqrt[5]{20}.$

15. $\sqrt{243a^5x^{10}} = \sqrt{81a^4x^{10}} \times \sqrt{3a} = 9a^2x^5\sqrt{3a}.$

16. $\sqrt[3]{128a^6b^4} = \sqrt[3]{64a^6b^3} \times \sqrt[3]{2b} = 4a^2b\sqrt[3]{2b}.$

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17. $(245\,a^6y^{-4})^{\frac{1}{2}} = (49\,a^6y^{-4})^{\frac{1}{2}} \times 5^{\frac{1}{2}} = 7\,a^3y^{-2}\sqrt{5}.$

18. $(a^3 + 5\,a^2)^{\frac{1}{2}} = \sqrt{a^2(a+5)} = \sqrt{a^2} \times \sqrt{a+5} = a\sqrt{a+5}.$

19. $\sqrt{18\,x-9} = \sqrt{9\,(2\,x-1)} = \sqrt{9} \times \sqrt{2\,x-1} = 3\sqrt{2\,x-1}.$

20. $\sqrt[3]{x^6 - 2\,x^3} = \sqrt[3]{x^3(x^3-2)} = \sqrt[3]{x^3} \times \sqrt[3]{x^3-2} = x\sqrt[3]{x^3-2}.$

21. $\sqrt{5\,x^2 - 10\,xy + 5\,y^2} = \sqrt{5(x-y)^2} = \sqrt{(x-y)^2} \times \sqrt{5} = (x-y)\sqrt{5}.$

22. $(3\,am^2 + 6\,am + 3\,a)^{\frac{1}{2}} = \sqrt{3\,a(m+1)^2} = \sqrt{(m+1)^2} \times \sqrt{3\,a}$
$= (m+1)\sqrt{3\,a}.$

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24. $\sqrt{\frac{1}{2}} = \sqrt{\frac{2}{4}} = \sqrt{\frac{1}{4}} \times \sqrt{2} = \frac{1}{2}\sqrt{2}.$

25. $\sqrt{\frac{1}{5}} = \sqrt{\frac{5}{25}} = \sqrt{\frac{1}{25}} \times \sqrt{5} = \frac{1}{5}\sqrt{5}.$

26. $\sqrt{\frac{2}{3}} = \sqrt{\frac{6}{9}} = \sqrt{\frac{1}{9}} \times \sqrt{6} = \frac{1}{3}\sqrt{6}.$

27. $\sqrt{\frac{3}{8}} = \sqrt{\frac{6}{16}} = \sqrt{\frac{1}{16}} \times \sqrt{6} = \frac{1}{4}\sqrt{6}.$

28. $\sqrt[3]{\frac{5}{12}} = \sqrt[3]{\frac{90}{216}} = \sqrt[3]{\frac{1}{216}} \times \sqrt[3]{90} = \frac{1}{6}\sqrt[3]{90}.$

29. $\sqrt[3]{\frac{4}{5}} = \sqrt[3]{\frac{100}{125}} = \sqrt[3]{\frac{1}{125}} \times \sqrt[3]{100} = \frac{1}{5}\sqrt[3]{100}.$

30. $\sqrt{\frac{2\,a^3}{b}} = \sqrt{\frac{2\,a^3b}{b^2}} = \sqrt{\frac{a^2}{b^2}} \times \sqrt{2\,ab} = \frac{a}{b}\sqrt{2\,ab}.$

31. $\sqrt{\frac{5\,x^4y^2}{2\,a^2}} = \sqrt{\frac{10\,x^4y^2}{4\,a^2}} = \sqrt{\frac{x^4y^2}{4\,a^2}} \times \sqrt{10} = \frac{x^2y}{2\,a}\sqrt{10}.$

32. $\sqrt[4]{\frac{x}{y}} = \sqrt[4]{\frac{xy^3}{y^4}} = \sqrt[4]{\frac{1}{y^4}} \times \sqrt[4]{xy^3} = \frac{1}{y}\sqrt[4]{xy^3}.$

33. $\sqrt{\frac{2}{3\,y^5}} = \sqrt{\frac{6\,y}{9\,y^6}} = \sqrt{\frac{1}{9\,y^6}} \times \sqrt{6\,y} = \frac{1}{3\,y^3}\sqrt{6\,y}.$

34. $\sqrt{\frac{4\,a}{3\,x^2}} = \sqrt{\frac{12\,a}{9\,x^2}} = \sqrt{\frac{4}{9\,x^2}} \times \sqrt{3\,a} = \frac{2}{3\,x}\sqrt{3\,a}.$

35. $\sqrt{\frac{3\,x}{50\,a^3y}} = \sqrt{\frac{6\,axy}{100\,a^4y^2}} = \sqrt{\frac{1}{100\,a^4y^2}} \times \sqrt{6\,axy} = \frac{1}{10\,a^2y}\sqrt{6\,axy}.$

36. $(a+b)\sqrt{\frac{a+b}{a-b}} = (a+b)\sqrt{\frac{a^2-b^2}{(a-b)^2}}$
$= (a+b)\sqrt{\frac{1}{(a-b)^2}} \times \sqrt{a^2-b^2} = \frac{a+b}{a-b}\sqrt{a^2-b^2}.$

37. $\frac{(a+b)^2}{a-b}\sqrt[3]{\frac{a+b}{(a-b)^2}} = \frac{(a+b)^2}{a-b}\sqrt[3]{\frac{(a-b)(a+b)}{(a-b)^3}}$
$= \frac{(a+b)^2}{a-b}\sqrt[3]{\frac{1}{(a-b)^3}} \times \sqrt[3]{a^2-b^2}$
$= \frac{(a+b)^2}{(a-b)^2}\sqrt[3]{a^2-b^2}.$

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3. $\sqrt[4]{36} = \sqrt[4]{6^2} = 6^{\frac{2}{4}} = 6^{\frac{1}{2}} = \sqrt{6}.$

4. $\sqrt[4]{25} = \sqrt[4]{5^2} = 5^{\frac{2}{4}} = 5^{\frac{1}{2}} = \sqrt{5}.$

5. $\sqrt[4]{1600} = \sqrt[4]{40^2} = 40^{\frac{2}{4}} = 40^{\frac{1}{2}} = \sqrt{4 \times 10} = 2\sqrt{10}.$

6. $\sqrt[6]{27\,a^3} = \sqrt[6]{(3\,a)^3} = (3\,a)^{\frac{3}{6}} = (3\,a)^{\frac{1}{2}} = \sqrt{3\,a}.$

7. $\sqrt[4]{9\,a^2b^2c^6} = \sqrt[4]{9\,a^2b^2c^2 \cdot c^4} = c\sqrt[4]{(3\,abc)^2} = c(3\,abc)^{\frac{2}{4}}$
$= c(3\,abc)^{\frac{1}{2}} = c\sqrt{3\,abc}.$

8. $\sqrt[4]{121\,a^6x^4} = \sqrt[4]{121\,a^2 \cdot a^4x^4} = ax\sqrt[4]{(11\,a)^2} = ax(11\,a)^{\frac{2}{4}}$
$= ax(11\,a)^{\frac{1}{2}} = ax\sqrt{11\,a}.$

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1. $\sqrt{600} = \sqrt{100 \times 6} = \sqrt{100} \times \sqrt{6} = 10\sqrt{6}.$

2. $\sqrt{500} = \sqrt{100 \times 5} = \sqrt{100} \times \sqrt{5} = 10\sqrt{5}.$

3. $\sqrt[5]{160} = \sqrt[5]{32 \times 5} = \sqrt[5]{32} \times \sqrt[5]{5} = 2\sqrt[5]{5}.$

4. $\sqrt[3]{3000} = \sqrt[3]{1000 \times 3} = \sqrt[3]{1000} \times \sqrt[3]{3} = 10\sqrt[3]{3}.$

5. $\sqrt[3]{189} = \sqrt[3]{27 \times 7} = \sqrt[3]{27} \times \sqrt[3]{7} = 3\sqrt[3]{7}.$

6. $\sqrt{84} = \sqrt{4 \times 21} = \sqrt{4} \times \sqrt{21} = 2\sqrt{21}.$

7. $\sqrt[3]{72} = \sqrt[3]{8 \times 9} = \sqrt[3]{8} \times \sqrt[3]{9} = 2\sqrt[3]{9}.$

8. $\sqrt[3]{192} = \sqrt[3]{64 \times 3} = \sqrt[3]{64} \times \sqrt[3]{3} = 4\sqrt[3]{3}.$

9. $\sqrt[4]{144} = \sqrt[4]{12^2} = 12^{\frac{2}{4}} = 12^{\frac{1}{2}} = \sqrt{4 \times 3} = 2\sqrt{3}.$

10. $\sqrt[6]{81} = \sqrt[6]{9^2} = 9^{\frac{2}{6}} = 9^{\frac{1}{3}} = \sqrt[3]{9}.$

11. $\sqrt[6]{343} = \sqrt[6]{7^3} = 7^{\frac{3}{6}} = 7^{\frac{1}{2}} = \sqrt{7}.$

12. $\sqrt[4]{289} = \sqrt[4]{17^2} = 17^{\frac{2}{4}} = 17^{\frac{1}{2}} = \sqrt{17}.$

13. $\sqrt{\frac{1}{3}} = \sqrt{\frac{3}{9}} = \sqrt{\frac{1}{9}} \times \sqrt{3} = \frac{1}{3}\sqrt{3}.$

14. $\sqrt{\frac{1}{x^3}} = \sqrt{\frac{x}{x^4}} = \sqrt{\frac{1}{x^4}} \times \sqrt{x} = \frac{1}{x^2}\sqrt{x}.$

15. $\sqrt[3]{\frac{a}{3\,b^2}} = \sqrt[3]{\frac{9\,ab}{27\,b^3}} = \sqrt[3]{\frac{1}{27\,b^3}} \times \sqrt[3]{9\,ab} = \frac{1}{3\,b}\sqrt[3]{9\,ab}.$

16. $\sqrt{405\,a^5y^2} = \sqrt{81\,a^4y^2} \times \sqrt{5\,a} = 9\,a^2y\sqrt{5\,a}.$

17. $(135\,x^4y^5)^{\frac{1}{3}} = (27\,x^3y^3)^{\frac{1}{3}} \times (5\,xy^2)^{\frac{1}{3}} = 3\,xy\sqrt[3]{5\,xy^2}.$

18. $\sqrt{8 - 20\,b^2} = \sqrt{4(2 - 5\,b^2)} = \sqrt{4} \times \sqrt{2 - 5\,b^2} = 2\sqrt{2 - 5\,b^2}.$

19. $5\sqrt{4\,a^2 + 4} = 5\sqrt{4\,(a^2 + 1)} = 5\sqrt{4} \times \sqrt{a^2 + 1} = 10\sqrt{a^2 + 1}.$

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20. $\sqrt[6]{a^4b^2c^4d^8} = \sqrt[6]{a^4b^2c^4d^2 \cdot d^6} = d\sqrt[6]{(a^2bc^2d)^2}$
$= d(a^2bc^2d)^{\frac{2}{6}} = d(a^2bc^2d)^{\frac{1}{3}} = d\sqrt[3]{a^2bc^2d}.$

21. $(16\,x - 16)^{\frac{1}{2}} = \sqrt{16\,(x-1)} = \sqrt{16} \times \sqrt{x-1} = 4\sqrt{x-1}.$

22. $$\frac{2\,y}{x-2\,y}\sqrt{\frac{x-2\,y}{2\,y}} = \frac{2\,y}{x-2\,y}\sqrt{\frac{2\,y(x-2\,y)}{4\,y^2}}$$
$$= \frac{2\,y}{x-2\,y}\sqrt{\frac{1}{4\,y^2}} \times \sqrt{2\,y(x-2\,y)}$$
$$= \frac{1}{x-2\,y}\sqrt{2\,y(x-2\,y)}.$$

23. $\sqrt{27\,c^2 - 36\,c + 12} = \sqrt{3(3\,c-2)^2} = \sqrt{(3\,c-2)^2} \times \sqrt{3}$
$= (3\,c - 2)\sqrt{3}.$

24. $\sqrt[4]{(x^2 - 2\,xy + y^2)} = \sqrt[4]{(x-y)^2} = (x-y)^{\frac{2}{4}} = (x-y)^{\frac{1}{2}} = \sqrt{x-y}.$

25. $$(1-x^3)\sqrt{\frac{1-x+x^2}{1+x+x^2}} = (1-x^3)\sqrt{\frac{(1+x+x^2)(1-x+x^2)}{(1+x+x^2)^2}}$$
$$= (1-x)(1+x+x^2)\sqrt{\frac{1}{(1+x+x^2)^2}} \times \sqrt{1+x^2+x^4}$$
$$= (1-x)\sqrt{1+x^2+x^4}.$$

26. $\sqrt{4\,a^3 - 24\,a^2x + 36\,ax^2} = \sqrt{4\,a(a-3\,x)^2} = \sqrt{4\,(a-3\,x)^2} \times \sqrt{a}$
$= 2\,(a - 3\,x)\sqrt{a}.$

27. $(x^4y - 3\,x^3y^2 + 3\,x^2y^3 - xy^4)^{\frac{1}{3}} = \sqrt[3]{xy(x-y)^3} = \sqrt[3]{(x-y)^3} \times \sqrt[3]{xy}$
$= (x-y)\sqrt[3]{xy}.$

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2. $2\sqrt{2} = \sqrt{4}\sqrt{2} = \sqrt{8}.$

3. $3\sqrt{5} = \sqrt{9}\sqrt{5} = \sqrt{45}.$

4. $5\sqrt{2} = \sqrt{25}\sqrt{2} = \sqrt{50}.$

5. $3\sqrt[4]{2} = \sqrt[4]{81}\sqrt[4]{2} = \sqrt[4]{162}.$

6. $3\sqrt[3]{3} = \sqrt[3]{27}\sqrt[3]{3} = \sqrt[3]{81}.$

7. $4\sqrt{5} = \sqrt{16}\sqrt{5} = \sqrt{80}.$

8. $\frac{1}{2}\sqrt{8} = \sqrt{\frac{1}{4}}\sqrt{8} = \sqrt{2}.$

9. $a^2\sqrt[3]{b} = \sqrt[3]{a^6}\sqrt[3]{b} = \sqrt[3]{a^6b}.$

10. $\frac{1}{2}\sqrt{2} = \sqrt{\frac{1}{4}}\sqrt{2} = \sqrt{\frac{1}{2}}.$

11. $\frac{3}{4}\sqrt{x^5} = \sqrt{\frac{9}{16}}\sqrt{x^5} = \sqrt{\frac{9}{16}x^5}.$

12. $\frac{1}{2}\sqrt{bc} = \sqrt{\frac{1}{4}}\sqrt{bc} = \sqrt{\frac{1}{4}bc}.$

13. $\frac{3}{4}\sqrt{\frac{7}{9}} = \sqrt{\frac{9}{16}}\sqrt{\frac{7}{9}} = \sqrt{\frac{7}{16}}.$

14. $\frac{4}{5}\sqrt{4\frac{3}{8}} = \sqrt{\frac{16}{25}}\sqrt{\frac{35}{8}} = \sqrt{\frac{14}{5}}.$

15. $\frac{3}{2}\sqrt{\frac{33}{27}a^2} = \sqrt{\frac{9}{4}}\sqrt{\frac{33}{27}a^2} = \sqrt{\frac{11}{4}a^2}.$

16. $\frac{2}{3}\sqrt[3]{1\frac{1}{8}} = \sqrt[3]{\frac{8}{27}}\sqrt[3]{\frac{9}{8}} = \sqrt[3]{\frac{1}{3}}.$

17. $\frac{2}{3}\sqrt[4]{3\frac{3}{8}} = \sqrt[4]{\frac{16}{81}}\sqrt[4]{\frac{27}{8}} = \sqrt[4]{\frac{2}{3}}.$

18. $$\frac{x+y}{x-y}\sqrt{\frac{x-y}{x+y}} = \sqrt{\frac{(x+y)^2}{(x-y)^2}}\sqrt{\frac{x-y}{x+y}} = \sqrt{\frac{x+y}{x-y}}.$$

19. $$\frac{a+4}{a-4}\sqrt{1-\frac{8}{a+4}} = \sqrt{\frac{(a+4)^2}{(a-4)^2}}\sqrt{\frac{a-4}{a+4}} = \sqrt{\frac{a+4}{a-4}}.$$

20. $$\frac{1}{ab}(a-b)^{\frac{1}{2}} = \left(\frac{1}{a^2b^2}\right)^{\frac{1}{2}}(a-b)^{\frac{1}{2}} = \sqrt{\frac{a-b}{a^2b^2}}.$$

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2. $\sqrt{2} = 2^{\frac{1}{2}} = 2^{\frac{2}{4}} = \sqrt[4]{4}$.
$\sqrt[4]{3} = \sqrt[4]{3}$.

3. $\sqrt{5} = 5^{\frac{1}{2}} = 5^{\frac{3}{6}} = \sqrt[6]{125}$.
$\sqrt[3]{6} = 6^{\frac{1}{3}} = 6^{\frac{2}{6}} = \sqrt[6]{36}$.

4. See next column.

5. $\sqrt[6]{10} = \sqrt[6]{10}$.
$\sqrt{2} = 2^{\frac{1}{2}} = 2^{\frac{3}{6}} = \sqrt[6]{8}$.
$\sqrt[3]{5} = 5^{\frac{1}{3}} = 5^{\frac{2}{6}} = \sqrt[6]{25}$.

6. $\sqrt[6]{4} = 4^{\frac{1}{6}} = 4^{\frac{2}{12}} = \sqrt[12]{16}$.
$\sqrt[4]{2} = 2^{\frac{1}{4}} = 2^{\frac{3}{12}} = \sqrt[12]{8}$.
$\sqrt{3} = 3^{\frac{1}{2}} = 3^{\frac{6}{12}} = \sqrt[12]{729}$.

7. $\sqrt[10]{13} = \sqrt[10]{13}$.
$\sqrt{5} = 5^{\frac{1}{2}} = 5^{\frac{5}{10}} = \sqrt[10]{3125}$.
$\sqrt[5]{4} = 4^{\frac{1}{5}} = 4^{\frac{2}{10}} = \sqrt[10]{16}$.

4.
$\sqrt[4]{7} = \sqrt[4]{7}$.
$\sqrt{10} = 10^{\frac{1}{2}} = 10^{\frac{2}{4}} = \sqrt[4]{100}$.

8.
$\sqrt{3} = 3^{\frac{1}{2}} = 3^{\frac{6}{12}} = \sqrt[12]{729}$.
$\sqrt[3]{5} = 5^{\frac{1}{3}} = 5^{\frac{4}{12}} = \sqrt[12]{625}$.
$\sqrt[4]{\frac{1}{27}} = \frac{1}{3}(3)^{\frac{1}{4}} = \frac{1}{3}(3)^{\frac{3}{12}} = \frac{1}{3}\sqrt[12]{27}$.

9.
$\sqrt{ab} = (ab)^{\frac{1}{2}} = (ab)^{\frac{6}{12}} = \sqrt[12]{a^6b^6}$.
$\sqrt[3]{ab^2} = (ab^2)^{\frac{1}{3}} = (ab^2)^{\frac{4}{12}} = \sqrt[12]{a^4b^8}$.
$\sqrt[4]{2} = 2^{\frac{1}{4}} = 2^{\frac{3}{12}} = \sqrt[12]{8}$.

10.
$\sqrt{a} = a^{\frac{1}{2}} = a^{\frac{6}{12}} = \sqrt[12]{a^6}$.
$\sqrt[3]{b} = b^{\frac{1}{3}} = b^{\frac{4}{12}} = \sqrt[12]{b^4}$.
$\sqrt[4]{x} = x^{\frac{1}{4}} = x^{\frac{3}{12}} = \sqrt[12]{x^3}$.
$\sqrt[6]{y} = y^{\frac{1}{6}} = y^{\frac{2}{12}} = \sqrt[12]{y^2}$.

11. $\sqrt[3]{a+b} = (a+b)^{\frac{1}{3}} = (a+b)^{\frac{2}{6}} = \sqrt[6]{(a+b)^2}$.
$\sqrt{x+y} = (x+y)^{\frac{1}{2}} = (x+y)^{\frac{3}{6}} = \sqrt[6]{(x+y)^3}$.

12. $\sqrt{\frac{2}{3}} = \frac{1}{3}\sqrt{6} = \frac{1}{3}(6)^{\frac{1}{2}} = \frac{1}{3}(6)^{\frac{3}{6}} = \frac{1}{3}\sqrt[6]{216}$.
$\sqrt[3]{\frac{1}{16}x} = \frac{1}{4}\sqrt[3]{4x} = \frac{1}{4}(4x)^{\frac{1}{3}} = \frac{1}{4}(4x)^{\frac{2}{6}} = \frac{1}{4}\sqrt[6]{16x^2}$.
$2\sqrt{5} = 2(5)^{\frac{1}{2}} = 2(5)^{\frac{3}{6}} = 2\sqrt[6]{125}$.

13. $\sqrt[n]{x} = x^{\frac{1}{n}} = x^{\frac{2}{2n}} = \sqrt[2n]{x^2}$.
$\sqrt{xy} = (xy)^{\frac{1}{2}} = (xy)^{\frac{n}{2n}} = \sqrt[2n]{x^ny^n}$.
$\sqrt[n]{x^2y^2} = (xy)^{\frac{2}{n}} = (xy)^{\frac{4}{2n}} = \sqrt[2n]{x^4y^4}$.

14. $(a+b)\sqrt{a-b} = (a+b)(a-b)^{\frac{1}{2}} = (a+b)(a-b)^{\frac{3}{6}} = (a+b)\sqrt[6]{(a-b)^3}$.
$\sqrt[3]{a-b} = (a-b)^{\frac{1}{3}} = (a-b)^{\frac{2}{6}} = \sqrt[6]{(a-b)^2}$.

15. $\sqrt{a+b} = (a+b)^{\frac{1}{2}} = (a+b)^{\frac{2}{4}} = \sqrt[4]{(a+b)^2}$.
$\sqrt[4]{a^2+b^2} = \sqrt[4]{a^2+b^2}$.
$\sqrt{a-b} = (a-b)^{\frac{1}{2}} = (a-b)^{\frac{2}{4}} = \sqrt[4]{(a-b)^2}$.

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16. $\sqrt[5]{5}=(5)^{\frac{1}{5}}=(5)^{\frac{2}{10}}=\sqrt[10]{25}.$

$\sqrt{2}=(2)^{\frac{1}{2}}=(2)^{\frac{5}{10}}=\sqrt[10]{32}.$

$\therefore\ \sqrt{2}$ is greater than $\sqrt[5]{5}.$

$\sqrt[3]{4}=(4)^{\frac{1}{3}}=(4)^{\frac{2}{6}}=\sqrt[6]{16}.$

$\sqrt{3}=(3)^{\frac{1}{2}}=(3)^{\frac{3}{6}}=\sqrt[6]{27}.$

$\therefore\ \sqrt{3}$ is greater than $\sqrt[3]{4}.$

17. $\sqrt[3]{3}=(3)^{\frac{1}{3}}=(3)^{\frac{4}{12}}=\sqrt[12]{81}.$

$\sqrt[4]{4}=(4)^{\frac{1}{4}}=(4)^{\frac{3}{12}}=\sqrt[12]{64}.$

$\therefore\ \sqrt[3]{3}$ is greater than $\sqrt[4]{4}.$

$3\sqrt{2}=3(2)^{\frac{1}{2}}=3(2)^{\frac{3}{6}}=3\sqrt[6]{8}.$

$3\sqrt[3]{4}=3(4)^{\frac{1}{3}}=3(4)^{\frac{2}{6}}=3\sqrt[6]{16}.$

$\therefore\ 3\sqrt[3]{4}$ is greater than $3\sqrt{2}.$

18. $\sqrt[3]{3}=(3)^{\frac{1}{3}}=(3)^{\frac{2}{6}}=\sqrt[6]{9}.$

$\sqrt{2}=(2)^{\frac{1}{2}}=(2)^{\frac{3}{6}}=\sqrt[6]{8}.$

$\sqrt[6]{7}=(7)^{\frac{1}{6}}=(7)^{\frac{1}{6}}=\sqrt[6]{7}.$

$\therefore$ order is $\sqrt[3]{3},\ \sqrt{2},\ \sqrt[6]{7}.$

19. $\sqrt{2}=(2)^{\frac{1}{2}}=(2)^{\frac{6}{12}}=\sqrt[12]{64}.$

$\sqrt[3]{4}=(4)^{\frac{1}{3}}=(4)^{\frac{4}{12}}=\sqrt[12]{256}.$

$\sqrt[4]{5}=(5)^{\frac{1}{4}}=(5)^{\frac{3}{12}}=\sqrt[12]{125}.$

Hence, the order is $\sqrt[3]{4},\ \sqrt[4]{5},\ \sqrt{2}.$

20.

$$\sqrt[3]{2}=(2)^{\frac{1}{3}}=(2)^{\frac{5}{15}}=\sqrt[15]{32}.$$
$$\sqrt[5]{3}=(3)^{\frac{1}{5}}=(3)^{\frac{3}{15}}=\sqrt[15]{27}.$$
$$\sqrt[15]{30}=(30)^{\frac{1}{15}}=\sqrt[15]{30}.$$

Hence, the order is $\sqrt[3]{2},\ \sqrt[15]{30},\ \sqrt[5]{3}.$

21.

$$\sqrt{2}=(2)^{\frac{1}{2}}=(2)^{\frac{3}{6}}=\sqrt[6]{8}.$$
$$\sqrt[3]{5}=(5)^{\frac{1}{3}}=(5)^{\frac{2}{6}}=\sqrt[6]{25}.$$
$$\sqrt{2\tfrac{1}{2}}=(2\tfrac{1}{2})^{\frac{1}{2}}=(2\tfrac{1}{2})^{\frac{3}{6}}=\sqrt[6]{15\tfrac{5}{8}}.$$
$$\sqrt[3]{4}=(4)^{\frac{1}{3}}=(4)^{\frac{2}{6}}=\sqrt[6]{16}.$$

Hence, the order is $\sqrt[3]{5},\ \sqrt[3]{4},\ \sqrt{2\tfrac{1}{2}},\ \sqrt{2}.$

22.

$$\sqrt{7}=(7)^{\frac{1}{2}}=(7)^{\frac{6}{12}}=\sqrt[12]{7^6}=\sqrt[12]{117,649}.$$
$$\sqrt[4]{48}=(48)^{\frac{1}{4}}=(48)^{\frac{3}{12}}=\sqrt[12]{48^3}=\sqrt[12]{110,592}.$$
$$\sqrt[3]{4}=(4)^{\frac{1}{3}}=(4)^{\frac{4}{12}}=\sqrt[12]{4^4}=\sqrt[12]{256}.$$
$$\sqrt[6]{63}=(63)^{\frac{1}{6}}=(63)^{\frac{2}{12}}=\sqrt[12]{63^2}=\sqrt[12]{3969}.$$

Hence, the order is $\sqrt{7},\ \sqrt[4]{48},\ \sqrt[6]{63},\ \sqrt[3]{4}.$

23.

$$\sqrt[3]{4}=(4)^{\frac{1}{3}}=(4)^{\frac{4}{12}}=\sqrt[12]{256}.$$
$$\sqrt{2}=(2)^{\frac{1}{2}}=(2)^{\frac{6}{12}}=\sqrt[12]{64}.$$
$$\sqrt[4]{5}=(5)^{\frac{1}{4}}=(5)^{\frac{3}{12}}=\sqrt[12]{125}.$$
$$\sqrt[6]{13}=(13)^{\frac{1}{6}}=(13)^{\frac{2}{12}}=\sqrt[12]{169}.$$
$$\sqrt[12]{150}=\sqrt[12]{150}.$$

Hence, the order is $\sqrt[3]{4},\ \sqrt[6]{13},\ \sqrt[12]{150},\ \sqrt[4]{5},\ \sqrt{2}.$

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2. $\sqrt{50}+\sqrt{18}+\sqrt{98}=5\sqrt{2}+3\sqrt{2}+7\sqrt{2}=15\sqrt{2}.$

3. $\sqrt{27}+\sqrt{12}+\sqrt{75}=3\sqrt{3}+2\sqrt{3}+5\sqrt{3}=10\sqrt{3}.$

4. $\sqrt{20}+\sqrt{80}+\sqrt{45}=2\sqrt{5}+4\sqrt{5}+3\sqrt{5}=9\sqrt{5}.$

5. $\sqrt{28}+\sqrt{63}+\sqrt{700}=2\sqrt{7}+3\sqrt{7}+10\sqrt{7}=15\sqrt{7}.$

6. $\sqrt[3]{250}+\sqrt[3]{16}+\sqrt[3]{54}=5\sqrt[3]{2}+2\sqrt[3]{2}+3\sqrt[3]{2}=10\sqrt[3]{2}.$

7. $\sqrt[3]{128}+\sqrt[3]{686}+\sqrt[3]{\frac{1}{4}}=4\sqrt[3]{2}+7\sqrt[3]{2}+\frac{1}{2}\sqrt[3]{2}=\frac{23}{2}\sqrt[3]{2}.$

8. $\sqrt[3]{135}+\sqrt[3]{320}+\sqrt[3]{625}=3\sqrt[3]{5}+4\sqrt[3]{5}+5\sqrt[3]{5}=12\sqrt[3]{5}.$

9. $\sqrt[3]{500}+\sqrt[3]{108}+\sqrt[3]{-32}=5\sqrt[3]{4}+3\sqrt[3]{4}-2\sqrt[3]{4}=6\sqrt[3]{4}.$

10. $\sqrt{\frac{1}{2}}+\sqrt{12\frac{1}{2}}+\sqrt{\frac{1}{8}}+\sqrt{1\frac{1}{8}}=\frac{1}{2}\sqrt{2}+\frac{5}{2}\sqrt{2}+\frac{1}{4}\sqrt{2}+\frac{3}{4}\sqrt{2}=4\sqrt{2}.$

11. $\sqrt{\frac{1}{3}}+\sqrt{75}+\frac{2}{3}\sqrt{3}+\sqrt{12}=\frac{1}{3}\sqrt{3}+5\sqrt{3}+\frac{2}{3}\sqrt{3}+2\sqrt{3}=8\sqrt{3}.$

12. $\sqrt{\frac{3}{4}}+\frac{1}{3}\sqrt{3}+\frac{7}{6}\sqrt[4]{9}+\sqrt{147}=\frac{1}{2}\sqrt{3}+\frac{1}{3}\sqrt{3}+\frac{7}{6}\sqrt{3}+7\sqrt{3}=9\sqrt{3}.$

13. $\sqrt[3]{40}+\sqrt{28}+\sqrt[6]{25}+\sqrt{175}=2\sqrt[3]{5}+2\sqrt{7}+\sqrt[3]{5}+5\sqrt{7}=3\sqrt[3]{5}+7\sqrt{7}.$

14. $\sqrt[3]{375}+\sqrt{44}+\sqrt[3]{192}+\sqrt{99}=5\sqrt[3]{3}+2\sqrt{11}+4\sqrt[3]{3}+3\sqrt{11}$
$=9\sqrt[3]{3}+5\sqrt{11}.$

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15. $\sqrt{245}-\sqrt{405}+\sqrt{45}=7\sqrt{5}-9\sqrt{5}+3\sqrt{5}=\sqrt{5}.$

16. $\sqrt{12}+3\sqrt{75}-2\sqrt{27}=2\sqrt{3}+15\sqrt{3}-6\sqrt{3}=11\sqrt{3}.$

17. $5\sqrt{72}+3\sqrt{18}-\sqrt{50}=30\sqrt{2}+9\sqrt{2}-5\sqrt{2}=34\sqrt{2}.$

18. $\sqrt[3]{128}+\sqrt[3]{686}-\sqrt[3]{54}=4\sqrt[3]{2}+7\sqrt[3]{2}-3\sqrt[3]{2}=8\sqrt[3]{2}.$

19. $\sqrt{112}-\sqrt{343}+\sqrt{448}=4\sqrt{7}-7\sqrt{7}+8\sqrt{7}=5\sqrt{7}.$

20. $\sqrt[3]{135}-\sqrt[3]{625}+\sqrt[3]{320}=3\sqrt[3]{5}-5\sqrt[3]{5}+4\sqrt[3]{5}=2\sqrt[3]{5}.$

21. $\sqrt[3]{\frac{8}{5}}+\sqrt[3]{\frac{1}{5}}+\sqrt[3]{5\frac{2}{5}}=2\sqrt[3]{\frac{1}{5}}+\sqrt[3]{\frac{1}{5}}+3\sqrt[3]{\frac{1}{5}}=6\sqrt[3]{\frac{1}{5}}=\frac{6}{5}\sqrt[3]{25}.$

22. $\sqrt[3]{864}-\sqrt[3]{4000}+\sqrt[3]{32}=6\sqrt[3]{4}-10\sqrt[3]{4}+2\sqrt[3]{4}=-2\sqrt[3]{4}.$

23. $\sqrt[3]{128x}+\sqrt[3]{375x}-\sqrt[3]{54x}=4\sqrt[3]{2x}+5\sqrt[3]{3x}-3\sqrt[3]{2x}=\sqrt[3]{2x}+5\sqrt[3]{3x}.$

24. $\sqrt{\frac{a}{x^2}}+\sqrt{\frac{a}{y^2}}-\sqrt{\frac{a}{z^2}}=\frac{1}{x}\sqrt{a}+\frac{1}{y}\sqrt{a}-\frac{1}{z}\sqrt{a}=\left(\frac{1}{x}+\frac{1}{y}-\frac{1}{z}\right)\sqrt{a}.$

25. $\sqrt{\frac{ax^4}{by^2}}-\sqrt{\frac{16ax^2}{by^2}}+\sqrt{\frac{4ax^2}{by^2}}=\sqrt{\frac{abx^4}{b^2y^2}}-\sqrt{\frac{16abx^2}{b^2y^2}}+\sqrt{\frac{4abx^2}{b^2y^2}}$
$=\frac{x^2}{by}\sqrt{ab}-\frac{4x}{by}\sqrt{ab}+\frac{2x}{by}\sqrt{ab}=\frac{x^2-2x}{by}\sqrt{ab}.$

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26. $\sqrt{\frac{a}{bc}}+\sqrt{\frac{b}{ac}}+\sqrt{\frac{c}{ab}}=\frac{1}{bc}\sqrt{abc}+\frac{1}{ac}\sqrt{abc}+\frac{1}{ab}\sqrt{abc}$

$$=\left(\frac{1}{bc}+\frac{1}{ac}+\frac{1}{ab}\right)\sqrt{abc}=\frac{a+b+c}{abc}\sqrt{abc}.$$

27. $\sqrt{(a+b)^2c}-\sqrt{(a-b)^2c}=(a+b)\sqrt{c}-(a-b)\sqrt{c}=2\,b\sqrt{c}.$

28. $6\sqrt[3]{\frac{40}{27}}+4\sqrt[3]{\frac{10}{16}}-8\sqrt[3]{\frac{675}{320}}=6\sqrt[3]{\frac{40}{27}}+4\sqrt[3]{\frac{5}{8}}-8\sqrt[3]{\frac{135}{64}}$

$$=4\sqrt[3]{5}+2\sqrt[3]{5}-6\sqrt[3]{5}=0.$$

29. $\sqrt[5]{-96\,x^4}+2\sqrt[5]{3\,x^4}-\sqrt[3]{5\,x}+\sqrt[3]{40\,x^4}=-2\sqrt[5]{3\,x^4}+2\sqrt[5]{3\,x^4}-\sqrt[3]{5\,x}$

$$+2\,x\sqrt[3]{5\,x}=(2\,x-1)\sqrt[3]{5\,x}.$$

30. $\sqrt[3]{abx}-\sqrt[6]{a^2b^2x^2}+\sqrt[9]{8\,a^3b^3x^3}=\sqrt[3]{abx}-\sqrt[3]{abx}+\sqrt[3]{2\,abx}=\sqrt[3]{2\,abx}.$

31. $\sqrt{3\,x^3+30\,x^2+75\,x}-\sqrt{3\,x^3-6\,x^2+3\,x}$

$=\sqrt{3\,x\,(x+5)^2}-\sqrt{3\,x\,(x-1)^2}$

$=[x+5-(x-1)]\sqrt{3\,x}=6\sqrt{3\,x}.$

32. $\sqrt{5\,a^5+30\,a^4+45\,a^3}-\sqrt{5\,a^5-40\,a^4+80\,a^3}$

$=\sqrt{5\,a^3\,(a+3)^2}-\sqrt{5\,a^3(a-4)^2}$

$=[a+3-(a-4)]\sqrt{5\,a^3}$

$=7\sqrt{5\,a^3}=7\,a\sqrt{5\,a}.$

33. $\sqrt{50}+\sqrt[6]{9}-4\sqrt{\frac{1}{2}}+\sqrt[3]{-24}+\sqrt[9]{27}-\sqrt[4]{64}$

$=5\sqrt{2}+\sqrt[3]{3}-2\sqrt{2}-2\sqrt[3]{3}+\sqrt[3]{3}-2\sqrt{2}=\sqrt{2}.$

34. $\sqrt{\frac{2}{3}}+6\sqrt{\frac{5}{4}}-\frac{1}{5}\sqrt{18}+\sqrt[4]{36}-\sqrt[8]{\frac{16}{81}}+\sqrt[6]{125}-2\sqrt{\frac{2}{25}}$

$=\frac{1}{3}\sqrt{6}+3\sqrt{5}-\frac{3}{5}\sqrt{2}+\sqrt{6}-\frac{1}{3}\sqrt{6}+\sqrt{5}-\frac{2}{5}\sqrt{2}$

$=\sqrt{6}+4\sqrt{5}-\sqrt{2}.$

35. $(\frac{8}{3})^{\frac{1}{2}}-(\frac{2}{3})^{-\frac{1}{2}}+\sqrt{(\frac{8}{27})^{-1}}+\sqrt{1.35}-\sqrt[4]{(1\frac{2}{3})^{-2}}$

$=2\sqrt{\frac{2}{3}}-\sqrt{\frac{3}{2}}+\frac{3}{2}\sqrt{\frac{3}{2}}+\frac{3}{2}\sqrt{\frac{3}{5}}-\sqrt{\frac{3}{5}}$

$=2\sqrt{\frac{2}{3}}+\frac{1}{2}\sqrt{\frac{3}{2}}+\frac{1}{2}\sqrt{\frac{3}{5}}$

$=\frac{2}{3}\sqrt{6}+\frac{1}{4}\sqrt{6}+\frac{1}{10}\sqrt{15}$

$=\frac{11}{12}\sqrt{6}+\frac{1}{10}\sqrt{15}.$

36. $5\cdot2^{-\frac{2}{3}}+2^{-\frac{5}{3}}+3\cdot2^{-\frac{8}{3}}+3\cdot5^{-1}\cdot2^{\frac{1}{5}}+\sqrt[5]{\frac{64}{3125}}$

$=\frac{5}{2^{\frac{2}{3}}}+\frac{1}{2^{\frac{5}{3}}}+\frac{3}{2^{\frac{8}{3}}}+\frac{3\cdot2^{\frac{1}{5}}}{5}+\frac{2}{5}\sqrt[5]{2}$

$=\frac{5}{\sqrt[3]{4}}+\frac{1}{2\sqrt[3]{4}}+\frac{3}{4\sqrt[3]{4}}+\sqrt[5]{2}$

$=\frac{5\sqrt[3]{2}}{2}+\frac{\sqrt[3]{2}}{4}+\frac{3\sqrt[3]{2}}{8}+\sqrt[5]{2}$

$=\frac{25}{8}\sqrt[3]{2}+\sqrt[5]{2}.$

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4. $\sqrt{2} \times \sqrt{8} = \sqrt{16} = 4.$

5. $\sqrt{2} \times \sqrt{6} = \sqrt{12} = 2\sqrt{3}.$

6. $\sqrt{3} \times \sqrt{15} = \sqrt{45} = 3\sqrt{5}.$

7. $2\sqrt{5} \times 3\sqrt{10} = 6\sqrt{50} = 30\sqrt{2}.$

8. $3\sqrt{20} \times 2\sqrt{2} = 6\sqrt{40} = 12\sqrt{10}.$

9. $\sqrt{2} \times 3\sqrt[3]{3} = \sqrt[6]{8} \times 3\sqrt[6]{9} = 3\sqrt[6]{72}.$

10. $2\sqrt[3]{3} \times 3\sqrt[3]{45} = 6\sqrt[3]{135} = 18\sqrt[3]{5}.$

11. $2\sqrt[4]{6} \times 3\sqrt{6} = 2\sqrt[4]{6} \times 3\sqrt[4]{36} = 6\sqrt[4]{216}.$

12. $3\sqrt{3} \times 2\sqrt[3]{5} = 3\sqrt[6]{27} \times 2\sqrt[6]{25} = 6\sqrt[6]{675}.$

13. $\sqrt[4]{5} \times \sqrt[6]{10} = \sqrt[12]{125} \times \sqrt[12]{100} = \sqrt[12]{12500}.$

14. $2\sqrt[3]{250} \times \sqrt{2} = 10\sqrt[6]{4} \times \sqrt[6]{8} = 10\sqrt[6]{32}.$

15. $2\sqrt[3]{24} \times \sqrt[3]{18} = 4\sqrt[3]{3} \times \sqrt[3]{18} = 4\sqrt[3]{54} = 12\sqrt[3]{2}.$

16. $2\sqrt[5]{2} \times \sqrt[10]{512} = 2(2)^{\frac{2}{10}} \times (2)^{\frac{9}{10}} = 2 \times 2 \times 2^{\frac{1}{10}} = 4\sqrt[10]{2}.$

17. $$\sqrt{2\,xy} \times 3\sqrt[3]{x^2y^3} = (2\,xy)^{\frac{3}{6}} \times 3(x^2y^3)^{\frac{2}{6}} = \sqrt[6]{8\,x^3y^3} \times 3\sqrt[6]{x^4y^6}$$
$$= 3\sqrt[6]{8\,x^7y^9} = 3\,xy\sqrt[6]{8\,xy^3}.$$

18. $$\sqrt{mn} \times \sqrt[4]{m^2n} \times \sqrt[8]{mn^4} = \sqrt[8]{m^4n^4} \times \sqrt[8]{m^4n^2} \times \sqrt[8]{mn^4}$$
$$= \sqrt[8]{m^9n^{10}} = mn\sqrt[8]{mn^2}.$$

19. $$\sqrt{2\,axy} \times \sqrt[3]{xy} \times \sqrt[4]{a^2xy} = \sqrt[12]{64\,a^6x^6y^6} \times \sqrt[12]{x^4y^4} \times \sqrt[12]{a^6x^3y^3}$$
$$= \sqrt[12]{64\,a^{12}x^{13}y^{13}} = axy\sqrt[12]{64\,xy}.$$

20. $$\sqrt{x^{-1}y} \times \sqrt[3]{x^{-2}y^2} \times \sqrt{x^{-3}y^3} = \sqrt{x^{-1}y} \times \sqrt{x^{-3}y^3} \times \sqrt[3]{x^{-2}y^2}$$
$$= \sqrt{x^{-4}y^4} \times \sqrt[3]{x^{-2}y^2}$$
$$= x^{-2}y^2\sqrt[3]{xy^2 \cdot x^{-3}}$$
$$= x^{-3}y^2\sqrt[3]{xy^2}.$$

21. $$\sqrt{a-b} \times \sqrt[4]{a^2b^2} \times \sqrt[4]{(a-b)^{-2}} = \sqrt{a-b} \times \sqrt{(a-b)^{-1}} \times \sqrt{ab}$$
$$= \sqrt{(a-b)^0} \times \sqrt{ab} = \sqrt{ab}.$$

22. $\sqrt{\frac{2}{3}} \times \sqrt{\frac{4}{5}} \times \sqrt{\frac{3}{4}} = \sqrt{\frac{2}{5}} = \frac{1}{5}\sqrt{10}.$

23. $\sqrt{\frac{1}{2}} \times \sqrt{\frac{4}{5}} \times \sqrt{\frac{7}{2}} = \sqrt{\frac{7}{5}} = \frac{1}{5}\sqrt{35}.$

24. $\sqrt[3]{\frac{2}{3}} \times \sqrt[3]{\frac{3}{4}} \times \sqrt{\frac{1}{2}} = \sqrt[3]{\frac{1}{2}} \times \sqrt{\frac{1}{2}} = \sqrt[6]{\frac{1}{4}} \times \sqrt[6]{\frac{1}{8}} = \sqrt[6]{\frac{1}{32}} = \frac{1}{2}\sqrt[6]{2}.$

25. $\sqrt[3]{\frac{1}{3}} \times \sqrt[6]{\frac{3}{2}} \times \sqrt{\frac{2}{3}} = \sqrt[6]{\frac{1}{9}} \times \sqrt[6]{\frac{3}{2}} \times \sqrt[6]{\frac{8}{27}} = \sqrt[6]{\frac{4}{81}} = \sqrt[3]{\frac{2}{9}} = \frac{1}{3}\sqrt[3]{6}.$

26. $16^{\frac{1}{3}} \times 2^{\frac{1}{2}} \times 32^{\frac{5}{6}} = 2^{\frac{4}{3}} \times 2^{\frac{1}{2}} \times 2^{\frac{25}{6}} = 2^{\frac{8}{6}} \times 2^{\frac{3}{6}} \times 2^{\frac{25}{6}} = 2^{\frac{36}{6}} = 2^6 = 64.$

27. $$27^{\frac{1}{4}} \times 9^{\frac{1}{3}} \times 81^{\frac{1}{3}} = 3^{\frac{3}{4}} \times 3^{\frac{2}{3}} \times 3^{\frac{4}{3}} = 3^{\frac{9}{12}} \times 3^{\frac{8}{12}} \times 3^{\frac{16}{12}} = 3^{\frac{33}{12}} = 3^{\frac{11}{4}}$$
$$= 9\sqrt[4]{27}.$$

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29.
$$\begin{array}{l} \sqrt{5}+\sqrt{3} \\ \sqrt{5}-\sqrt{3} \\ \hline 5+\sqrt{15} \\ \qquad -\sqrt{15}-3 \\ \hline 5 \qquad\qquad -3 \\ =2. \end{array}$$

30.
$$\begin{array}{l} \sqrt{7}+\sqrt{2} \\ \sqrt{7}-\sqrt{2} \\ \hline 7+\sqrt{14} \\ \qquad -\sqrt{14}-2 \\ \hline 7 \qquad\qquad -2 \\ =5. \end{array}$$

31.
$$\begin{array}{l} \sqrt{6}-\sqrt{5} \\ \sqrt{6}-\sqrt{5} \\ \hline 6-\sqrt{30} \\ \qquad -\sqrt{30}+5 \\ \hline 6-2\sqrt{30}+5 \\ =11-2\sqrt{30}. \end{array}$$

32.
$$\begin{array}{l} 5-\sqrt{5} \\ 1+\sqrt{5} \\ \hline 5-\sqrt{5} \\ \qquad 5\sqrt{5}-5 \\ \hline 5+4\sqrt{5}-5 \\ =4\sqrt{5}. \end{array}$$

33.
$$\begin{array}{l} 4\sqrt{7}+1 \\ 4\sqrt{7}-1 \\ \hline 112+4\sqrt{7} \\ \qquad -4\sqrt{7}-1 \\ \hline 112 \qquad\qquad -1 \\ =111. \end{array}$$

34.
$$\begin{array}{l} 2\sqrt{2}+\sqrt{3} \\ 4\sqrt{2}+\sqrt{3} \\ \hline 16+4\sqrt{6} \\ \qquad 2\sqrt{6}+3 \\ \hline 16+6\sqrt{6}+3 \\ =19+6\sqrt{6}. \end{array}$$

35.
$$\begin{array}{l} 2\sqrt{3}+3\sqrt{5} \\ 3\sqrt{3}+2\sqrt{5} \\ \hline 18+9\sqrt{15} \\ \qquad 4\sqrt{15}+30 \\ \hline 18+13\sqrt{15}+30 \\ =48+13\sqrt{15}. \end{array}$$

36.
$$\begin{array}{l} 3a+\sqrt{5} \\ 2a-\sqrt{5} \\ \hline 6a^2+2a\sqrt{5} \\ \qquad -3a\sqrt{5}-5 \\ \hline 6a^2-\ a\sqrt{5}-5. \end{array}$$

37.
$$\begin{array}{l} 2\sqrt{6}-3\sqrt{5} \\ 4\sqrt{3}-\sqrt{10} \\ \hline 24\sqrt{2}-12\sqrt{15} \\ \qquad -\ 4\sqrt{15}+15\sqrt{2} \\ \hline 24\sqrt{2}-16\sqrt{15}+15\sqrt{2} \\ =39\sqrt{2}-16\sqrt{15}. \end{array}$$

38.
$$\begin{array}{l} a^2-ab\sqrt{2}+b^2 \\ a^2+ab\sqrt{2}+b^2 \\ \hline a^4-a^3b\sqrt{2}+a^2b^2 \\ \qquad a^3b\sqrt{2}-2a^2b^2+ab^3\sqrt{2} \\ \qquad\qquad a^2b^2-ab^3\sqrt{2}+b^4 \\ \hline a^4 \qquad\qquad\qquad\qquad +b^4 \end{array}$$

39.
$$\begin{array}{l} x-\sqrt{xyz}+yz \\ \sqrt{x}+\sqrt{yz} \\ \hline x\sqrt{x}-x\sqrt{yz}+yz\sqrt{x} \\ \qquad x\sqrt{yz}-yz\sqrt{x}+yz\sqrt{yz} \\ \hline x\sqrt{x} \qquad\qquad\qquad +yz\sqrt{yz} \end{array}$$

40.
$$\begin{array}{l} x\sqrt{x}-x\sqrt{y}+y\sqrt{x}-y\sqrt{y} \\ \sqrt{x}+\ \sqrt{y} \\ \hline x^2-x\sqrt{xy}+xy-y\sqrt{xy} \\ \qquad x\sqrt{xy}-xy+y\sqrt{xy}-y^2 \\ \hline x^2 \qquad\qquad\qquad\qquad -y^2 \end{array}$$

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41. $(\sqrt{3+\sqrt{5}})(\sqrt{3-\sqrt{5}}) = \sqrt{9-5} = 2.$

42. $(\sqrt{9+\sqrt{6}})(\sqrt{9-\sqrt{6}}) = \sqrt{81-6} = \sqrt{75} = 5\sqrt{3}.$

43. $(\sqrt{6+\sqrt{11}})(\sqrt{6-\sqrt{11}}) = \sqrt{36-11} = \sqrt{25} = 5.$

44. $(\sqrt{5\,a + a\sqrt{5}})(\sqrt{5\,a - a\sqrt{5}}) = \sqrt{25\,a^2 - 5\,a^2} = \sqrt{20\,a^2} = 2\,a\sqrt{5}.$

45. $(\sqrt{7\,c + \sqrt{5\,c^2}})(\sqrt{7\,c - \sqrt{5\,c^2}}) = \sqrt{49\,c^2 - 5\,c^2} = \sqrt{44\,c^2} = 2\,c\sqrt{11}.$

46. $(\sqrt{14\,x + x\sqrt{27}})(\sqrt{14\,x - x\sqrt{27}}) = \sqrt{196\,x^2 - 27\,x^2} = \sqrt{169\,x^2} = 13\,x.$

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4. $\sqrt{50} \div \sqrt{8} = \sqrt{\frac{50}{8}} = \sqrt{\frac{25}{4}} = \frac{5}{2}.$

5. $\sqrt{72} \div 2\sqrt{6} = \frac{1}{2}\sqrt{12} = \sqrt{3}.$

6. $4\sqrt{5} \div \sqrt{40} = 4\sqrt{\frac{1}{8}} = \sqrt{2}.$

7. $6\sqrt{7} \div \sqrt{126} = 6\sqrt{\frac{1}{18}} = \sqrt{2}.$

8. $\sqrt[3]{4} \div \sqrt{2} = \sqrt[6]{16} \div \sqrt[6]{8} = \sqrt[6]{2}.$

10. $7\sqrt{75} \div 5\sqrt{28} = 35\sqrt{3} \div 10\sqrt{7} = \frac{7}{2}\sqrt{\frac{3}{7}} = \frac{1}{2}\sqrt{21}.$

11. $\sqrt[3]{16} \div \sqrt[6]{32} = \sqrt[6]{256} \div \sqrt[6]{32} = \sqrt[6]{8} = \sqrt{2}.$

12. $2\sqrt[3]{12} \div \sqrt{8} = 2\sqrt[6]{144} \div 2\sqrt{2}$
$= \sqrt[6]{144} \div \sqrt[6]{8} = \sqrt[6]{18}.$

13. $\sqrt[3]{ax} \div \sqrt{xy} = \sqrt[6]{a^2x^2} \div \sqrt[6]{x^3y^3}$
$= \sqrt[6]{\frac{a^2}{xy^3}} = \frac{1}{xy}\sqrt[6]{a^2x^5y^3}.$

14. $\sqrt{2\,ab^3} \div \sqrt[4]{a^4b^4} = b\sqrt{2\,ab} \div ab = \frac{1}{a}\sqrt{2\,ab}.$

15. $\sqrt[3]{a^2x^2} \div \sqrt{2\,ax} = \sqrt[6]{a^4x^4} \div \sqrt[6]{8\,a^3x^3} = \sqrt[6]{\frac{1}{8}\,ax} = \frac{1}{2}\sqrt[6]{8\,ax}.$

16. $\sqrt[3]{9\,a^2b^2} \div \sqrt{3\,ab} = \sqrt[6]{81\,a^4b^4} \div \sqrt[6]{27\,a^3b^3} = \sqrt[6]{3\,ab}.$

17. $\sqrt[4]{4\,x^2y^2} \div \sqrt[3]{2\,xy} = \sqrt[6]{8\,x^3y^3} \div \sqrt[6]{4\,x^2y^2} = \sqrt[6]{2\,xy}.$

18. $\sqrt{a-b} \div \sqrt{a+b} = \sqrt{\frac{a-b}{a+b}} = \sqrt{\frac{a^2-b^2}{(a+b)^2}} = \frac{1}{a+b}\sqrt{a^2-b^2}.$

19. $3\sqrt[3]{\frac{2}{3}} \div \sqrt{\frac{3}{4}} = 3\sqrt[6]{\frac{4}{9}} \div \frac{1}{2}\sqrt[6]{27} = 6\sqrt[6]{\frac{4}{243}} = 2\sqrt[6]{12}.$

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20. $\begin{array}{l|l} \sqrt{3} & \sqrt{15} - \sqrt{3} \\ \hline & \sqrt{5} - \quad 1 \end{array}$

21. $\begin{array}{l|l} \sqrt{2} & \sqrt{6} - 2\sqrt{3} + 4 \\ \hline & \sqrt{3} - 2\sqrt{\frac{3}{2}} + \sqrt{8} \end{array}$
$= \sqrt{3} - \sqrt{6} + 2\sqrt{2}.$

22. $\begin{array}{l|l} \frac{1}{3}\sqrt{6} & \sqrt{2} + 2 + \frac{1}{3}\sqrt{42} \\ \hline & 3\sqrt{\frac{1}{3}} + 6\sqrt{\frac{1}{6}} + \sqrt{7} \end{array}$
$= \sqrt{3} + \sqrt{6} + \sqrt{7}.$

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23. $(5\sqrt{2}+5\sqrt{3})\div(\sqrt{10}+\sqrt{15})=(\sqrt{50}+\sqrt{75})\div(\sqrt{10}+\sqrt{15})=\sqrt{5}.$

24.
$$\begin{array}{r|l} 5+5\sqrt{30}+36 & \sqrt{5}+2\sqrt{6} \\ \underline{5+2\sqrt{30}} & \overline{\sqrt{5}+3\sqrt{6}} \\ 3\sqrt{30}+36 & \\ \underline{3\sqrt{30}+36} & \end{array}$$

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4. $(3\sqrt{ab})^2=3^2(ab)^{\frac{2}{2}}=9\,ab.$

5. $(2\sqrt[3]{3\,x})^2=2^2\cdot3^{\frac{2}{3}}x^{\frac{2}{3}}=4\sqrt[3]{9\,x^2}.$

6. $(x\sqrt[3]{2\,x^3})^2=(x^2\sqrt[3]{2})^2=x^4\cdot2^{\frac{2}{3}}=x^4\sqrt[3]{4}.$

7. $(n^2\sqrt{4\,b})^2=n^4(4\,b)^{\frac{2}{2}}=4\,bn^4.$

8. $(a\sqrt[4]{a^2b})^2=a^2(a^2)^{\frac{2}{4}}b^{\frac{2}{4}}=a^2\cdot a\cdot b^{\frac{1}{2}}=a^3\sqrt{b}.$

9. $(2\sqrt{5})^3=2^3\cdot5^{\frac{3}{2}}=8\cdot5\cdot5^{\frac{1}{2}}=40\sqrt{5}.$

10. $(3\sqrt{2})^3=3^3\cdot2^{\frac{3}{2}}=27\cdot2\cdot2^{\frac{1}{2}}=54\sqrt{2}.$

11. $(2\sqrt[3]{a^2})^3=2^3(a^2)^{\frac{3}{3}}=8\,a^2.$

12. $(\sqrt[4]{a^2b^3})^3=(a^2b^3)^{\frac{3}{4}}=a^{\frac{3}{2}}b^{\frac{9}{4}}=ab^2\cdot a^{\frac{2}{4}}b^{\frac{1}{4}}=ab^2\sqrt[4]{a^2b}.$

13. $(\sqrt[6]{4\,n^3})^3=(4\,n^3)^{\frac{3}{6}}=(4\,n^3)^{\frac{1}{2}}=\sqrt{4\,n^3}=2\,n\sqrt{n}.$

14. $(-2\sqrt{2\,ab})^4=16\,(2\,ab)^{\frac{4}{2}}=16\,(2\,ab)^2=64\,a^2b^2.$

15. $(-\sqrt{2}\sqrt[6]{x})^3=-(2)^{\frac{3}{2}}x^{\frac{3}{6}}=-2\cdot2^{\frac{1}{2}}x^{\frac{1}{2}}=-2\sqrt{2\,x}.$

16. $(-\sqrt{2}\sqrt[3]{ax^2})^4=2^{\frac{4}{2}}(ax^2)^{\frac{4}{3}}=4\,ax^2\,(ax^2)^{\frac{1}{3}}=4\,ax^2\sqrt[3]{ax^2}.$

17. $(-2\sqrt{x}\sqrt[3]{y})^5=(-2)^5x^{\frac{5}{2}}y^{\frac{5}{3}}=-32\,x^2y\cdot x^{\frac{1}{2}}y^{\frac{2}{3}}$
$=-32\,x^2y\cdot x^{\frac{3}{6}}y^{\frac{4}{6}}=-32\,x^2y\sqrt[6]{x^3y^4}.$

18. $(-3\,a^{\frac{n}{2}}x^{\frac{n}{3}})^6=(-3)^6\,a^{3n}x^{2n}=729\,a^{3n}x^{2n}.$

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19. $(2+\sqrt{6})^2=2^2+2\cdot2\sqrt{6}+(\sqrt{6})^2$
$=4+4\sqrt{6}+6$
$=10+4\sqrt{6}.$

20. $(2+\sqrt{2})^2=2^2+2\cdot2\sqrt{2}+(\sqrt{2})^2$
$=4+4\sqrt{2}+2$
$=6+4\sqrt{2}.$

21. $(2+\sqrt{5})^3=2^3+3\,(2)^2\sqrt{5}+3\cdot2\,(\sqrt{5})^2+(\sqrt{5})^3$
$=8+12\sqrt{5}+30+5\sqrt{5}$
$=38+17\sqrt{5}.$

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22. $$\begin{aligned}(2-\sqrt{3})^3 &= 2^3 - 3(2)^2\sqrt{3} + 3\cdot 2(\sqrt{3})^2 - (\sqrt{3})^3\\ &= 8 - 12\sqrt{3} + 18 - 3\sqrt{3}\\ &= 26 - 15\sqrt{3}.\end{aligned}$$

23. $$\begin{aligned}(\sqrt{7}-\sqrt{6})^2 &= (\sqrt{7})^2 - 2\sqrt{7}\sqrt{6} + (\sqrt{6})^2\\ &= 7 - 2\sqrt{42} + 6\\ &= 13 - 2\sqrt{42}.\end{aligned}$$

24. $$\begin{aligned}(2\sqrt{2}-\sqrt{3})^2 &= (2\sqrt{2})^2 - 2\cdot 2\sqrt{2}\sqrt{3} + (\sqrt{3})^2\\ &= 8 - 4\sqrt{6} + 3\\ &= 11 - 4\sqrt{6}.\end{aligned}$$

25. $$\begin{aligned}(\sqrt{x}\pm 1)^2 &= (\sqrt{x})^2 \pm 2\sqrt{x}\cdot 1 + 1^2\\ &= x \pm 2\sqrt{x} + 1.\end{aligned}$$

26. $$\begin{aligned}(\sqrt{a}-\sqrt{b})^3 &= (\sqrt{a})^3 - 3(\sqrt{a})^2\sqrt{b} + 3\sqrt{a}(\sqrt{b})^2 - (\sqrt{b})^3\\ &= a\sqrt{a} - 3a\sqrt{b} + 3b\sqrt{a} - b\sqrt{b}.\end{aligned}$$

27. $$\begin{aligned}(\sqrt{x}\pm 1)^3 &= (\sqrt{x})^3 \pm 3(\sqrt{x})^2\cdot 1 + 3\sqrt{x}(1)^2 \pm (1)^3\\ &= x\sqrt{x} \pm 3x + 3\sqrt{x} \pm 1.\end{aligned}$$

29. $\sqrt{\sqrt{2}} = (2^{\frac12})^{\frac12} = 2^{\frac14} = \sqrt[4]{2}$.

31. $\sqrt{\sqrt[6]{x^2}} = (x^{\frac26})^{\frac12} = x^{\frac16} = \sqrt[6]{x}$.

30. $\sqrt{\sqrt[3]{5}} = (5^{\frac13})^{\frac12} = 5^{\frac16} = \sqrt[6]{5}$.

32. $\sqrt{\sqrt[3]{x^n}} = (x^{\frac n3})^{\frac12} = x^{\frac n6} = \sqrt[6]{x^n}$.

33. $\sqrt{\sqrt[5]{x^{12}}} = (x^{\frac{12}{5}})^{\frac12} = x^{\frac65} = x\cdot x^{\frac15} = x\sqrt[5]{x}$.

34. $\sqrt{\sqrt[n]{a^nx^2}} = [(a^nx^2)^{\frac1n}]^{\frac12} = (a^nx^2)^{\frac1{2n}} = \sqrt[2n]{a^nx^2}$.

35. $\sqrt[3]{\sqrt{2x}} = [(2x)^{\frac12}]^{\frac13} = (2x)^{\frac16} = \sqrt[6]{2x}$.

36. $\sqrt[3]{\sqrt{7a^3}} = [(7a^3)^{\frac12}]^{\frac13} = (7a^3)^{\frac16} = \sqrt[6]{7a^3}$.

37. $\sqrt[3]{\sqrt[4]{8m^3x^3}} = (8^{\frac14}m^{\frac34}x^{\frac34})^{\frac13} = (2^{\frac34}m^{\frac34}x^{\frac34})^{\frac13} = 2^{\frac14}m^{\frac14}x^{\frac14} = \sqrt[4]{2mx}$.

38. $\sqrt[3]{-27\sqrt{x^6}} = \sqrt[3]{-27x^3} = -3x$.

39. $\sqrt[3]{-\sqrt{a^nb^n}} = -[(ab)^{\frac n2}]^{\frac13} = -(ab)^{\frac n6} = -\sqrt[6]{a^nb^n}$.

40. $\sqrt[3]{-64\sqrt[5]{a^3y^3}} = (-64)^{\frac13}[(ay)^{\frac35}]^{\frac13} = -4(ay)^{\frac15} = -4\sqrt[5]{ay}$.

41. $\sqrt{\sqrt[3]{4a^2x^4}} = \sqrt{\sqrt[3]{(2ax^2)^2}} = [(2ax^2)^{\frac23}]^{\frac12} = (2ax^2)^{\frac13} = \sqrt[3]{2ax^2}$.

42. $\sqrt[3]{\sqrt{a^{12}x^4}} = \sqrt[3]{a^6x^2} = a^2\sqrt[3]{x^2}$.

43. $(\sqrt{8a^3x^3})^{\frac13} = [\sqrt{(2ax)^3}]^{\frac13} = [(2ax)^{\frac32}]^{\frac13} = (2ax)^{\frac12} = \sqrt{2ax}$.

44. $(\sqrt{x^my^m})^{\frac1{mn}} = [(xy)^{\frac m2}]^{\frac1{mn}} = (xy)^{\frac1{2n}} = \sqrt[2n]{xy}$.

45. $\sqrt{\left(\dfrac{x^nb^2}{a^{-2}y^n}\right)^{\frac2n}} = \left(\dfrac{x^nb^2}{a^{-2}y^n}\right)^{\frac1n} = \left(\dfrac{x^na^2b^2}{y^n}\right)^{\frac1n} = \sqrt[n]{\dfrac{x^n}{y^n}\cdot a^2b^2} = \dfrac{x}{y}\sqrt[n]{a^2b^2}$.

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1. $\dfrac{5}{\sqrt{2}}=\dfrac{5\sqrt{2}}{2}=\dfrac{5\times 1.41421}{2}=3.5355.$

2. $\dfrac{2}{\sqrt{5}}=\dfrac{2\sqrt{5}}{5}=\dfrac{2\times 2.23607}{5}=.8944.$

3. $\dfrac{6}{\sqrt{8}}=\dfrac{6\sqrt{2}}{4}=\dfrac{6\times 1.41421}{4}=\dfrac{3\times 1.41421}{2}=2.1213.$

4. $\dfrac{10}{\sqrt{45}}=\dfrac{10\sqrt{5}}{15}=\dfrac{10\times 2.23607}{15}=\dfrac{2\times 2.23607}{3}=1.4907.$

5. $\dfrac{15}{\sqrt{50}}=\dfrac{15\sqrt{2}}{10}=\dfrac{15\times 1.41421}{10}=\dfrac{3\times 1.41421}{2}=2.1213.$

6. $\dfrac{1}{\sqrt{125}}=\dfrac{\sqrt{5}}{25}=\dfrac{2.23607}{25}=.0894.$

9. $\dfrac{\sqrt[3]{6}}{\sqrt{12}}=\dfrac{\sqrt[3]{6}\times\sqrt{12}}{\sqrt{12}\times\sqrt{12}}=\dfrac{\sqrt[6]{36}\times\sqrt[6]{1728}}{12}=\dfrac{\sqrt[6]{62208}}{12}=\dfrac{2\sqrt[6]{972}}{12}=\dfrac{\sqrt[6]{972}}{6}$

10. $\dfrac{\sqrt{a}}{\sqrt[3]{ax^2}}=\dfrac{\sqrt{a}\sqrt[3]{a^2x}}{\sqrt[3]{ax^2}\sqrt[3]{a^2x}}=\dfrac{\sqrt[6]{a^3}\sqrt[6]{a^4x^2}}{\sqrt[3]{a^3x^3}}=\dfrac{a\sqrt[6]{ax^2}}{ax}=\dfrac{\sqrt[6]{ax^2}}{x}.$

11. $\dfrac{\sqrt{a+b}}{\sqrt{a-b}}=\dfrac{\sqrt{a+b}\sqrt{a-b}}{\sqrt{(a-b)^2}}=\dfrac{\sqrt{a^2-b^2}}{a-b}.$

12. $\sqrt{1-\dfrac{4}{x+2}}=\sqrt{\dfrac{x+2-4}{x+2}}=\sqrt{\dfrac{x-2}{x+2}\times\dfrac{x+2}{x+2}}=\dfrac{\sqrt{x^2-4}}{x+2}.$

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3. $\dfrac{3}{2+\sqrt{3}}=\dfrac{3(2-\sqrt{3})}{(2+\sqrt{3})(2-\sqrt{3})}=\dfrac{3(2-\sqrt{3})}{4-3}=6-3\sqrt{3}.$

4. $\dfrac{5}{\sqrt{5}-\sqrt{3}}=\dfrac{5(\sqrt{5}+\sqrt{3})}{(\sqrt{5}-\sqrt{3})(\sqrt{5}+\sqrt{3})}=\dfrac{5(\sqrt{5}+\sqrt{3})}{5-3}$

$=\dfrac{5(\sqrt{5}+\sqrt{3})}{2}.$

5. $\dfrac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}=\dfrac{(\sqrt{3}+\sqrt{2})(\sqrt{3}+\sqrt{2})}{(\sqrt{3}-\sqrt{2})(\sqrt{3}+\sqrt{2})}=\dfrac{3+2\sqrt{6}+2}{3-2}=5+2\sqrt{6}.$

6. $\dfrac{5-3\sqrt{2}}{2-\sqrt{2}}=\dfrac{(5-3\sqrt{2})(2+\sqrt{2})}{(2-\sqrt{2})(2+\sqrt{2})}=\dfrac{4-\sqrt{2}}{4-2}=\dfrac{4-\sqrt{2}}{2}.$

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7. $\dfrac{a-2\sqrt{b}}{a+2\sqrt{b}}=\dfrac{(a-2\sqrt{b})(a-2\sqrt{b})}{(a+2\sqrt{b})(a-2\sqrt{b})}=\dfrac{a^2-4a\sqrt{b}+4b}{a^2-4b}.$

8. $\dfrac{\sqrt{x}+\sqrt{y}}{\sqrt{x}-\sqrt{y}}=\dfrac{(\sqrt{x}+\sqrt{y})(\sqrt{x}+\sqrt{y})}{(\sqrt{x}-\sqrt{y})(\sqrt{x}+\sqrt{y})}=\dfrac{x+2\sqrt{xy}+y}{x-y}.$

9. $\dfrac{4\sqrt{2}+6\sqrt{3}}{3\sqrt{3}-2\sqrt{2}}=\dfrac{2(3\sqrt{3}+2\sqrt{2})(3\sqrt{3}+2\sqrt{2})}{(3\sqrt{3}-2\sqrt{2})(3\sqrt{3}+2\sqrt{2})}=\dfrac{2(27+12\sqrt{6}+8)}{27-8}$

$=\dfrac{70+24\sqrt{6}}{19}.$

10. $\dfrac{x-\sqrt{x^2-1}}{x+\sqrt{x^2-1}}=\dfrac{(x-\sqrt{x^2-1})(x-\sqrt{x^2-1})}{(x+\sqrt{x^2-1})(x-\sqrt{x^2-1})}=\dfrac{x^2-2x\sqrt{x^2-1}+x^2-1}{x^2-(x^2-1)}$

$=2x^2-2x\sqrt{x^2-1}-1.$

11. $\dfrac{\sqrt{a^2+a+1}-1}{\sqrt{a^2+a+1}+1}=\dfrac{(\sqrt{a^2+a+1}-1)(\sqrt{a^2+a+1}-1)}{(\sqrt{a^2+a+1}+1)(\sqrt{a^2+a+1}-1)}$

$=\dfrac{a^2+a+1-2\sqrt{a^2+a+1}+1}{a^2+a+1-1}=\dfrac{a^2+a+2-2\sqrt{a^2+a+1}}{a^2+a.}.$

12. $\dfrac{\sqrt{x+y}-\sqrt{x-y}}{\sqrt{x+y}+\sqrt{x-y}}=\dfrac{(\sqrt{x+y}-\sqrt{x-y})(\sqrt{x+y}-\sqrt{x-y})}{(\sqrt{x+y}+\sqrt{x-y})(\sqrt{x+y}-\sqrt{x-y})}$

$=\dfrac{x+y-2\sqrt{x^2-y^2}+x-y}{x+y-(x-y)}=\dfrac{2x-2\sqrt{x^2-y^2}}{2y}=\dfrac{x-\sqrt{x^2-y^2}}{y}.$

13. $\dfrac{8-\sqrt{3}}{2-\sqrt{3}}=\dfrac{(8-\sqrt{3})(2+\sqrt{3})}{(2-\sqrt{3})(2+\sqrt{3})}=\dfrac{13+6\sqrt{3}}{4-3}=13+6\sqrt{3}$

$=13+6\times 1.732=23.392.$

14. $\dfrac{4}{3+\sqrt{5}}=\dfrac{4(3-\sqrt{5})}{(3+\sqrt{5})(3-\sqrt{5})}=\dfrac{12-4\sqrt{5}}{9-5}=3-\sqrt{5}=3-2.236=.764$

15. $\dfrac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}=\dfrac{(\sqrt{3}+\sqrt{2})(\sqrt{3}+\sqrt{2})}{(\sqrt{3}-\sqrt{2})(\sqrt{3}+\sqrt{2})}=\dfrac{3+2\sqrt{6}+2}{3-2}=5+2\sqrt{6}$

$=5+2\times 2.449=9.898.$

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17. $\dfrac{\sqrt{2}-\sqrt{5}-\sqrt{7}}{\sqrt{2}+\sqrt{5}+\sqrt{7}}=\dfrac{(\sqrt{2}-\sqrt{7})-\sqrt{5}}{(\sqrt{2}+\sqrt{5})+\sqrt{7}}\times\dfrac{(\sqrt{2}-\sqrt{7})+\sqrt{5}}{(\sqrt{2}+\sqrt{5})-\sqrt{7}}$

$=\dfrac{2-2\sqrt{14}+7-5}{2+2\sqrt{10}+5-7}=\dfrac{4-2\sqrt{14}}{2\sqrt{10}}$

$=\dfrac{2-\sqrt{14}}{\sqrt{10}}=\dfrac{2\sqrt{10}-2\sqrt{35}}{10}=\dfrac{\sqrt{10}-\sqrt{35}}{5}.$

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18. $$\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}-\sqrt{6}}=\frac{\sqrt{3}+\sqrt{2}}{(\sqrt{3}+\sqrt{2})-\sqrt{6}}\times\frac{\sqrt{3}+\sqrt{2}+\sqrt{6}}{(\sqrt{3}+\sqrt{2})+\sqrt{6}}$$
$$=\frac{3+2\sqrt{6}+2+3\sqrt{2}+2\sqrt{3}}{3+2\sqrt{6}+2-6}=\frac{2\sqrt{6}+5+3\sqrt{2}+2\sqrt{3}}{2\sqrt{6}-1}$$
$$=\frac{(2\sqrt{6}+5+3\sqrt{2}+2\sqrt{3})(2\sqrt{6}+1)}{(2\sqrt{6}-1)(2\sqrt{6}+1)}$$
$$=\frac{24+10\sqrt{6}+12\sqrt{3}+12\sqrt{2}+2\sqrt{6}+5+3\sqrt{2}+2\sqrt{3}}{24-1}$$
$$=\frac{29+12\sqrt{6}+14\sqrt{3}+15\sqrt{2}}{23}.$$

19. $$\frac{1}{\sqrt{2}+\sqrt{3}+\sqrt{5}}=\frac{\sqrt{2}+\sqrt{3}-\sqrt{5}}{(\sqrt{2}+\sqrt{3}+\sqrt{5})(\sqrt{2}+\sqrt{3}-\sqrt{5})}$$
$$=\frac{\sqrt{2}+\sqrt{3}-\sqrt{5}}{2+2\sqrt{6}+3-5}=\frac{\sqrt{2}+\sqrt{3}-\sqrt{5}}{2\sqrt{6}}$$
$$=\frac{2\sqrt{3}+3\sqrt{2}-\sqrt{30}}{2\sqrt{6}\cdot\sqrt{6}}=\frac{2\sqrt{3}+3\sqrt{2}-\sqrt{30}}{12}.$$

20. $$\frac{2\sqrt{2}-3\sqrt{3}+4\sqrt{5}}{\sqrt{2}+\sqrt{3}-\sqrt{5}}=\frac{2\sqrt{2}-3\sqrt{3}+4\sqrt{5}}{(\sqrt{2}+\sqrt{3})-\sqrt{5}}\times\frac{\sqrt{2}+\sqrt{3}+\sqrt{5}}{(\sqrt{2}+\sqrt{3})+\sqrt{5}}$$
$$=\frac{15-\sqrt{6}+6\sqrt{10}+\sqrt{15}}{2\sqrt{6}}$$
$$=\frac{15\sqrt{6}-6+12\sqrt{15}+3\sqrt{10}}{12}$$
$$=\frac{5\sqrt{6}-2+4\sqrt{15}+\sqrt{10}}{4}.$$

22. $$\frac{\sqrt[3]{ab}}{\sqrt[3]{a}-\sqrt[3]{b}}=\frac{a^{\frac{1}{3}}b^{\frac{1}{3}}(a^{\frac{2}{3}}+a^{\frac{1}{3}}b^{\frac{1}{3}}+b^{\frac{2}{3}})}{(a^{\frac{1}{3}}-b^{\frac{1}{3}})(a^{\frac{2}{3}}+a^{\frac{1}{3}}b^{\frac{1}{3}}+b^{\frac{2}{3}})}=\frac{a^{\frac{1}{3}}b^{\frac{1}{3}}(a^{\frac{2}{3}}+a^{\frac{1}{3}}b^{\frac{1}{3}}+b^{\frac{2}{3}})}{a-b}.$$

23. $$\frac{2}{\sqrt[3]{x}+\sqrt{y}}=\frac{2(x^{\frac{5}{3}}-x^{\frac{4}{3}}y^{\frac{1}{2}}+xy-x^{\frac{2}{3}}y^{\frac{3}{2}}+x^{\frac{1}{3}}y^2-y^{\frac{5}{2}})}{(x^{\frac{1}{3}}+y^{\frac{1}{2}})(x^{\frac{5}{3}}-x^{\frac{4}{3}}y^{\frac{1}{2}}+xy-x^{\frac{2}{3}}y^{\frac{3}{2}}+x^{\frac{1}{3}}y^2-y^{\frac{5}{2}})}$$
$$=\frac{2(x^{\frac{5}{3}}-x^{\frac{4}{3}}y^{\frac{1}{2}}+xy-x^{\frac{2}{3}}y^{\frac{3}{2}}+x^{\frac{1}{3}}y^2-y^{\frac{5}{2}})}{x^2-y^3}.$$

24. $$\frac{\sqrt[3]{ab^2}}{\sqrt[3]{a^2}-\sqrt{b^3}}=\frac{a^{\frac{1}{3}}b^{\frac{2}{3}}(a^{\frac{10}{3}}+a^{\frac{8}{3}}b^{\frac{3}{2}}+a^2b^3+a^{\frac{4}{3}}b^{\frac{9}{2}}+a^{\frac{2}{3}}b^6+b^{\frac{15}{2}})}{(a^{\frac{2}{3}}-b^{\frac{3}{2}})(a^{\frac{10}{3}}+a^{\frac{8}{3}}b^{\frac{3}{2}}+a^2b^3+a^{\frac{4}{3}}b^{\frac{9}{2}}+a^{\frac{2}{3}}b^6+b^{\frac{15}{2}})}$$
$$=\frac{a^{\frac{1}{3}}b^{\frac{2}{3}}(a^{\frac{10}{3}}+a^{\frac{8}{3}}b^{\frac{3}{2}}+a^2b^3+a^{\frac{4}{3}}b^{\frac{9}{2}}+a^{\frac{2}{3}}b^6+b^{\frac{15}{2}})}{a^4-b^9}.$$

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25. $$\frac{\sqrt{a}+b}{\sqrt[4]{a}-\sqrt{b}}=\frac{(a^{\frac{1}{2}}+b)(a^{\frac{3}{4}}+a^{\frac{1}{2}}b^{\frac{1}{2}}+a^{\frac{1}{4}}b+b^{\frac{3}{2}})}{(a^{\frac{1}{4}}-b^{\frac{1}{2}})(a^{\frac{3}{4}}+a^{\frac{1}{2}}b^{\frac{1}{2}}+a^{\frac{1}{4}}b+b^{\frac{3}{2}})}$$
$$=\frac{(a^{\frac{1}{2}}+b)(a^{\frac{3}{4}}+a^{\frac{1}{2}}b^{\frac{1}{2}}+a^{\frac{1}{4}}b+b^{\frac{3}{2}})}{a-b^2}.$$

26. $$\frac{\sqrt{ax}}{\sqrt[3]{a}-\sqrt[5]{x}}$$
$$=\frac{a^{\frac{1}{2}}x^{\frac{1}{2}}(a^{\frac{14}{3}}+a^{\frac{13}{3}}x^{\frac{1}{5}}+a^4x^{\frac{2}{5}}+a^{\frac{11}{3}}x^{\frac{3}{5}}+\cdots+ax^{\frac{11}{5}}+a^{\frac{2}{3}}x^{\frac{12}{5}}+a^{\frac{1}{3}}x^{\frac{13}{5}}+x^{\frac{14}{5}})}{(a^{\frac{1}{3}}-x^{\frac{1}{5}})(a^{\frac{14}{3}}+a^{\frac{13}{3}}x^{\frac{1}{5}}+a^4x^{\frac{2}{5}}+a^{\frac{11}{3}}x^{\frac{3}{5}}+\cdots+ax^{\frac{11}{5}}+a^{\frac{2}{3}}x^{\frac{12}{5}}+a^{\frac{1}{3}}x^{\frac{13}{5}}+x^{\frac{14}{5}})}$$
$$=\frac{a^{\frac{1}{2}}x^{\frac{1}{2}}(a^{\frac{14}{3}}+a^{\frac{13}{3}}x^{\frac{1}{5}}+a^4x^{\frac{2}{5}}+a^{\frac{11}{3}}x^{\frac{3}{5}}+\cdots+ax^{\frac{11}{5}}+a^{\frac{2}{3}}x^{\frac{12}{5}}+a^{\frac{1}{3}}x^{\frac{13}{5}}+x^{\frac{14}{5}})}{a^5-x^3}.$$

27. $$\frac{\sqrt[3]{xy^2}}{\sqrt{x}+\sqrt[5]{y^3}}$$
$$=\frac{x^{\frac{1}{3}}y^{\frac{2}{3}}(x^{\frac{9}{2}}-x^4y^{\frac{3}{5}}+x^{\frac{7}{2}}y^{\frac{6}{5}}-x^3y^{\frac{9}{5}}+x^{\frac{5}{2}}y^{\frac{12}{5}}-\cdots+x^{\frac{3}{2}}y^{\frac{18}{5}}-xy^{\frac{21}{5}}+x^{\frac{1}{2}}y^{\frac{24}{5}}-y^{\frac{27}{5}})}{(x^{\frac{1}{2}}+y^{\frac{3}{5}})(x^{\frac{9}{2}}-x^4y^{\frac{3}{5}}+x^{\frac{7}{2}}y^{\frac{6}{5}}-x^3y^{\frac{9}{5}}+x^{\frac{5}{2}}y^{\frac{12}{5}}-\cdots+x^{\frac{3}{2}}y^{\frac{18}{5}}-xy^{\frac{21}{5}}+x^{\frac{1}{2}}y^{\frac{24}{5}}-y^{\frac{27}{5}})}$$
$$=\frac{x^{\frac{1}{3}}y^{\frac{2}{3}}(x^{\frac{9}{2}}-x^4y^{\frac{3}{5}}+x^{\frac{7}{2}}y^{\frac{6}{5}}-x^3y^{\frac{9}{5}}+x^{\frac{5}{2}}y^{\frac{12}{5}}-\cdots+x^{\frac{3}{2}}y^{\frac{18}{5}}-xy^{\frac{21}{5}}+x^{\frac{1}{2}}y^{\frac{24}{5}}-y^{\frac{27}{5}})}{x^5-y^6}.$$

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9. $\sqrt{12-6\sqrt{3}}=\sqrt{12-2\sqrt{27}}=\sqrt{9}-\sqrt{3}=3-\sqrt{3}.$

10. $\sqrt{17+12\sqrt{2}}=\sqrt{17+2\sqrt{72}}=\sqrt{9}+\sqrt{8}=3+2\sqrt{2}.$

11. $\sqrt{12+4\sqrt{5}}=\sqrt{12+2\sqrt{20}}=\sqrt{10}+\sqrt{2}.$

12. $\sqrt{11+4\sqrt{7}}=\sqrt{11+2\sqrt{28}}=\sqrt{7}+\sqrt{4}=\sqrt{7}+2.$

13. $\sqrt{15-6\sqrt{6}}=\sqrt{15-2\sqrt{54}}=\sqrt{9}-\sqrt{6}=3-\sqrt{6}.$

14. $\sqrt{18+6\sqrt{5}}=\sqrt{18+2\sqrt{45}}=\sqrt{15}+\sqrt{3}.$

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17. $\sqrt{a^2+b+2a\sqrt{b}}=\sqrt{a^2+b+2\sqrt{a^2b}}=\sqrt{a^2}+\sqrt{b}=a+\sqrt{b}.$

18. $$\sqrt{2a-2\sqrt{a^2-b^2}}=\sqrt{(a+b)+(a-b)-2\sqrt{(a+b)(a-b)}}$$
$$=\sqrt{a+b}-\sqrt{a-b}.$$

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2. Let $\sqrt{x}+\sqrt{y}=\sqrt{25+10\sqrt{6}}.$ (1)

Then, $\sqrt{x}-\sqrt{y}=\sqrt{25-10\sqrt{6}}.$ (2)

Multiplying (1) by (2), $x-y=\sqrt{625-600}=\sqrt{25}=5.$ (3)

Squaring (1), $x+y+2\sqrt{xy}=25+10\sqrt{6}.$

$\therefore x+y=25.$ (4)

Solving (4) and (3), $x=15,\ y=10.$

$\therefore \sqrt{x}=\sqrt{15},\ \sqrt{y}=\sqrt{10}.$

Hence, $\sqrt{25+10\sqrt{6}}=\sqrt{15}+\sqrt{10}.$

3. Let $\sqrt{x}+\sqrt{y}=\sqrt{19+6\sqrt{2}}.$ (1)

Then, $\sqrt{x}-\sqrt{y}=\sqrt{19-6\sqrt{2}}.$ (2)

Multiplying (1) by (2), $x-y=\sqrt{361-72}=\sqrt{289}=17.$ (3)

Squaring (1), $x+y+2\sqrt{xy}=19+6\sqrt{2}.$

$\therefore x+y=19.$ (4)

Solving (4) and (3), $x=18,\ y=1.$

$\therefore \sqrt{x}=3\sqrt{2},\ \sqrt{y}=1.$

Hence, $\sqrt{19+6\sqrt{2}}=3\sqrt{2}+1.$

4. Let $\sqrt{x}+\sqrt{y}=\sqrt{45+30\sqrt{2}}.$ (1)

Then, $\sqrt{x}-\sqrt{y}=\sqrt{45-30\sqrt{2}}.$ (2)

Multiplying (1) by (2), $x-y=\sqrt{2025-1800}=\sqrt{225}=15.$ (3)

Squaring (1), $x+y+2\sqrt{xy}=45+30\sqrt{2}.$

$\therefore x+y=45.$ (4)

Solving (4) and (3), $x=30,\ y=15.$

$\therefore \sqrt{x}=\sqrt{30},\ \sqrt{y}=\sqrt{15}.$

Hence, $\sqrt{45+30\sqrt{2}}=\sqrt{30}+\sqrt{15}.$

5. Let $\sqrt{x}-\sqrt{y}=\sqrt{35-14\sqrt{6}}.$ (1)

Then, $\sqrt{x}+\sqrt{y}=\sqrt{35+14\sqrt{6}}.$ (2)

Multiplying (1) by (2), $x-y=\sqrt{1225-1176}=\sqrt{49}=7.$ (3)

Squaring (2), $x+y+2\sqrt{xy}=35+14\sqrt{6}.$

$\therefore x+y=35.$ (4)

Solving (4) and (3), $x=21,\ y=14.$

$\therefore \sqrt{x}=\sqrt{21},\ \sqrt{y}=\sqrt{14}.$

Hence, $\sqrt{35-14\sqrt{6}}=\sqrt{21}-\sqrt{14}.$

6. Let $\sqrt{x}+\sqrt{y}=\sqrt{11+6\sqrt{2}}.$ (1)

Then, $\sqrt{x}-\sqrt{y}=\sqrt{11-6\sqrt{2}}.$ (2)

Multiplying (1) by (2), $x-y=\sqrt{121-72}=\sqrt{49}=7.$ (3)

Squaring (1), $x+y+2\sqrt{xy}=11+6\sqrt{2}.$

$\therefore x+y=11.$ (4)

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Solving (4) and (3), $x = 9,\ y = 2.$

$\therefore\ \sqrt{x} = 3,\ \sqrt{y} = \sqrt{2}.$

Hence, $\sqrt{11 + 6\sqrt{2}} = 3 + \sqrt{2}.$

7. Let $\sqrt{x} - \sqrt{y} = \sqrt{24 - 8\sqrt{5}}.$ (1)

Then, $\sqrt{x} + \sqrt{y} = \sqrt{24 + 8\sqrt{5}}.$ (2)

Multiplying (1) by (2), $x - y = \sqrt{576 - 320} = \sqrt{256} = 16.$ (3)

Squaring (2), $x + y + 2\sqrt{xy} = 24 + 8\sqrt{5}.$

$\therefore\ x + y = 24.$ (4)

Solving (4) and (3), $x = 20,\ y = 4.$

$\therefore\ \sqrt{x} = 2\sqrt{5},\ \sqrt{y} = 2.$

Hence, $\sqrt{24 - 8\sqrt{5}} = 2\sqrt{5} - 2.$

8. Let $\sqrt{x} + \sqrt{y} = \sqrt{16 + 6\sqrt{7}}.$ (1)

Then, $\sqrt{x} - \sqrt{y} = \sqrt{16 - 6\sqrt{7}}.$ (2)

Multiplying (1) by (2), $x - y = \sqrt{256 - 252} = \sqrt{4} = 2.$ (3)

Squaring (1), $x + y + 2\sqrt{xy} = 16 + 6\sqrt{7}.$

$\therefore\ x + y = 16.$ (4)

Solving (4) and (3), $x = 9,\ y = 7.$

$\therefore\ \sqrt{x} = 3,\ \sqrt{y} = \sqrt{7}.$

Hence, $\sqrt{16 + 6\sqrt{7}} = 3 + \sqrt{7}.$

9. Let $\sqrt{x} - \sqrt{y} = \sqrt{21 - 8\sqrt{5}}.$ (1)

Then, $\sqrt{x} + \sqrt{y} = \sqrt{21 + 8\sqrt{5}}.$ (2)

Multiplying (1) by (2), $x - y = \sqrt{441 - 320} = \sqrt{121} = 11.$ (3)

Squaring (2), $x + y + 2\sqrt{xy} = 21 + 8\sqrt{5}.$

$\therefore\ x + y = 21.$ (4)

Solving (4) and (3), $x = 16,\ y = 5.$

$\therefore\ \sqrt{x} = 4,\ \sqrt{y} = \sqrt{5}.$

Hence, $\sqrt{21 - 8\sqrt{5}} = 4 - \sqrt{5}.$

10. Let $\sqrt{x} - \sqrt{y} = \sqrt{47 - 12\sqrt{11}}.$ (1)

Then, $\sqrt{x} + \sqrt{y} = \sqrt{47 + 12\sqrt{11}}.$ (2)

Multiplying (1) by (2), $x - y = \sqrt{2209 - 1584} = \sqrt{625} = 25.$ (3)

Squaring (2), $x + y + 2\sqrt{xy} = 47 + 12\sqrt{11}.$

$\therefore\ x + y = 47.$ (4)

Solving (4) and (3), $x = 36,\ y = 11.$

$\therefore\ \sqrt{x} = 6,\ \sqrt{y} = \sqrt{11}.$

Hence, $\sqrt{47 - 12\sqrt{11}} = 6 - \sqrt{11}.$

11. Let $\sqrt{x} + \sqrt{y} = \sqrt{56 + 32\sqrt{3}}.$ (1)

Then, $\sqrt{x} - \sqrt{y} = \sqrt{56 - 32\sqrt{3}}.$ (2)

Multiplying (1) by (2), $x - y = \sqrt{3136 - 3072} = \sqrt{64} = 8.$ (3)

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Squaring (1), $x + y + 2\sqrt{xy} = 56 + 32\sqrt{3}.$

$\therefore x + y = 56.$ (4)

Solving (4) and (3), $x = 32,\ y = 24.$

$\therefore \sqrt{x} = 4\sqrt{2},\ \sqrt{y} = 2\sqrt{6}.$

Hence, $\sqrt{56 + 32\sqrt{3}} = 4\sqrt{2} + 2\sqrt{6}.$

12. Let $\sqrt{x} - \sqrt{y} = \sqrt{35 - 12\sqrt{6}}.$ (1)

Then, $\sqrt{x} + \sqrt{y} = \sqrt{35 + 12\sqrt{6}}.$ (2)

Multiplying (1) by (2), $x - y = \sqrt{1225 - 864} = \sqrt{361} = 19.$ (3)

Squaring (2), $x + y + 2\sqrt{xy} = 35 + 12\sqrt{6}.$

$\therefore x + y = 35.$ (4)

Solving (4) and (3), $x = 27,\ y = 8.$

$\therefore \sqrt{x} = 3\sqrt{3},\ \sqrt{y} = 2\sqrt{2}.$

Hence, $\sqrt{35 - 12\sqrt{6}} = 3\sqrt{3} - 2\sqrt{2}.$

13. Let $\sqrt{x} - \sqrt{y} = \sqrt{56 - 12\sqrt{3}}.$ (1)

Then, $\sqrt{x} + \sqrt{y} = \sqrt{56 + 12\sqrt{3}}.$ (2)

Multiplying (1) by (2), $x - y = \sqrt{3136 - 432} = \sqrt{2704} = 52.$ (3)

Squaring (2), $x + y + 2\sqrt{xy} = 56 + 12\sqrt{3}.$

$\therefore x + y = 56.$ (4)

Solving (4) and (3), $x = 54,\ y = 2.$

$\therefore \sqrt{x} = 3\sqrt{6},\ \sqrt{y} = \sqrt{2}.$

Hence, $\sqrt{56 - 12\sqrt{3}} = 3\sqrt{6} - \sqrt{2}.$

14. Let $\sqrt{x} + \sqrt{y} = \sqrt{2 + \sqrt{3}}.$ (1)

Then, $\sqrt{x} - \sqrt{y} = \sqrt{2 - \sqrt{3}}.$ (2)

Multiplying (1) by (2), $x - y = \sqrt{4 - 3} = 1.$ (3)

Squaring (1), $x + y + 2\sqrt{xy} = 2 + \sqrt{3}.$

$\therefore x + y = 2.$ (4)

Solving (4) and (3), $x = \frac{3}{2},\ y = \frac{1}{2}.$

$\therefore \sqrt{x} = \frac{1}{2}\sqrt{6},\ \sqrt{y} = \frac{1}{2}\sqrt{2}.$

Hence, $\sqrt{2 + \sqrt{3}} = \frac{1}{2}\sqrt{6} + \frac{1}{2}\sqrt{2}.$

15. Let $\sqrt{x} + \sqrt{y} = \sqrt{6 + \sqrt{35}}.$ (1)

Then, $\sqrt{x} - \sqrt{y} = \sqrt{6 - \sqrt{35}}.$ (2)

Multiplying (1) by (2), $x - y = \sqrt{36 - 35} = 1.$ (3)

Squaring (1), $x + y + 2\sqrt{xy} = 6 + \sqrt{35}.$

$\therefore x + y = 6.$ (4)

Solving (4) and (3), $x = \frac{7}{2},\ y = \frac{5}{2}.$

$\therefore \sqrt{x} = \frac{1}{2}\sqrt{14},\ \sqrt{y} = \frac{1}{2}\sqrt{10}.$

Hence, $\sqrt{6 + \sqrt{35}} = \frac{1}{2}\sqrt{14} + \frac{1}{2}\sqrt{10}.$

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16. Let $\sqrt{x}+\sqrt{y}=\sqrt{1+\frac{2}{3}\sqrt{2}}.$ (1)

Then, $\sqrt{x}-\sqrt{y}=\sqrt{1-\frac{2}{3}\sqrt{2}}.$ (2)

Multiplying (1) by (2), $x-y=\sqrt{1-\frac{8}{9}}=\sqrt{\frac{1}{9}}=\frac{1}{3}.$ (3)

Squaring (1), $x+y+2\sqrt{xy}=1+\frac{2}{3}\sqrt{2}.$

$\therefore x+y=1.$ (4)

Solving (4) and (3), $x=\frac{2}{3},\ y=\frac{1}{3}.$

$\therefore \sqrt{x}=\frac{1}{3}\sqrt{6},\ \sqrt{y}=\frac{1}{3}\sqrt{3}.$

Hence, $\sqrt{1+\frac{2}{3}\sqrt{2}}=\frac{1}{3}\sqrt{6}+\frac{1}{3}\sqrt{3}.$

17. Let $\sqrt{x}+\sqrt{y}=\sqrt{2+\frac{4}{5}\sqrt{6}}.$ (1)

Then, $\sqrt{x}-\sqrt{y}=\sqrt{2-\frac{4}{5}\sqrt{6}}.$ (2)

Multiplying (1) by (2), $x-y=\sqrt{4-\frac{96}{25}}=\sqrt{\frac{4}{25}}=\frac{2}{5}.$ (3)

Squaring (1), $x+y+2\sqrt{xy}=2+\frac{4}{5}\sqrt{6}.$

$\therefore x+y=2.$ (4)

Solving (4) and (3), $x=\frac{6}{5},\ y=\frac{4}{5}.$

$\therefore \sqrt{x}=\frac{1}{5}\sqrt{30},\ \sqrt{y}=\frac{2}{5}\sqrt{5}.$

Hence, $\sqrt{2+\frac{4}{5}\sqrt{6}}=\frac{1}{5}\sqrt{30}+\frac{2}{5}\sqrt{5}.$

18. Let $\sqrt{x}+\sqrt{y}=\sqrt{30+20\sqrt{2}}.$ (1)

Then, $\sqrt{x}-\sqrt{y}=\sqrt{30-20\sqrt{2}}.$ (2)

Multiplying (1) by (2), $x-y=\sqrt{900-800}=\sqrt{100}=10.$ (3)

Squaring (1), $x+y+2\sqrt{xy}=30+20\sqrt{2}.$

$\therefore x+y=30.$ (4)

Solving (4) and (3), $x=20,\ y=10.$

$\therefore \sqrt{x}=2\sqrt{5},\ \sqrt{y}=\sqrt{10}.$

Hence, $\sqrt{30+20\sqrt{2}}=2\sqrt{5}+\sqrt{10}.$

19. Let $\sqrt{x}-\sqrt{y}=\sqrt{18-6\sqrt{5}}.$ (1)

Then, $\sqrt{x}+\sqrt{y}=\sqrt{18+6\sqrt{5}}.$ (2)

Multiplying (1) by (2), $x-y=\sqrt{324-180}=\sqrt{144}=12.$ (3)

Squaring (2), $x+y+2\sqrt{xy}=18+6\sqrt{5}.$

$\therefore x+y=18.$ (4)

Solving (4) and (3), $x=15,\ y=3.$

$\therefore \sqrt{x}=\sqrt{15},\ \sqrt{y}=\sqrt{3}.$

Hence, $\sqrt{18-6\sqrt{5}}=\sqrt{15}-\sqrt{3}.$

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12. $\sqrt{x+16}-\sqrt{x}=2.$

Transposing, $\sqrt{x+16}=\sqrt{x}+2.$

Squaring, $x+16=x+4\sqrt{x}+4.$

Canceling, etc., $\sqrt{x}=3.$

Squaring, $x=9.$

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13. $\sqrt{2x} - \sqrt{2x-15} = 1.$

Transposing, $\sqrt{2x} - 1 = \sqrt{2x-15}.$

Squaring, $2x - 2\sqrt{2x} + 1 = 2x - 15.$

Canceling, etc., $\sqrt{2x} = 8.$

Squaring, etc., $x = 32.$

14. $\sqrt{x^2+x+1} = 2 - x.$

Squaring, $x^2 + x + 1 = 4 - 4x + x^2.$

Canceling $x^2 = x^2$, $x + 1 = 4 - 4x.$

Solving, $x = \frac{3}{5}.$

15. $3\sqrt{x^2-9} = 3x - 3.$

Dividing by 3, $\sqrt{x^2-9} = x - 1.$

Squaring, $x^2 - 9 = x^2 - 2x + 1.$

Canceling $x^2 = x^2$, $-9 = -2x + 1.$

Solving, $x = 5.$

16. $\sqrt{x} + 2 = \sqrt{x+32}.$

Squaring, $x + 4\sqrt{x} + 4 = x + 32.$

Simplifying, $4\sqrt{x} = 28.$

Dividing by 4 and squaring, $x = 49.$

17. $5 - \sqrt{x+5} = \sqrt{x}.$

Transposing, $-\sqrt{x+5} = \sqrt{x} - 5.$

Squaring, § 276, Prin. 1, $x + 5 = x - 10\sqrt{x} + 25.$

Canceling, etc., $10\sqrt{x} = 20.$

Dividing by 10 and squaring, $x = 4.$

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18. $\sqrt{x^2-5x+7} + 2 = x.$

Transposing, $\sqrt{x^2-5x+7} = x - 2.$

Squaring, $x^2 - 5x + 7 = x^2 - 4x + 4.$

Canceling $x^2 = x^2$, $-5x + 7 = -4x + 4.$

Solving, $x = 3.$

19. $\sqrt{9x+8} + \sqrt{9x} - 4 = 0.$

Transposing, $\sqrt{9x} - 4 = -\sqrt{9x+8}.$

Squaring, § 276, Prin. 1, $9x - 8\sqrt{9x} + 16 = 9x + 8.$

Canceling $9x = 9x$, $-8\sqrt{9x} + 16 = 8.$

Dividing by -8, transposing, etc., $\sqrt{9x} = 1.$

Squaring, $9x = 1.$

$\therefore x = \frac{1}{9}.$

20. $4 - \sqrt{4-8x+9x^2} = 3x.$

Transposing, $-\sqrt{4-8x+9x^2} = 3x - 4.$

Squaring, § 276, Prin. 1, $4 - 8x + 9x^2 = 9x^2 - 24x + 16.$

Canceling $9x^2 = 9x^2$, $4 - 8x = -24x + 16.$

Solving, $x = \frac{3}{4}.$

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21. $\sqrt{2(1-x)(3-2x)}-1=2x.$

Transposing, $\sqrt{2(1-x)(3-2x)}=2x+1.$

Squaring, $2(1-x)(3-2x)=4x^2+4x+1.$

Expanding, $6-10x+4x^2=4x^2+4x+1.$

Canceling $4x^2=4x^2$, $6-10x=4x+1.$

Solving, $x=\frac{5}{14}.$

22. $\sqrt{2x-1}+\sqrt{2x+4}=5.$

Transposing, $\sqrt{2x-1}=5-\sqrt{2x+4}.$

Squaring, $2x-1=25-10\sqrt{2x+4}+(2x+4).$

Canceling $2x=2x$, transposing and uniting terms,

$$10\sqrt{2x+4}=30.$$

Dividing by 10 and squaring, $2x+4=9.$

Solving, $x=\frac{5}{2}.$

23. $\sqrt{3x-5}+\sqrt{3x+7}=6.$

Transposing, $\sqrt{3x+7}=6-\sqrt{3x-5}.$

Squaring, $3x+7=36-12\sqrt{3x-5}+(3x-5).$

Canceling $3x=3x$, transposing and uniting terms,

$$12\sqrt{3x-5}=24.$$

Dividing by 12 and squaring, $3x-5=4.$

Solving, $x=3.$

24. $\sqrt{16x+3}+\sqrt{16x+8}=5.$

Transposing, $\sqrt{16x+8}=5-\sqrt{16x+3}.$

Squaring, $16x+8=25-10\sqrt{16x+3}+(16x+3).$

Canceling $16x=16x$, transposing and uniting terms,

$$10\sqrt{16x+3}=20.$$

Dividing by 10 and squaring, $16x+3=4.$

Solving, $x=\frac{1}{16}.$

25. $\sqrt{1+x\sqrt{x^2+12}}=1+x.$

Squaring, $1+x\sqrt{x^2+12}=1+2x+x^2.$

Canceling $1=1$ and dividing by x, $\sqrt{x^2+12}=2+x.$

Squaring, $x^2+12=4+4x+x^2.$

Canceling $x^2=x^2$, $12=4+4x.$

Solving, $x=2.$

NOTE. — The given equation is satisfied also for $x=0.$

26. $\sqrt{7+3\sqrt{5x-16}}-4=0.$

Transposing, $\sqrt{7+3\sqrt{5x-16}}=4.$

Squaring, $7+3\sqrt{5x-16}=16.$

Transposing, uniting terms, and dividing by 3,

$$\sqrt{5x-16}=3.$$

Squaring, $5x-16=9.$

Solving, $x=5.$

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27. $2x - \sqrt{4x^2 - \sqrt{16x^2 - 7}} = 1.$

Transposing, $-\sqrt{4x^2 - \sqrt{16x^2 - 7}} = 1 - 2x.$

Squaring, $4x^2 - \sqrt{16x^2 - 7} = 1 - 4x + 4x^2.$

Canceling $4x^2 = 4x^2$, $-\sqrt{16x^2 - 7} = 1 - 4x.$

Squaring, § 276, Prin. 1, $16x^2 - 7 = 1 - 8x + 16x^2.$

Canceling $16x^2 = 16x^2$, $-7 = 1 - 8x.$

Solving, $x = 1.$

28. $2\sqrt{x} - \sqrt{4x - 22} - \sqrt{2} = 0.$

Dividing by $\sqrt{2}$, $\sqrt{2x} - \sqrt{2x - 11} - 1 = 0.$

Transposing, $-\sqrt{2x - 11} = 1 - \sqrt{2x}.$

Squaring, $2x - 11 = 1 - 2\sqrt{2x} + 2x.$

Canceling $2x = 2x$, etc., $-12 = -2\sqrt{2x}.$

Dividing by -2, $6 = \sqrt{2x}.$

Squaring, etc., $x = 18.$

29. $\sqrt{2(x+1)} + \sqrt{2x - 1} = \sqrt{8x + 1}.$

Squaring, $2x + 2 + 2\sqrt{4x^2 + 2x - 2} + 2x - 1 = 8x + 1.$

Transposing and uniting terms, $2\sqrt{4x^2 + 2x - 2} = 4x.$

Squaring, $16x^2 + 8x - 8 = 16x^2.$

Canceling $16x^2 = 16x^2$, $8x - 8 = 0.$

Solving, $x = 1.$

30. $\sqrt{3x + 7} + \sqrt{4x - 3} = \sqrt{4x + 4} + \sqrt{3x}.$

Squaring,

$$3x + 7 + 2\sqrt{12x^2 + 19x - 21} + 4x - 3 = 4x + 4 + 2\sqrt{12x^2 + 12x} + 3x.$$

Uniting terms, $2\sqrt{12x^2 + 19x - 21} = 2\sqrt{12x^2 + 12x}.$

Dividing by 2, and squaring,

$$12x^2 + 19x - 21 = 12x^2 + 12x.$$

Canceling $12x^2 = 12x^2$, $19x - 21 = 12x.$

Solving, $x = 3.$

31. $\sqrt{\sqrt{\sqrt{2x + 56}}} = 2.$ (1)

Squaring (1), $\sqrt{\sqrt{2x + 56}} = 4.$ (2)

Squaring (2), $\sqrt{2x + 56} = 16.$ (3)

Squaring (3), $2x + 56 = 256.$

Solving, $x = 100.$

32. $\sqrt{7 + \sqrt{1 + \sqrt{4 + \sqrt{1 + 2\sqrt{x}}}}} = 3.$

Squaring, $7 + \sqrt{1 + \sqrt{4 + \sqrt{1 + 2\sqrt{x}}}} = 9.$

Transposing, etc., $\sqrt{1 + \sqrt{4 + \sqrt{1 + 2\sqrt{x}}}} = 2.$

Squaring, $1 + \sqrt{4 + \sqrt{1 + 2\sqrt{x}}} = 4.$

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Transposing, etc., $\sqrt{4+\sqrt{1+2\sqrt{x}}}=3.$

Squaring, $4+\sqrt{1+2\sqrt{x}}=9.$

Transposing, etc., $\sqrt{1+2\sqrt{x}}=5.$

Squaring, $1+2\sqrt{x}=25.$

Transposing, etc., $2\sqrt{x}=24.$

Dividing by 2 and squaring, $x=144.$

33. $$\frac{5}{\sqrt{3x+2}}=\sqrt{3x+2}+\sqrt{3x-1}.$$

Clearing of fractions, $5=3x+2+\sqrt{9x^2+3x-2}.$

Transposing, $-\sqrt{9x^2+3x-2}=3x-3.$

Squaring, $9x^2+3x-2=9x^2-18x+9.$

Canceling $9x^2=9x^2$, $21x=11.$

Solving, $x=\frac{11}{21}.$

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35. $$\frac{\sqrt{2x}+9}{\sqrt{2x}-7}=\frac{\sqrt{2x}+20}{\sqrt{2x}-12}.$$

Reducing to mixed numbers, $1+\dfrac{16}{\sqrt{2x}-7}=1+\dfrac{32}{\sqrt{2x}-12}.$

Canceling $1=1$ and dividing by 16, $\dfrac{1}{\sqrt{2x}-7}=\dfrac{2}{\sqrt{2x}-12}.$

Clearing of fractions, etc., $\sqrt{2x}-2\sqrt{2x}=12-14.$

$$-\sqrt{2x}=-2.$$

Squaring, etc., $x=2.$

36. $$\frac{\sqrt{x}+18}{\sqrt{x}+2}=\frac{32}{\sqrt{x}+6}+1.$$

$$1+\frac{16}{\sqrt{x}+2}=\frac{32}{\sqrt{x}+6}+1.$$

Canceling $1=1$ and dividing by 16, $\dfrac{1}{\sqrt{x}+2}=\dfrac{2}{\sqrt{x}+6}.$

Clearing of fractions, etc., $\sqrt{x}-2\sqrt{x}=-6+4.$

$$-\sqrt{x}=-2.$$

Squaring, $x=4.$

37. $$\frac{\sqrt{s-1}}{\sqrt{s+5}}=\frac{\sqrt{s-3}}{\sqrt{s-1}}.$$

Clearing of fractions, $s-1=\sqrt{s^2+2s-15}.$

Squaring, $s^2-2s+1=s^2+2s-15.$

Canceling $s^2=s^2$, $-2s+1=2s-15.$

Solving, $s=4.$

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38. $$\frac{\sqrt{v}-6}{\sqrt{v}-1}=\frac{\sqrt{v}-8}{\sqrt{v}-5}.$$

Reducing to mixed numbers, $$1-\frac{5}{\sqrt{v}-1}=1-\frac{3}{\sqrt{v}-5}.$$

Canceling $1=1$, changing signs, $$\frac{5}{\sqrt{v}-1}=\frac{3}{\sqrt{v}-5}.$$

Clearing of fractions, etc., $$5\sqrt{v}-3\sqrt{v}=25-3.$$
$$2\sqrt{v}=22.$$

Dividing by 2 and squaring, $$v=121.$$

39. $$\frac{\sqrt{t-3}}{\sqrt{t+1}}=\frac{\sqrt{t-4}}{\sqrt{t-2}}.$$

Clearing of fractions, $$\sqrt{t^2-5t+6}=\sqrt{t^2-3t-4}.$$
Squaring, $$t^2-5t+6=t^2-3t-4.$$
Canceling $t^2=t^2$, $$-5t+6=-3t-4.$$
Solving, $$t=5.$$

40. $$\frac{\sqrt{2r}+6}{\sqrt{2r}+4}=\frac{\sqrt{2r}+2}{\sqrt{2r}+1}.$$

Reducing to mixed numbers, $$1+\frac{2}{\sqrt{2r}+4}=1+\frac{1}{\sqrt{2r}+1}.$$

Canceling $1=1$, $$\frac{2}{\sqrt{2r}+4}=\frac{1}{\sqrt{2r}+1}.$$

Clearing of fractions, $$2\sqrt{2r}+2=\sqrt{2r}+4.$$
$$\sqrt{2r}=2.$$

Squaring, etc., $$r=2.$$

41. $$\frac{\sqrt{11n}+\sqrt{2n+3}}{\sqrt{11n}-\sqrt{2n+3}}=\frac{8}{3}.$$

Clearing of fractions, $$3\sqrt{11n}+3\sqrt{2n+3}=8\sqrt{11n}-8\sqrt{2n+3}.$$
Transposing, etc., $$11\sqrt{2n+3}=5\sqrt{11n}.$$
Dividing by $\sqrt{11}$, $$\sqrt{11}\sqrt{2n+3}=5\sqrt{n}.$$
Squaring, $$22n+33=25n.$$
Solving, $$n=11.$$

42. $$\frac{2\sqrt{2x}+4}{2\sqrt{2x}-4}=\frac{3\sqrt{x+1}+9}{3\sqrt{x+1}-9}.$$

Reducing to lowest terms, $$\frac{\sqrt{2x}+2}{\sqrt{2x}-2}=\frac{\sqrt{x+1}+3}{\sqrt{x+1}-3}.$$

Reducing to mixed numbers, $$1+\frac{4}{\sqrt{2x}-2}=1+\frac{6}{\sqrt{x+1}-3}.$$

Canceling and dividing by 2, $$\frac{2}{\sqrt{2x}-2}=\frac{3}{\sqrt{x+1}-3}.$$

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Clearing of fractions, etc., $2\sqrt{x+1} - 3\sqrt{2x} = 6 - 6 = 0.$

Transposing and squaring, $18x = 4x + 4.$

Solving, $x = \frac{2}{7}.$

43. $$\frac{\sqrt{m+1} - \sqrt{m-1}}{\sqrt{m+1} + \sqrt{m-1}} = \frac{1}{2}.$$

Clearing of fractions, etc., $\sqrt{m+1} = 3\sqrt{m-1}.$

Squaring, $m + 1 = 9(m-1).$

Solving, $m = \frac{5}{4}.$

44. $$\frac{\sqrt{4z+3} + 2\sqrt{z-1}}{\sqrt{4z+3} - 2\sqrt{z-1}} = 5.$$

Clearing of fractions, etc., $12\sqrt{z-1} = 4\sqrt{4z+3}.$

Dividing by 4 and squaring, $9(z-1) = 4z + 3.$

Solving, $z = \frac{12}{5}.$

45. $$\frac{\sqrt{\sqrt{5x} - 9}}{\sqrt{\sqrt{5x} + 11}} = \frac{\sqrt{\sqrt{5x} - 21}}{\sqrt{\sqrt{5x} - 16}}.$$

Squaring, $$\frac{\sqrt{5x} - 9}{\sqrt{5x} + 11} = \frac{\sqrt{5x} - 21}{\sqrt{5x} - 16}.$$

Reducing to mixed numbers, $1 - \dfrac{20}{\sqrt{5x} + 11} = 1 - \dfrac{5}{\sqrt{5x} - 16}.$

Canceling $1 = 1$ and dividing by -5,
$$\frac{4}{\sqrt{5x} + 11} = \frac{1}{\sqrt{5x} - 16}.$$

Clearing of fractions, etc., $4\sqrt{5x} - \sqrt{5x} = 64 + 11.$

$\sqrt{5x} = 25.$

Squaring, etc., $x = 125.$

46. $$\frac{x-3}{\sqrt{x} - \sqrt{3}} = \frac{\sqrt{x} + \sqrt{3}}{2} + 2\sqrt{3}.$$

Reducing the first member, $\sqrt{x} + \sqrt{3} = \dfrac{\sqrt{x} + \sqrt{3}}{2} + 2\sqrt{3}.$

Clearing of fractions, $2\sqrt{x} + 2\sqrt{3} = \sqrt{x} + \sqrt{3} + 4\sqrt{3}.$

Transposing, etc., $\sqrt{x} = 3\sqrt{3}.$

Squaring, $x = 27.$

47. $$\sqrt{2x} - \sqrt{2x-7} = \frac{3}{\sqrt{2x-7}}.$$

Clearing of fractions, $\sqrt{4x^2 - 14x} - (2x - 7) = 3.$

Transposing, etc., $\sqrt{4x^2 - 14x} = 2x - 4.$

Squaring, $4x^2 - 14x = 4x^2 - 16x + 16.$

Canceling $4x^2 = 4x^2$, $-14x = -16x + 16.$

Solving, $x = 8.$

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48. $$\frac{\sqrt{x+a}+\sqrt{x-a}}{\sqrt{x+a}-\sqrt{x-a}}=2+\frac{\sqrt{x^2-a^2}}{a}.$$

Rationalizing denominator,

$$\frac{x+a+2\sqrt{x^2-a^2}+x-a}{x+a-x+a}=2+\frac{\sqrt{x^2-a^2}}{a}.$$

Uniting terms, $$\frac{x+\sqrt{x^2-a^2}}{a}=2+\frac{\sqrt{x^2-a^2}}{a}.$$

Canceling $\frac{\sqrt{x^2-a^2}}{a}=\frac{\sqrt{x^2-a^2}}{a}$, $x=2a.$

49. $$\sqrt{x}+\sqrt{x-(a-b)^2}=a+b.$$

Transposing, $\sqrt{x-(a-b)^2}=a+b-\sqrt{x}.$

Squaring, etc., $x-a^2+2ab-b^2=a^2+2ab+b^2-2(a+b)\sqrt{x}+x.$

Canceling, etc., $(a+b)\sqrt{x}=a^2+b^2.$

Dividing by $a+b$ and squaring, $x=\left(\frac{a^2+b^2}{a+b}\right)^2.$

50. $$a\sqrt{x}-b\sqrt{x}=a^2+b^2-2ab.$$

Dividing by $a-b$, $\sqrt{x}=a-b.$

Squaring, $x=(a-b)^2.$

51. $$\sqrt{5ax-9a^2}+a=\sqrt{5ax}.$$

Transposing, $\sqrt{5ax-9a^2}=\sqrt{5ax}-a.$

Squaring, $5ax-9a^2=5ax-2a\sqrt{5ax}+a^2.$

Canceling, etc., $-10a^2=-2a\sqrt{5ax}.$

Dividing by $-2a$ and squaring, $25a^2=5ax.$

$\therefore x=5a.$

52. $$\sqrt{x+3a}=\frac{6a}{\sqrt{x+3a}}-\sqrt{x}.$$

Clearing of fractions, $x+3a=6a-\sqrt{x(x+3a)}.$

Transposing, etc., $x-3a=-\sqrt{x^2+3ax}.$

Squaring, $x^2-6ax+9a^2=x^2+3ax.$

Canceling $x^2=x^2$, $-6ax+9a^2=3ax.$

Solving, $x=a.$

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54. $$\sqrt{2x}+\sqrt{3x}+\sqrt{5x}=\sqrt{m}.$$

Factoring, $\sqrt{x}(\sqrt{2}+\sqrt{3}+\sqrt{5})=\sqrt{m}.$

Multiplying by $\sqrt{2}+\sqrt{3}-\sqrt{5}$,

$$\sqrt{x}(2+2\sqrt{6}+3-5)=\sqrt{m}(\sqrt{2}+\sqrt{3}-\sqrt{5}).$$

$$\sqrt{x}\cdot 2\sqrt{6}=\sqrt{m}(\sqrt{2}+\sqrt{3}-\sqrt{5}).$$

Squaring, $24x=m(\sqrt{2}+\sqrt{3}-\sqrt{5})^2.$

$$\therefore x=\frac{m(\sqrt{2}+\sqrt{3}-\sqrt{5})^2}{24}.$$

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55. $$\sqrt{2x}+\sqrt{3x}-\sqrt{5x}=\sqrt{c}.$$
Factoring, $$\sqrt{x}(\sqrt{2}+\sqrt{3}-\sqrt{5})=\sqrt{c}.$$
Multiplying by $\sqrt{2}+\sqrt{3}+\sqrt{5}$,
$$\sqrt{x}(2+2\sqrt{6}+3-5)=\sqrt{c}(\sqrt{2}+\sqrt{3}+\sqrt{5}).$$
$$\sqrt{x}\cdot 2\sqrt{6}=\sqrt{c}(\sqrt{2}+\sqrt{3}+\sqrt{5}).$$
Squaring, $$24x=c(\sqrt{2}+\sqrt{3}+\sqrt{5})^2.$$
$$\therefore x=\frac{c(\sqrt{2}+\sqrt{3}+\sqrt{5})^2}{24}.$$

56. $$\sqrt{x-a}+\sqrt{2(x-a)}=\sqrt{3x+a\sqrt{2}}.$$
Factoring, $$\sqrt{x-a}(1+\sqrt{2})=\sqrt{3x+a\sqrt{2}}.$$
Squaring, $$(x-a)(3+2\sqrt{2})=3x+a\sqrt{2}.$$
$$3x+2x\sqrt{2}-3a-2a\sqrt{2}=3x+a\sqrt{2}.$$
Transposing, etc., $$2x\sqrt{2}=3a+3a\sqrt{2}.$$
$$\therefore x=\frac{3a(1+\sqrt{2})}{2\sqrt{2}}=\frac{3a}{4}(\sqrt{2}+2).$$

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2. $$\sqrt{2x}+\sqrt{2x-3}=1.$$
Transposing, $$\sqrt{2x-3}=1-\sqrt{2x}.$$
Squaring, $$2x-3=1-2\sqrt{2x}+2x.$$
Canceling $2x=2x$, $$-3=1-2\sqrt{2x}.$$
Transposing, squaring, etc., $$x=2.$$
Verifying, $\sqrt{4}+\sqrt{4-3}=2+1\neq 1$. Hence, the equation is impossible.
But $x=2$ satisfies the equation $\sqrt{2x}-\sqrt{2x-3}=1$.

3. $$\sqrt{3x+7}+\sqrt{3x}=7.$$
Transposing, $$\sqrt{3x+7}=7-\sqrt{3x}.$$
Squaring, $$3x+7=49-14\sqrt{3x}+3x.$$
Canceling $3x=3x$, $$14\sqrt{3x}=42.$$
Dividing by 14 and squaring, $$3x=9.$$
$$x=3.$$
Verifying, $\sqrt{9+7}+\sqrt{9}=4+3=7$. Hence, the given equation is satisfied.

4. $$2\sqrt{x}+\sqrt{4x-11}=1.$$
Transposing, $$\sqrt{4x-11}=1-2\sqrt{x}.$$
Squaring, $$4x-11=1-4\sqrt{x}+4x.$$
Canceling $4x=4x$, $$4\sqrt{x}=12.$$
Dividing by 4 and squaring, $$x=9.$$
Verifying, $2\sqrt{9}+\sqrt{36-11}=6+5\neq 1$. Hence, the equation is impossible.
But $x=9$ satisfies the equation $2\sqrt{x}-\sqrt{4x-11}=1$.

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5. $\sqrt{4x+5} - 2\sqrt{x-1} = 9.$

Transposing, $-2\sqrt{x-1} = 9 - \sqrt{4x+5}.$

Squaring, $4x - 4 = 81 - 18\sqrt{4x+5} + 4x + 5.$

Canceling $4x = 4x$, uniting terms, etc.,

$$18\sqrt{4x+5} = 90.$$

Dividing by 18 and squaring, $4x + 5 = 25.$

Solving, $x = 5.$

Verifying, $\sqrt{4 \cdot 5 + 5} - 2\sqrt{5-1} = 5 - 4 \neq 9.$ Hence, the equation is impossible.

But $x = 5$ satisfies the equation $\sqrt{4x+5} + 2\sqrt{x-1} = 9.$

6. $\sqrt{4x} - \sqrt{x} = \sqrt{9x - 32}.$

Simplifying, $2\sqrt{x} - \sqrt{x} = \sqrt{9x-32}.$

$$\sqrt{x} = \sqrt{9x-32}.$$

Squaring, $x = 9x - 32.$

Solving, $x = 4.$

Verifying, $\sqrt{4 \cdot 4} - \sqrt{4} = \sqrt{36-32}$, or $2 = 2$. Hence, the equation is satisfied.

7. $\sqrt{5x-1} - 1 = \sqrt{5x+16}.$

Squaring, $5x - 1 - 2\sqrt{5x-1} + 1 = 5x + 16.$

Canceling $5x = 5x$, $-2\sqrt{5x-1} = 16.$

Dividing by 2, and squaring, $5x - 1 = 64.$

Solving, $x = 13.$

Verifying, $\sqrt{65-1} - 1 = 8 - 1 \neq 9.$ Hence, the equation is impossible.

But $x = 13$ satisfies the equation $\sqrt{5x-1} + 1 = \sqrt{5x+16}.$

8. $\sqrt{x+1} + \sqrt{x+2} - \sqrt{4x+5} = 0.$

Transposing, $\sqrt{x+1} + \sqrt{x+2} = \sqrt{4x+5}.$

Squaring, $x + 1 + 2\sqrt{x^2+3x+2} + x + 2 = 4x + 5.$

Uniting terms, $2\sqrt{x^2+3x+2} = 2x + 2.$

Dividing by 2, and squaring, $x^2 + 3x + 2 = x^2 + 2x + 1.$

Canceling $x^2 = x^2$ and solving, $x = -1.$

Verifying, $\sqrt{-1+1} + \sqrt{-1+2} - \sqrt{-4+5} = 0 + 1 - 1 = 0.$ Hence, the equation is satisfied.

9. $\sqrt{2(x^2+3x-5)} = (x+2)\sqrt{2}.$

Squaring, $2(x^2 + 3x - 5) = 2(x^2 + 4x + 4).$

Dividing by 2, and canceling $x^2 = x^2$, $3x - 5 = 4x + 4.$

$$\therefore x = -9.$$

Verifying, $\sqrt{2(81 - 27 - 5)} = 7 \cdot \sqrt{2} \neq -7\sqrt{2}.$ Hence, the equation is impossible.

But $x = -9$ satisfies the equation $\sqrt{2(x^2+3x-5)} = -(x+2)\sqrt{2}.$

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10. $$\frac{\sqrt{x-5}}{\sqrt{x-4}}+\frac{\sqrt{x+1}}{\sqrt{x+8}}=0.$$

Transposing, $$\frac{\sqrt{x-5}}{\sqrt{x-4}}=-\frac{\sqrt{x+1}}{\sqrt{x+8}}.$$

Squaring, $$\frac{x-5}{x-4}=\frac{x+1}{x+8}.$$

Clearing of fractions, $x^2+3x-40=x^2-3x-4.$

Canceling $x^2=x^2$, transposing, etc., $6x=36.$

$\therefore x=6.$

Verifying, $\dfrac{\sqrt{6-5}}{\sqrt{6-4}}+\dfrac{\sqrt{6+1}}{\sqrt{6+8}}=\dfrac{1}{\sqrt{2}}+\dfrac{1}{\sqrt{2}}\neq 0.$ Hence, the equation is impossible.

But $x=6$ satisfies the equation $\dfrac{\sqrt{x-5}}{\sqrt{x-4}}-\dfrac{\sqrt{x+1}}{\sqrt{x+8}}=0.$

11. $$\frac{\sqrt{19x}+\sqrt{2x+11}}{\sqrt{19x}-\sqrt{2x+11}}=2\tfrac{1}{6}.$$

Clearing of fractions, $6\sqrt{19x}+6\sqrt{2x+11}=13\sqrt{19x}-13\sqrt{2x+11}.$

Transposing, etc., $19\sqrt{2x+11}=7\sqrt{19x}.$

Dividing by $\sqrt{19}$, $\sqrt{19}\sqrt{2x+11}=7\sqrt{x}.$

Squaring, $19(2x+11)=49x.$

$38x+209=49x.$

Solving, $x=19.$

Verifying, $\dfrac{\sqrt{19\cdot 19}+\sqrt{38+11}}{\sqrt{19\cdot 19}-\sqrt{38+11}}=\dfrac{19+7}{19-7}=\dfrac{26}{12}=2\tfrac{1}{6}.$ Hence, the equation is satisfied.

IMAGINARY NUMBERS

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10. $\sqrt{-4}+\sqrt{-49}=2\sqrt{-1}+7\sqrt{-1}=9\sqrt{-1}.$

11. $\sqrt{-9}+\sqrt{-64}=3\sqrt{-1}+8\sqrt{-1}=11\sqrt{-1}.$

12. $2\sqrt{-4}+3\sqrt{-1}=2\cdot 2\sqrt{-1}+3\sqrt{-1}=7\sqrt{-1}.$

13. $\sqrt{-12}+4\sqrt{-3}=2\sqrt{-3}+4\sqrt{-3}=6\sqrt{-3}.$

14. $5\sqrt{-18}-\sqrt{-72}=5\cdot 3\sqrt{-2}-6\sqrt{-2}=9\sqrt{-2}.$

15. $3\sqrt{-20}-\sqrt{-80}=3\cdot 2\sqrt{-5}-4\sqrt{-5}=2\sqrt{-5}.$

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16. $\sqrt{-16a^2x^2}+\sqrt{-a^2x^2}-\sqrt{-9a^2x^2}$
$=4ax\sqrt{-1}+ax\sqrt{-1}-3ax\sqrt{-1}=2ax\sqrt{-1}.$

17. $(\sqrt{-a}+3\sqrt{-b})+(\sqrt{-a}-3\sqrt{-b})=2\sqrt{-a}.$

18. $(\sqrt{-9xy}-\sqrt{-xy})-(\sqrt{-4xy}+\sqrt{-xy})$
$=3\sqrt{-xy}-\sqrt{-xy}-2\sqrt{-xy}-\sqrt{-xy}=-\sqrt{-xy}.$

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19. $\sqrt{-x^2}+\sqrt{-4x^2}-\sqrt{-x^3}+3x\sqrt{-x}$
$$=x\sqrt{-1}+2x\sqrt{-1}-x\sqrt{-x}+3x\sqrt{-x}$$
$$=3x\sqrt{-1}+2x\sqrt{-x}.$$

20. $\sqrt{-16}-3\sqrt{-4}+\sqrt{-18}+\sqrt{-50}+\sqrt{-25}$
$$=4\sqrt{-1}-3\cdot 2\sqrt{-1}+3\sqrt{-2}+5\sqrt{-2}+5\sqrt{-1}$$
$$=3\sqrt{-1}+8\sqrt{-2}.$$

21. $\sqrt{-8}+a\sqrt{-2}-\sqrt{-98}-5\sqrt{-2a^2}$
$$=2\sqrt{-2}+a\sqrt{-2}-7\sqrt{-2}-5\cdot a\sqrt{-2}$$
$$=(-5-4a)\sqrt{-2}=-(4a+5)\sqrt{-2}.$$

22. $\sqrt{1-5}-3\sqrt{1-10}+2\sqrt{5-30}$
$$=2\sqrt{-1}-3\cdot 3\sqrt{-1}+2\cdot 5\sqrt{-1}=3\sqrt{-1}.$$

25. $3\sqrt{-5}\times 2\sqrt{-15}=3\sqrt{5}\sqrt{-1}\times 2\sqrt{15}\sqrt{-1}$
$$=6\sqrt{5}\sqrt{5}\sqrt{3}(-1)=-6\times 5\sqrt{3}=-30\sqrt{3}.$$

26. $4\sqrt{-27}\times\sqrt{-12}=4\sqrt{27}\sqrt{-1}\times\sqrt{12}\sqrt{-1}$
$$=4\times 3\sqrt{3}\times 2\sqrt{3}(-1)=-24\times 3=-72.$$

27. $2\sqrt{-8}\times 5\sqrt{-3}=2\times 2\sqrt{2}\sqrt{-1}\times 5\sqrt{3}\sqrt{-1}$
$$=20\sqrt{2}\sqrt{3}(-1)=-20\sqrt{6}.$$

28. $8\sqrt{-1}\times\sqrt{-b^3}=8\sqrt{-1}\times b\sqrt{b}\sqrt{-1}=8b\sqrt{b}(-1)$
$$=-8b\sqrt{b}.$$

29. $\sqrt{-125}\times\sqrt{-108}=5\sqrt{5}\sqrt{-1}\times 6\sqrt{3}\sqrt{-1}=30\sqrt{15}(-1)$
$$=-30\sqrt{15}.$$

30. $\sqrt{-100}\times\sqrt{-30}=10\sqrt{-1}\sqrt{30}\sqrt{-1}=-10\sqrt{30}.$

31. $(\sqrt{-6}+\sqrt{-3})(\sqrt{-6}-\sqrt{-3})=(\sqrt{-6})^2-(\sqrt{-3})^2$
$$=-6-(-3)=-6+3=-3.$$

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32. $(\sqrt{-ab}+\sqrt{-a})(\sqrt{-ab}-\sqrt{-a})=(\sqrt{-ab})^2-(\sqrt{-a})^2$
$$=-ab-(-a)=a-ab.$$

33. $(\sqrt{-xy}+\sqrt{-x})(\sqrt{-xy}+\sqrt{-x})$
$$=(\sqrt{-xy})^2+2\sqrt{-xy}\sqrt{-x}+(\sqrt{-x})^2$$
$$=-xy+2\sqrt{x^2y}(\sqrt{-1})^2+(-x)$$
$$=-xy-2x\sqrt{y}-x=-x(y+2\sqrt{y}+1).$$

34.
$$\begin{array}{rrcr}
\sqrt{-50}-\sqrt{-12}= & 5\sqrt{-2} & - & 2\sqrt{-3} \\
\sqrt{-8}-\sqrt{-75}= & 2\sqrt{-2} & - & 5\sqrt{-3} \\
\hline
 & -20 & + & 4\sqrt{6} \\
 & -30 & + & 25\sqrt{6} \\
\hline
 & -50 & + & 29\sqrt{6}
\end{array}$$

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35.
$$\begin{array}{llll}
\sqrt{-a}+\sqrt{-b}+\sqrt{-c} & & & \\
\underline{\sqrt{-a}+\sqrt{-b}-\sqrt{-c}} & & & \\
-a-\sqrt{ab}-\sqrt{ac} & & & \\
\quad -\sqrt{ab} & -b-\sqrt{bc} & & \\
\quad\quad +\sqrt{ac} & +\sqrt{bc} & +c & \\
\hline
-a-2\sqrt{ab} & -b & +c &
\end{array}$$

39. $\sqrt{-18} \div \sqrt{-3} = \sqrt{6}\sqrt{-3} \div \sqrt{-3} = \sqrt{6}.$

40. $\dfrac{\sqrt{27}}{\sqrt{-3}} = \dfrac{3\sqrt{3}}{\sqrt{3}\sqrt{-1}} = \dfrac{3}{\sqrt{-1}} = \dfrac{3\sqrt{-1}}{-1} = -3\sqrt{-1}.$

41. $\dfrac{14\sqrt{-5}}{2\sqrt{-7}} = \dfrac{7\sqrt{5}\sqrt{-1}}{\sqrt{7}\sqrt{-1}} = \sqrt{7}\sqrt{5} = \sqrt{35}.$

42. $\dfrac{-\sqrt{-a^2}}{\sqrt{-b^2}} = \dfrac{-a\sqrt{-1}}{b\sqrt{-1}} = -\dfrac{a}{b}.$ **43.** $\dfrac{1}{\sqrt{-1}} = \dfrac{1\sqrt{-1}}{-1} = -\sqrt{-1}.$

44. $\dfrac{\sqrt{8}+3\sqrt{14}}{\sqrt{-2}} = \dfrac{(2+3\sqrt{7})\sqrt{2}\sqrt{-1}}{\sqrt{2}(-1)} = -(2+3\sqrt{7})\sqrt{-1}.$

45. $\dfrac{\sqrt{12}+\sqrt{3}}{\sqrt{-3}} = \dfrac{(2+1)\sqrt{3}\sqrt{-1}}{\sqrt{3}(-1)} = -3\sqrt{-1}.$

46. $\dfrac{-2}{\sqrt{-1}} = \dfrac{-2\sqrt{-1}}{-1} = 2\sqrt{-1}.$

47. $\dfrac{(\sqrt{-1})^5}{\frac{1}{3}\sqrt{-1}} = 3(\sqrt{-1})^4 = 3(+1) = 3.$

48. $\dfrac{(\sqrt{-1})^3}{(\sqrt{-1})^{15}} = \dfrac{1}{(\sqrt{-1})^{12}} = \dfrac{1}{\{(\sqrt{-1})^4\}^3} = \dfrac{1}{1^3} = 1.$

49. $\dfrac{\sqrt{4ab}}{\sqrt{-bc}} = \dfrac{2\sqrt{ab}\sqrt{c}\sqrt{-1}}{\sqrt{b}\cdot c(-1)} = -\dfrac{2\sqrt{-ac}}{c}.$

50. $\dfrac{(\sqrt{-1})^{14}}{-\frac{1}{2}\sqrt{-1}} = -2(\sqrt{-1})^{13} = -2(\sqrt{-1})^{12}\sqrt{-1} = -2\sqrt{-1}.$

51. $(\sqrt{-1})^{10} \div (\sqrt{-1})^{-2} = (\sqrt{-1})^{12} = \{(\sqrt{-1})^4\}^3 = (+1)^3 = 1.$

52. $\dfrac{\sqrt{-a^2}+b\sqrt{-1}}{\sqrt{-ab}} = \dfrac{(a+b)\sqrt{-1}}{\sqrt{ab}\sqrt{-1}} = \dfrac{a+b}{ab}\sqrt{ab}.$

53. $\dfrac{\sqrt{-4}}{\sqrt{-2}\cdot\sqrt{-2}\cdot\sqrt{-1}} = \dfrac{2\sqrt{-1}}{-2\sqrt{-1}} = -1.$

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REVIEW

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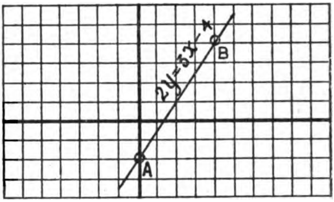

6. Solving for y, $y = \frac{3}{2}x - 2$.

When $x = 0$, $y = -2$;
when $x = 4$, $y = 4$.

Locate $A = (0, -2)$, $B = (4, 4)$.

A straight line drawn through A and B is the graph of $2y = 3x - 4$.

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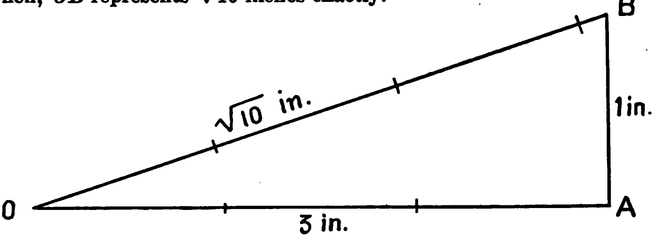
16. Proceeding as in § 324, since $10 = 3^2 + 1^2$, make the sides of the right angle 3 inches and 1 inch, respectively.

Then, OB represents $\sqrt{10}$ inches exactly.

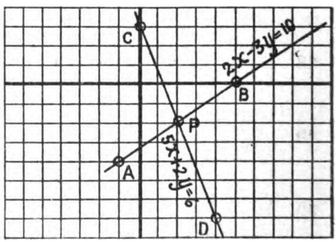


27. Solving $2x - 3y = 10$ for y, $y = \frac{1}{3}(2x - 10)$.

When $x = -1$, $y = -4$;
when $x = 5$, $y = 0$.

Locate $A = (-1, -4)$, $B = (5, 0)$.

A straight line drawn through A and B is the graph of $2x - 3y = 10$.



Solving $5x + 2y = 6$ for y,

$$y = 3 - \frac{5}{2}x.$$

When $x = 0$, $y = 3$;
when $x = 4$, $y = -7$.

Locate $C = (0, 3)$, $D = (4, -7)$.

A straight line drawn through C and D is the graph of $5x + 2y = 6$.

These two graphs intersect at $P = (2, -2)$. Hence, $x = 2$ and $y = -2$.

30. $2\sqrt{-4} + 3\sqrt{-9} = 4\sqrt{-1} + 9\sqrt{-1} = 13\sqrt{-1}$.

$2\sqrt{-4} \times 3\sqrt{-9} = 4\sqrt{-1} \times 9\sqrt{-1} = 36(-1) = -36$.

31. $\sqrt{-81} - \sqrt{-9} = 9\sqrt{-1} - 3\sqrt{-1} = 6\sqrt{-1}$.

$\sqrt{-81} \div \sqrt{-9} = 9\sqrt{-1} \div 3\sqrt{-1} = 3$.

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1. $\dfrac{6x^3-7x^2-5x}{9x^3-25x}=\dfrac{x(2x+1)(3x-5)}{x(3x+5)(3x-5)}=\dfrac{2x+1}{3x+5}.$

2. $\dfrac{8x^2+18x-5}{12x^2+5x-2}=\dfrac{(4x-1)(2x+5)}{(4x-1)(3x+2)}=\dfrac{2x+5}{3x+2}.$

3. $\dfrac{a^2x^2-a\sqrt{x}+x}{\sqrt{x}}=a^2x\sqrt{x}-a+\sqrt{x}.$

4. $\dfrac{a^2-2a\sqrt{b}+b}{a-\sqrt{b}}=\dfrac{(a-\sqrt{b})^2}{a-\sqrt{b}}=a-\sqrt{b}.$

5. $\dfrac{\sqrt{2}-\sqrt{3}-\sqrt{5}}{\sqrt{2}+\sqrt{3}+\sqrt{5}}=\dfrac{(\sqrt{2}-\sqrt{5})-\sqrt{3}}{(\sqrt{2}+\sqrt{3})+\sqrt{5}}\times\dfrac{(\sqrt{2}-\sqrt{5})+\sqrt{3}}{(\sqrt{2}+\sqrt{3})-\sqrt{5}}$

$$=\frac{2-2\sqrt{10}+5-3}{2+2\sqrt{6}+3-5}=\frac{4-2\sqrt{10}}{2\sqrt{6}}$$

$$=\frac{2-\sqrt{10}}{\sqrt{6}}=\frac{2\sqrt{6}-2\sqrt{15}}{6}=\frac{\sqrt{6}-\sqrt{15}}{3}.$$

6. $\dfrac{2-\sqrt{5}}{2+\sqrt{5}}+\dfrac{2\sqrt{3}}{\sqrt{243}}=\dfrac{(2-\sqrt{5})(2-\sqrt{5})}{(2+\sqrt{5})(2-\sqrt{5})}+\dfrac{2\sqrt{3}}{9\sqrt{3}}$

$$=\frac{4-4\sqrt{5}+5}{4-5}+\frac{2}{9}=-9+4\sqrt{5}+\frac{2}{9}$$

$$=4\sqrt{5}-8\tfrac{7}{9}.$$

7. $\dfrac{x-y}{x+y}-\dfrac{y+x}{y-x}-\dfrac{4x^2y^2}{x^4-y^4}=\dfrac{x-y}{x+y}+\dfrac{x+y}{x-y}-\dfrac{4x^2y^2}{x^4-y^4}$

$$=\frac{2(x^2+y^2)}{x^2-y^2}-\frac{4x^2y^2}{x^4-y^4}$$

$$=\frac{2x^4+4x^2y^2+2y^4-4x^2y^2}{x^4-y^4}=\frac{2(x^4+y^4)}{x^4-y^4}.$$

8. $\dfrac{\sqrt{2}-\sqrt{3}}{\sqrt{2}+\sqrt{3}}-\dfrac{\sqrt{2}+\sqrt{3}}{\sqrt{2}-\sqrt{3}}=\dfrac{(\sqrt{2}-\sqrt{3})^2-(\sqrt{2}+\sqrt{3})^2}{(\sqrt{2}+\sqrt{3})(\sqrt{2}-\sqrt{3})}$

$$=\frac{2-2\sqrt{6}+3-2-2\sqrt{6}-3}{2-3}=4\sqrt{6}.$$

9. $\dfrac{x+\sqrt{xy}+y}{\sqrt{x}+\sqrt{y}}+\dfrac{x\sqrt{x}+y\sqrt{y}}{x+y}=\dfrac{(x+\sqrt{xy}+y)(\sqrt{x}-\sqrt{y})}{(\sqrt{x}+\sqrt{y})(\sqrt{x}-\sqrt{y})}+\dfrac{x\sqrt{x}+y\sqrt{y}}{x+y}$

$$=\frac{x\sqrt{x}-y\sqrt{y}}{x-y}+\frac{x\sqrt{x}+y\sqrt{y}}{x+y}$$

$$=\frac{x^2\sqrt{x}-xy\sqrt{y}+xy\sqrt{x}-y^2\sqrt{y}+x^2\sqrt{x}+xy\sqrt{y}-xy\sqrt{x}-y^2\sqrt{y}}{x^2-y^2}$$

$$=\frac{2(x^2\sqrt{x}-y^2\sqrt{y})}{x^2-y^2}.$$

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10. $$\frac{1}{\sqrt{a}+\sqrt{b}}-1+\frac{1}{\sqrt{a}-\sqrt{b}}=\frac{\sqrt{a}-\sqrt{b}-(a-b)+\sqrt{a}+\sqrt{b}}{(\sqrt{a}+\sqrt{b})(\sqrt{a}-\sqrt{b})}$$
$$=\frac{2\sqrt{a}-a+b}{a-b}.$$

11. $$\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}=\frac{(\sqrt{1+x^2}-\sqrt{1-x^2})(\sqrt{1+x^2}-\sqrt{1-x^2})}{(\sqrt{1+x^2}+\sqrt{1-x^2})(\sqrt{1+x^2}-\sqrt{1-x^2})}$$
$$=\frac{1+x^2-2\sqrt{1-x^4}+1-x^2}{1+x^2-(1-x^2)}=\frac{2-2\sqrt{1-x^4}}{2x^2}=\frac{1-\sqrt{1-x^4}}{x^2}.$$

12. $$\frac{1}{1-\sqrt{2x}}+\frac{1}{1+\sqrt{2x}}+\frac{1}{1-2x}=\frac{1+\sqrt{2x}+1-\sqrt{2x}+1}{1-2x}=\frac{3}{1-2x}.$$

13. $$\frac{x+\sqrt{x^2-a^2}}{x-\sqrt{x^2-a^2}}-\frac{x-\sqrt{x^2-a^2}}{x+\sqrt{x^2-a^2}}=\frac{(x+\sqrt{x^2-a^2})^2-(x-\sqrt{x^2-a^2})^2}{x^2-(x^2-a^2)}$$
$$=\frac{4x\sqrt{x^2-a^2}}{a^2}.$$

14. $$\frac{\sqrt{a+1}+\sqrt{a-1}}{\sqrt{a+1}-\sqrt{a-1}}-\frac{\sqrt{a+1}-\sqrt{a-1}}{\sqrt{a+1}+\sqrt{a-1}}$$
$$=\frac{(\sqrt{a+1}+\sqrt{a-1})^2-(\sqrt{a+1}-\sqrt{a-1})^2}{a+1-(a-1)}=\frac{4\sqrt{a^2-1}}{2}=2\sqrt{a^2-1}.$$

15. $$\frac{a^2-b}{a^2-2a\sqrt{b}+b}\times\frac{a^2-4a\sqrt{b}+4b}{a^2+2a\sqrt{b}+b}$$
$$=\frac{(a+\sqrt{b})(a-\sqrt{b})}{(a-\sqrt{b})(a-\sqrt{b})}\times\frac{(a-2\sqrt{b})(a-2\sqrt{b})}{(a+\sqrt{b})(a+\sqrt{b})}=\frac{(a-2\sqrt{b})^2}{a^2-b}.$$

16. $$\left(1-\frac{a}{b}\right)\div\left(1+\sqrt{\frac{a}{b}}\right)=\left[1^2-\left(\sqrt{\frac{a}{b}}\right)^2\right]\div\left[1+\sqrt{\frac{a}{b}}\right]=1-\sqrt{\frac{a}{b}}$$
$$=1-\frac{\sqrt{ab}}{b}.$$

17. $$\frac{\dfrac{1+a+a^2}{1+\sqrt{a}+a}}{\dfrac{1-\sqrt{a}+a}{1-\sqrt{a}}}=\frac{1+a+a^2}{1+\sqrt{a}+a}\times\frac{1-\sqrt{a}}{1-\sqrt{a}+a}$$
$$=\frac{(1+\sqrt{a}+a)(1-\sqrt{a}+a)}{1+\sqrt{a}+a}\times\frac{1-\sqrt{a}}{1-\sqrt{a}+a}=1-\sqrt{a}.$$

18. $$1\div\left(\sqrt{\frac{a}{b}}-\sqrt{\frac{b}{a}}\right)+\frac{\sqrt{ab}}{a+b}=1\div\frac{a-b}{\sqrt{ab}}+\frac{\sqrt{ab}}{a+b}$$
$$=\frac{\sqrt{ab}}{a-b}+\frac{\sqrt{ab}}{a+b}$$
$$=\frac{(a+b+a-b)\sqrt{ab}}{a^2-b^2}=\frac{2a\sqrt{ab}}{a^2-b^2}.$$

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19. $\dfrac{\left(\dfrac{a}{\sqrt{x}}+\dfrac{\sqrt{x}}{a}\right)\left(\dfrac{a}{\sqrt{x}}-\dfrac{\sqrt{x}}{a}\right)}{\left(\dfrac{a}{\sqrt{x}}-\dfrac{\sqrt{x}}{a}\right)\left(\dfrac{a}{\sqrt{x}}-\dfrac{\sqrt{x}}{a}\right)}=\dfrac{\dfrac{a}{\sqrt{x}}+\dfrac{\sqrt{x}}{a}}{\dfrac{a}{\sqrt{x}}-\dfrac{\sqrt{x}}{a}}=\dfrac{\dfrac{a^2+x}{a\sqrt{x}}}{\dfrac{a^2-x}{a\sqrt{x}}}=\dfrac{a^2+x}{a^2-x}.$

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20. $(a^3-b^2)^3=(a^3)^3-3\,(a^3)^2(b^2)+3\,(a^3)(b^2)^2-(b^2)^3$
$= a^9-3\,a^6b^2+3\,a^3b^4-b^6.$

21. $(2\,a-3\,b)^4=(2\,a)^4-4(2\,a)^3(3\,b)+6(2\,a)^2(3\,b)^2-4(2\,a)(3\,b)^3+(3\,b)^4$
$=16\,a^4-96\,a^3b+216\,a^2b^2-216\,ab^3+81\,b^4.$

22. $\left(\dfrac{a}{3}-\dfrac{b}{2}\right)^3=\left(\dfrac{a}{3}\right)^3-3\left(\dfrac{a}{3}\right)^2\left(\dfrac{b}{2}\right)+3\left(\dfrac{a}{3}\right)\left(\dfrac{b}{2}\right)^2-\left(\dfrac{b}{2}\right)^3$
$=\dfrac{a^3}{27}-\dfrac{a^2b}{6}+\dfrac{ab^2}{4}-\dfrac{b^3}{8}.$

23. $\left(ax+\dfrac{1}{a}\right)^5=(ax)^5+5(ax)^4\left(\dfrac{1}{a}\right)+10(ax)^3\left(\dfrac{1}{a}\right)^2+10(ax)^2\left(\dfrac{1}{a}\right)^3+5(ax)\left(\dfrac{1}{a}\right)^4$
$+\left(\dfrac{1}{a}\right)^5=a^5x^5+5\,a^3x^4+10\,ax^3+\dfrac{10\,x^2}{a}+\dfrac{5\,x}{a^3}+\dfrac{1}{a^5}.$

24. $(a^{-2}+a^{-1})^2=(a^{-2})^2+2\,(a^{-2})(a^{-1})+(a^{-1})^2=a^{-4}+2\,a^{-3}+a^{-2}.$

25. $(a^{-1}+b)^4=(a^{-1})^4+4\,(a^{-1})^3b+6\,(a^{-1})^2b^2+4\,(a^{-1})b^3+b^4$
$=a^{-4}+4\,a^{-3}b+6\,a^{-2}b^2+4\,a^{-1}b^3+b^4.$

26. $(a^{\frac{1}{2}}-b^{\frac{1}{2}})^6=(a^{\frac{1}{2}})^6-6\,(a^{\frac{1}{2}})^5\,(b^{\frac{1}{2}})+15\,(a^{\frac{1}{2}})^4\,(b^{\frac{1}{2}})^2-20\,(a^{\frac{1}{2}})^3\,(b^{\frac{1}{2}})^3$
$+15\,(a^{\frac{1}{2}})^2\,(b^{\frac{1}{2}})^4-6\,(a^{\frac{1}{2}})(b^{\frac{1}{2}})^5+(b^{\frac{1}{2}})^6$
$=a^3-6\,a^{\frac{5}{2}}b^{\frac{1}{2}}+15\,a^2b-20\,a^{\frac{3}{2}}b^{\frac{3}{2}}+15\,ab^2-6\,a^{\frac{1}{2}}b^{\frac{5}{2}}+b^3.$

27. $(a^{\frac{1}{2}}-b^{-\frac{1}{2}})^4=(a^{\frac{1}{2}})^4-4(a^{\frac{1}{2}})^3(b^{-\frac{1}{2}})+6(a^{\frac{1}{2}})^2(b^{-\frac{1}{2}})^2-4(a^{\frac{1}{2}})(b^{-\frac{1}{2}})^3+(b^{-\frac{1}{2}})^4$
$=a^2-4\,a^{\frac{3}{2}}b^{-\frac{1}{2}}+6\,ab^{-1}-4\,a^{\frac{1}{2}}b^{-\frac{3}{2}}+b^{-2}.$

28. $(a^{-\frac{1}{3}}-b^{-\frac{1}{3}})^6=(a^{-\frac{1}{3}})^6-6(a^{-\frac{1}{3}})^5(b^{-\frac{1}{3}})+15(a^{-\frac{1}{3}})^4(b^{-\frac{1}{3}})^2-20(a^{-\frac{1}{3}})^3(b^{-\frac{1}{3}})^3$
$+15\,(a^{-\frac{1}{3}})^2(b^{-\frac{1}{3}})^4-6\,(a^{-\frac{1}{3}})(b^{-\frac{1}{3}})^5+(b^{-\frac{1}{3}})^6$
$=a^{-2}-6\,a^{-\frac{5}{3}}b^{-\frac{1}{3}}+15\,a^{-\frac{4}{3}}b^{-\frac{2}{3}}-20\,a^{-1}b^{-1}+15\,a^{-\frac{2}{3}}b^{-\frac{4}{3}}-6\,a^{-\frac{1}{3}}b^{-\frac{5}{3}}+b^{-2}.$

29. $(a^{\frac{1}{3}}+b^{\frac{1}{2}})^6=(a^{\frac{1}{3}})^6+6(a^{\frac{1}{3}})^5(b^{\frac{1}{2}})+15(a^{\frac{1}{3}})^4(b^{\frac{1}{2}})^2+20(a^{\frac{1}{3}})^3(b^{\frac{1}{2}})^3$
$+15(a^{\frac{1}{3}})^2(b^{\frac{1}{2}})^4+6(a^{\frac{1}{3}})(b^{\frac{1}{2}})^5+(b^{\frac{1}{2}})^6$
$=a^2+6\,a^{\frac{5}{3}}b^{\frac{1}{2}}+15\,a^{\frac{4}{3}}b+20\,ab^{\frac{3}{2}}+15\,a^{\frac{2}{3}}b^2+6\,a^{\frac{1}{3}}b^{\frac{5}{2}}+b^3.$

30. $(a-\sqrt{b})^4=a^4-4\,a^3\sqrt{b}+6\,a^2(\sqrt{b})^2-4\,a(\sqrt{b})^3+(\sqrt{b})^4$
$=a^4-4\,a^3\sqrt{b}+6\,a^2b-4\,ab\sqrt{b}+b^2.$

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31. $(\sqrt{x}+\sqrt{y})^6$

$$\begin{aligned}&=(\sqrt{x})^6+6(\sqrt{x})^5\sqrt{y}+15(\sqrt{x})^4(\sqrt{y})^2+20(\sqrt{x})^3(\sqrt{y})^3\\&\quad+15(\sqrt{x})^2(\sqrt{y})^4+6\sqrt{x}(\sqrt{y})^5+(\sqrt{y})^6\\&=x^3+6x^2\sqrt{xy}+15x^2y+20xy\sqrt{xy}+15xy^2+6y^2\sqrt{xy}+y^3.\end{aligned}$$

32. $(\sqrt{2}-\sqrt{3})^4$

$$\begin{aligned}&=(\sqrt{2})^4-4(\sqrt{2})^3\sqrt{3}+6(\sqrt{2})^2(\sqrt{3})^2-4\sqrt{2}(\sqrt{3})^3+(\sqrt{3})^4\\&=2^2-4\cdot 2\sqrt{2\cdot 3}+6\cdot 2\cdot 3-4\cdot 3\sqrt{2\cdot 3}+3^2\\&=4-8\sqrt{6}+36-12\sqrt{6}+9=49-20\sqrt{6}.\end{aligned}$$

33. $(\sqrt{5}-2)^6$

$$\begin{aligned}&=(\sqrt{5})^6-6(\sqrt{5})^5\cdot 2+15(\sqrt{5})^4\cdot 2^2-20(\sqrt{5})^3\cdot 2^3\\&\quad+15(\sqrt{5})^2\cdot 2^4-6\sqrt{5}\cdot 2^5+2^6\\&=5^3-6\cdot 5^2\sqrt{5}\cdot 2+15\cdot 5^2\cdot 2^2-20\cdot 5\sqrt{5}\cdot 2^3+15\cdot 5\cdot 2^4-6\sqrt{5}\cdot 2^5+2^6\\&=125-300\sqrt{5}+1500-800\sqrt{5}+1200-192\sqrt{5}+64=2889-1292\sqrt{5}.\end{aligned}$$

34. $(\sqrt[3]{4}-\sqrt[3]{2})^3=(\sqrt[3]{4})^3-3(\sqrt[3]{4})^2(\sqrt[3]{2})+3(\sqrt[3]{4})(\sqrt[3]{2})^2-(\sqrt[3]{2})^3$

$$=4-3\cdot 2\sqrt[3]{4}+3\cdot 2\sqrt[3]{2}-2=2-6\sqrt[3]{4}+6\sqrt[3]{2}.$$

35. $(\sqrt{2}-\sqrt[3]{2})^6$

$$\begin{aligned}&=(\sqrt{2})^6-6(\sqrt{2})^5(\sqrt[3]{2})+15(\sqrt{2})^4(\sqrt[3]{2})^2-20(\sqrt{2})^3(\sqrt[3]{2})^3\\&\quad+15(\sqrt{2})^2(\sqrt[3]{2})^4-6\sqrt{2}(\sqrt[3]{2})^5+(\sqrt[3]{2})^6\\&=2^3-6\cdot 2^2\sqrt{2}\cdot\sqrt[3]{2}+15\cdot 2^2\cdot\sqrt[3]{4}-20\cdot 2\sqrt{2}\cdot 2\\&\quad+15\cdot 2\cdot 2\sqrt[3]{2}-6\sqrt{2}\cdot 2\sqrt[3]{4}+2^2\\&=8-24\sqrt[6]{32}+60\sqrt[3]{4}-80\sqrt{2}+60\sqrt[3]{2}-24\sqrt[6]{2}+4\\&=12-24\sqrt[6]{32}+60\sqrt[3]{4}-80\sqrt{2}+60\sqrt[3]{2}-24\sqrt[6]{2}.\end{aligned}$$

36.

$$\begin{array}{l|l}
 & \frac{9x^4}{4}+3x^3-x^2-\frac{4x}{3}+\frac{4}{9}\;\Big|\;\underline{\frac{3x^2}{2}+x-\frac{2}{3}}\\
 & \frac{9x^4}{4}\\ \hline
\frac{3x^2}{2} & 3x^3\\
\frac{3x^2}{2}+x & 3x^3+x^2\\ \hline
3x^2+2x & -2x^2\\
3x^2+2x-\frac{2}{3} & -2x^2-\frac{4x}{3}+\frac{4}{9}\\ \hline
\end{array}$$

37.

$$\begin{array}{l|l}
 & \frac{x^2}{4}-2xy+\frac{xz}{4}+4y^2-yz+\frac{z^2}{16}\;\Big|\;\underline{\frac{x}{2}-2y+\frac{z}{4}}\\
 & \frac{x^2}{4}\\ \hline
x & -2xy\\
x-2y & -2xy+4y^2\\ \hline
x-4y & \frac{xz}{4}-yz+\frac{z^2}{16}\\
x-4y+\frac{z}{4} & \frac{xz}{4}-yz+\frac{z^2}{16}\\ \hline
\end{array}$$

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38.

$$\begin{array}{l|l}
 & a^2 + 12a^{\frac32}b^{\frac12} + 54ab + 108a^{\frac12}b^{\frac32} + 81b^2 \;\big|\; \underline{a + 6a^{\frac12}b^{\frac12} + 9b} \\
 & a^2 \\ \hline
2a & 12a^{\frac32}b^{\frac12} \\
2a + 6a^{\frac12}b^{\frac12} & 12a^{\frac32}b^{\frac12} + 36ab \\ \hline
2a + 12a^{\frac12}b^{\frac12} & 18ab \\
2a + 12a^{\frac12}b^{\frac12} + 9b & 18ab + 108a^{\frac12}b^{\frac32} + 81b^2 \\ \hline
\end{array}$$

39.

$$\begin{array}{l|l}
 & 1 + 2\sqrt{x} - x - 2x\sqrt{x} + x^2 \;\big|\; \underline{1 + \sqrt{x} - x} \\
 & 1 \\ \hline
2 & 2\sqrt{x} \\
2 + \sqrt{x} & 2\sqrt{x} + x \\ \hline
2 + 2\sqrt{x} & -2x \\
2 + 2\sqrt{x} - x & -2x - 2x\sqrt{x} + x^2 \\ \hline
\end{array}$$

40.

$$\begin{array}{l|l}
 & a - 4\sqrt{ab} + 4b + 6\sqrt{ac} - 12\sqrt{bc} + 9c \;\big|\; \underline{\sqrt{a} - 2\sqrt{b} + 3\sqrt{c}} \\
 & a \\ \hline
2\sqrt{a} & -4\sqrt{ab} + 4b \\
2\sqrt{a} - 2\sqrt{b} & -4\sqrt{ab} + 4b \\ \hline
2\sqrt{a} - 4\sqrt{b} & 6\sqrt{ac} - 12\sqrt{bc} + 9c \\
2\sqrt{a} - 4\sqrt{b} + 3\sqrt{c} & 6\sqrt{ac} - 12\sqrt{bc} + 9c \\ \hline
\end{array}$$

41.

$$\begin{array}{l|l}
 & x^2 - 4x\sqrt{xy} + 6xy - 4y\sqrt{xy} + y^2 \;\big|\; \underline{x - 2\sqrt{xy} + y} \\
 & x^2 \\ \hline
2x & -4x\sqrt{xy} \\
2x - 2\sqrt{xy} & -4x\sqrt{xy} + 4xy \\ \hline
2x - 4\sqrt{xy} & 2xy \\
2x - 4\sqrt{xy} + y & 2xy - 4y\sqrt{xy} + y^2 \\ \hline
\end{array}$$

42.

$$\begin{array}{r|l}
 & 81'23'41'69 \;\big|\; \underline{9013} \\
 & 81 \\ \hline
900 \times 2 = 1800 & 23\ 41 \\
1800 + 1 = 1801 & 18\ 01 \\ \hline
9010 \times 2 = 18020 & 5\ 40\ 69 \\
18020 + 3 = 18023 & 5\ 40\ 69 \\ \hline
\end{array}$$

43.

$$\begin{array}{r|l}
 & 64'06'40'16 \;\big|\; \underline{8004} \\
 & 64 \\ \hline
8000 \times 2 = 16000 & 6\ 40\ 16 \\
16000 + 4 = 16004 & 6\ 40\ 16 \\ \hline
\end{array}$$

44.

$$\begin{array}{r|l}
 & .00'02'28'01 \;\big|\; \underline{.0151} \\
 & 1 \\ \hline
10 \times 2 = 20 & 1\ 28 \\
20 + 5 = 25 & 1\ 25 \\ \hline
150 \times 2 = 300 & 3\ 01 \\
300 + 1 = 301 & 3\ 01 \\ \hline
\end{array}$$

45.

$$\begin{array}{r|l}
 & .10'00'00'00 \;\big|\; \underline{.3162} \\
 & 9 \\ \hline
30 \times 2 = 60 & 1\ 00 \\
60 + 1 = 61 & 61 \\ \hline
310 \times 2 = 620 & 39\ 00 \\
620 + 6 = 626 & 37\ 56 \\ \hline
3160 \times 2 = 6320 & 1\ 44\ 00 \\
6320 + 2 = 6322 & 1\ 26\ 44 \\ \hline
\end{array}$$

46.

$$\sqrt{56 + 14\sqrt{15}} = \sqrt{56 + 2\sqrt{7 \times 7 \times 3 \times 5}}$$

$$= \sqrt{35 + 21 + 2\sqrt{35 \times 21}} = \sqrt{35} + \sqrt{21}.$$

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47. $\sqrt{47-12\sqrt{15}} = \sqrt{47-2\sqrt{2\times3\times2\times3\times3\times5}}$

$= \sqrt{27+20-2\sqrt{27\times20}} = \sqrt{27}-\sqrt{20} = 3\sqrt{3}-2\sqrt{5}.$

48. $\sqrt{62+20\sqrt{6}} = \sqrt{62+2\sqrt{600}} = \sqrt{50+12+2\sqrt{50\times12}}$

$= \sqrt{50}+\sqrt{12} = 5\sqrt{2}+2\sqrt{3}.$

49. $\sqrt{51-36\sqrt{2}} = \sqrt{51-2\sqrt{3\times3\times2\times3\times3\times2\times2}}$

$= \sqrt{27+24-2\sqrt{27\times24}} = \sqrt{27}-\sqrt{24} = 3\sqrt{3}-2\sqrt{6}.$

50.

$$\begin{array}{l|l|l}
 & x^3-9x+27x^{-1}-27x^{-3} & x-3x^{-1} \\
 & x^3 & \\
3x^2 & -9x & \\
3x^2-9+9x^{-2} & -9x+27x^{-1}-27x^{-3} &
\end{array}$$

51.

$$\begin{array}{l|l|l}
 & 27x^3+27x^2-5+\frac{1}{3x^2}-\frac{1}{27x^3} & 3x+1-\frac{1}{3x} \\
 & 27x^3 & \\
27x^2 & 27x^2 & \\
27x^2+9x+1 & 27x^2+9x+1 & \\
27x^2+18x+3 & -9x-6+\frac{1}{3x^2}-\frac{1}{27x^3} & \\
27x^2+18x \quad -\frac{1}{x}+\frac{1}{9x^2} & -9x-6+\frac{1}{3x^2}-\frac{1}{27x^3} &
\end{array}$$

52.

$$\begin{array}{l|l|l}
 & x^3+3x^2\sqrt{x}-5x\sqrt{x}+3\sqrt{x}-1 & x+\sqrt{x}-1 \\
 & x^3 & \\
3x^2 & 3x^2\sqrt{x} & \\
3x^2+3x\sqrt{x}+x & 3x^2\sqrt{x}+3x^2+x\sqrt{x} & \\
3x^2+6x\sqrt{x}+3x & -3x^2-6x\sqrt{x} & \\
3x^2+6x\sqrt{x} \quad -3\sqrt{x}+1 & -3x^2-6x\sqrt{x}+3\sqrt{x}-1 &
\end{array}$$

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53.

$$\begin{array}{l|l|l}
 & 2\sqrt{2}-6\sqrt[3]{2}+3\sqrt{2}\sqrt[3]{4}-2 & \sqrt{2}-\sqrt[3]{2} \\
 & 2\sqrt{2} & \\
6 & -6\sqrt[3]{2} & \\
6-3\sqrt{2}\sqrt[3]{2}+\sqrt[3]{4} & -6\sqrt[3]{2}+3\sqrt{2}\sqrt[3]{4}-2 &
\end{array}$$

54.

$$\begin{array}{lcr|l|l}
 & & & 510'082'399 & 799 \\
 & & & 343 & \\
70^2\times3 & = & 14700 & 167\ 082 & \\
70\times9\times3 & = & 1890 & & \\
9^2 & = & 81 & & \\
 & & 16671 & 150\ 039 & \\
790^2\times3 & = & 1872300 & 17\ 043\ 399 & \\
790\times9\times3 & = & 21330 & & \\
9^2 & = & 81 & & \\
 & & 1893711 & 17\ 043\ 399 &
\end{array}$$

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55.

$$
\begin{array}{lrl|l}
 & & 1'042'590'744 & \underline{1014} \\
 & & \underline{1\ 000} & \\
100^2 \times 3 & = & 30000 & 42\ 590 \\
100 \times 3 & = & 300 & \\
1^2 & = & 1 & \\
\hline
 & & 30301 & \underline{30\ 301} \\
1010^2 \times 3 & = & 3060300 & 12\ 289\ 744 \\
1010 \times 4 \times 3 & = & 12120 & \\
4^2 & = & 16 & \\
\hline
 & & \underline{3072436} & \underline{12\ 289\ 744}
\end{array}
$$

56.

$$
\begin{array}{lrl|l}
 & & 2.000'000'000 & \underline{1.259} \\
 & & \underline{1} & \\
3(10)^2 & = & 300 & 1\ 000 \\
3(10 \times 2) & = & 60 & \\
2^2 & = & 4 & \\
\hline
 & & 364 & \underline{728} \\
3(120)^2 & = & 43200 & 272\ 000 \\
3(120 \times 5) & = & 1800 & \\
5^2 & = & 25 & \\
\hline
 & & 45025 & \underline{225\ 125} \\
3(1250)^2 & = & 4687500 & 46\ 875\ 000 \\
3(1250 \times 9) & = & 33750 & \\
9^2 & = & 81 & \\
\hline
 & & \underline{4721331} & \underline{42\ 491\ 979}
\end{array}
$$

57.

$$
\begin{array}{l|l}
 & 1 + x - x^2 \\
 & \underline{1} \\
2 & x \\
2 + \frac{x}{2} & x + \frac{x^2}{4} \\
\hline
2 + x & -\frac{5x^2}{4} \\
2 + x - \frac{5x^2}{8} & -\frac{5x^2}{4} - \frac{5x^3}{8} + \frac{25x^4}{64} \\
\hline
2 + x - \frac{5x^2}{4} & \frac{5x^3}{8} - \frac{25x^4}{64}
\end{array}
\qquad
\left| \underline{1 + \frac{x}{2} - \frac{5x^2}{8} + \frac{5x^3}{16}} \right.
$$

58.

$$
\begin{array}{l|l}
 & 1 + x^3 \\
 & \underline{1} \\
3 & x^3 \\
3 + x^3 + \frac{x^6}{9} & x^3 + \frac{x^6}{3} + \frac{x^9}{27} \\
\hline
3 + 2x^3 + \frac{x^6}{3} & -\frac{x^6}{3} - \frac{x^9}{27} \\
\hline
\end{array}
\qquad
\left| \underline{1 + \frac{x^3}{3} - \frac{x^6}{9}} \right.
$$

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59.

$$
\begin{array}{l|l}
 & a^6 - 4a^4\sqrt{ab^{-1}} + 6a^3b^{-1} - 4ab^{-1}\sqrt{ab^{-1}} + b^{-2} \;\big|\; \underline{a^3 - 2a\sqrt{ab^{-1}} + b^{-1}} \\
 & \underline{a^6} \\
2a^3 & -4a^4\sqrt{ab^{-1}} \\
2a^3 - 2a\sqrt{ab^{-1}} & \underline{-4a^4\sqrt{ab^{-1}} + 4a^3b^{-1}} \\
2a^3 - 4a\sqrt{ab^{-1}} & 2a^3b^{-1} \\
2a^3 - 4a\sqrt{ab^{-1}} + b^{-1} & \underline{2a^3b^{-1} - 4ab^{-1}\sqrt{ab^{-1}} + b^{-2}}
\end{array}
$$

Since the square root of the given polynomial is $a^3 - 2a\sqrt{ab^{-1}} + b^{-1}$ and the square root of $a^3 - 2a\sqrt{ab^{-1}} + b^{-1}$ is $a\sqrt{a} - \sqrt{b^{-1}}$, the fourth root of the given polynomial is $a\sqrt{a} - \sqrt{b^{-1}}$.

60. $8 - 48\sqrt{a} + 120a - 160a\sqrt{a} + 120a^2 - 48a^2\sqrt{a} + 8a^3$
$= 8(1 - 6\sqrt{a} + 15a - 20a\sqrt{a} + 15a^2 - 6a^2\sqrt{a} + a^3).$

$$
\begin{array}{l|l}
 & 1 - 6\sqrt{a} + 15a - 20a\sqrt{a} + 15a^2 - 6a^2\sqrt{a} + a^3 \;\big|\; \underline{1 - 3\sqrt{a} + 3a - a\sqrt{a}} \\
 & \underline{1} \\
2 & -6\sqrt{a} \\
2 - 3\sqrt{a} & \underline{-6\sqrt{a} + 9a} \\
2 - 6\sqrt{a} & 6a \\
2 - 6\sqrt{a} + 3a & \underline{6a - 18a\sqrt{a} + 9a^2} \\
2 - 6\sqrt{a} + 6a & -2a\sqrt{a} + 6a^2 \\
2 - 6\sqrt{a} + 6a - a\sqrt{a} & \underline{-2a\sqrt{a} + 6a^2 - 6a^2\sqrt{a} + a^3}
\end{array}
$$

$$
\begin{array}{l|l}
 & 1 - 3\sqrt{a} + 3a - a\sqrt{a} \;\big|\; \underline{1 - \sqrt{a}} \\
 & \underline{1} \\
3 & -3\sqrt{a} \\
3 - 3\sqrt{a} + a & \underline{-3\sqrt{a} + 3a - a\sqrt{a}}
\end{array}
$$

Since the sixth root of 8 is equal to the square root of 2, or to $\sqrt{2}$, and the sixth root of $1 - 6\sqrt{a} + 15a - 20a\sqrt{a} + 15a^2 - 6a^2\sqrt{a} + a^3$ is equal to the cube root of the square root of this polynomial factor, or to $1 - \sqrt{a}$, the sixth root of the given polynomial is equal to $\sqrt{2}(1 - \sqrt{a})$.

61. $a^m \times a^n = a^{m+n}$ for all values of m and n. (1)
If $n = 0$, $a^m \times a^0 = a^{m+0} = a^m$. (2)
Dividing by a^m, $a^0 = 1$. (3)
Since (1) is true for all values of m and n, let $m = -2$ and $n = 2$.
Then, $a^{-2} \times a^2 = a^{-2+2} = a^0$.
Therefore, by (3), Ax. 5, $a^{-2} \times a^2 = 1$.
Dividing by a^2, Ax. 4, $a^{-2} = \dfrac{1}{a^2}$.

62. $a^m \times a^n = a^{m+n}$ for all values of m and n. (1)
Let $m = \frac{3}{2}$ and $n = \frac{3}{2}$.
Then, by (1), $a^{\frac{3}{2}} \times a^{\frac{3}{2}} = a^{\frac{3}{2}+\frac{3}{2}} = a^3$. (2)

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Taking the square root of both members of (2),

Ax. 7, § 28, $a^{\frac{3}{2}} = \sqrt{a^3}.$ (3)

Again, let $m = \frac{1}{2}$ and $n = \frac{1}{2}$.

Then, by (1), $a^{\frac{1}{2}} \times a^{\frac{1}{2}} = a^{\frac{1}{2}+\frac{1}{2}} = a^1.$ (4)

If $m=1$ and $n=\frac{1}{2}$, $a^{\frac{1}{2}} \times a^{\frac{1}{2}} \times a^{\frac{1}{2}} = a^1 \times a^{\frac{1}{2}} = a^{1+\frac{1}{2}} = a^{\frac{3}{2}}.$ (5)

Hence, § 26, $a^{\frac{3}{2}} = (a^{\frac{1}{2}})^3 = (\sqrt{a})^3.$ (6)

From (3) and (6), Ax. 5, $a^{\frac{3}{2}} = \sqrt{a^3} = (\sqrt{a})^3.$

63. $a^m \times a^n = a^{m+n}$ for all values of m and n. (1)

Let $m = -\frac{1}{3}$ and $n = 1$.

Then, by (1), $a^{-\frac{1}{3}} \times a^1 = a^{-\frac{1}{3}+1} = a^{\frac{2}{3}}$

by Ex. 62, (3), $= \sqrt[3]{a^2}.$ (2)

Hence, Ax. 3 and 4, $\frac{2}{a}(a^{-\frac{1}{3}} \times a^1) = \frac{2}{a}\sqrt[3]{a^2};$

that is, $2a^{-\frac{1}{3}} = \frac{2\sqrt[3]{a^2}}{a}.$

64. $a^m \times a^n = a^{m+n}$ for all values of m and n. (1)

Then, $(ab)^m \times (ab)^n = (ab)^{m+n}$ for all values of m and n, (2)
since a in (1) represents any number.

Let $n = 0$.

Then, in (2), $(ab)^m \times (ab)^0 = (ab)^{m+0} = (ab)^m.$ (3)

Dividing both members of (3) by $(ab)^m$, Ax. 4,

$$(ab)^0 = 1.$$

65. $(abc)^3 = abc \cdot abc \cdot abc$

§ 82, $= aaa \cdot bbb \cdot ccc$

§ 26, $= a^3b^3c^3.$

66. $\left(\frac{a}{b}\right)^3 = \frac{a}{b} \cdot \frac{a}{b} \cdot \frac{a}{b}.$ (1)

$b^3 = b \cdot b \cdot b.$ (2)

Multiplying (1) by (2), Ax. 3, $\left(\frac{a}{b}\right)^3 \cdot b^3 = \left(\frac{a}{b} \times b\right)\left(\frac{a}{b} \times b\right)\left(\frac{a}{b} \times b\right)$

§§ 121, 26, $= a \cdot a \cdot a = a^3.$

Dividing by b^3, Ax. 4, $\left(\frac{a}{b}\right)^3 = \frac{a^3}{b^3}.$

67. $16^{\frac{3}{4}} = (\sqrt[4]{16})^3 = 2^3 = 8.$

68. $27^{\frac{2}{3}} = (\sqrt[3]{27})^2 = 3^2 = 9.$

69. $8^{-\frac{2}{3}} = 1 \div 8^{\frac{2}{3}} = 1 \div (\sqrt[3]{8})^2 = 1 \div 2^2 = \frac{1}{4}.$

70. $(a^4x^4)^{\frac{3}{2}} = a^{4\left(\frac{3}{2}\right)}x^{4\left(\frac{3}{2}\right)} = a^6x^6.$

71. $(b^2y^4)^{-\frac{3}{2}} = b^{2\left(-\frac{3}{2}\right)}y^{4\left(-\frac{3}{2}\right)} = b^{-3}y^{-6} = \frac{1}{b^3y^6}.$

72. $(a^nb^n)^{-\frac{1}{n}} = a^{n\left(-\frac{1}{n}\right)}b^{n\left(-\frac{1}{n}\right)} = a^{-1}b^{-1} = \frac{1}{ab}.$

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73. $(\frac{32}{243})^{-\frac{3}{5}} = 1 \div (\frac{32}{243})^{\frac{3}{5}} = (1 \div \frac{32}{243})^{\frac{3}{5}} = (\frac{243}{32})^{\frac{3}{5}} = (\sqrt[5]{\frac{243}{32}})^3 = (\frac{3}{2})^3 = \frac{27}{8}.$

74. $(\frac{36}{49})^{-\frac{3}{2}} = 1 \div (\frac{36}{49})^{\frac{3}{2}} = (1 \div \frac{36}{49})^{\frac{3}{2}} = (\frac{49}{36})^{\frac{3}{2}} = (\sqrt{\frac{49}{36}})^3 = (\frac{7}{6})^3 = \frac{343}{216}.$

75. $(-\frac{8}{27})^{-\frac{2}{3}} = 1 \div (-\frac{8}{27})^{\frac{2}{3}} = 1 \div (\sqrt[3]{-\frac{8}{27}})^2 = 1 \div (-\frac{2}{3})^2 = 1 \div \frac{4}{9} = \frac{9}{4}.$

76. Since $b-a = -1(a-b)$, $(b-a)^n = [-1(a-b)]^n = (-1)^n(a-b)^n$. If n is *even*, $(-1)^n = 1$ and $(-1)^n(a-b)^n$, or $(b-a)^n = (a-b)^n$.

77. $(36\,a^{-3} \div 25\,a^{-2})^{-\frac{1}{2}} = (\frac{36}{25}\,a^{-1})^{-\frac{1}{2}} = (\frac{36}{25})^{-\frac{1}{2}}a^{\frac{1}{2}} = a^{\frac{1}{2}} \div (\frac{36}{25})^{\frac{1}{2}}$

$$= a^{\frac{1}{2}} \div (\tfrac{6}{5}) = \tfrac{5}{6}\,a^{\frac{1}{2}}.$$

78. $(8\,a^3x^6 \times 64\,a^{-4}x^{-5})^{-\frac{1}{3}} = (2^3 \cdot 2^6 \cdot a^{-1}x)^{-\frac{1}{3}} = 2^{-\frac{9}{3}}a^{\frac{1}{3}}x^{-\frac{1}{3}} = \dfrac{a^{\frac{1}{3}}}{8\,x^{\frac{1}{3}}}.$

79. $(a^{\frac{1}{2}}b^{\frac{1}{4}})^{\frac{4}{3}} \div (a^{\frac{1}{6}}b^{\frac{1}{4}})^2 = a^{\frac{2}{3}}b^{\frac{1}{3}} \div a^{\frac{1}{3}}b^{\frac{1}{2}} = a^{\frac{1}{3}}b^{-\frac{1}{6}} = \dfrac{a^{\frac{1}{3}}}{b^{\frac{1}{6}}}.$

80. $(\sqrt{a^3x^{-3}} \div \sqrt[3]{a^2x^{-2}})^{\frac{2}{5}} = (a^{\frac{3}{2}}x^{-\frac{3}{2}} \div a^{\frac{2}{3}}x^{-\frac{2}{3}})^{\frac{2}{5}} = (a^{\frac{3}{2}-\frac{2}{3}}x^{-\frac{3}{2}+\frac{2}{3}})^{\frac{2}{5}}$

$$= (a^{\frac{5}{6}}x^{-\frac{5}{6}})^{\frac{2}{5}} = a^{\frac{1}{3}}x^{-\frac{1}{3}} = \frac{a^{\frac{1}{3}}}{x^{\frac{1}{3}}}.$$

81. $(\sqrt{a^{-1}b^4} \div \sqrt{a^2b})^{-\frac{1}{3}} = (a^{-\frac{1}{2}}b^2 \div ab^{\frac{1}{2}})^{-\frac{1}{3}} = (a^{-\frac{1}{2}-1}b^{2-\frac{1}{2}})^{-\frac{1}{3}}$

$$= (a^{-\frac{3}{2}}b^{\frac{3}{2}})^{-\frac{1}{3}} = a^{\frac{1}{2}}b^{-\frac{1}{2}} = \frac{a^{\frac{1}{2}}}{b^{\frac{1}{2}}}.$$

82. $(\sqrt{a} \div \sqrt[3]{a}) \div \sqrt[4]{a} = (a^{\frac{1}{2}} \div a^{\frac{1}{3}}) \div a^{\frac{1}{4}} = a^{\frac{1}{6}} \div a^{\frac{1}{4}} = a^{-\frac{1}{12}} = \dfrac{1}{a^{\frac{1}{12}}}.$

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83. $\dfrac{a-b}{a^{\frac{1}{3}}-b^{\frac{1}{3}}} - \dfrac{a+b}{a^{\frac{1}{3}}+b^{\frac{1}{3}}} + \dfrac{2\,a^{\frac{1}{3}}b}{a^{\frac{2}{3}}-b^{\frac{2}{3}}}$

$$= (a^{\frac{2}{3}} + a^{\frac{1}{3}}b^{\frac{1}{3}} + b^{\frac{2}{3}}) - (a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}} + b^{\frac{2}{3}}) + \frac{2\,a^{\frac{1}{3}}b}{a^{\frac{2}{3}}-b^{\frac{2}{3}}}$$

$$= 2\,a^{\frac{1}{3}}b^{\frac{1}{3}} + \frac{2\,a^{\frac{1}{3}}b}{a^{\frac{2}{3}}-b^{\frac{2}{3}}} = \frac{2\,ab^{\frac{1}{3}} - 2\,a^{\frac{1}{3}}b + 2\,a^{\frac{1}{3}}b}{a^{\frac{2}{3}}-b^{\frac{2}{3}}} = \frac{2\,ab^{\frac{1}{3}}}{a^{\frac{2}{3}}-b^{\frac{2}{3}}}.$$

84. $\dfrac{1+a^{-1}b}{1-a^{-1}b} \div \left(\dfrac{1+ab^{-1}+a^2b^{-2}}{1-ab^{-1}+a^2b^{-2}} \times \dfrac{1+a^{-3}b^3}{1-a^{-3}b^3}\right).$

Multiplying both terms of the first fraction by a, of the second fraction by b^2, and of the third by a^3,

$$= \frac{a+b}{a-b} \div \left(\frac{b^2+ab+a^2}{b^2-ab+a^2} \times \frac{a^3+b^3}{a^3-b^3}\right)$$

$$= \frac{a+b}{a-b} \div \frac{a+b}{a-b} = 1.$$

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85.
$$\frac{x+1}{x-1}-\frac{x-1}{x+1}=\frac{3-5x}{1-x^2}.$$

Multiplying by x^2-1,
$$x^2+2x+1-(x^2-2x+1)=-(3-5x).$$
$$4x=5x-3.$$
$$\therefore x=3.$$

86.
$$\frac{7-2x}{10}-\frac{2x-1}{5}+\frac{x}{2}=\frac{5x-6\frac{1}{5}}{2x}-\frac{17+3x}{30}.$$
$$\frac{7}{10}-\frac{x}{5}-\frac{2x}{5}+\frac{1}{5}+\frac{x}{2}=\frac{5}{2}-\frac{31}{10x}-\frac{17}{30}-\frac{x}{10}.$$

Transposing, etc.,
$$\frac{31}{10x}=\frac{31}{30}.$$
$$\therefore x=3.$$

87.
$$\frac{4x-17}{9}-\frac{3\frac{2}{3}-22x}{33}=x-\frac{6}{x}\left(1-\frac{x^2}{54}\right).$$
$$\frac{4x}{9}-\frac{17}{9}-\frac{1}{9}+\frac{2x}{3}=x-\frac{6}{x}+\frac{x}{9}.$$

Canceling,
$$-2=-\frac{6}{x}.$$
$$\therefore x=3.$$

88.
$$\begin{cases}\dfrac{3x-5y}{3}-\dfrac{2x-8y-9}{12}=\dfrac{y}{2}+\dfrac{7}{12}, & (1)\\[2ex] \dfrac{7}{2}\left(\dfrac{x}{7}+\dfrac{y}{4}+1\frac{1}{3}\right)-\dfrac{10}{3}\left(4x-\dfrac{y}{8}-24\right)=0. & (2)\end{cases}$$

Reducing (1), $\qquad 5x-9y=-1. \qquad$ (3)

Expanding (2), $\qquad \dfrac{x}{2}+\dfrac{7y}{8}+\dfrac{14}{3}-\dfrac{40x}{3}+\dfrac{5y}{12}+80=0;$

$\qquad \therefore 308x-31y=2032. \qquad$ (4)

Multiplying (3) by 31, $\qquad 155x-279y=-31. \qquad$ (5)

Multiplying (4) by 9, $\qquad 2772x-279y=18288. \qquad$ (6)

Subtracting (5) from (6), $\qquad 2617x=18319.$

$\qquad \therefore x=7. \qquad$ (7)

Substituting (7) in (3), $\qquad y=4.$

89.
$$\begin{cases}3x+1=2y, & (1)\\ (x+5)(y+7)=(x+1)(y-9)+112. & (2)\end{cases}$$

Reducing (2), $\qquad 4x+y=17. \qquad$ (3)

Multiplying (3) by 2, $\qquad 8x+2y=34. \qquad$ (4)

Adding (1) and (4), $\qquad 11x+2y+1=2y+34.$

$\qquad \therefore x=3. \qquad$ (5)

Substituting (5) in (1), $\qquad y=5.$

90.
$$\frac{\left(\dfrac{a}{27}\div\dfrac{a^{-2}}{8}\right)^{-\frac{2}{3}}-x^2}{\dfrac{3a^{-1}+2x}{2}}=\frac{\left(\dfrac{a}{27}\times 8a^2\right)^{-\frac{2}{3}}-x^2}{\dfrac{3+2ax}{2a}}=\frac{\dfrac{9}{4a^2}-x^2}{\dfrac{3+2ax}{2a}}$$
$$=\frac{9-4a^2x^2}{2a(3+2ax)}=\frac{3-2ax}{2a}=\frac{3}{2a}-x.$$

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91. $\{a^{-2}[a^{\frac{3}{5}}(a^{\frac{3}{5}})^{\frac{1}{3}}]^5\}^{\frac{5}{2}} = \{a^{-2}[a^{\frac{3}{5}}a^{\frac{1}{5}}]^5\}^{\frac{5}{2}} = \{a^{-2} \cdot a^4\}^{\frac{5}{2}} = (a^2)^{\frac{5}{2}} = a^5.$

92.
$$\left[\frac{\left(\dfrac{a^3b}{x^2y}\right)^{\frac{1}{2}}}{\left(\dfrac{a^3b^2}{xy^2}\right)^{\frac{1}{3}}}\right]^6 - \frac{(ax^{-1})^3}{b} = \left[\frac{\left(\dfrac{a^9b^3}{x^6y^3}\right)^{\frac{1}{6}}}{\left(\dfrac{a^6b^4}{x^2y^4}\right)^{\frac{1}{6}}}\right]^6 - \frac{a^3}{bx^3}$$
$$= \frac{a^9b^3}{x^6y^3} \times \frac{x^2y^4}{a^6b^4} - \frac{a^3}{bx^3}$$
$$= \frac{a^3y}{bx^4} - \frac{a^3x}{bx^4} = \frac{a^3(y-x)}{bx^4}.$$

93. $\left[\dfrac{x^{-\frac{1}{2}}y^{-\frac{2}{3}}}{x^{-\frac{1}{6}}y^{-1}} \div \dfrac{x^{-2}y^2}{(xy)^{-3}}\right]^{-3} = (x^{-\frac{1}{3}}y^{\frac{1}{3}} \div xy^5)^{-3} = (x^{-\frac{4}{3}}y^{-\frac{14}{3}})^{-3} = x^4y^{14}.$

94. $\{(a^{\frac{1}{2}}b^{\frac{5}{2}})^{\frac{1}{5}} \div (a^{-\frac{1}{2}}b)^{-2}\}^{\frac{1}{9}} = \{a^{\frac{1}{10}}b^{\frac{1}{2}} \div ab^{-2}\}^{\frac{1}{9}} = (a^{-\frac{9}{10}}b^{\frac{5}{2}})^{\frac{1}{9}}$
$$= a^{-\frac{1}{10}}b^{\frac{5}{18}} = \frac{b^{\frac{5}{18}}}{a^{\frac{1}{10}}}.$$

95.
$$\frac{a+b}{a^{\frac{1}{3}} - b^{\frac{1}{3}}} - \frac{a-b}{a^{\frac{1}{3}} + b^{\frac{1}{3}}} = \frac{a^{\frac{4}{3}} + ab^{\frac{1}{3}} + a^{\frac{1}{3}}b + b^{\frac{4}{3}} - (a^{\frac{4}{3}} - ab^{\frac{1}{3}} - a^{\frac{1}{3}}b + b^{\frac{4}{3}})}{(a^{\frac{1}{3}} - b^{\frac{1}{3}})(a^{\frac{1}{3}} + b^{\frac{1}{3}})}$$
$$= \frac{2\,ab^{\frac{1}{3}} + 2\,a^{\frac{1}{3}}b}{a^{\frac{2}{3}} - b^{\frac{2}{3}}} = \frac{2\,a^{\frac{1}{3}}b^{\frac{1}{3}}(a^{\frac{2}{3}} + b^{\frac{2}{3}})}{a^{\frac{2}{3}} - b^{\frac{2}{3}}}.$$

96.
$$\frac{\left[-\dfrac{a^{-1} + b^{-1}}{a^{-1} - b^{-1}} \times (a^2 - b^2)\right]^{\frac{1}{2}}}{\dfrac{b+a}{ab}} = \frac{\left[-\dfrac{b+a}{b-a} \times (a^2 - b^2)\right]^{\frac{1}{2}}}{\dfrac{b+a}{ab}}$$
$$= \left[\frac{a+b}{a-b} \times (a+b)(a-b)\right]^{\frac{1}{2}} \times \frac{ab}{a+b}$$
$$= (a+b) \times \frac{ab}{a+b} = ab.$$

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98. From § 276, Prin. 2, $(a^m)^n = a^{mn}$ for all values of m and n.

Substituting $\dfrac{1}{m}$ for m and $\dfrac{1}{n}$ for n, $(a^{\frac{1}{m}})^{\frac{1}{n}} = a^{\frac{1}{mn}}$.

From § 88, $a^m \times a^n = a^{m+n}$ for all values of m and n.

Substituting $(-n)$ for n, $a^m \times a^{-n} = a^{m+(-n)} = a^{m-n}$.

99. $2^{\frac{7}{3}} = \sqrt[3]{128}$, $5 = \sqrt[3]{125}$. Hence, $2^{\frac{7}{3}}$ is greater than 5.

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100. $\dfrac{1}{2+\sqrt{5}-\sqrt{2}}=\dfrac{1}{2+2.23-1.41}=\dfrac{1}{2\ 82}=.35.$

101. $x(x-\sqrt{2})(x-\sqrt{8})(x-\sqrt{18})+4.$

$$\begin{array}{l|l}
 & x^4-6\sqrt{2}\,x^3+22\,x^2-12\sqrt{2}\,x+4 \quad \underline{\mid\, x^2-3\sqrt{2}\,x+2} \\
 & x^4 \\ \hline
2\,x^2 & -6\sqrt{2}\,x^3+22\,x^2 \\
2\,x^2-3\sqrt{2}\,x & -6\sqrt{2}\,x^3+18\,x^2 \\ \hline
2\,x^2-6\sqrt{2}\,x & 4\,x^2-12\sqrt{2}\,x+4 \\
2\,x^2-6\sqrt{2}\,x+2 & 4\,x^2-12\sqrt{2}\,x+4 \\ \hline
\end{array}$$

102. By § 134, $x^{\frac{2}{3}}+y^{\frac{3}{2}}$ is exactly contained in the difference of any like even powers of $x^{\frac{2}{3}}$ and $y^{\frac{3}{2}}$. Hence, $x^{\frac{2}{3}}+y^{\frac{3}{2}}$ is exactly contained in $(x^{\frac{2}{3}})^6-(y^{\frac{3}{2}})^6$, or x^4-y^9.

Dividing x^4-y^9 by $x^{\frac{2}{3}}+y^{\frac{3}{2}}$, the rationalizing factor is found to be $x^{\frac{10}{3}}-x^{\frac{8}{3}}y^{\frac{3}{2}}+x^2y^3-x^{\frac{4}{3}}y^{\frac{9}{2}}+x^{\frac{2}{3}}y^6-y^{\frac{15}{2}}$.

103. $16^{\frac{1}{3}}\cdot 2^{\frac{1}{2}}\cdot 32^{\frac{5}{6}}=2^{\frac{4}{3}}\cdot 2^{\frac{1}{2}}\cdot 2^{\frac{25}{6}}=2^{\frac{8}{6}}\cdot 2^{\frac{3}{6}}\cdot 2^{\frac{25}{6}}=2^{\frac{36}{6}}=2^6=64.$

104. $\sqrt{38-12\sqrt{10}}=\sqrt{38-2(6\sqrt{10})}=\sqrt{38-2\sqrt{360}}.$

$$\sqrt{360}=\sqrt{20}\times\sqrt{18}\text{ and }38=20+18.$$

$$\sqrt{38-12\sqrt{10}}=\sqrt{20}-\sqrt{18}=2\sqrt{5}-3\sqrt{2}.$$

105. $\dfrac{\sqrt{x}-\sqrt{x-2}}{\sqrt{x}+\sqrt{x-2}}=\dfrac{(\sqrt{x}-\sqrt{x-2})(\sqrt{x}-\sqrt{x-2})}{(\sqrt{x}+\sqrt{x-2})(\sqrt{x}-\sqrt{x-2})}$

$$=\frac{x-2\sqrt{x^2-2x}+x-2}{x-x+2}=x-1-\sqrt{x^2-2x}.$$

106. $\sqrt[3]{4^2}\cdot\sqrt[9]{8}\cdot 3\sqrt[3]{4}=2^{\frac{4}{3}}\cdot 2^{\frac{1}{3}}\cdot 3(2)^{\frac{2}{3}}=3(2)^{\frac{7}{3}}=3\sqrt[3]{128}=12\sqrt[3]{2}.$

107. $\sqrt[3]{-27}+(\sqrt{-1})^6+8^{-\frac{1}{3}}=-3-1+\frac{1}{2}=-\frac{8}{2}+\frac{1}{2}=-\frac{7}{2}.$

108. $\sqrt{\dfrac{\sqrt{23}+\sqrt{7}}{\sqrt{23}-\sqrt{7}}}=\sqrt{\dfrac{(\sqrt{23}+\sqrt{7})(\sqrt{23}+\sqrt{7})}{(\sqrt{23}-\sqrt{7})(\sqrt{23}+\sqrt{7})}}=\sqrt{\dfrac{23+2\sqrt{161}+7}{23-7}}$

$$=\tfrac{1}{4}\sqrt{30+2\sqrt{161}}=\tfrac{1}{4}(\sqrt{23}+\sqrt{7}).$$

109. $12^0+4^{\frac{1}{2}}-9^{-1}+\dfrac{1}{\sqrt{-64}}+27^{\frac{2}{3}}=1+2-\dfrac{1}{9}+\dfrac{1}{8\sqrt{-1}}+9$

$$=\frac{107}{9}+\frac{1}{8\sqrt{-1}}=11\tfrac{8}{9}-\frac{1}{8}\sqrt{-1}.$$

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110. $$\frac{\sqrt{3}+\sqrt{2}}{2-\sqrt{3}} \div \frac{7+4\sqrt{3}}{\sqrt{3}-\sqrt{2}} = \frac{(\sqrt{3}+\sqrt{2})(2+\sqrt{3})}{(2-\sqrt{3})(2+\sqrt{3})} \div \frac{(7+4\sqrt{3})(\sqrt{3}+\sqrt{2})}{(\sqrt{3}-\sqrt{2})(\sqrt{3}+\sqrt{2})}$$

$$= \frac{(\sqrt{3}+\sqrt{2})(2+\sqrt{3})}{4-3} \div \frac{(7+4\sqrt{3})(\sqrt{3}+\sqrt{2})}{3-2}$$

$$= (\sqrt{3}+\sqrt{2})(2+\sqrt{3}) \times \frac{1}{(7+4\sqrt{3})(\sqrt{3}+\sqrt{2})}$$

$$= \frac{2+\sqrt{3}}{7+4\sqrt{3}} = \frac{(2+\sqrt{3})(7-4\sqrt{3})}{(7+4\sqrt{3})(7-4\sqrt{3})}$$

$$= \frac{2-\sqrt{3}}{49-48} = 2-\sqrt{3}.$$

111. $$\frac{\sqrt{3+\sqrt{5}}-\sqrt{3-\sqrt{5}}}{\sqrt{3+\sqrt{5}}+\sqrt{3-\sqrt{5}}}$$

$$= \frac{(\sqrt{3+\sqrt{5}}-\sqrt{3-\sqrt{5}})(\sqrt{3+\sqrt{5}}-\sqrt{3-\sqrt{5}})}{(\sqrt{3+\sqrt{5}}+\sqrt{3-\sqrt{5}})(\sqrt{3+\sqrt{5}}-\sqrt{3-\sqrt{5}})}$$

$$= \frac{3+\sqrt{5}-2\sqrt{(3+\sqrt{5})(3-\sqrt{5})}+3-\sqrt{5}}{3+\sqrt{5}-(3-\sqrt{5})}$$

$$= \frac{6-2\sqrt{9-5}}{2\sqrt{5}} = \frac{1}{\sqrt{5}} = \frac{1}{5}\sqrt{5}.$$

112. $$\frac{2\sqrt{15}+8}{5+\sqrt{15}} \div \frac{8\sqrt{3}-6\sqrt{5}}{5\sqrt{3}-3\sqrt{5}}$$

$$= \frac{(2\sqrt{15}+8)(5-\sqrt{15})}{(5+\sqrt{15})(5-\sqrt{15})} \div \frac{(8\sqrt{3}-6\sqrt{5})(5\sqrt{3}+3\sqrt{5})}{(5\sqrt{3}-3\sqrt{5})(5\sqrt{3}+3\sqrt{5})}$$

$$= \frac{2\sqrt{15}+10}{25-15} \div \frac{30-6\sqrt{15}}{75-45}$$

$$= \frac{2(\sqrt{15}+5)}{10} \times \frac{30}{6(5-\sqrt{15})}$$

$$= \frac{\sqrt{15}+5}{5-\sqrt{15}} = \frac{(\sqrt{15}+5)(5+\sqrt{15})}{(5-\sqrt{15})(5+\sqrt{15})} = \frac{25+10\sqrt{15}+15}{25-15} = 4+\sqrt{15}.$$

113. $$\frac{b}{\sqrt{a}} \times \sqrt[3]{\frac{c}{a^{-1}}} \times \frac{\sqrt[4]{c^3}}{\sqrt{b}} \div \frac{a^{-\frac{1}{6}}}{b^{-\frac{1}{2}}} = \frac{b}{a^{\frac{1}{2}}} \times c^{\frac{1}{3}} \times a^{\frac{1}{3}} \times \frac{c^{\frac{3}{4}}}{b^{\frac{1}{2}}} \times \frac{a^{\frac{1}{6}}}{b^{\frac{1}{2}}}$$

$$= c^{\frac{1}{3}} \times c^{\frac{3}{4}} = c^{1\frac{1}{12}} = c\sqrt[12]{c}.$$

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114. $$\frac{\dfrac{1}{1+x}-\dfrac{2}{x^2+3x+2}}{(x+2)^{-1}-(x+1)^{-1}(2+x)^{-1}}=\frac{\dfrac{2+x-2}{(1+x)(2+x)}}{\dfrac{1}{(x+2)}-\dfrac{1}{(x+1)(2+x)}}$$

$$=\frac{\dfrac{x}{(1+x)(2+x)}}{\dfrac{x}{(1+x)(2+x)}}=1.$$

QUADRATIC EQUATIONS

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3. $3x^2-5=22.$

Transposing, $3x^2=27.$

Dividing by 3, $x^2=9.$

Extracting the square root, $x=\pm 3.$

4. $2x^2+3x^2=80.$

Uniting terms, $5x^2=80.$

Dividing by 5, $x^2=16.$

Extracting the square root, $x=\pm 4.$

5. $4x^2=\frac{1}{9}.$

Clearing of fractions, $36x^2=1.$

Extracting the square root, $6x=\pm 1.$

$\therefore x=\pm\frac{1}{6}.$

6. $\frac{3}{4}x^2-5=22.$

Transposing, $\frac{3}{4}x^2=27.$

Dividing by $\frac{3}{4}$, $x^2=36.$

Extracting the square root, $x=\pm 6.$

7. $x^2-b=0.$

Transposing, $x^2=b.$

Extracting the square root, $x=\pm\sqrt{b}.$

8. $6ax^2-54a^5=0.$

Transposing and dividing by $6a$, $x^2=9a^4.$

Extracting the square root, $x=\pm 3a^2.$

9. $7x^2-25=5x^2+73.$

Transposing, etc., $2x^2=98.$

Dividing by 2, $x^2=49.$

Extracting the square root, $x=\pm 7.$

10. $(x+4)^2=8x+25.$

Expanding, $x^2+8x+16=8x+25.$

Canceling, etc., $x^2=9.$

Extracting the square root, $x=\pm 3.$

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11. $(a-x)^2 = (3x+a)(x-a).$

Expanding, $a^2 - 2ax + x^2 = 3x^2 - 2ax - a^2.$

Canceling, etc., $2x^2 = 2a^2.$

$x^2 = a^2.$

Extracting the square root, $x = \pm a.$

12. $ax^2 = (a-b)(a^2-b^2) - bx^2.$

Transposing, etc., $(a+b)x^2 = (a-b)(a^2-b^2).$

Dividing by $a+b$, $x^2 = (a-b)(a-b).$

Extracting the square root, $x = \pm (a-b).$

13. $a^2x^2 + 2ax^2 = (a^2-1)^2 - x^2.$

Transposing, etc., $(a^2+2a+1)x^2 = (a^2-1)^2.$

Extracting the square root, $(a+1)x = \pm (a^2-1).$

Dividing by $a+1$, $x = \pm (a-1).$

14. $(x+2)^2 - 4(x+2) = 4.$

Expanding, $x^2 + 4x + 4 - 4x - 8 = 4.$

Canceling, etc., $x^2 = 8.$

Extracting the square root, $x = \pm 2\sqrt{2}.$

15. $\dfrac{x-8}{6} = \dfrac{6}{x+8}.$

Clearing of fractions, $x^2 - 64 = 36.$

Transposing, etc., $x^2 = 100.$

Extracting the square root, $x = \pm 10.$

16. $\dfrac{1}{1-x} + \dfrac{1}{1+x} = \frac{8}{3}.$

Uniting terms, $\dfrac{2}{1-x^2} = \dfrac{8}{3}.$

Dividing by 2 and clearing of fractions, $3 = 4 - 4x^2.$

Transposing, etc., $x^2 = \frac{1}{4}.$

Extracting the square root, $x = \pm \frac{1}{2}.$

17. $\dfrac{x}{12} + \dfrac{x^2-15}{5x} = \dfrac{x}{5}.$

$$\frac{x}{12} + \frac{x}{5} - \frac{3}{x} = \frac{x}{5}.$$

Canceling and clearing of fractions, $x^2 - 36 = 0.$

Transposing and extracting the square root, $x = \pm 6.$

18. $\dfrac{x+3}{x-3} + \dfrac{x-3}{x+3} = 4.$

Clearing of fractions, $(x+3)^2 + (x-3)^2 = 4x^2 - 36.$

Expanding, etc., $2x^2 + 18 = 4x^2 - 36.$

Transposing, etc., $x^2 = 27.$

Extracting the square root, $x = \pm 3\sqrt{3}.$

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19. $$\frac{x-2}{x+1}+\frac{x+2}{x-1}=-1.$$

Clearing of fractions,
$$x^2-3x+2+x^2+3x+2=-x^2+1.$$
$$3x^2=-3.$$
$$x^2=-1.$$

Extracting the square root, $$x=\pm\sqrt{-1}.$$

20. $$\frac{x-3}{x-2}+\frac{x+3}{x+2}=1\tfrac{7}{8}.$$

Reducing to mixed numbers and subtracting 2 from each side,
$$-\frac{1}{x-2}+\frac{1}{x+2}=-\tfrac{1}{8}.$$

Clearing of fractions, $$-8x-16+8x-16=-x^2+4.$$
Transposing, etc., $$x^2=36.$$
Extracting the square root, $$x=\pm 6.$$

21. $$\frac{a}{x}+\frac{x}{a}=\frac{ab}{x}.$$

Transposing, $$\frac{x}{a}=\frac{ab}{x}-\frac{a}{x}=\frac{a(b-1)}{x}.$$

Clearing of fractions, $$x^2=a^2(b-1).$$
Extracting the square root, $$x=\pm a\sqrt{b-1}.$$

22. $$\frac{x}{a+b}-\frac{a-b}{x}=0.$$

Clearing of fractions, $$x^2-(a^2-b^2)=0.$$
Transposing, $$x^2=a^2-b^2.$$
Extracting the square root, $$x=\pm\sqrt{a^2-b^2}.$$

23. $$\frac{x-2}{x+2}-\frac{x+2}{2-x}=\frac{40}{x^2-4}.$$

Clearing of fractions, $$x^2-4x+4+x^2+4x+4=40.$$
Uniting terms, $$2x^2=32.$$
Dividing by 2, $$x^2=16.$$
Extracting square root, $$x=\pm 4.$$

24. $$\frac{\sqrt{x^2+1}-\sqrt{x^2-1}}{\sqrt{x^2+1}+\sqrt{x^2-1}}=\frac{1}{2}.$$

Clearing of fractions, $$2\sqrt{x^2+1}-2\sqrt{x^2-1}=\sqrt{x^2+1}+\sqrt{x^2-1}.$$
Transposing, etc., $$\sqrt{x^2+1}=3\sqrt{x^2-1}.$$
Squaring, $$x^2+1=9x^2-9.$$
$$\therefore x^2=\tfrac{5}{4}.$$

Extracting the square root, $$x=\pm\tfrac{1}{2}\sqrt{5}.$$

25. $$\frac{x+a}{x-a}+\frac{x-a}{x+a}=\frac{2a}{1-a}.$$

Uniting terms, $$\frac{2(x^2+a^2)}{x^2-a^2}=\frac{2a}{1-a}.$$

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Dividing by 2 and clearing of fractions,

$$x^2 + a^2 - ax^2 - a^3 = ax^2 - a^3.$$

Transposing, etc., $x^2(1 - 2a) = -a^2.$

Dividing by $1 - 2a$, $x^2 = \dfrac{a^2}{2a - 1}.$

Extracting the square root, $x = \pm \dfrac{a}{2a - 1}\sqrt{2a - 1}.$

26. $$\frac{x + a}{x + b} + \frac{x - a}{x - b} = \frac{a^2 + b^2}{x^2 - b^2}.$$

Clearing of fractions,

$$x^2 + ax - bx - ab + x^2 - ax + bx - ab = a^2 + b^2.$$

Uniting terms, etc., $2x^2 = a^2 + 2ab + b^2.$

$$x^2 = \tfrac{1}{2}(a + b)^2.$$

Extracting the square root, $x = \pm \tfrac{1}{2}(a + b)\sqrt{2}.$

27. $$\sqrt{(x + 3)(x - 5)} = \sqrt{49 - 2x}.$$

Squaring, $x^2 - 2x - 15 = 49 - 2x.$

Canceling $-2x = -2x$, etc., $x^2 = 64.$

Taking the square root, $x = \pm 8.$

28. $$\sqrt{25 - 6x} + \sqrt{25 + 6x} = 8.$$

Squaring, $25 - 6x + 2\sqrt{625 - 36x^2} + 25 + 6x = 64.$

Canceling, etc., $\sqrt{625 - 36x^2} = 7.$

Squaring, $625 - 36x^2 = 49.$

$$\therefore x^2 = 16.$$

Extracting the square root, $x = \pm 4.$

29. $$\frac{x + 7}{x^2 - 7x} - \frac{x - 7}{x^2 + 7x} = \frac{7}{x^2 - 73}.$$

Uniting terms, etc., $\dfrac{4}{x^2 - 49} = \dfrac{1}{x^2 - 73}.$

Clearing of fractions, $4x^2 - 292 = x^2 - 49.$

Transposing, etc., $x^2 = 81.$

Extracting the square root, $x = \pm 9.$

30. $$\frac{\sqrt{x + 2a} - \sqrt{x - 2a}}{\sqrt{x - 2a} + \sqrt{x + 2a}} = \frac{x}{2a}.$$

Multiplying both terms of the first fraction by $\sqrt{x + 2a} - \sqrt{x - 2a}$,

$$\frac{x + 2a - 2\sqrt{x^2 - 4a^2} + x - 2a}{(x + 2a) - (x - 2a)} = \frac{x}{2a}.$$

Uniting terms, $\dfrac{2x - 2\sqrt{x^2 - 4a^2}}{4a} = \dfrac{x}{2a},$

that is, $\dfrac{x - \sqrt{x^2 - 4a^2}}{2a} = \dfrac{x}{2a}.$

$$\therefore x - \sqrt{x^2 - 4a^2} = x.$$

Canceling, $-\sqrt{x^2 - 4a^2} = 0.$

Squaring, $x^2 - 4a^2 = 0.$

Transposing, $x^2 = 4a^2.$

Extracting the square root, $x = \pm 2a.$

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31. $$\frac{2}{x+\sqrt{2-x^2}}+\frac{2}{x-\sqrt{2-x^2}}=x.$$

Clearing of fractions,

$$2x-2\sqrt{2-x^2}+2x+2\sqrt{2-x^2}=2x^3-2x.$$

Canceling, etc., $6x=2x^3.$

Dividing by $2x$, $3=x^2.$

$$\therefore x=\pm\sqrt{3}.$$

NOTE. — The given equation is satisfied also for $x=0$.

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2. Let $x=$ the number.

Then, $x^2+25=13^2.$

$$x^2=169-25=144.$$

$$\therefore x=\pm 12.$$

Hence, the number is 12 or -12.

3. Let $x=$ the number.

Then, $x^2=25^2-20^2.$

$$=625-400=225.$$

$$\therefore x=\pm 15.$$

Hence, the number is 15 or -15.

4. Let $x=$ the number.

Then, $(x+5)(x-5)=75.$

$$x^2-25=75.$$

$$x^2=100.$$

$$\therefore x=\pm 10.$$

Hence, the number is 10 or -10.

5. Let $x=$ number of rods in each side of garden.

Then, $4x=$ number of rods of fence,

and $x^2=$ number of square rods in area of garden.

$$\therefore x^2=160\times 2\tfrac{1}{2}=400.$$

Solving, $x=\pm 20,$

whence, rejecting the negative value, $4x=80$, the number of rods of fence required.

6. Let $x=$ number of rods in length of field.

Then, $\frac{3}{4}x=$ number of rods in width of field,

and $x^2+(\frac{3}{4}x)^2=$ the square of the number of rods in the length of the path.

$$\therefore x^2+(\tfrac{3}{4}x)^2=20^2.$$

$$x^2+\tfrac{9}{16}x^2=400.$$

Combining, etc., $x^2=256.$

Solving, $x=\pm 16,$

and $\frac{3}{4}x=\pm 12.$

Hence, rejecting the negative values, the field is 16 rods long and 12 rods wide.

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7. Let $5 + x =$ one number,
and $5 - x =$ the other number.

Then, $(5 + x)(5 - x) = 21.$

$25 - x^2 = 21.$

Solving, $x = \pm 2,$
whence, $5 + x = 7$ or 3, and $5 - x = 3$ or 7.

Hence, the numbers are 7 and 3.

8. Let $8 + x =$ one number,
and $8 - x =$ the other number.

Then, $(8 + x)(8 - x) = 55.$

$64 - x^2 = 55.$

Solving, $x = \pm 3,$
whence, $8 + x = 11$ or 5, and $8 - x = 5$ or 11.

Hence, the numbers are 11 and 5.

9. Let $\frac{5}{2} + x =$ one number,
and $\frac{5}{2} - x =$ the other number.

Then, $(\frac{5}{2} + x)(\frac{5}{2} - x) = - 14.$

$\frac{25}{4} - x^2 = - 14.$

Solving, $x = \pm \frac{9}{2},$
whence, $\frac{5}{2} + x = 7$ or $- 2$, and $\frac{5}{2} - x = - 2$ or 7.

Hence, the numbers are 7 and $- 2$.

10. Let $\frac{17}{2} + x$ and $\frac{17}{2} - x$ represent the two factors of 60 whose algebraic sum is 17.

Then, $(\frac{17}{2} + x)(\frac{17}{2} - x) = 60.$

$\frac{289}{4} - x^2 = 60.$

Solving, $x = \pm \frac{7}{2},$
whence, $\frac{17}{2} + x = 12$ or 5, and $\frac{17}{2} - x = 5$ or 12.

Since the two factors of 60, whose algebraic sum is 17, are 12 and 5,

$$a^2 + 17\,a + 60 = (a + 12)(a + 5).$$

11. Let $1 + x$ and $1 - x$, whose sum is 2, be the two factors of $- 2$.

Then, $(1 + x)(1 - x) = - 2.$

$1 - x^2 = - 2.$

Solving, $x = \pm \sqrt{3},$
whence, $1 + x = 1 + \sqrt{3}$ or $1 - \sqrt{3}$ and $1 - x = 1 - \sqrt{3}$ or $1 + \sqrt{3}$.

Since the two factors of $- 2$, whose algebraic sum is 2, are $1 + \sqrt{3}$ and $1 - \sqrt{3}$, $a^2 + 2\,a - 2 = (a + 1 + \sqrt{3})(a + 1 - \sqrt{3}).$

12. Let $- 1 + x$ and $- 1 - x$, whose sum is $- 2$, be the two factors of $- 1$.

Then, $(- 1 + x)(- 1 - x) = - 1.$

$1 - x^2 = - 1.$

Solving, $x = \pm \sqrt{2},$
whence, $- 1 + x = - 1 \pm \sqrt{2}$ and $- 1 - x = - 1 \mp \sqrt{2}$.

Since the two factors of $- 1$, whose algebraic sum is $- 2$, are $- 1 + \sqrt{2}$ and $- 1 - \sqrt{2}$, $x^2 - 2\,x - 1 = (x - 1 + \sqrt{2})(x - 1 - \sqrt{2}).$

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13. Let $12 + x =$ first part,
and $12 - x =$ second part.
Then, $(12 + x)(12 - x) = 143.$
$144 - x^2 = 143.$
Solving, $x = \pm 1,$
whence, $12+x = 13$ or 11, and $12 - x = 11$ or 13.
Hence, the parts are 13 and 11.

14. Let $x =$ one number.
Then, $x^2 =$ its square,
and $x^2 + 56 =$ square of the other number,
whence, $\sqrt{x^2 + 56} =$ the other number.
$\therefore x^2 + x^2 + 56 = 394.$
Solving, $x = \pm 13,$
whence, $\sqrt{x^2 + 56} = \pm 15.$
Hence, the numbers are 13 and 15, 13 and -15, -13 and 15, or -13 and -15.

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15. Let $x =$ number of rods in width.
Then, $4x =$ number of rods in length.
$\therefore 4x \cdot x = 160 \cdot 10.$
Solving, $x = \pm 20,$
whence, $4x = \pm 80.$
Hence, rejecting negative values, the length of the field was 80 rods, and its width was 20 rods.

16. Let $x =$ number of yards in width.
Then, $3x =$ number of yards in length.
$\therefore 3x^2 =$ number of square yards in area.
$\frac{3}{4} \cdot 3x^2 = 36.$
Solving, $9x^2 = 144,$
whence, $x = \pm 4,$
and $3x = \pm 12.$
Hence, rejecting negative values, the length of the floor was 12 yards, and its width was 4 yards.

17. Let $x =$ number of feet in width.
Then, $8x =$ number of feet in length.
$\therefore 8x \cdot x = 80{,}000.$
Dividing by 8, $x^2 = 10{,}000.$
Extracting square root, $x = \pm 100,$
and $8x = \pm 800.$
Neglecting the negative values, the length is 800 feet and the width is 100 feet.

18. Let $50 + x =$ number of rods in a side of one,
and $50 - x =$ number of rods in a side of the other.
Then, $(50 + x)^2 + (50 - x)^2 = 160 \times \frac{205}{4}.$
$5000 + 2x^2 = 8200.$
Solving, $x = \pm 40,$
whence, $50 + x = 90$ or 10, and $50 - x = 10$ or 90.
Hence, the larger field is 90 rods square, and the smaller is 10 rods square.

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19. Let $3x =$ number of rods in length of field,
and $2x =$ number of rods in width.
Then, $2x =$ number of rods in length and in width of the square field.

$$\therefore 4x^2 = 160 \times 10 = 1600.$$

Solving, $x = \pm 20,$
whence, $3x = 60$ or -60, and $2x = 40$ or -40.

Rejecting the negative values, the dimensions of the original field were 60 rods by 40 rods.

20. Let $x =$ number of board feet in one tie.
Then, $250x =$ number of ties.

$$250x \cdot x = 400{,}000.$$

Dividing by 250, $x^2 = 1600.$

$$\therefore x = 40,$$

and $250x = 10{,}000.$

Hence, there were 10,000 ties and 40 board feet in 1 tie.

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1. $s = \frac{1}{2}gt^2.$

Dividing by $\frac{1}{2}g$, etc., $t^2 = \dfrac{2s}{g}.$

Extracting the square root, $t = \pm\sqrt{\dfrac{2s}{g}}.$

2. $E = \frac{1}{2}Mv^2.$

Dividing by $\frac{1}{2}M$, etc., $v^2 = \dfrac{2E}{M}.$

Extracting the square root, $v = \pm\sqrt{\dfrac{2E}{M}}.$

3. $P = I^2R.$

Dividing by R, etc., $I^2 = \dfrac{P}{R}.$

Extracting the square root, $I = \pm\sqrt{\dfrac{P}{R}}.$

4. $F = \dfrac{mv^2}{R}.$

Dividing by $\dfrac{m}{R}$, etc., $v^2 = \dfrac{FR}{m}.$

Extracting the square root, $v = \pm\sqrt{\dfrac{FR}{m}}.$

5. $G = \dfrac{mm'}{d^2}.$

Multiplying by $\dfrac{d^2}{G}$, $d^2 = \dfrac{mm'}{G}.$

Extracting the square root, $d = \pm\sqrt{\dfrac{mm'}{G}}.$

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6. From formula 1, $t = \pm\sqrt{\frac{2s}{g}}.$

Substituting values for g and s, $t = \pm\sqrt{\frac{201}{32.16}} = \pm\sqrt{6.25} = 2.5.$

Hence, it will take 2.5 seconds for the brick to fall to the sidewalk.

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7. From formula 1, $t = \sqrt{\frac{2s}{g}}.$

Substituting given values, $t = \sqrt{\frac{5000}{32.16}} = \sqrt{155.47} = 12.5.$

Hence, it requires 12.5 seconds to reach the earth.

8. $c^2 = a^2 + b^2.$

Transposing, $b^2 = c^2 - a^2.$

Extracting the square root, $b = \pm\sqrt{c^2 - a^2}.$

9. $4m^2 = 2(a^2 + b^2) - c^2.$

Extracting the square root, $m = \pm\frac{1}{2}\sqrt{2(a^2 + b^2) - c^2}.$

10. $A = .7854\, d^2.$

Dividing by .7854, etc. $d^2 = \frac{A}{.7854}.$

Extracting the square root, $d = \pm\sqrt{\frac{A}{.7854}}.$

11. $V = \frac{1}{3}\pi r^2 h.$

Dividing by $\frac{1}{3}\pi h$, etc., $r^2 = \frac{3V}{\pi h}.$

Extracting the square root, $r = \pm\sqrt{\frac{3V}{\pi h}}.$

12. From 8, $c = \sqrt{a^2 + b^2}.$

Substituting given values, $c = \sqrt{64 + 36} = \sqrt{100} = 10.$

13. From formula 8, $b = \sqrt{c^2 - a^2}.$

Substituting the given values, $b = \sqrt{25 - 9} = \sqrt{16} = 4.$

14. Substituting value for c in 8, $100 = a^2 + b^2.$

But $a = b$, $\therefore 100 = 2a^2.$

Transposing and dividing by 2, $a^2 = 50.$

Extracting the square root, $a = 5\sqrt{2} = 7.1.$

15. The diameter of the log is equal to the value of c in formula 8, and since the timber is to be square $a = b$ in the formula.

Substituting the given values in 8, $324 = 2a^2.$

Solving, $a = 9\sqrt{2} = 12.7.$

Since the length of the timber is the same as the length of the log, the dimensions are 12 ft. × 12.7 in. × 12.7 in.

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16. $$c^2 = a^2 + b^2.$$

From figure $a = \frac{1}{2}c$, $b = h$, $$\therefore c^2 = \frac{c^2}{4} + h^2.$$

Transposing, $$h^2 = c^2 - \frac{c^2}{4} = \frac{3c^2}{4}.$$

Extracting the square root, $$h = \pm\frac{c}{2}\sqrt{3}.$$

17. From formula 9, $$m = \tfrac{1}{2}\sqrt{2(a^2 + b^2) - c^2}.$$

Substituting the given values, $$m = \tfrac{1}{2}\sqrt{2(121 + 64) - 81}.$$

$$\therefore m = \tfrac{1}{2}\sqrt{289} = \tfrac{17}{2} = 8\tfrac{1}{2}.$$

18. From formula 10, $$d = \sqrt{\frac{A}{.7854}}.$$

Substituting the given value, $$d = \sqrt{\frac{1000}{.7854}} = \sqrt{1273.23} = 35.7.$$

Hence, the diameter is 35.7 feet.

19. From formula 11, $$r = \sqrt{\frac{3V}{\pi h}}.$$

Substituting given values, $$r = \sqrt{\frac{3000}{3.1416 \times 20}} = \sqrt{47.74}.$$

$$\therefore r = 6.9.$$

Hence, the radius is 6.9 centimeters.

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12. $$y^2 = 10 - 3y.$$

Transposing, $$y^2 + 3y = 10.$$

Completing the square, $$y^2 + 3y + (\tfrac{3}{2})^2 = 10 + \tfrac{9}{4} = \tfrac{49}{4}.$$

Extracting the square root, $$y + \tfrac{3}{2} = \pm\tfrac{7}{2}.$$

Taking the upper sign, $$y = -\tfrac{3}{2} + \tfrac{7}{2} = 2.$$

Taking the lower sign, $$y = -\tfrac{3}{2} - \tfrac{7}{2} = -5.$$

13. $$z^2 - 180 = 3z.$$

Transposing, $$z^2 - 3z = 180.$$

Completing the square, $$z^2 - 3z + (\tfrac{3}{2})^2 = 180 + \tfrac{9}{4} = \tfrac{729}{4}.$$

Extracting the square root, $$z - \tfrac{3}{2} = \pm\tfrac{27}{2}.$$

Taking the upper sign, $$z = \tfrac{3}{2} + \tfrac{27}{2} = 15.$$

Taking the lower sign, $$z = \tfrac{3}{2} - \tfrac{27}{2} = -12.$$

14. $$v^2 + 15v = 54.$$

Completing the square, $$v^2 + 15v + (\tfrac{15}{2})^2 = 54 + \tfrac{225}{4} = \tfrac{441}{4}.$$

Extracting the square root, $$v + \tfrac{15}{2} = \pm\tfrac{21}{2}.$$

Taking the upper sign, $$v = -\tfrac{15}{2} + \tfrac{21}{2} = 3.$$

Taking the lower sign, $$v = -\tfrac{15}{2} - \tfrac{21}{2} = -18.$$

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15. $v^2 + 21v = -54.$
Completing the square, $v^2 + 21v + (\frac{21}{2})^2 = -54 + \frac{441}{4} = \frac{225}{4}.$
Extracting the square root, $v + \frac{21}{2} = \pm \frac{15}{2}.$
Taking the upper sign, $v = -\frac{21}{2} + \frac{15}{2} = -3.$
Taking the lower sign, $v = -\frac{21}{2} - \frac{15}{2} = -18.$

16. $n(n-1) = 930.$
Completing the square, $n^2 - n + (\frac{1}{2})^2 = 930 + \frac{1}{4} = \frac{3721}{4}.$
Extracting the square root, $n - \frac{1}{2} = \pm \frac{61}{2}.$
Taking the upper sign, $n = \frac{1}{2} + \frac{61}{2} = 31.$
Taking the lower sign, $n = \frac{1}{2} - \frac{61}{2} = -30.$

17. $r^2 + 27r + 140 = 0.$
Completing the square, $r^2 + 27r + (\frac{27}{2})^2 = -140 + \frac{729}{4} = \frac{169}{4}.$
Extracting the square root, $r + \frac{27}{2} = \pm \frac{13}{2}.$
Taking the upper sign, $r = -\frac{27}{2} + \frac{13}{2} = -7.$
Taking the lower sign, $r = -\frac{27}{2} - \frac{13}{2} = -20.$

18. $l^2 - 11l + 28 = 0.$
Completing the square, $l^2 - 11l + (\frac{11}{2})^2 = -28 + \frac{121}{4} = \frac{9}{4}.$
Extracting the square root, $l - \frac{11}{2} = \pm \frac{3}{2}.$
Taking the upper sign, $l = \frac{11}{2} + \frac{3}{2} = 7.$
Taking the lower sign, $l = \frac{11}{2} - \frac{3}{2} = 4.$

19. $5x^2 - 3x - 2 = 0.$
Dividing by 5, etc., $x^2 - \frac{3}{5}x = \frac{2}{5}.$
Completing the square, $x^2 - \frac{3}{5}x + (\frac{3}{10})^2 = \frac{2}{5} + \frac{9}{100} = \frac{49}{100}.$
Extracting the square root, $x - \frac{3}{10} = \pm \frac{7}{10}.$
Taking the upper sign, $x = \frac{3}{10} + \frac{7}{10} = 1.$
Taking the lower sign, $x = \frac{3}{10} - \frac{7}{10} = -\frac{2}{5}.$

20. $6x^2 - 5x - 6 = 0.$
Dividing by 6, etc., $x^2 - \frac{5}{6}x = 1.$
Completing the square, $x^2 - \frac{5}{6}x + (\frac{5}{12})^2 = 1 + \frac{25}{144} = \frac{169}{144}.$
Extracting the square root, $x - \frac{5}{12} = \pm \frac{13}{12}.$
Taking the upper sign, $x = \frac{5}{12} + \frac{13}{12} = \frac{3}{2}.$
Taking the lower sign, $x = \frac{5}{12} - \frac{13}{12} = -\frac{2}{3}.$

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21. $.2x^2 + .9x = 3.5.$
Clearing of decimals, $2x^2 + 9x = 35.$
Dividing by 2, $x^2 + \frac{9}{2}x = \frac{35}{2}.$
Completing the square, $x^2 + \frac{9}{2}x + (\frac{9}{4})^2 = \frac{35}{2} + \frac{81}{16} = \frac{361}{16}.$
Extracting the square root, $x + \frac{9}{4} = \pm \frac{19}{4}.$
Taking the upper sign, $x = -\frac{9}{4} + \frac{19}{4} = \frac{5}{2}.$
Taking the lower sign, $x = -\frac{9}{4} - \frac{19}{4} = -7.$

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22. $.03\,x^2 - .07\,x = .1.$

Clearing of decimals, $3\,x^2 - 7\,x = 10.$

Dividing by 3, $x^2 - \frac{7}{3}x = \frac{10}{3}.$

Completing the square, $x^2 - \frac{7}{3}x + (\frac{7}{6})^2 = \frac{10}{3} + \frac{49}{36} = \frac{169}{36}.$

Extracting the square root, $x - \frac{7}{6} = \pm\,\frac{13}{6}.$

Taking the upper sign, $x = \frac{7}{6} + \frac{13}{6} = \frac{10}{3}.$

Taking the lower sign, $x = \frac{7}{6} - \frac{13}{6} = -1.$

23. $2\,x^2 - \frac{19}{2}x = \frac{5}{2}.$

Dividing by 2, $x^2 - \frac{19}{4}x = \frac{5}{4}.$

Completing the square, $x^2 - \frac{19}{4}x + (\frac{19}{8})^2 = \frac{5}{4} + \frac{361}{64} = \frac{441}{64}.$

Extracting the square root, $x - \frac{19}{8} = \pm\,\frac{21}{8}.$

Taking the upper sign, $x = \frac{19}{8} + \frac{21}{8} = 5.$

Taking the lower sign, $x = \frac{19}{8} - \frac{21}{8} = -\frac{1}{4}.$

24. $\dfrac{1}{x+1} + \dfrac{3}{x-1} = \dfrac{10}{3}.$

Clearing of fractions, $3\,x - 3 + 9\,x + 9 = 10\,x^2 - 10.$

Transposing, etc., $10\,x^2 - 12\,x = 16.$

Dividing by 10, $x^2 - \frac{6}{5}x = \frac{8}{5}.$

Completing the square, $x^2 - \frac{6}{5}x + (\frac{3}{5})^2 = \frac{8}{5} + \frac{9}{25} = \frac{49}{25}.$

Extracting the square root, $x - \frac{3}{5} = \pm\,\frac{7}{5}.$

Taking the upper sign, $x = \frac{3}{5} + \frac{7}{5} = 2.$

Taking the lower sign, $x = \frac{3}{5} - \frac{7}{5} = -\frac{4}{5}.$

25. $\dfrac{x^2}{x-2} - \dfrac{3\,x-5}{2} = \dfrac{x+2}{5}.$

Clearing of fractions, $10\,x^2 - 15\,x^2 + 55\,x - 50 = 2\,x^2 - 8.$

Transposing, etc., $-7\,x^2 + 55\,x = 42.$

Dividing by -7, $x^2 - \frac{55}{7}x = -6.$

Completing the square, $x^2 - \frac{55}{7}x + (\frac{55}{14})^2 = -6 + \frac{3025}{196} = \frac{1849}{196}.$

Extracting the square root, $x - \frac{55}{14} = \pm\,\frac{43}{14}.$

Taking the upper sign, $x = \frac{55}{14} + \frac{43}{14} = 7.$

Taking the lower sign, $x = \frac{55}{14} - \frac{43}{14} = \frac{6}{7}.$

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2. $2\,x^2 - 5\,x = 42.$

Multiplying by 2, $4\,x^2 - 10\,x = 84.$

Completing the square, $4\,x^2 - 10\,x + (\frac{5}{2})^2 = 84 + \frac{25}{4} = \frac{361}{4}.$

Extracting the square root, $2\,x - \frac{5}{2} = \pm\frac{19}{2}.$

$$2\,x = \tfrac{5}{2} \pm \tfrac{19}{2} = 12 \text{ or } -7.$$

$$\therefore x = 6 \text{ or } -\tfrac{7}{2}.$$

3. $6\,x^2 - 5\,x + 1 = 0.$

Multiplying by 6, etc., $36\,x^2 - 30\,x = -6.$

Completing the square, $36\,x^2 - 30\,x + (\frac{5}{2})^2 = -6 + \frac{25}{4} = \frac{1}{4}.$

Extracting the square root, $6\,x - \frac{5}{2} = \pm\,\frac{1}{2}.$

$$6\,x = \tfrac{5}{2} \pm \tfrac{1}{2} = 3 \text{ or } 2.$$

$$\therefore x = \tfrac{1}{2} \text{ or } \tfrac{1}{3}.$$

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4. $$4x^2 - 12x = 27.$$
Completing the square, $$4x^2 - 12x + 3^2 = 27 + 9 = 36.$$
Extracting the square root, $$2x - 3 = \pm 6.$$
$$2x = 3 \pm 6 = 9 \text{ or } -3.$$
$$\therefore x = \tfrac{9}{2} \text{ or } -\tfrac{3}{2}.$$

5. $$18x^2 + 6x = 4.$$
Multiplying by 2, $$36x^2 + 12x = 8.$$
Completing the square, $$36x^2 + 12x + 1^2 = 8 + 1 = 9.$$
Extracting the square root, $$6x + 1 = \pm 3.$$
$$6x = -1 \pm 3 = 2 \text{ or } -4.$$
$$\therefore x = \tfrac{1}{3} \text{ or } -\tfrac{2}{3}.$$

6. $$2x^2 - 11x + 12 = 0.$$
Multiplying by 2, $$4x^2 - 22x = -24.$$
Completing the square, $$4x^2 - 22x + (\tfrac{11}{2})^2 = -24 + \tfrac{121}{4} = \tfrac{25}{4}.$$
Extracting the square root, $$2x - \tfrac{11}{2} = \pm \tfrac{5}{2}.$$
$$2x = \tfrac{11}{2} \pm \tfrac{5}{2} = 8 \text{ or } 3.$$
$$\therefore x = 4 \text{ or } \tfrac{3}{2}.$$

7. $$3x^2 + 4x = 95.$$
Multiplying by 3, $$9x^2 + 12x = 285.$$
Completing the square, $$9x^2 + 12x + 2^2 = 285 + 4 = 289.$$
Extracting the square root, $$3x + 2 = \pm 17.$$
$$3x = -2 \pm 17 = 15 \text{ or } -19.$$
$$\therefore x = 5 \text{ or } -\tfrac{19}{3}.$$

8. $$7v^2 + 2v = 32.$$
Multiplying by 7, $$49v^2 + 14v = 224.$$
Completing the square, $$49v^2 + 14v + 1^2 = 224 + 1 = 225.$$
Extracting the square root, $$7v + 1 = \pm 15.$$
$$7v = -1 \pm 15 = 14 \text{ or } -16.$$
$$\therefore v = 2 \text{ or } -\tfrac{16}{7}.$$

9. $$8x^2 - 18x = 5.$$
Multiplying by 2, $$16x^2 - 36x = 10.$$
Completing the square, $$16x^2 - 36x + (\tfrac{9}{2})^2 = 10 + \tfrac{81}{4} = \tfrac{121}{4}.$$
Extracting the square root, $$4x - \tfrac{9}{2} = \pm \tfrac{11}{2}.$$
$$\therefore x = \tfrac{5}{2} \text{ or } -\tfrac{1}{4}.$$

10. $$6m^2 + 5m = 4.$$
Multiplying by 6, $$36m^2 + 30m = 24.$$
Completing the square, $$36m^2 + 30m + (\tfrac{5}{2})^2 = 24 + \tfrac{25}{4} = \tfrac{121}{4}.$$
Extracting the square root, $$6m + \tfrac{5}{2} = \pm \tfrac{11}{2}.$$
$$6m = -\tfrac{5}{2} \pm \tfrac{11}{2} = 3 \text{ or } -8.$$
$$\therefore m = \tfrac{1}{2} \text{ or } -\tfrac{4}{3}.$$

11. $$5n^2 - 14n = -8.$$
Multiplying by 5, $$25n^2 - 70n = -40.$$
Completing the square, $$25n^2 - 70n + 49 = -40 + 49 = 9.$$

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Extracting the square root, $5n-7=\pm 3.$

$$5n=7\pm 3=10 \text{ or } 4.$$

$$\therefore n=2 \text{ or } \tfrac{4}{5}.$$

13. $2x^2+3x=27.$

Multiplying by 8 and adding 9 to each member,

$$16x^2+24x+9=216+9=225.$$

Extracting the square root, $4x+3=\pm 15.$

$$4x=-3\pm 15=12 \text{ or } -18.$$

$$\therefore x=3 \text{ or } -\tfrac{9}{2}.$$

14. $2x^2+5x=7.$

Multiplying by 8 and adding 25 to each member,

$$16x^2+40x+25=56+25=81.$$

Extracting the square root, $4x+5=\pm 9.$

$$4x=-5\pm 9=4 \text{ or } -14.$$

$$\therefore x=1 \text{ or } -\tfrac{7}{2}.$$

15. $2x^2+7x=-6.$

Multiplying by 8 and adding 49 to each member,

$$16x^2+56x+49=-48+49=1.$$

Extracting the square root, $4x+7=\pm 1.$

$$4x=-7\pm 1=-6 \text{ or } -8.$$

$$\therefore x=-\tfrac{3}{2} \text{ or } -2.$$

16. $3x^2-7x=-2.$

Multiplying by 12, $36x^2-84x=-24.$

Adding 49 to each member, $36x^2-84x+49=-24+49=25.$

Extracting the square root, $6x-7=\pm 5.$

$$6x=7\pm 5=12 \text{ or } 2.$$

$$\therefore x=2 \text{ or } \tfrac{1}{3}.$$

17. $4x^2-17x=-4.$

Multiplying by 16, $64x^2-272x=-64.$

Adding 289 to each member,

$$64x^2-272x+289=-64+289=225.$$

Extracting the square root, $8x-17=\pm 15.$

$$8x=17\pm 15=32 \text{ or } 2.$$

$$\therefore x=4 \text{ or } \tfrac{1}{4}.$$

18. $4x^2-x-3=0.$

Transposing, multiplying by 16, and adding 1 to each member,

$$64x^2-16x+1=48+1=49.$$

Extracting the square root, $8x-1=\pm 7.$

$$8x=1\pm 7=8 \text{ or } -6.$$

$$\therefore x=1 \text{ or } -\tfrac{3}{4}.$$

19. $5x^2-2x-16=0.$

Transposing, multiplying by 20, and adding 4 to each member,

$$100x^2-40x+4=320+4=324.$$

Extracting the square root, $10x-2=\pm 18.$

$$10x=2\pm 18=20 \text{ or } -16.$$

$$\therefore x=2 \text{ or } -\tfrac{8}{5}.$$

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20. $$3x^2 + 7x - 110 = 0.$$
Transposing, multiplying by 12, and adding 49 to each member,
$$36x^2 + 84x + 49 = 1320 + 49 = 1369.$$
Extracting the square root, $$6x + 7 = \pm 37.$$
$$6x = -7 \pm 37 = 30 \text{ or } -44.$$
$$\therefore x = 5 \text{ or } -\tfrac{22}{3}.$$

21. $$2x^2 - 5x - 150 = 0.$$
Transposing, multiplying by 8, and adding 25 to each member,
$$16x^2 - 40x + 25 = 1200 + 25 = 1225.$$
Extracting the square root, $$4x - 5 = \pm 35.$$
$$4x = 5 \pm 35 = 40 \text{ or } -30.$$
$$\therefore x = 10 \text{ or } -\tfrac{15}{2}.$$

22. $$3x^2 + x - 200 = 0.$$
Transposing, multiplying by 12, and adding 1 to each member,
$$36x^2 + 12x + 1 = 2400 + 1 = 2401.$$
Extracting the square root, $$6x + 1 = \pm 49.$$
$$6x = -1 \pm 49 = 48 \text{ or } -50.$$
$$\therefore x = 8 \text{ or } -\tfrac{25}{3}.$$

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23. $$5x^2 - 7x = -2.$$
Multiplying by 20 and adding 49 to each member,
$$100x^2 - 140x + 49 = -40 + 49 = 9.$$
Extracting the square root, $$10x - 7 = \pm 3.$$
$$10x = 7 \pm 3 = 10 \text{ or } 4.$$
$$\therefore x = 1 \text{ or } \tfrac{2}{5}.$$

24. $$6x^2 + 5x = -1.$$
Multiplying by 24, and adding 25 to each member,
$$144x^2 + 120x + 25 = -24 + 25 = 1.$$
Extracting the square root, $$12x + 5 = \pm 1.$$
$$12x = -5 \pm 1 = -4 \text{ or } -6.$$
$$\therefore x = -\tfrac{1}{3} \text{ or } -\tfrac{1}{2}.$$

25. $$15x^2 - 7x - 2 = 0.$$
Transposing, multiplying by 60, and adding 49 to each member,
$$900x^2 - 420x + 49 = 120 + 49 = 169.$$
Extracting the square root, $$30x - 7 = \pm 13.$$
$$30x = 7 \pm 13 = 20 \text{ or } -6.$$
$$\therefore x = \tfrac{2}{3} \text{ or } -\tfrac{1}{5}.$$

26. $$7x^2 - 20x - 32 = 0.$$
Transposing, multiplying by 28, and adding 400 to each member,
$$196x^2 - 560x + 400 = 896 + 400 = 1296.$$
Extracting the square root, $$14x - 20 = \pm 36.$$
$$14x = 20 \pm 36 = 56 \text{ or } -16.$$
$$\therefore x = 4 \text{ or } -\tfrac{8}{7}.$$

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2. $2x^2 + 5x + 2 = 0.$

$$\therefore x = \frac{-5 \pm \sqrt{25 - 4 \cdot 2 \cdot 2}}{2 \cdot 2} = \frac{-5 \pm 3}{4} = -\frac{1}{2} \text{ or } -2.$$

3. $3x^2 + 11x + 6 = 0.$

$$\therefore x = \frac{-11 \pm \sqrt{121 - 4 \cdot 3 \cdot 6}}{2 \cdot 3} = \frac{-11 \pm 7}{6} = -\frac{2}{3} \text{ or } -3.$$

4. $6x^2 + 2 = 7x.$

$6x^2 - 7x + 2 = 0.$

$$\therefore x = \frac{7 \pm \sqrt{49 - 4 \cdot 6 \cdot 2}}{2 \cdot 6} = \frac{7 \pm 1}{12} = \frac{2}{3} \text{ or } \frac{1}{2}.$$

5. $4x^2 + 4x = 15.$

$$\therefore x = \frac{-4 \pm \sqrt{16 - 4 \cdot 4(-15)}}{2 \cdot 4} = \frac{-4 \pm 16}{8} = \frac{3}{2} \text{ or } -\frac{5}{2}.$$

6. $2x^2 = 9 - 3x.$

$2x^2 + 3x - 9 = 0.$

$$\therefore x = \frac{-3 \pm \sqrt{9 - 4 \cdot 2(-9)}}{2 \cdot 2} = \frac{-3 \pm 9}{4} = \frac{3}{2} \text{ or } -3.$$

7. $x(2x + 3) = -1.$

$2x^2 + 3x + 1 = 0.$

$$\therefore x = \frac{-3 \pm \sqrt{9 - 4 \cdot 2 \cdot 1}}{2 \cdot 2} = \frac{-3 \pm 1}{4} = -\frac{1}{2} \text{ or } -1.$$

8. $13x = 3x^2 - 10.$

$3x^2 - 13x - 10 = 0.$

$$\therefore x = \frac{13 \pm \sqrt{169 - 4 \cdot 3(-10)}}{2 \cdot 3} = \frac{13 \pm 17}{6} = 5 \text{ or } -\frac{2}{3}.$$

9. $7x^2 + 9x = 10.$

$$\therefore x = \frac{-9 \pm \sqrt{81 - 4 \cdot 7(-10)}}{2 \cdot 7} = \frac{-9 \pm 19}{14} = \frac{5}{7} \text{ or } -2.$$

10. $1 - 3x = 2x^2.$

$2x^2 + 3x - 1 = 0.$

$$\therefore x = \frac{-3 \pm \sqrt{9 - 4 \cdot 2 \cdot (-1)}}{2 \cdot 2}$$

$$x = \frac{-3 \pm \sqrt{17}}{4} = \frac{1}{4}(-3 \pm \sqrt{17}).$$

11. $4 = x(3x + 2)$

$3x^2 + 2x - 4 = 0.$

$$\therefore x = \frac{-2 \pm \sqrt{4 - 4 \cdot 3 \cdot (-4)}}{2 \cdot 3} = \frac{-2 \pm 2\sqrt{13}}{6} = \frac{1}{3}(-1 \pm \sqrt{13}).$$

12. $x^2 - 5x = -3.$

$$\therefore x = \frac{5 \pm \sqrt{25 - 4 \cdot 1 \cdot 3}}{2 \cdot 1} = \frac{5 \pm \sqrt{13}}{2} = \frac{1}{2}(5 \pm \sqrt{13}).$$

13. $3x^2 - 6x = -2.$

$$\therefore x = \frac{6 \pm \sqrt{36 - 4 \cdot 3 \cdot 2}}{2 \cdot 3} = \frac{6 \pm 2\sqrt{3}}{6} = 1 \pm \frac{1}{3}\sqrt{3}.$$

14. $4x^2 - 3x - 2 = 0.$

$$\therefore x = \frac{3 \pm \sqrt{9 - 4 \cdot 4(-2)}}{2 \cdot 4} = \frac{3 \pm \sqrt{41}}{8} = \frac{1}{8}(3 \pm \sqrt{41}).$$

15. $x^2 + 10 = 6x.$

$x^2 - 6x + 10 = 0.$

$$\therefore x = \frac{6 \pm \sqrt{36 - 4 \cdot 1 \cdot 10}}{2 \cdot 1} = \frac{6 \pm 2\sqrt{-1}}{2} = 3 \pm \sqrt{-1}.$$

16. $x^2 = -4(x + 3).$

$x^2 + 4x + 12 = 0.$

$$x = \frac{-4 \pm \sqrt{16 - 4 \cdot 1 \cdot 12}}{2 \cdot 1}.$$

$$x = \frac{-4 \pm 4\sqrt{-2}}{2} = -2 \pm 2\sqrt{-2}.$$

17. $4(2x - 5) = x^2.$

$x^2 - 8x + 20 = 0.$

$$\therefore x = \frac{8 \pm \sqrt{64 - 4 \cdot 1 \cdot 20}}{2 \cdot 1}.$$

$$x = \frac{8 \pm 4\sqrt{-1}}{2} = 4 \pm 2\sqrt{-1}.$$

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3.
$$x^2-6x+5=0.$$
$$(x-1)(x-5)=0.$$
$$\therefore x=1 \text{ or } 5.$$

4.
$$2x^2-5x=0.$$
$$x(2x-5)=0.$$
$$x=0 \text{ or } x=\tfrac{5}{2}.$$

5.
$$7x^2+2x=32.$$
$$49x^2+14x+1=224+1=225.$$
$$7x+1=\pm 15.$$
$$7x=14 \text{ or } -16.$$
$$\therefore x=2 \text{ or } -\tfrac{16}{7}.$$

6.
$$x^2=3x+10.$$
$$x^2-3x-10=0.$$
$$(x-5)(x+2)=0.$$
$$\therefore x=5 \text{ or } -2.$$

7.
$$x^2-30=13x.$$
$$x^2-13x-30=0.$$
$$(x-15)(x+2)=0.$$
$$\therefore x=15 \text{ or } -2.$$

8.
$$x^2-12x=28.$$
$$x^2-12x-28=0.$$
$$(x-14)(x+2)=0.$$
$$\therefore x=14 \text{ or } -2.$$

9.
$$x^2-12x=0.$$
$$x(x-12)=0.$$
$$x=0 \text{ or } 12.$$

10.
$$18x^2+6x=0.$$
$$6x(3x+1)=0.$$
$$x=0 \text{ or } -\tfrac{1}{3}.$$

11.
$$4x^2-12x=0.$$
$$4x(x-3)=0.$$
$$x=0 \text{ or } 3.$$

12.
$$x^2-4.3x=27.3.$$
$$10x^2-43x=273.$$
$$\therefore x=\frac{43\pm\sqrt{1849-40\cdot(-273)}}{20}$$
$$=\frac{43\pm\sqrt{12769}}{20}=\frac{43\pm 113}{20}$$
$$=7.8 \text{ or } -3.5.$$

13.
$$x^2+.25x=.015. \qquad (1)$$
$$(1)\times 400, \qquad 400x^2+100x=6.$$
$$400x^2+100x+(\tfrac{5}{2})^2=6+\tfrac{25}{4}=\tfrac{49}{4}.$$
$$20x+\tfrac{5}{2}=\pm\tfrac{7}{2}.$$
$$20x=1 \text{ or } -6.$$
$$x=.05 \text{ or } -.3.$$

14.
$$x+\frac{1}{x}-\frac{5}{2}=0.$$
$$2x^2-5x+2=0.$$
$$(x-2)(2x-1)=0.$$
$$\therefore x=2 \text{ or } \tfrac{1}{2}.$$

15.
$$\frac{x^2}{9}+\frac{x^2-2x}{3x-6}=\frac{35}{4}.$$
$$\frac{x^2}{9}+\frac{x}{3}=\frac{35}{4}.$$
$$4x^2+12x=315.$$
$$4x^2+12x+9=315+9=324.$$
$$2x+3=\pm 18.$$
$$\therefore x=\tfrac{15}{2} \text{ or } -\tfrac{21}{2}.$$

16.
$$\frac{x}{9(x-1)}=\frac{x-2}{6}.$$
$$2x=3x^2-9x+6.$$
$$3x^2-11x=-6.$$
$$36x^2-132x+121=-72+121=49.$$
$$6x-11=\pm 7.$$
$$6x=18 \text{ or } 4.$$
$$\therefore x=3 \text{ or } \tfrac{2}{3}.$$

17.
$$\frac{4}{x^2-2x+1}=\frac{1}{4}.$$
Extracting the square root,
$$\frac{2}{x-1}=\pm\frac{1}{2}.$$
$$4=\pm(x-1)$$
$$=x-1 \text{ or } -x+1.$$
$$\therefore x=5 \text{ or } -3.$$

18.
$$\frac{x^2}{4}-\frac{2x}{3}=28.$$
$$\frac{x^2}{4}-\frac{2x}{3}+\frac{4}{9}=\frac{252}{9}+\frac{4}{9}=\frac{256}{9}.$$
$$\frac{x}{2}-\frac{2}{3}=\pm\frac{16}{3}.$$
$$\frac{x}{2}=6 \text{ or } -\frac{14}{3}.$$
$$\therefore x=12 \text{ or } -\tfrac{28}{3}.$$

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19.
$$\frac{9x}{2x^2+x}+\frac{3}{x-3}=4.$$
$$\frac{9}{2x+1}+\frac{3}{x-3}=4.$$
$$9x-27+6x+3=8x^2-20x-12.$$
$$8x^2-35x+12=0.$$
$$\therefore x=\frac{35\pm\sqrt{1225-4\cdot 8\cdot 12}}{2\cdot 8}$$
$$=\frac{35\pm 29}{16}=4 \text{ or } \frac{3}{8}.$$

20.
$$\frac{1+x}{x-3}-\frac{x-1}{x-2}=\frac{4}{5}.$$
$$5x^2-5x-10-5x^2+20x-15=4x^2-20x+24.$$
$$4x^2-35x+49=0.$$
$$\therefore x=\frac{35\pm\sqrt{1225-4\cdot 4\cdot 49}}{2\cdot 4}$$
$$=\frac{35\pm 21}{8}=7 \text{ or } \frac{7}{4}.$$

21.
$$\frac{x}{x-5}-\frac{x-5}{x}=\frac{3}{2}.$$
$$2x^2-2x^2+20x-50=3x^2-15x.$$
$$3x^2-35x+50=0.$$
$$(x-10)(3x-5)=0.$$
$$\therefore x=10 \text{ or } \tfrac{5}{3}.$$

22.
$$\frac{x+7}{x+5}+\frac{x+12}{x+6}=7.$$
$$x^2+13x+42+x^2+17x+60=7x^2+77x+210.$$
$$5x^2+47x+108=0.$$
$$(x+4)(5x+27)=0.$$
$$\therefore x=-4 \text{ or } -\tfrac{27}{5}.$$

23.
$$\frac{x+4}{x-2}+3=\frac{(x+3)^2}{(x^2-9)}$$
$$\frac{x+4}{x-2}+3=\frac{(x+3)(x+3)}{(x+3)(x-3)}$$
$$\frac{x+4}{x-2}+3=\frac{x+3}{x-3}.$$
$$x^2+x-12+3x^2-15x+18=x^2+x-6.$$
$$3x^2-15x+12=0.$$
$$x^2-5x+4=0.$$
$$(x-4)(x-1)=0.$$
$$\therefore x=4 \text{ or } 1.$$

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24.
$$\frac{x^2}{x-2}=\frac{4}{x-2}+5.$$
$$x+2+\frac{4}{x-2}=\frac{4}{x-2}+5.$$
Canceling,
$$\frac{4}{x-2}=\frac{4}{x-2}.$$
$$x=3.$$

25.
$$\frac{x}{x+2}+\frac{1}{2}=\frac{x+2}{2x}.$$
$$2x^2+x^2+2x=x^2+4x+4.$$
$$2x^2-2x-4=0.$$
$$x^2-x-2=0.$$
$$(x-2)(x+1)=0.$$
$$\therefore x=2 \text{ or } -1.$$

26.
$$\frac{5x}{x+7}+\frac{x+6}{x+3}=3.$$
$$5x^2+15x+x^2+13x+42=3x^2+30x+63.$$
$$3x^2-2x-21=0.$$
$$(x-3)(3x+7)=0.$$
$$\therefore x=3 \text{ or } -\tfrac{7}{3}.$$

27.
$$\frac{x+2}{x-7}-\frac{x+5}{x-5}=1.$$
$$x^2-3x-10-x^2+2x+35=x^2-12x+35.$$
$$x^2-11x+10=0.$$
$$(x-1)(x-10)=0.$$
$$\therefore x=1 \text{ or } 10.$$

28.
$$\frac{x-3}{x+4}+\frac{x+2}{x-2}=\frac{23}{10}.$$
$$10x^2-50x+60+10x^2+60x+80=23x^2+46x-184.$$
$$3x^2+36x-324=0.$$
$$x^2+12x-108=0.$$
$$(x-6)(x+18)=0.$$
$$\therefore x=6 \text{ or } -18.$$

29.
$$\frac{2x+1}{1-2x}-\frac{5}{7}=\frac{x-8}{2}.$$
$$28x+14-10+20x=-14x^2+119x-56.$$
$$14x^2-71x=-60.$$
$$56\cdot 14x^2-56\cdot 71x+71^2=56(-60)+71^2=1681.$$
$$28x-71=\pm 41.$$
$$28x=112 \text{ or } 30.$$
$$\therefore x=4 \text{ or } \tfrac{15}{14}.$$

30.
$$\frac{2x-3}{x^2-3x}=2-\frac{3}{x^2-3x}.$$
$$\frac{2x-3+3}{x^2-3x}=2.$$
$$\frac{2}{x-3}=2.$$
$$2=2x-6.$$
$$\therefore x=4.$$

31.
$$x^2-4x-1=0.$$
Transposing,
$$x^2-4x=1.$$
Completing the square,
$$x^2-4x+4=1+4=5.$$
Extracting the square root,
$$x-2=\pm\sqrt{5}=\pm 2.236.$$
$$\therefore x=4.236 \text{ or } -.236.$$

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32. $v^2 + 6v + 7 = 0.$

Transposing, $v^2 + 6v = -7.$

Completing the square, $v^2 + 6v + 9 = -7 + 9 = 2.$

Extracting the square root, $v + 3 = \pm\sqrt{2} = \pm 1.414.$

$\therefore v = -1.586$ or $-4.414.$

33. $u^2 + 5u + 5.5 = 0.$

Transposing, $u^2 + 5u = -5.5.$

Completing the square, $u^2 + 5u + (\frac{5}{2})^2 = -5.5 + \frac{25}{4} = \frac{3}{4}.$

Extracting the square root, $u + \frac{5}{2} = \pm\frac{1}{2}\sqrt{3} = \pm .866.$

$\therefore u = -1.634$ or $-3.366.$

34. $t^2 - 12t + 16.5 = 0.$

Transposing, $t^2 - 12t = -16.5.$

Completing the square, $t^2 - 12t + (6)^2 = -16.5 + 36 = 19.5.$

Extracting the square root, $t - 6 = \pm 4.416.$

$\therefore t = 10.416$ or $1.584.$

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1. $x^2 - ax = ab - bx.$
$x(x-a) = -b(x-a).$
$(x+b)(x-a) = 0.$
$\therefore x = -b$ or $a.$

2. $x^2 + ax = ac + cx.$
$x(x+a) = c(x+a).$
$(x-c)(x+a) = 0.$
$\therefore x = c$ or $-a.$

3. $x^2 = (m-n)x + mn.$
$x^2 - (m-n)x - mn = 0.$
$(x-m)(x+n) = 0.$
$\therefore x = m$ or $-n.$

4. $5x - 2ax = x^2 - 10a.$
$x^2 - 5x + 2ax - 10a = 0.$
$(x-5)(x+2a) = 0.$
$\therefore x = 5$ or $-2a.$

5. $x^2 + 3bx = 5cx + 15bc.$
$x(x+3b) = 5c(x+3b).$
$(x-5c)(x+3b) = 0.$
$\therefore x = 5c$ or $-3b.$

6. $6x^2 + 3ax = 2bx + ab.$
$3x(2x+a) = b(2x+a).$
$(3x-b)(2x+a) = 0.$
$3x - b = 0$ or $2x + a = 0.$
$\therefore x = \frac{b}{3}$ or $-\frac{a}{2}.$

7. $acx^2 - bcx - bd + adx = 0.$
$cx(ax-b) + d(-b+ax) = 0.$
$(cx+d)(ax-b) = 0.$
$ax - b = 0$ or $cx + d = 0.$
$\therefore x = \frac{b}{a}$ or $-\frac{d}{c}.$

8. $x^2 + 4mx + 3nx + 12mn = 0.$
$x(x+4m) + 3n(x+4m) = 0.$
$(x+3n)(x+4m) = 0.$
$\therefore x = -3n$ or $-4m.$

9. $x^2 = 4ax - 2a^2.$
$x^2 - 4ax + 4a^2 = 4a^2 - 2a^2 = 2a^2.$
$x - 2a = \pm a\sqrt{2}.$
$\therefore x = 2a \pm a\sqrt{2} = a(2 \pm \sqrt{2}).$

10. $x^2 - ax + a^2 = 0.$

$$\therefore x = \frac{a \pm \sqrt{a^2 - 4 \cdot 1 \cdot a^2}}{2}$$

$$= \frac{a \pm a\sqrt{-3}}{2}$$

$$= \frac{a}{2}(1 \pm \sqrt{-3}).$$

11. $4ax - x^2 = 3a^2.$
$x^2 - 4ax = -3a^2.$
$x^2 - 4ax + 4a^2 = a^2.$
$x - 2a = \pm a.$
$\therefore x = 3a$ or $a.$

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12. $5ax + 6a^2 = 6x^2.$

$6x^2 - 5ax - 6a^2 = 0.$

$(2x - 3a)(3x + 2a) = 0.$

$2x - 3a = 0 \text{ or } 3x + 2a = 0.$

$\therefore x = \frac{3a}{2} \text{ or } -\frac{2a}{3}.$

13. $21b^2 - 4bx = x^2.$

$x^2 + 4bx - 21b^2 = 0.$

$(x - 3b)(x + 7b) = 0.$

$\therefore x = 3b \text{ or } -7b.$

14. $\frac{7m^2}{12} - mx = \frac{x^2}{3}.$

$7m^2 - 12mx = 4x^2.$

$x^2 + 3mx = \frac{7}{4}m^2.$

$x^2 + 3mx + \left(\frac{3}{2}m\right)^2 = \frac{7}{4}m^2 + \frac{9}{4}m^2$

$= \frac{16m^2}{4}.$

$x + \frac{3}{2}m = \pm 2m.$

$x = \frac{m}{2} \text{ or } -\frac{7m}{2}.$

15. $\frac{x^2}{3b} = \frac{5x}{4} + \frac{b}{3}.$

$4x^2 = 15bx + 4b^2.$

$x^2 - \frac{15}{4}bx = b^2.$

$x^2 - \frac{15b}{4}x + \left(\frac{15b}{8}\right)^2 = b^2 + \left(\frac{15b}{8}\right)^2$

$= \frac{289b^2}{64}.$

$x - \frac{15b}{8} = \pm\frac{17b}{8}.$

$x = 4b \text{ or } -\frac{1}{4}b.$

16. $\frac{x}{x-1} - \frac{x}{x+1} = m.$

$x^2 + x - x^2 + x = mx^2 - m.$

$mx^2 - 2x = m.$

$x^2 - \frac{2}{m}x + \left(\frac{1}{m}\right)^2 = 1 + \frac{1}{m^2} = \frac{1+m^2}{m^2}.$

$x - \frac{1}{m} = \pm\frac{1}{m}\sqrt{1+m^2}.$

$\therefore x = \frac{1}{m}(1 \pm \sqrt{1+m^2}).$

17. $x + \frac{a^2}{x} = \frac{a^2}{b} + b.$

$bx^2 + a^2b = a^2x + b^2x.$

$x^2 - \frac{a^2+b^2}{b}x = -a^2.$

$x^2 - \frac{a^2+b^2}{b}x + \left(\frac{a^2+b^2}{2b}\right)^2$

$= \frac{-4a^2b^2 + a^4 + 2a^2b^2 + b^4}{4b^2}.$

$x - \frac{a^2+b^2}{2b} = \pm\frac{a^2-b^2}{2b}.$

$x = \frac{a^2}{b} \text{ or } b.$

18. $2x - \frac{3x^2}{a} = a - 2x.$

$2ax - 3x^2 = a^2 - 2ax.$

$3x^2 - 4ax = -a^2.$

$x = \frac{4a \pm \sqrt{16a^2 - 4 \cdot 3 \cdot a^2}}{6}.$

$x = \frac{4a \pm 2a}{6} = a \text{ or } \frac{a}{3}.$

19. $\frac{1}{ax+4} = 1 - \frac{ax-4}{16}.$

$16 = 16ax + 64 - a^2x^2 + 16.$

$a^2x^2 - 16ax = 64.$

$a^2x^2 - 16ax + 64 = 64 + 64 = 128.$

$ax - 8 = \pm 8\sqrt{2}.$

$\therefore x = \frac{8}{a}(1 \pm \sqrt{2}).$

20. $x^2 + \frac{a}{b}x = \frac{a+b}{b}.$

$x^2 + \frac{a}{b}x + \left(\frac{a}{2b}\right)^2 = \frac{4ab + 4b^2}{4b^2} + \frac{a^2}{4b^2}$

$= \frac{a^2 + 4ab + 4b^2}{4b^2}.$

$x + \frac{a}{2b} = \pm\frac{a+2b}{2b}.$

$\therefore x = 1 \text{ or } -\frac{a+b}{b}.$

21. $x^2 + 2 = \frac{(2a^2+1)x}{a}.$

$ax^2 + 2a = 2a^2x + x.$

$ax^2 - x = 2a^2x - 2a.$

$x(ax - 1) = 2a(ax - 1).$

$(x - 2a)(ax - 1) = 0.$

$\therefore x = 2a \text{ or } \frac{1}{a}.$

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22.
$$x^2 - \frac{2x}{ab} = \frac{4(ab-1)}{ab}.$$
$$x^2 - \frac{2x}{ab} + \left(\frac{1}{ab}\right)^2 = \frac{4ab(ab-1)}{a^2b^2} + \frac{1}{a^2b^2} = \frac{4a^2b^2 - 4ab + 1}{a^2b^2}.$$
$$x - \frac{1}{ab} = \pm\frac{2ab-1}{ab}.$$
$$x = \frac{1 \pm (2ab-1)}{ab} = 2 \text{ or } \frac{2(1-ab)}{ab}.$$

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23.
$$x^2 - 2(a-b)x = 4ab.$$
$$x^2 - 2(a-b)x + (a-b)^2 = (a-b)^2 + 4ab$$
$$= a^2 + 2ab + b^2.$$
$$x - (a-b) = \pm(a+b).$$
$$\therefore x = a - b + a + b \text{ or } a - b - a - b$$
$$= 2a \text{ or } -2b.$$

24.
$$x^2 - 2x(m-n) = 2mn.$$
$$x^2 - 2(m-n)x + (m-n)^2 = (m-n)^2 + 2mn$$
$$= m^2 + n^2.$$
$$x - (m-n) = \pm\sqrt{m^2+n^2}.$$
$$\therefore x = m - n \pm \sqrt{m^2+n^2}.$$

25.
$$x^2 + 2(a+8)x = -32a.$$
$$x^2 + 2(a+8)x + (a+8)^2 = (a+8)^2 - 32a$$
$$= a^2 - 16a + 64.$$
$$x + a + 8 = \pm(a-8).$$
$$\therefore x = -a - 8 + a - 8 \text{ or } -a - 8 - a + 8$$
$$= -16 \text{ or } -2a.$$

26.
$$x^2 + x + bx + b = a(x+1).$$
$$x(x+1) + b(x+1) = a(x+1).$$
$$(x-a+b)(x+1) = 0.$$
$$x - a + b = 0 \text{ or } x + 1 = 0.$$
$$\therefore x = a - b \text{ or } -1.$$

27.
$$a(2x-1) + 2bx - b = x(2x-1).$$
$$2ax - a + 2bx - b = 2x^2 - x.$$
$$a(2x-1) + b(2x-1) = x(2x-1).$$
$$(a+b-x)(2x-1) = 0.$$
$$a + b - x = 0 \text{ or } 2x - 1 = 0.$$
$$\therefore x = a + b \text{ or } \tfrac{1}{2}.$$

28.
$$x^2 + 4(a-1)x = 8a - 4a^2.$$
$$x^2 + 4(a-1)x + 4(a-1)^2 = 4(a-1)^2 + 8a - 4a^2 = 4.$$
$$x + 2a - 2 = \pm 2.$$
$$\therefore x = -2a + 2 + 2 \text{ or } -2a + 2 - 2$$
$$= -2a + 4 \text{ or } -2a.$$

29.
$$\frac{1}{a+b+x} = \frac{1}{a} + \frac{1}{b} + \frac{1}{x}.$$
$$abx = abx + b^2x + bx^2 + a^2x + abx + ax^2 + a^2b + ab^2 + abx.$$
$$(a+b)x^2 + (a^2 + 2ab + b^2)x = -ab(a+b).$$
$$x^2 + (a+b)x = -ab.$$
$$x^2 + (a+b)x + \frac{(a+b)^2}{4} = \frac{a^2 + 2ab + b^2}{4} - ab = \frac{(a-b)^2}{4}.$$
$$x + \frac{a+b}{2} = \pm\frac{a-b}{2}.$$
$$x = -a \text{ or } -b.$$

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30.
$$\frac{x^2+1}{n^2x-2n}-\frac{1}{2-nx}=\frac{x}{n}.$$
$$\frac{x^2+1}{n(nx-2)}+\frac{1}{nx-2}=\frac{x}{n}.$$
$$x^2+1+n=nx^2-2x.$$
$$(n-1)x^2-2x=n+1.$$
$$x^2-\frac{2}{n-1}x+\left(\frac{1}{n-1}\right)^2=\frac{n+1}{n-1}+\frac{1}{(n-1)^2}=\frac{n^2}{(n-1)^2}.$$
$$x-\frac{1}{n-1}=\pm\frac{n}{n-1}.$$
$$\therefore x=\frac{n+1}{n-1}\text{ or }\frac{1-n}{n-1}.$$
$$x=\frac{n+1}{n-1}\text{ or }-1.$$

31.
$$\frac{2a+x}{2a-x}+\frac{a-2x}{a+2x}=\frac{8}{3}.$$
$$1+\frac{2x}{2a-x}+1-\frac{4x}{a+2x}=2\tfrac{2}{3}.$$
$$\frac{x}{2a-x}-\frac{2x}{a+2x}=\frac{1}{3}.$$
$$3ax+6x^2-12ax+6x^2=2a^2+3ax-2x^2.$$
$$14x^2-12ax-2a^2=0.$$
$$7x^2-6ax-a^2=0.$$
$$(x-a)(7x+a)=0.$$
$$\therefore x=a\text{ or }-\frac{a}{7}.$$

32.
$$\frac{1}{a-x}-\frac{1}{a+x}=\frac{3+x^2}{a^2-x^2}.$$
$$a+x-a+x=3+x^2.$$
$$x^2-2x=-3.$$
$$x^2-2x+1=-2.$$
$$x-1=\pm\sqrt{-2}.$$
$$\therefore x=1\pm\sqrt{-2}.$$

33. $a(x-2a+b)+a(x+a-b)=x^2-(a-b)^2.$
$$a(2x-a)=x^2-a^2+2ab-b^2.$$
$$2ax-a^2=x^2-a^2+2ab-b^2.$$
$$x^2-2ax=-2ab+b^2.$$
$$x^2-2ax+a^2=a^2-2ab+b^2.$$
$$x-a=a-b\text{ or }-a+b.$$
$$\therefore x=2a-b\text{ or }b.$$

34.
$$\frac{x^2}{a+b}-\left(1+\frac{1}{ab}\right)x+\frac{1}{a}+\frac{1}{b}=0.$$
$$abx^2-(a^2b+a+ab^2+b)x+(a+b)^2=0.$$
$$abx^2-(ab+1)(a+b)x+(a+b)^2=0.$$
$$x=\frac{(ab+1)(a+b)\pm\sqrt{(ab+1)^2(a+b)^2-4ab(a+b)^2}}{2ab}.$$

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$$x=\frac{(ab+1)(a+b)\pm\sqrt{(a+b)^2(ab-1)^2}}{2ab}.\quad \textbf{That is,}$$

$$x=\frac{a^2b+a+ab^2+b+a^2b-a+ab^2-b}{2ab}=\frac{2a^2b+2ab^2}{2ab}=a+b$$

$$\text{or } x=\frac{a^2b+a+ab^2+b-a^2b+a-ab^2+b}{2ab}=\frac{2a+2b}{2ab}=\frac{a+b}{ab}.$$

35.
$$\frac{x^2+1}{x}-\frac{a+b}{c}=\frac{c}{a+b}.$$

$$\frac{x^2+1}{x}-\frac{(a+b)^2+c^2}{c(a+b)}=0.$$

$$x^2+1-\frac{(a+b)^2+c^2}{c(a+b)}x=0.$$

$$x^2-\frac{(a+b)^2+c^2}{c(a+b)}x=-1.$$

$$x^2-\frac{(a+b)^2+c^2}{c(a+b)}x+\left[\frac{(a+b)^2+c^2}{2c(a+b)}\right]^2=\frac{(a+b)^4+2c^2(a+b)^2+c^4}{4c^2(a+b)^2}-\frac{4c^2(a+b)^2}{4c^2(a+b)^2}$$

$$=\frac{(a+b)^4-2c^2(a+b)^2+c^4}{4c^2(a+b)^2}.$$

$$x-\frac{(a+b)^2+c^2}{2c(a+b)}=\pm\frac{(a+b)^2-c^2}{2c(a+b)}.$$

$$\therefore x=\frac{(a+b)^2+c^2+(a+b)^2-c^2}{2c(a+b)}\text{ or }\frac{(a+b)^2+c^2-(a+b)^2+c^2}{2c(a+b)}$$

$$=\frac{a+b}{c}\text{ or }\frac{c}{a+b}.$$

36.
$$\frac{2x-a}{b}+3=\frac{4a}{2x-b}.$$

$$4x^2-2ax-2bx+ab+6bx-3b^2=4ab.$$

$$4x^2-2ax+4bx=3b^2+3ab.$$

$$4x^2-2x(a-2b)=3b^2+3ab.$$

$$16x^2-8x(a-2b)+(a-2b)^2=12b^2+12ab+a^2-4ab+4b^2$$

$$=a^2+8ab+16b^2.$$

$$4x-(a-2b)=\pm(a+4b).$$

$$\therefore x=\frac{a-2b+a+4b}{4}\text{ or }\frac{a-2b-a-4b}{4}=\frac{a+b}{2}\text{ or }\frac{-3b}{2}.$$

37.
$$\frac{bx}{a-x}+b=\frac{a(x+2b)}{a+b}.$$

$$abx+b^2x+a^2b-abx+ab^2-b^2x=a^2x+2a^2b-ax^2-2abx.$$

$$ax^2-a^2x+2abx=a^2b-ab^2.$$

$$x^2-ax+2bx=ab-b^2.$$

$$4x^2-4x(a-2b)+(a-2b)^2=4ab-4b^2+a^2-4ab+4b^2=a^2.$$

$$2x-(a-2b)=\pm a.$$

$$\therefore x=\frac{a-2b+a}{2}\text{ or }\frac{a-2b-a}{2}=a-b\text{ or }-b.$$

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38.
$$\frac{3x+b}{x+b}=\frac{b}{2x-a}+\frac{1}{1+\dfrac{x-a}{a+b}}.$$
$$3-\frac{2b}{x+b}=\frac{b}{2x-a}+\frac{a+b}{x+b}.$$
$$6x^2-3ax+6bx-3ab-4bx+2ab=bx+b^2+2ax-a^2+2bx-ab.$$
$$6x^2-(5a+b)x=b^2-a^2.$$
$$x^2-\frac{5a+b}{6}x+\left(\frac{5a+b}{12}\right)^2=\frac{b^2-a^2}{6}+\frac{25a^2+10ab+b^2}{144}=\frac{(a+5b)^2}{144}.$$
$$x-\frac{5a+b}{12}=\pm\frac{a+5b}{12}.$$
$$\therefore\ x=\frac{a+b}{2}\text{ or }\frac{a-b}{3}.$$

39.
$$\frac{x^4}{a^2}+\left(x+\frac{ab}{x}\right)^2-\left(\frac{x^2}{a}+\frac{ab}{x}\right)^2=ax.$$
$$\frac{x^4}{a^2}+\frac{x^4+2abx^2+a^2b^2}{x^2}-\frac{x^6+2a^2bx^3+a^4b^2}{a^2x^2}=ax.$$
$$\frac{x^4}{a^2}+x^2+2ab+\frac{a^2b^2}{x^2}-\frac{x^4}{a^2}-2bx-\frac{a^2b^2}{x^2}=ax.$$
$$x^2-(a+2b)x=-2ab.$$
$$x^2-(a+2b)x+\frac{(a+2b)^2}{4}=\frac{a^2+4ab+4b^2}{4}-2ab=\frac{(a-2b)^2}{4}.$$
$$x-\frac{a+2b}{2}=\pm\frac{a-2b}{2}.$$
$$\therefore\ x=a\text{ or }2b.$$

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3.
$$8\sqrt{x}-8x=\tfrac{3}{2}.$$
$$16\sqrt{x}=16x+3.$$
$$256x=256x^2+96x+9.$$
$$256x^2-160x+25=-9+25=16.$$
$$16x-5=\pm 4.$$
$$16x=9\text{ or }1.$$
$$x=\tfrac{9}{16}\text{ or }\tfrac{1}{16}.$$

VERIFICATION. — Substituting $\frac{9}{16}$ for x in the given equation,
$$8\sqrt{\tfrac{9}{16}}-\tfrac{9}{2}=\tfrac{3}{2},$$
which reduces to $\frac{3}{2}=\frac{3}{2}$.

Hence, $\frac{9}{16}$ is a root of the equation.

Substituting $\frac{1}{16}$ for x in the given equation,
$$8\sqrt{\tfrac{1}{16}}-\tfrac{1}{2}=\tfrac{3}{2},$$
which reduces to $\frac{3}{2}=\frac{3}{2}$.

Hence, $\frac{1}{16}$ is a root of the equation.

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4. $$3x + \sqrt{x} = 5\sqrt{4x}.$$

Since $\sqrt{x}$ is a factor of both members, $x = 0$ derived from $\sqrt{x} = 0$, satisfies the equation, and hence 0 is a root of the equation.

Dividing by $\sqrt{x}$, $$3\sqrt{x} + 1 = 5\sqrt{4}.$$

$$3\sqrt{x} = 10 - 1 = 9.$$

$$\sqrt{x} = 3.$$

$$\therefore\ x = 9.$$

VERIFICATION. — Substituting 9 for x in the given equation,

$$27 + \sqrt{9} = 5\sqrt{36},$$

which reduces to $$30 = 30.$$

Hence, 9 is a root of the equation.

5. $$x - 1 + \sqrt{x+5} = 0.$$

$$x - 1 = -\sqrt{x+5}.$$

$$x^2 - 2x + 1 = +(x+5).$$

$$x^2 - 3x - 4 = 0.$$

$$(x+1)(x-4) = 0.$$

$$\therefore\ x = -1 \text{ or } 4.$$

VERIFICATION. — Substituting -1 for x in the given equation,

$$-1 - 1 + \sqrt{-1+5} = 0,$$

which reduces to $$0 = 0.$$

Hence, -1 is a root of the equation.

Substituting 4 for x in the given equation,

$$4 - 1 + \sqrt{4+5} = 3 + 3 = 6 \neq 0.$$

Hence, 4 is not a root of the given equation.

6. $$x - 5 - \sqrt{x-3} = 0.$$

$$x - 5 = \sqrt{x-3}.$$

$$x^2 - 10x + 25 = x - 3.$$

$$x^2 - 11x + 28 = 0.$$

$$(x-4)(x-7) = 0.$$

$$\therefore\ x = 4 \text{ or } 7.$$

VERIFICATION. — Substituting 4 for x in the given equation,

$$4 - 5 - \sqrt{4-3} = -1 - 1 = -2 \neq 0.$$

Hence, 4 is not a root of the given equation.

Substituting 7 for x in the given equation,

$$7 - 5 - \sqrt{7-3} = 0,$$

which reduces to $$0 = 0.$$

Hence, 7 is a root of the equation.

7. $$\sqrt{4x+17} + \sqrt{x+1} - 4 = 0.$$

$$\sqrt{4x+17} = 4 - \sqrt{x+1}.$$

$$4x + 17 = 16 - 8\sqrt{x+1} + x + 1.$$

$$3x = -8\sqrt{x+1}.$$

$$9x^2 = 64x + 64.$$

$$81x^2 - 576x = 576.$$

$$81x^2 - 576x + 1024 = 1600.$$

$$9x - 32 = \pm 40.$$

$$9x = 72 \text{ or } -8.$$

$$\therefore\ x = 8 \text{ or } -\tfrac{8}{9}.$$

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VERIFICATION. — Substituting 8 for x in the given equation,

$$\sqrt{49}+\sqrt{9}-4=7+3-4=6\neq 0.$$

Hence, 8 is not a root of the given equation.

Substituting $-\frac{8}{9}$ for x in the given equation,

$$\sqrt{\tfrac{121}{9}}+\sqrt{\tfrac{1}{9}}-4=0,$$

which reduces to $$0=0.$$

Hence, $-\frac{8}{9}$ is a root of the equation.

8. $$1+\sqrt{(3-5x)^2+16}=2(3-x).$$

$$\sqrt{(3-5x)^2+16}=5-2x.$$

$$9-30x+25x^2+16=25-20x+4x^2.$$

$$21x^2-10x=0.$$

$$x(21x-10)=0.$$

$$\therefore\ x=0 \text{ or } \tfrac{10}{21}.$$

VERIFICATION. — Substituting 0 for x in the given equation,

$$1+\sqrt{3^2+16}=2(3).$$

$$1+5=6.$$

Hence, 0 is a root of the given equation.

Substituting $\frac{10}{21}$ for x in the given equation,

$$1+\sqrt{(3-\tfrac{50}{21})^2+16}=2(3-\tfrac{10}{21})$$

$$\tfrac{106}{21}=\tfrac{106}{21}.$$

Hence, $\frac{10}{21}$ is a root of the given equation.

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9. $$\sqrt{1+x\sqrt{x^2+12}}=1+x. \qquad (1)$$

Squaring, $$1+x\sqrt{x^2+12}=1+2x+x^2. \qquad (2)$$

Canceling $1=1$ and dividing by x, $$\sqrt{x^2+12}=2+x. \qquad (3)$$

Squaring, $$x^2+12=4+4x+x^2.$$

Canceling $x^2=x^2$, $$12=4+4x$$

Solving, $$x=2.$$

Since equation (3) was divided by x, $x=0$ is another value.

VERIFICATION. — Substituting 2 for x in the given equation,

$$\sqrt{1+2\sqrt{4+12}}=1+2.$$

$$3=3.$$

Hence, 2 is a root of the given equation.

Substituting 0 for x in the given equation,

$$\sqrt{1+0}=1+0.$$

$$1=1.$$

Hence, 0 is a root of the given equation.

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10. $$\sqrt{x-1}+\sqrt{2x-1}-\sqrt{5x}=0.$$
$$\sqrt{x-1}-\sqrt{5x}=-\sqrt{2x-1}.$$
$$x-1-2\sqrt{5x^2-5x}+5x=+(2x-1).$$
$$\sqrt{5x^2-5x}=2x.$$
$$5x^2-5x=4x^2.$$
$$x(x-5)=0.$$
$$\therefore x=0 \text{ or } 5.$$

VERIFICATION. — Substituting 0 for x in the given equation,
$$\sqrt{-1}+\sqrt{-1}\neq 0.$$
Hence, 0 is not a root of the given equation.

Substituting 5 for x in the given equation,
$$\sqrt{4}+\sqrt{9}-\sqrt{25}=0,$$
which reduces to $$0=0.$$
Hence, 5 is a root of the equation.

11. $$\sqrt{2x-7}-\sqrt{2x}+\sqrt{x-7}=0.$$
$$\sqrt{x-7}-\sqrt{2x}=-\sqrt{2x-7}.$$
$$x-7-2\sqrt{2x^2-14x}+2x=+(2x-7).$$
$$2\sqrt{2x^2-14x}=x.$$
$$8x^2-56x=x^2.$$
$$7x(x-8)=0.$$
$$\therefore x=0 \text{ or } 8.$$

VERIFICATION. — Substituting 0 for x in the given equation,
$$\sqrt{-7}+\sqrt{-7}\neq 0.$$
Hence, 0 is not a root of the given equation.

Substituting 8 for x in the given equation,
$$\sqrt{9}-\sqrt{16}+\sqrt{1}=0,$$
which reduces to $$0=0.$$
Hence, 8 is a root of the equation.

12. $$\sqrt{x+3}+\sqrt{4x+1}-\sqrt{10x+4}=0.$$
$$\sqrt{x+3}+\sqrt{4x+1}=\sqrt{10x+4}.$$
$$x+3+2\sqrt{4x^2+13x+3}+4x+1=10x+4.$$
$$2\sqrt{4x^2+13x+3}=5x.$$
$$16x^2+52x+12=25x^2.$$
$$9x^2-52x=12.$$
$$81x^2-468x+26^2=108+676=784.$$
$$9x-26=\pm 28.$$
$$\therefore x=6 \text{ or } -\tfrac{2}{9}.$$

VERIFICATION. — Substituting 6 for x in the given equation,
$$\sqrt{9}+\sqrt{25}-\sqrt{64}=0,$$
which reduces to $$0=0.$$
Hence, 6 is a root of the equation.

Substituting $-\frac{2}{9}$ for x in the given equation,
$$\sqrt{\tfrac{25}{9}}+\sqrt{\tfrac{1}{9}}-\sqrt{\tfrac{16}{9}}=\tfrac{2}{3}\neq 0.$$
Hence, $-\frac{2}{9}$ is not a root of the given equation.

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13.
$$\sqrt{a+x}-\sqrt{a-x}=\sqrt{2x}.$$
$$a+x-2\sqrt{a^2-x^2}+a-x=2x.$$
$$2a-2x=2\sqrt{a^2-x^2}.$$
$$a-x=\sqrt{a^2-x^2}.$$
$$a^2-2ax+x^2=a^2-x^2.$$
$$2x^2-2ax=0.$$
$$x(x-a)=0.$$
$$\therefore x=0 \text{ or } a.$$

VERIFICATION. — Substituting 0 for x in the given equation,
$$\sqrt{a}-\sqrt{a}=0.$$
Hence, 0 is a root of the given equation.
Substituting a for x in the given equation,
$$\sqrt{2a}-\sqrt{a-a}=\sqrt{2a}.$$
$$\sqrt{2a}=\sqrt{2a}.$$
Hence, a is a root of the given equation.

14.
$$\sqrt{x-a}+\sqrt{b-x}=\sqrt{b-a}.$$
$$x-a+2\sqrt{(x-a)(b-x)}+b-x=b-a.$$
$$2\sqrt{(x-a)(b-x)}=0.$$
$$\sqrt{(x-a)(b-x)}=0.$$
$$(x-a)(b-x)=0.$$
$$\therefore x=a \text{ or } b.$$

VERIFICATION. — Substituting a for x in the given equation,
$$\sqrt{a-a}+\sqrt{b-a}=\sqrt{b-a}.$$
$$\sqrt{b-a}=\sqrt{b-a}.$$
Hence, a is a root of the given equation.
Substituting b for x in the given equation,
$$\sqrt{b-a}=\sqrt{b-a}.$$
Hence, b is a root of the given equation.

15.
$$\sqrt{x^2-b^2}=\sqrt{x+b}\sqrt{a+b}.$$
$$x^2-b^2=(x+b)(a+b).$$
$$(x+b)(x-b)=(x+b)(a+b).$$
$$(x-b-a-b)(x+b)=0.$$
$$(x-a-2b)(x+b)=0.$$
$$\therefore x=a+2b \text{ or } -b.$$

VERIFICATION. — Substituting $a+2b$ for x in the given equation,
$$\sqrt{a^2+4ab+4b^2-b^2}=\sqrt{a+3b}\sqrt{a+b}.$$
$$\sqrt{a^2+4ab+3b^2}=\sqrt{a^2+4ab+3b^2}.$$
Hence, $a+2b$ is a root of the given equation.
Substituting $-b$ for x in the given equation,
$$\sqrt{b^2-b^2}=\sqrt{-b+b}\sqrt{a+b} \text{ or } 0=0.$$
Hence, $-b$ is a root of the given equation.

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16. $$\sqrt{2x+\sqrt{10x+1}}=\sqrt{2x}+1.$$

Squaring, $$2x+\sqrt{10x+1}=2x+2\sqrt{2x}+1.$$

Canceling $2x=2x$, $$\sqrt{10x+1}=2\sqrt{2x}+1.$$

Squaring, $$10x+1=8x+4\sqrt{2x}+1.$$

Transposing, etc., $$2x=4\sqrt{2x}.$$

Squaring, $$4x^2=32x.$$

Factoring, $$4x(x-8)=0.$$

$$\therefore x=0 \text{ or } 8.$$

VERIFICATION. — Substituting 0 for x in the given equation,

$$1=1.$$

Hence, 0 is a root of the given equation.

Substituting 8 for x in the given equation,

$$\sqrt{16+\sqrt{81}}=\sqrt{16}+1.$$

$$5=5.$$

Hence, 8 is a root of the given equation.

17. $$\sqrt{6+x}+\sqrt{x}-\sqrt{10-4x}=0.$$

$$\sqrt{6+x}+\sqrt{x}=\sqrt{10-4x}.$$

$$6+x+2\sqrt{6x+x^2}+x=10-4x.$$

$$\sqrt{6x+x^2}=2-3x.$$

$$6x+x^2=4-12x+9x^2.$$

$$4x^2-9x+2=0.$$

$$(x-2)(4x-1)=0.$$

$$\therefore x=2 \text{ or } \tfrac{1}{4}.$$

VERIFICATION. — Substituting 2 for x in the given equation,

$$\sqrt{8}+\sqrt{2}-\sqrt{2}\neq 0.$$

Hence, 2 is not a root of the given equation.

Substituting $\frac{1}{4}$ for x in the given equation,

$$\sqrt{\tfrac{25}{4}}+\sqrt{\tfrac{1}{4}}-\sqrt{9}=0,$$

which reduces to $$0=0.$$

Hence, $\frac{1}{4}$ is a root of the equation.

18. $$\sqrt{4x-3}-\sqrt{2x+2}=\sqrt{x-6}.$$

$$4x-3-2\sqrt{8x^2+2x-6}+2x+2=x-6.$$

$$2\sqrt{8x^2+2x-6}=5x+5.$$

$$32x^2+8x-24=25x^2+50x+25.$$

$$7x^2-42x-49=0.$$

$$x^2-6x-7=0.$$

$$(x-7)(x+1)=0.$$

$$\therefore x=7 \text{ or } -1.$$

VERIFICATION. — Substituting 7 for x in the given equation,

$$\sqrt{25}-\sqrt{16}=\sqrt{1},$$

which reduces to $$1=1.$$

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Hence, 7 is a root of the equation.

Substituting -1 for x in the given equation,

$$\sqrt{-7}-\sqrt{0}=\sqrt{-7}.$$

Hence, -1 is also a root of the equation.

19. $$\sqrt{2x+3}-\sqrt{x+1}=\sqrt{5x-14}.$$

$$2x+3-2\sqrt{2x^2+5x+3}+x+1=5x-14.$$

$$-\sqrt{2x^2+5x+3}=x-9.$$

$$2x^2+5x+3=x^2-18x+81.$$

$$x^2+23x-78=0.$$

$$(x-3)(x+26)=0.$$

$$\therefore x=3 \text{ or } -26.$$

VERIFICATION.—Substituting 3 for x in the given equation,

$$\sqrt{9}-\sqrt{4}=\sqrt{1},$$

which reduces to $$1=1.$$

Hence, 3 is a root of the equation.

Substituting -26 for x in each member of the given equation, we find $$\sqrt{-49}-\sqrt{-25}=2\sqrt{-1}\neq 12\sqrt{-1}.$$

Hence, -26 is not a root of the given equation.

20. $$\sqrt{3x-5}+\sqrt{x-9}=\sqrt{4x-4}.$$

$$3x-5+2\sqrt{3x^2-32x+45}+x-9=4x-4.$$

$$\sqrt{3x^2-32x+45}=5.$$

$$3x^2-32x+45=25.$$

$$3x^2-32x=-20.$$

$$9x^2-96x+256=-60+256=196.$$

$$3x-16=\pm 14.$$

$$\therefore x=10 \text{ or } \tfrac{2}{3}.$$

VERIFICATION.—Substituting 10 for x in the given equation,

$$\sqrt{25}+\sqrt{1}=\sqrt{36},$$

which reduces to $$6=6.$$

Hence, 10 is a root of the equation.

Substituting $\frac{2}{3}$ for x in the given equation,

$$\sqrt{-3}+\tfrac{5}{3}\sqrt{-3}=\tfrac{8}{3}\sqrt{-3}\neq\tfrac{2}{3}\sqrt{-3}.$$

Hence, $\frac{2}{3}$ is not a root of the given equation.

21. $$\sqrt{x^2+8}-\frac{6}{\sqrt{x^2+8}}=x.$$

Clearing of fractions, $$x^2+8-6=x\sqrt{x^2+8}.$$

$$x^2+2=x\sqrt{x^2+8}.$$

Squaring, $$x^4+4x^2+4=x^4+8x^2.$$

$$4x^2=4.$$

$$\therefore x=1 \text{ or } -1.$$

VERIFICATION.—Substituting 1 for x in the given equation,

$$\sqrt{1+8}-\frac{6}{\sqrt{1+8}}=1.$$

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Hence, 1 is a root of the given equation.

Substituting -1 for x in the given equation,

$$\sqrt{1+8}-\frac{6}{\sqrt{1+8}}=1\neq-1.$$

Hence, -1 is not a root of the given equation.

22.
$$x+\sqrt{x^2+m^2}=\frac{2m^2}{\sqrt{x^2+m^2}}.$$

Clearing of fractions, $x\sqrt{x^2+m^2}+x^2+m^2=2m^2.$

Transposing, $x\sqrt{x^2+m^2}=m^2-x^2.$

Squaring, $x^4+x^2m^2=m^4-2m^2x^2+x^4.$

$$3m^2x^2=m^4.$$

$$\therefore x=\pm\frac{m}{3}\sqrt{3}.$$

VERIFICATION. — Substituting $\frac{m}{3}\sqrt{3}$ for x in the given equation,

$$\frac{m}{3}\sqrt{3}+\sqrt{\frac{m^2}{3}+m^2}=\frac{2m^2}{\sqrt{\frac{m^2}{3}+m^2}}.$$

$$m\sqrt{3}=m\sqrt{3}.$$

Hence, $\frac{m}{3}\sqrt{3}$ is a root of the given equation.

Substituting $-\frac{m}{3}\sqrt{3}$ for x in the given equation,

$$-\frac{m}{3}\sqrt{3}+\sqrt{\frac{4m^2}{3}}=\frac{m}{3}\sqrt{3}\neq m\sqrt{3}.$$

Hence, $-\frac{m}{3}\sqrt{3}$ is not a root of the given equation.

23.
$$x+\sqrt{x^2-a^2}=\frac{a^2}{\sqrt{x^2-a^2}}.$$

Clearing of fractions, $x\sqrt{x^2-a^2}+x^2-a^2=a^2.$

Transposing, etc., $x\sqrt{x^2-a^2}=2a^2-x^2.$

Squaring, $x^4-a^2x^2=4a^4-4a^2x^2+x^4.$

Canceling, etc., $3a^2x^2=4a^4.$

Dividing by $3a^2$, $x^2=\frac{4a^2}{3}.$

Extracting the square root, $x=\pm\frac{2}{3}a\sqrt{3}.$

VERIFICATION. — Substituting $\frac{2}{3}a\sqrt{3}$ for x in the given equation,

$$\frac{2}{3}a\sqrt{3}+\sqrt{\frac{4}{3}a^2-a^2}=\frac{a^2}{\sqrt{\frac{4}{3}a^2-a^2}}.$$

$$a\sqrt{3}=a\sqrt{3}.$$

Hence, $\frac{2}{3}a\sqrt{3}$ is a root of the given equation.

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Substituting $-\frac{2}{3}a\sqrt{3}$ for x in the given equation,

$$-\tfrac{2}{3}a\sqrt{3}+\sqrt{\tfrac{4}{3}a^2-a^2}=-\frac{a}{3}\sqrt{3}\neq a\sqrt{3}.$$

Hence, $-\frac{2}{3}a\sqrt{3}$ is not a root of the given equation.

24.
$$\frac{2x+\sqrt{4x^2-1}}{2x-\sqrt{4x^2-1}}=4.$$

Clearing of fractions, $\quad 2x+\sqrt{4x^2-1}=8x-4\sqrt{4x^2-1}.$

Transposing, etc., $\quad 5\sqrt{4x^2-1}=6x.$

Squaring, $\quad 100x^2-25=36x^2.$

$$\therefore x^2=\tfrac{25}{64}.$$

Extracting the square root, $\quad x=\pm\frac{5}{8}.$

VERIFICATION. — Substituting $\frac{5}{8}$ for x in the given equation,

$$\frac{\frac{5}{4}+\sqrt{\frac{25}{16}-1}}{\frac{5}{4}-\sqrt{\frac{25}{16}-1}}=4.$$
$$4=4.$$

Hence, $\frac{5}{8}$ is a root of the given equation.

Substituting $-\frac{5}{8}$ for x in the given equation,

$$\frac{-\frac{5}{4}+\sqrt{\frac{25}{16}-1}}{-\frac{5}{4}-\sqrt{\frac{25}{16}-1}}=\frac{1}{4}\neq 4.$$

Hence, $-\frac{5}{8}$ is not a root of the given equation.

25.
$$\sqrt{\frac{x-a}{x+a}}+\sqrt{\frac{x+a}{x-a}}=a^2.$$

Multiplying by $\sqrt{x+a}\sqrt{x-a}$, or $\sqrt{x^2-a^2}$,

$$x-a+x+a=a^2\sqrt{x^2-a^2}.$$
$$2x=a^2\sqrt{x^2-a^2}.$$

Squaring, $\quad 4x^2=a^4x^2-a^6.$

Transposing, etc., $\quad x^2(a^4-4)=a^6.$

$$\therefore x^2=\frac{a^6}{a^4-4}.$$

Extracting the square root, $\quad x=\pm\dfrac{a^3}{a^4-4}\sqrt{a^4-4}.$

VERIFICATION. — Substituting $\dfrac{a^3}{a^4-4}\sqrt{a^4-4}$ for x in the given equation,

$$\sqrt{\frac{\dfrac{a^3}{\sqrt{a^4-4}}-a}{\dfrac{a^3}{\sqrt{a^4-4}}+a}}+\sqrt{\frac{\dfrac{a^3}{\sqrt{a^4-4}}+a}{\dfrac{a^3}{\sqrt{a^4-4}}-a}},$$

$$\sqrt{\frac{a^2-\sqrt{a^4-4}}{a^2+\sqrt{a^4-4}}}+\sqrt{\frac{a^2+\sqrt{a^4-4}}{a^2-\sqrt{a^4-4}}},$$

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$$\sqrt{\frac{(a^2-\sqrt{a^4-4})(a^2-\sqrt{a^4-4})}{(a^2+\sqrt{a^4-4})(a^2-\sqrt{a^4-4})}}+\sqrt{\frac{(a^2+\sqrt{a^4-4})(a^2+\sqrt{a^4-4})}{(a^2+\sqrt{a^4-4})(a^2-\sqrt{a^4-4})}},$$

$$\frac{a^2-\sqrt{a^4-4}}{2}+\frac{a^2+\sqrt{a^4-4}}{2}=\frac{2a^2}{2}=a^2.$$

Hence, $\frac{a^8}{a^4-4}\sqrt{a^4-4}$ is a root of the given equation.

Similarly, it may be shown that $-\frac{a^8}{a^4-4}\sqrt{a^4-4}$ is a root of the equation.

26. $$\sqrt{a-x}+\sqrt{b-x}=\sqrt{a+b-2x}.$$

Squaring, $a-x+2\sqrt{ab-ax-bx+x^2}+b-x=a+b-2x.$

$$2\sqrt{ab-ax-bx+x^2}=0.$$

Squaring, $$ab-ax-bx+x^2=0.$$

$$x^2-(a+b)x+\left(\frac{a+b}{2}\right)^2=-ab+\left(\frac{a+b}{2}\right)^2=\frac{(a-b)^2}{4}.$$

$$x-\left(\frac{a+b}{2}\right)=\pm\frac{a-b}{2}.$$

$$\therefore x=a \text{ or } b.$$

Verification. — Substituting a for x in the given equation,

$$\sqrt{a-a}+\sqrt{b-a}=\sqrt{a+b-2a}.$$

$$\sqrt{b-a}=\sqrt{b-a}.$$

Hence, a is a root of the given equation.

Substituting b for x in the given equation,

$$\sqrt{a-b}+\sqrt{b-b}=\sqrt{a+b-2b}, \text{ or } \sqrt{a-b}=\sqrt{a-b}.$$

Hence, b is a root of the given equation.

27. $$\sqrt{x+a^2}-\sqrt{x-2a^2}=\sqrt{2x-5a^2}.$$

$$x+a^2-2\sqrt{x^2-a^2x-2a^4}+x-2a^2=2x-5a^2.$$

$$2\sqrt{x^2-a^2x-2a^4}=-4a^2.$$

$$x^2-a^2x-2a^4=4a^4.$$

$$x^2-a^2x+\left(\frac{a^2}{2}\right)^2=6a^4+\frac{a^4}{4}=\frac{25a^4}{4}.$$

$$x-\frac{a^2}{2}=\pm\frac{5a^2}{2}.$$

$$\therefore x=3a^2 \text{ or } -2a^2.$$

Verification. — Substituting $3a^2$ for x in the given equation,

$$\sqrt{4a^2}-\sqrt{a^2}=\sqrt{a^2}.$$

$$a=a.$$

Hence, $3a^2$ is a root of the given equation.

Substituting $-2a^2$ for x in the given equation,

$$\sqrt{-a^2}-\sqrt{-4a^2}=-a\sqrt{-1}\neq 3a\sqrt{-1}.$$

Hence, $-2a^2$ is not a root of the given equation.

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28. $$\sqrt{mn-x}-\sqrt{x}\sqrt{mn-1}=\sqrt{mn}\sqrt{1-x}. \quad (1)$$

Squaring,

$$mn-x-2\sqrt{x(mn-x)(mn-1)}+mnx-x=mn-mnx. \quad (2)$$

Transposing and uniting terms, and dividing by 2,

$$mnx-x=\sqrt{x(mn-x)(mn-1)}. \quad (3)$$

Dividing by $\sqrt{mnx-x}$, $\sqrt{mnx-x}=\sqrt{mn-x}$.

Squaring, $mnx-x=mn-x$.

$$\therefore x=1.$$

Since equation (3) was divided by $\sqrt{mnx-x}$, by placing this equal to 0 and squaring, $x=0$ is found to be another value.

VERIFICATION. — Substituting 1 for x in the given equation,

$$\sqrt{mn-1}-\sqrt{mn-1}=0, \text{ or } 0=0.$$

Hence, 1 is a root of the given equation.

Substituting 0 for x in the given equation,

$$\sqrt{mn}=\sqrt{mn}.$$

Hence, 0 is a root of the given equation.

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2. Let $x=$ one part.

Then, $20-x=$ the other part.

Since their product is 96, $20x-x^2=96$.

Solving, $x=12$ or 8;

whence, $20-x=8$ or 12.

Hence, the parts are 12 and 8.

3. Let $x=$ one part.

Then, $14-x=$ the other part.

Since their product is 45, $14x-x^2=45$.

Solving, $x=9$ or 5;

whence, $14-x=5$ or 9.

Hence, the parts are 9 and 5.

4. Let $x=$ smaller number.

Then, $x+1=$ larger number.

$$\therefore x^2+(x+1)^2=61.$$

Solving, $x=5$ or -6;

whence, $x+1=6$ or -5.

Hence, the numbers are 5 and 6, or -6 and -5.

5. Let $x=$ number of rods in the length.

Then $x-12=$ number of rods in the breadth.

$$\therefore x(x-12)=160.$$

Solving, $x=20$ or -8;

whence, $x-12=8$ or -20.

The second value of x is inadmissible, since the side of a rectangle cannot be a negative number.

Hence, the field is 20 rods long and 8 rods wide.

6. Let $x=$ number of cents plumber earned per hour.

Then, $x-20=$ number of hours plumber worked.

$$\therefore x(x-20)=2400.$$

Solving, $x=60$ or -40.

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The second value of x is inadmissible, since there could not be a negative number of cents earned.

Hence, the plumber earned 60 cents per hour.

7. Let $x =$ number of boxes.

Then, $\frac{1}{2}x - 1 =$ number of bunches in a box.

$$\therefore x(\tfrac{1}{2}x - 1) = 1860.$$

Solving, $x = 62$ or -60,

and $\frac{1}{2}x - 1 = 30$ or -31.

The second value of x and of $\frac{1}{2}x - 1$ are inadmissible, since there cannot be a negative number of boxes or bunches.

Hence, there are 30 bunches in one box.

8. Let $x =$ number of feet in width.

Then, $x + 30 =$ number of feet in length.

$$\therefore x(x + 30 - 10) = 3500.$$

Solving, $x = 50$ or -70,

and $x + 30 = 80$ or -40.

Since the length and width cannot be negative numbers, the second values are inadmissible. Considering the first values, then

$$x(x + 30) - x(x + 20) = 4000 - 3500 = 500.$$

Hence, the grounds were laid out 500 square feet too large.

9. Let $x =$ number of inches in middle dimension.

Then, $9 + x + 13 =$ number of inches in greatest dimension.

$$\therefore 9 \cdot x(9 + x + 13) = 2880.$$

Solving, $x = 10$ or -32.

$$\therefore 9 + x + 13 = 32 \text{ or } -10.$$

Since there cannot be a negative number of inches in any dimension, the negative values are inadmissible.

Hence, the three dimensions are 9 inches, 10 inches, and 32 inches.

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11. Let $x =$ number of persons in the club.

Then, $\frac{60}{x} =$ number of dollars each paid.

$$\therefore \frac{60}{x + 5} = \frac{60}{x} - 1.$$

Solving, $x = 15$ or -20.

The second value of x is inadmissible, since there could not be a negative number of persons.

Hence, there were 15 persons in the club.

12. Let $x =$ number of persons on the ride.

Then, $\frac{800}{x} =$ number of cents each person paid.

$$\therefore \left(\frac{800}{x} - 10\right)(x + 4) = 800.$$

Solving, $x = 16$ or -20.

The second value is inadmissible, since there could not be a negative number of persons.

Hence, there were 16 people on the ride.

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13. Let $x =$ number of pounds in 1 tub of dairy butter.
Then, $x + 20 =$ number of pounds in 1 tub of creamery butter.

$$\therefore \frac{360}{x} - 3 = \frac{360}{x+20}.$$

Solving, $x = 40$ or -60,
and $x + 20 = 60$ or -40.

The second values are inadmissible, since there could not be a negative number of pounds.

Hence, a tub of dairy butter weighs 40 pounds, and a tub of creamery butter weighs 60 pounds.

14. Let $x =$ number of inches in length of each picture.

Then, $\frac{1800}{x} =$ number of pictures.

$$\therefore \frac{1800}{x+\frac{1}{4}} + 600 = \frac{1800}{x}.$$

Solving, $x = \frac{3}{4}$ or -1.

The second value is inadmissible, since there could not be a negative number of inches in length of picture.

Hence, each picture is $\frac{3}{4}$ of an inch long.

15. Let $x =$ number of square yards 1 pound covers in first coat.

Then, $x + \frac{3}{2} =$ number of square yards 1 pound covers in second coat.

$$\therefore \frac{195}{x} + \frac{195}{x+\frac{3}{2}} = 69.$$

Solving, $x = 5$ or $-\frac{39}{46}$,
and $x + \frac{3}{2} = 6\frac{1}{2}$ or $\frac{15}{23}$.

The second values are inadmissible, since there could not be a negative number of square yards. Hence, 1 pound of paint covers 5 square yards in first coat and $6\frac{1}{2}$ square yards in second coat.

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16. Let $x =$ number of 4-inch spikes in 1 pound.
Then, $x - 6 =$ number of 5-inch spikes in 1 pound.

$$\therefore \frac{80}{x} + 3 = \frac{80}{x-6}.$$

Solving, $x = 16$ or -10,
and $x - 6 = 10$ or -16.

The second values are inadmissible, since there could not be a negative number of spikes.

Hence, 16 4-inch spikes weigh 1 pound, and 10 5-inch spikes weigh 1 pound.

17. Let $x =$ number of cents No. 9 wire costs per pound.
Then, $x + \frac{1}{2} =$ number of cents No. 14 wire costs per pound.

$$\frac{800}{x} - \frac{288}{x+\frac{1}{2}} = 224.$$

Solving, $x = 2\frac{1}{2}$ or $-\frac{5}{7}$,
and $x + \frac{1}{2} = 3$ or $-\frac{3}{14}$.

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The second values are inadmissible, since wire could not cost a negative number of cents.

Hence, No. 9 wire costs $2\frac{1}{2}$ cents per pound, and No. 14 wire 3 cents per pound.

18. Let $x =$ number of miles per hour at usual rate.

Then, $\frac{120}{x} =$ number of hours it takes to run 120 miles.

$$\therefore \frac{120}{x+5} + \frac{4}{15} = \frac{120}{x}.$$

Solving, $x = 45$ or -50.

The negative value is inadmissible, since the rate could not be a negative number of miles.

Hence, the usual rate was 45 miles per hour.

19. Let $x =$ number of seconds it took first man.

Then, $x + 5 =$ number of seconds it took second man.

$$\therefore \frac{1320}{x} - \frac{1320}{x+5} = 2.$$

Solving, $x = 55$ or -60,

and $x + 5 = 60$ or -55.

The negative values are inadmissible, since it could not take a negative number of seconds.

Hence, it took first man 55 seconds, and second man 60 seconds.

20. Let $x =$ number of hours it took first automobile.

Then, $x + \frac{5}{6} =$ number of hours it took second automobile.

$$\therefore \frac{60}{x} - \frac{60}{x+\frac{5}{6}} = 6.$$

Solving, $x = 2\frac{1}{2}$ or $-3\frac{1}{3}$,

and $x + \frac{5}{6} = 3\frac{1}{3}$ or $-2\frac{1}{2}$.

The second values are inadmissible, since it could not take a negative number of hours.

Hence, it took first automobile $2\frac{1}{2}$ hours, and second automobile $3\frac{1}{3}$ hours.

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22. Let $x =$ number of days it takes large pump.

Then, $x + 1\frac{3}{4} =$ number of days it takes small pump.

$$\therefore \frac{1}{x} + \frac{1}{x+1\frac{3}{4}} = \frac{1}{3}.$$

Solving, $x = 5\frac{1}{4}$ or -1,

and $x + 1\frac{3}{4} = 7$ or $\frac{3}{4}$.

The second value is inadmissible, since it could not take a negative number of days.

Hence, the large pump takes $5\frac{1}{4}$ days, and the small pump 7 days.

23. Let $x =$ number of days it takes the first plant.

Then, $x - 7 =$ number of days it takes the second plant.

$$\therefore \frac{1}{x} + \frac{1}{x-7} = \frac{1}{12}.$$

Solving, $x = 28$ or 3,

and $x - 7 = 21$ or -4.

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The second values are inadmissible, since both plants together require 12 days. Considering the first values,

$$\frac{25200}{x} = 900,$$

and
$$\frac{25200}{x-7} = 1200.$$

Hence, the daily capacity of the first plant was 900 blocks, and of the second plant 1200 blocks.

24. Let $x =$ number of strawberry baskets made per minute.
Then, $x - 12 =$ number of peach baskets made per minute.

$$\therefore \frac{2400}{x} + 45 = \frac{2400}{x-12}.$$

Solving, $x = 32$ or -20,
and $x - 12 = 20$ or -32.

The negative values are inadmissible, since there could not be a negative number of baskets.

Hence, 32 strawberry baskets were made per minute and 20 peach baskets.

25. Let $x =$ smaller number.
Then, $x + 1 =$ larger number.

$$\therefore \frac{1}{x} + \frac{1}{x+1} = \frac{9}{20}.$$

Solving, $x = 4$ or $-\frac{5}{9}$;
whence, $x + 1 = 5$ or $\frac{4}{9}$.

The second values are inadmissible, since they are not integers.

Hence, the numbers are 4 and 5.

26. Let $x =$ number of cents asked for 1 dozen eggs.

Then, $\frac{30}{x} =$ number of dozen eggs to be had for 30 cents;

whence, $\frac{360}{x} =$ number of eggs to be had for 30 cents.

Under the second supposition, $\frac{360}{x} - 2$, or $\frac{360 - 2x}{x}$, eggs can be had for 30 cents, and this raises the price 2 cents per dozen, or $\frac{1}{6}$ of a cent per egg. Hence, the price of 1 egg is

$$30 \div \frac{360 - 2x}{x}, \text{ or } \frac{15x}{180 - x} = \frac{x}{12} + \frac{1}{6}.$$

Solving, $x = 18$ or -20.

The negative value is inadmissible, since eggs could not cost a negative number of cents.

Hence, the price of eggs is 18 cents a dozen.

27. Let $x =$ his per cent of gain and also the number of dollars the coat cost him.

Then, $$11 = x + x\frac{x}{100}.$$

Solving, $x = 10$ or -110.

The second value is inadmissible, since his gain could not be a negative number.

Hence, he gained 10 %.

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28. Let $x =$ number of hundredths in the second discount.

Then, $5x =$ number of hundredths in the first discount.

$$\therefore 20 - \frac{5x}{100} \cdot 20 - \frac{x}{100}\left(20 - \frac{5x}{100} \cdot 20\right) = 9.$$

Solving, $x = 110$ or 10,

and $5x = 550$ or 50.

The first values are inadmissible, since they make the reduction greater than the list price.

Hence, the first discount is 50% and the second is 10%.

29. Let $x =$ number of bushels a basket holds.

Then, $x + \frac{1}{8} =$ number of bushels a crate holds.

$$\therefore \frac{187\frac{1}{2}}{x} - \frac{187\frac{1}{2}}{x + \frac{1}{8}} = 50.$$

Solving, $x = \frac{5}{8}$ or $-\frac{3}{4}$,

and $x + \frac{1}{8} = \frac{3}{4}$ or $-\frac{5}{8}$.

The second values are inadmissible, since there could not be a negative number of bushels.

Hence, a basket holds $\frac{5}{8}$ bushels and a crate holds $\frac{3}{4}$ bushels.

30. Let $x =$ number of inches margin was reduced.

Then, $(10 - 2x)(6 - 2x) =$ number of square inches in 1 page after reduction.

$$\therefore 200(10 - 2x)(6 - 2x) = 200 \cdot 60 - 1550.$$

Solving, $x = 7\frac{3}{4}$ or $\frac{1}{4}$.

The first value is inadmissible, since the margin cannot be reduced more than the width of the original page.

Hence, the margin was reduced $\frac{1}{4}$ of an inch.

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1. $c^2 = a^2 + b^2.$

Substituting the given values, $c^2 = (c - 2)^2 + (c - 4)^2.$

$$c^2 = c^2 - 4c + 4 + c^2 - 8c + 16.$$

$$\therefore c^2 - 12c + 20 = 0.$$

$$(c - 10)(c - 2) = 0.$$

$$c = 10 \text{ or } 2.$$

Then, $a = c - 2 = 10 - 2 = 8$, or $a = c - 2 = 2 - 2 = 0.$

$b = c - 4 = 10 - 4 = 6$, or $b = c - 4 = 2 - 4 = -2.$

The second set of values is inadmissible, since there could be no triangle with sides of those values.

Hence, the sides are $a = 8$, $b = 6$, and $c = 10$.

2. $A = \frac{1}{2}ah.$

Substituting the given values, $60 = \frac{1}{2} \cdot a \cdot (a + 2).$

$$120 = a^2 + 2a.$$

Completing the square, etc.,

$$a^2 + 2a + 1 = 121.$$

$$a + 1 = \pm 11.$$

$$\therefore a = 10 \text{ or } -12.$$

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The second value is inadmissible, since the base could not be a negative number of inches in length.

Hence, the base is 10 inches long.

3. $$a \cdot b = c \cdot d.$$

Substituting the given values, $a(a+5) = 4 \cdot 6.$

$$a^2 + 5a = 24.$$

Completing the square, $a^2 + 5a + (\frac{5}{2})^2 = 24 + \frac{25}{4} = \frac{121}{4}.$

$$a + \tfrac{5}{2} = \pm \tfrac{11}{2}.$$

$$\therefore a = 3 \text{ or } -8.$$

$$b = a + 5 = 3 + 5 = 8, \text{ or } b = a + 5 = -8 + 5 = -3.$$

The second set of values is inadmissible, since the length of a and b cannot be negative numbers.

Hence, $a = 3$ and $b = 8$.

4. $$h = a + vt - 16t^2.$$

$$16t^2 - vt = a - h.$$

Completing the square,

$$16t^2 - vt + \left(\frac{v}{8}\right)^2 = a - h + \frac{v^2}{64} = \frac{v^2 + 64(a-h)}{64}.$$

$$4t - \frac{v}{8} = \pm \frac{1}{8}\sqrt{v^2 + 64(a-h)}.$$

$$t = \frac{v \pm \sqrt{v^2 + 64(a-h)}}{32}.$$

Substituting the given values, $$t = \frac{224 \pm \sqrt{50{,}176 + 64(12 - 796)}}{32}$$

$$= \frac{224 \pm \sqrt{50{,}176 - 50{,}176}}{32} = 7.$$

Hence, it will take 7 seconds for the sky rocket to reach a height of 796 feet.

5. From formula in 4, $$t = \frac{v \pm \sqrt{v^2 + 64(a-h)}}{32}.$$

Substituting the given values, $$t = \frac{1280 \pm \sqrt{1{,}638{,}400 + 64(-25{,}600)}}{32}$$

$$= \frac{1280 \pm \sqrt{1{,}638{,}400 - 1{,}638{,}400}}{32} = 40.$$

6. $$h = a - vt - 16t^2.$$

Transposing, $16t^2 + vt = a - h.$

Completing the square,

$$16t^2 + vt + \left(\frac{v}{8}\right)^2 = a - h + \frac{v^2}{64} = \frac{v^2 + 64(a-h)}{64}.$$

$$4t + \frac{v}{8} = \pm \frac{1}{8}\sqrt{v^2 + 64(a-h)}.$$

$$\therefore t = \frac{-v \pm \sqrt{v^2 + 64(a-h)}}{32}.$$

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Substituting the given values, $t = \dfrac{-24 \pm \sqrt{576 + 64(984 - 368)}}{32}$

$$= \frac{-24 \pm 200}{32} = 5\tfrac{1}{2} \text{ or } -7.$$

The second value is inadmissible, since the number of seconds could not be a negative quantity.

Hence, in $5\frac{1}{2}$ seconds, the ball will be 368 feet above ground.

Substituting again in the formula,

$$t = \frac{-24 \pm \sqrt{576 + 64(984)}}{32}$$

$$= \frac{-24 \pm 252.09}{32} = 7.1 \text{ or } -8.6.$$

The second value is inadmissible, since the number of seconds could not be a negative quantity.

Hence, the time is 7 seconds.

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3. $x^4 - 13x^2 + 36 = 0.$
$(x^2 - 4)(x^2 - 9) = 0.$
$(x-2)(x+2)(x-3)(x+3) = 0.$
$\therefore x = 2 \text{ or } -2 \text{ or } 3 \text{ or } -3;$
that is, $x = \pm 2 \text{ or } \pm 3.$

4. $x^4 - 25x^2 + 144 = 0.$
$(x^2 - 9)(x^2 - 16) = 0.$
$(x-3)(x+3)(x-4)(x+4) = 0.$
$\therefore x = 3 \text{ or } -3 \text{ or } 4 \text{ or } -4;$
that is, $x = \pm 3 \text{ or } \pm 4.$

5. $x^4 - 18x^2 + 32 = 0.$
$(x^2 - 16)(x^2 - 2) = 0.$
$(x-4)(x+4)(x-\sqrt{2})(x+\sqrt{2}) = 0.$
$\therefore x = 4 \text{ or } -4 \text{ or } \sqrt{2} \text{ or } -\sqrt{2}.$
$x = \pm 4 \text{ or } \pm\sqrt{2}.$

6. $3x^4 + 5x^2 - 8 = 0.$
$(x^2 - 1)(3x^2 + 8) = 0.$
$(x-1)(x+1)(3x^2+8) = 0.$
Since $(x-1)(x+1) = 0, x = \pm 1.$
Also, $3x^2 + 8 = 0.$
$x^2 = -\frac{8}{3} = -\frac{24}{9}.$
$\therefore x = \pm\frac{2}{3}\sqrt{-6}.$
Hence, $x = \pm 1 \text{ or } \pm\frac{2}{3}\sqrt{-6}.$

7. $5x^4 + 6x^2 - 11 = 0.$
$(x^2 - 1)(5x^2 + 11) = 0.$
$(x-1)(x+1)(5x^2+11) = 0.$
Since $(x-1)(x+1) = 0, x = \pm 1.$
Also, $5x^2 + 11 = 0.$
$x^2 = -\frac{11}{5} = -\frac{55}{25}.$
$\therefore x = \pm\frac{1}{5}\sqrt{-55}.$
Hence, $x = \pm 1 \text{ or } \pm\frac{1}{5}\sqrt{-55}.$

8. $2x^4 - 8x^2 - 90 = 0.$
$x^4 - 4x^2 - 45 = 0.$
$(x^2 - 9)(x^2 + 5) = 0.$
$(x-3)(x+3)(x^2+5) = 0.$
Since $(x-3)(x+3) = 0, x = \pm 3.$
Also, $x^2 + 5 = 0.$
$x^2 = -5.$
$\therefore x = \pm\sqrt{-5}.$
Hence, $x = \pm 3 \text{ or } \pm\sqrt{-5}.$

9. $x^{\frac{1}{2}} - 5x^{\frac{1}{4}} + 6 = 0.$
Let $x^{\frac{1}{4}} = p$, then, $x^{\frac{1}{2}} = p^2$,
and $p^2 - 5p + 6 = 0.$
$(p-2)(p-3) = 0.$
$\therefore p = 2 \text{ or } 3;$
that is, $x^{\frac{1}{4}} = 2 \text{ or } 3.$
$\therefore x = 16 \text{ or } 81.$

10. $x^{\frac{1}{2}} + 3x^{\frac{1}{4}} - 28 = 0.$
Let $x^{\frac{1}{4}} = p$, then, $x^{\frac{1}{2}} = p^2$,
and $p^2 + 3p - 28 = 0.$
$(p-4)(p+7) = 0.$
$\therefore p = 4 \text{ or } -7;$
that is, $x^{\frac{1}{4}} = 4 \text{ or } -7.$
$\therefore x = 256 \text{ or } 2401.$
2401 does not verify and is rejected.

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11. $$x^{\frac{1}{3}} - 3x^{\frac{1}{6}} = -2.$$

Let $x^{\frac{1}{6}} = p$, then, $x^{\frac{1}{3}} = p^2$,

and $$p^2 - 3p + 2 = 0.$$

$$(p-1)(p-2) = 0.$$

$$\therefore p = 1 \text{ or } 2;$$

that is, $$x^{\frac{1}{6}} = 1 \text{ or } 2.$$

$$\therefore x = 1 \text{ or } 64.$$

12. $$x^{\frac{2}{3}} - x^{\frac{1}{3}} = 6.$$

$$x^{\frac{2}{3}} - x^{\frac{1}{3}} - 6 = 0.$$

$$(x^{\frac{1}{3}} - 3)(x^{\frac{1}{3}} + 2) = 0.$$

$$\therefore x^{\frac{1}{3}} = 3 \text{ or } -2.$$

$$x = 27 \text{ or } -8.$$

13. $$x + 2\sqrt{x} = 3.$$

$$x + 2\sqrt{x} + 1 = 4.$$

$$\sqrt{x} + 1 = \pm 2.$$

$$\therefore \sqrt{x} = 1 \text{ or } -3.$$

$$x = 1 \text{ or } 9.$$

9 does not verify and is rejected.

14. $$x^{\frac{2}{3}} - 2x^{\frac{1}{3}} = 3.$$

$$x^{\frac{2}{3}} - 2x^{\frac{1}{3}} + 1 = 4.$$

$$x^{\frac{1}{3}} - 1 = \pm 2.$$

$$\therefore x^{\frac{1}{3}} = 3 \text{ or } -1.$$

$$x = 27 \text{ or } -1.$$

15. $$x^{\frac{4}{3}} = 10x^{\frac{2}{3}} - 9.$$

Transposing and completing the square,

$$x^{\frac{4}{3}} - 10x^{\frac{2}{3}} + 25 = 16.$$

$$x^{\frac{2}{3}} - 5 = \pm 4.$$

$$\therefore x^{\frac{2}{3}} = 9 \text{ or } 1.$$

$$x^2 = 729 \text{ or } 1.$$

$$\therefore x = \pm 27 \text{ or } \pm 1.$$

16. $$(x-3)^2 + 2(x-3) = 3.$$

Put p for $x - 3$ and p^2 for $(x-3)^2$.

Then, transposing 3, $$p^2 + 2p - 3 = 0.$$

Factoring, $$(p-1)(p+3) = 0.$$

$$\therefore p = 1 \text{ or } -3;$$

that is, $$x - 3 = 1 \text{ or } -3.$$

$$\therefore x = 4 \text{ or } 0.$$

17. $$(x^2+1)^2 + 4(x^2+1) = 45.$$

Put p for $x^2 + 1$ and p^2 for $(x^2+1)^2$.

Then, transposing 45, $$p^2 + 4p - 45 = 0.$$

Factoring, $$(p-5)(p+9) = 0.$$

$$\therefore p = 5 \text{ or } -9;$$

that is, $$x^2 + 1 = 5 \text{ or } -9.$$

$$x^2 = 4 \text{ or } -10.$$

$$\therefore x = \pm 2 \text{ or } \pm\sqrt{-10}.$$

18. $$(x^2-4)^2 - 3(x^2-4) = 10.$$

Put p for $x^2 - 4$ and p^2 for $(x^2-4)^2$.

Then, transposing 10, $$p^2 - 3p - 10 = 0.$$

Factoring, $$(p-5)(p+2) = 0.$$

$$\therefore p = 5 \text{ or } -2;$$

that is, $$x^2 - 4 = 5 \text{ or } -2,$$

$$x^2 = 9 \text{ or } 2.$$

$$\therefore x = \pm 3 \text{ or } \pm\sqrt{2}.$$

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20. $$x^{\frac{3}{2}} - 4x - 5x^{\frac{1}{2}} = 0.$$
$$x^{\frac{1}{2}}(x^{\frac{1}{2}} + 1)(x^{\frac{1}{2}} - 5) = 0.$$
$$\therefore x^{\frac{1}{2}} = 0 \text{ or } -1 \text{ or } 5.$$
$$x = 0 \text{ or } 1 \text{ or } 25.$$
1 does not verify and is rejected.

21. $$x - 3x^{\frac{3}{4}} + 2x^{\frac{1}{2}} = 0.$$
$$x^{\frac{1}{2}}(x^{\frac{1}{4}} - 1)(x^{\frac{1}{4}} - 2) = 0.$$
$$\therefore x^{\frac{1}{2}} = 0 \text{ or } x^{\frac{1}{4}} = 1 \text{ or } 2.$$
$$x = 0 \text{ or } 1 \text{ or } 16.$$

22. $$x + 2x^{\frac{5}{3}} - 3x^{\frac{4}{3}} = 0.$$
$$x - 3x^{\frac{4}{3}} + 2x^{\frac{5}{3}} = 0.$$
$$x(1 - x^{\frac{1}{3}})(1 - 2x^{\frac{1}{3}}) = 0.$$
$$\therefore x = 0 \text{ or } x^{\frac{1}{3}} = 1 \text{ or } \tfrac{1}{2}.$$
$$x = 0 \text{ or } 1 \text{ or } \tfrac{1}{8}.$$

23. $$5x = x\sqrt{x} + 6\sqrt{x}.$$
$$x\sqrt{x} - 5x + 6\sqrt{x} = 0.$$
$$\sqrt{x}(\sqrt{x} - 2)(\sqrt{x} - 3) = 0.$$
$$\therefore \sqrt{x} = 0 \text{ or } 2 \text{ or } 3.$$
$$x = 0 \text{ or } 4 \text{ or } 9.$$

24. $$3x = x\sqrt[3]{x} + 2\sqrt[3]{x^2}.$$
$$x\sqrt[3]{x} - 3x + 2\sqrt[3]{x^2} = 0.$$
$$x^{\frac{4}{3}} - 3x + 2x^{\frac{2}{3}} = 0.$$
$$x^{\frac{2}{3}}(x^{\frac{1}{3}} - 1)(x^{\frac{1}{3}} - 2) = 0.$$
$$\therefore x^{\frac{2}{3}} = 0 \text{ or } x^{\frac{1}{3}} = 1 \text{ or } 2.$$
$$x = 0 \text{ or } 1 \text{ or } 8.$$

25. $$2x + \sqrt{x} = 15x\sqrt{x}.$$
$$\sqrt{x} + 2x - 15x\sqrt{x} = 0.$$
$$\sqrt{x}(1 - 3\sqrt{x})(1 + 5\sqrt{x}) = 0.$$
$$\therefore \sqrt{x} = 0 \text{ or } \tfrac{1}{3} \text{ or } -\tfrac{1}{5}.$$
$$x = 0 \text{ or } \tfrac{1}{9} \text{ or } \tfrac{1}{25}.$$
$\frac{1}{25}$ does not verify and is rejected.

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27. $$x + 2\sqrt{x + 3} = 21.$$
Adding 3, $$x + 3 + 2\sqrt{x + 3} = 24.$$
Put p for $\sqrt{x + 3}$ and p^2 for $x + 3$.
Then, $$p^2 + 2p = 24.$$
Completing the square, $$p^2 + 2p + 1 = 25.$$
$$p + 1 = \pm 5.$$
$$\therefore p = \sqrt{x + 3} = 4 \text{ or } -6.$$
$$x + 3 = 16 \text{ or } 36.$$
$$\therefore x = 13 \text{ or } 33.$$
33 does not verify and is rejected.

28. $$x^2 - 3x + 2\sqrt{x^2 - 3x + 6} = 18.$$
Adding 6, $$x^2 - 3x + 6 + 2\sqrt{x^2 - 3x + 6} = 24.$$
Put p for $\sqrt{x^2 - 3x + 6}$ and p^2 for $x^2 - 3x + 6$.
Then, $$p^2 + 2p = 24.$$
Completing the square, $$p^2 + 2p + 1 = 25.$$
$$p + 1 = \pm 5.$$
$$\therefore p = \sqrt{x^2 - 3x + 6} = 4 \text{ or } -6.$$
$$x^2 - 3x + 6 = 16 \text{ or } 36.$$
Solving these two equations, $$x = 5 \text{ or } -2 \text{ or } \tfrac{3}{2} \pm \tfrac{1}{2}\sqrt{129}.$$
$\frac{3}{2} \pm \frac{1}{2}\sqrt{129}$ does not verify and is rejected.

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30.
$$x^6 - 28x^3 + 27 = 0. \qquad (1)$$
Factoring,
$$(x^3 - 1)(x^3 - 27) = 0. \qquad (2)$$
$$x^3 - 1 = 0. \qquad (3)$$
$$x^3 - 27 = 0. \qquad (4)$$
Factoring (3),
$$(x - 1)(x^2 + x + 1) = 0, \qquad (5)$$
and (4),
$$(x - 3)(x^2 + 3x + 9) = 0. \qquad (6)$$
Writing each factor equal to zero, and solving,

From (5) $\qquad x = 1,\ \frac{1}{2}(-1 \pm \sqrt{-3})$.

From (6), $\qquad x = 3,\ \frac{3}{2}(-1 \pm \sqrt{-3})$.

31.
$$x^4 - 16 = 0. \qquad (1)$$
Factoring,
$$(x^2 + 4)(x^2 - 4) = 0. \qquad (2)$$
$$x^2 - 4 = 0. \qquad (3)$$
$$x^2 + 4 = 0. \qquad (4)$$
From (3), $\qquad x = \pm 2$.

From (4), $\qquad x = \pm 2\sqrt{-1}$.

32. Let $\qquad x^3 = -1$.

Transposing, $\qquad x^3 + 1 = 0$.

Factoring, $\qquad (x + 1)(x^2 - x + 1) = 0$.

Equating each factor to zero and solving, $\quad x = -1$ or $\frac{1}{2}(1 \pm \sqrt{-3})$.

Hence, the three cube roots of -1 are
$$-1,\ \tfrac{1}{2}(1 + \sqrt{-3}),\ \text{and}\ \tfrac{1}{2}(1 - \sqrt{-3}).$$

33. Let $\qquad x^3 = -8$.

Transposing, $\qquad x^3 + 8 = 0$.

Factoring, $\qquad (x + 2)(x^2 - 2x + 4) = 0$.

Equating each factor to zero and solving, $\quad x = -2$ or $1 \pm \sqrt{-3}$.

Hence, the three cube roots of -8 are -2, $1 + \sqrt{-3}$, and $1 - \sqrt{-3}$.

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35.
$$x^4 + 2x^3 - x = 30.$$
Extracting the square root of the first member as far as possible, it is found that the first member lacks $\frac{1}{4}$ of being the square of $x^2 + x - \frac{1}{2}$.

Adding $\frac{1}{4}$ to each member,
$$x^4 + 2x^3 - x + \tfrac{1}{4} = \tfrac{121}{4}.$$
Extracting the square root,
$$x^2 + x - \tfrac{1}{2} = \pm \tfrac{11}{2}.$$
$$\therefore x^2 + x = 6 \text{ or } -5.$$
Solving these two equations, $\qquad x = 2$ or -3 or $\frac{1}{2}(-1 \pm \sqrt{-19})$.

36.
$$x^4 - 4x^3 + 8x = -3.$$
Extracting the square root of the first member as far as possible, it is found that the first member lacks 4 of being the square of $x^2 - 2x - 2$.

Adding 4 to each member, $x^4 - 4x^3 + 8x + 4 = 1$.

Extracting the square root, $\qquad x^2 - 2x - 2 = \pm 1$.

Adding 3 to each member, $\qquad x^2 - 2x + 1 = 4$ or 2.

Solving these two equations, $\qquad x = 3$ or -1 or $1 \pm \sqrt{2}$.

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37. $$x^4 - 2x^3 + x = 132.$$

Extracting the square root of the first member as far as possible, it is found that the first member lacks $\frac{1}{4}$ of being the square of $x^2 - x - \frac{1}{2}$.

Adding $\frac{1}{4}$ to each member, $x^4 - 2x^3 + x + \frac{1}{4} = \frac{529}{4}$.

Extracting the square root, $x^2 - x - \frac{1}{2} = \pm \frac{23}{2}$.

$$\therefore x^2 - x = 12 \text{ or } -11.$$

Solving these two equations, $x = 4$ or -3 or $\frac{1}{2}(1 \pm \sqrt{-43})$.

38. $$x^4 - 6x^3 + 27x = 10.$$

Extracting the square root of the first member as far as possible, it is found that the first member lacks $\frac{81}{4}$ of being the square of $x^2 - 3x - \frac{9}{2}$.

Adding $\frac{81}{4}$ to each member,

$$x^4 - 6x^3 + 27x + \frac{81}{4} = \frac{121}{4}.$$

Extracting the square root, $x^2 - 3x - \frac{9}{2} = \pm \frac{11}{2}$.

$$x^2 - 3x = 10 \text{ or } -1.$$

Solving these two equations, $x = 5$ or -2 or $\frac{1}{2}(3 \pm \sqrt{5})$.

39. $$x^4 + 2x^3 + 5x^2 + 4x - 60 = 0.$$

Extracting the square root of the first member as far as possible, it is found that the first member lacks 64 of being the square of $x^2 + x + 2$.

Adding 64 to each member,

$$x^4 + 2x^3 + 5x^2 + 4x + 4 = 64.$$

Extracting the square root, $x^2 + x + 2 = \pm 8$.

$$x^2 + x = 6 \text{ or } -10.$$

Solving these two equations, $x = 2$ or -3 or $\frac{1}{2}(-1 \pm \sqrt{-39})$.

40. $$x^4 + 6x^3 + 7x^2 - 6x - 8 = 0.$$

Extracting the square root of the first member as far as possible, it is found that the first member lacks 9 of being the square of $x^2 + 3x - 1$.

Adding 9 to each member,

$$x^4 + 6x^3 + 7x^2 - 6x + 1 = 9.$$

Extracting the square root, $x^2 + 3x - 1 = \pm 3$.

$$x^2 + 3x = 4 \text{ or } -2.$$

Solving these two equations, $x = 1$ or -4 or -1 or -2.

41. $$x^4 - 6x^3 + 15x^2 - 18x + 8 = 0.$$

Extracting the square root of the first member as far as possible, it is found that the first member lacks 1 of being the square of $x^2 - 3x + 3$.

Adding 1 to each member,

$$x^4 - 6x^3 + 15x^2 - 18x + 9 = 1.$$

Extracting the square root, $x^2 - 3x + 3 = \pm 1$.

$$x^2 - 3x = -2 \text{ or } -4.$$

Solving these two equations, $x = 1$ or 2 or $\frac{1}{2}(3 \pm \sqrt{-7})$.

42. $$\frac{x^2}{x+1} + \frac{x+1}{x^2} = \frac{25}{12}. \quad (1)$$

Putting p for the first term and $\frac{1}{p}$ for the second,

$$p + \frac{1}{p} = \frac{25}{12}.$$

Clearing of fractions, etc., $12p^2 - 25p = -12$. (2)

Solving, $p = \frac{4}{3}$ or $\frac{3}{4}$.

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Since $\dfrac{x^2}{x+1}=p$, $\qquad \dfrac{x^2}{x+1}=\dfrac{4}{3}$, (3)

and $\qquad \dfrac{x^2}{x+1}=\dfrac{3}{4}$. (4)

From (3), $\qquad 3x^2-4x=4$. (5)

Solving (5), $\qquad x=2 \text{ or } -\tfrac{2}{3}$.

From (4), $\qquad 4x^2-3x=3$. (6)

Solving (6), $\qquad x=\tfrac{1}{8}(3\pm\sqrt{57})$.

Hence, $\qquad x=2 \text{ or } -\tfrac{2}{3} \text{ or } \tfrac{1}{8}(3\pm\sqrt{57})$.

43. $$\frac{x^2+x}{2}+\frac{2}{x^2+x}=2.$$

If $\dfrac{x^2+x}{2}=p$, $\qquad p+\dfrac{1}{p}=2$.

Clearing of fractions, etc., $\qquad p^2-2p+1=0$.

$$\therefore p=1\pm 0=1;$$

that is, $\qquad \dfrac{x^2+x}{2}=1$.

Clearing of fractions, etc., $\qquad x^2+x-2=0$.

Factoring, $\qquad (x-1)(x+2)=0$.

$$\therefore x=1 \text{ or } -2.$$

Each of these numbers occurs twice as a root.

44. $$\frac{x^2+1}{4}+\frac{4}{x^2+1}=\frac{5}{2}.$$

If $\dfrac{x^2+1}{4}=p$, $\qquad p+\dfrac{1}{p}=\dfrac{5}{2}$.

Clearing of fractions, etc., $\qquad 2p^2-5p+2=0$.

Factoring, $\qquad (p-2)(2p-1)=0$.

$$\therefore p=\frac{x^2+1}{4}=2 \text{ or } \frac{1}{2}.$$

Clearing of fractions, etc., $\qquad x^2=7 \text{ or } 1$.

$$\therefore x=\pm\sqrt{7} \text{ or } \pm 1.$$

45. $$\frac{x+2}{x^2+4}+\frac{2(x^2+4)}{x+2}=\frac{51}{5}. \qquad (1)$$

If $\dfrac{x+2}{x^2+4}=p$, $\qquad p+\dfrac{2}{p}=\dfrac{51}{5}$. (2)

Clearing of fractions, etc., $\qquad 5p^2-51p+10=0$.

Factoring, $\qquad (p-10)(5p-1)=0$.

$$\therefore p=\frac{x+2}{x^2+4}=10 \text{ or } \frac{1}{5}. \qquad (3)$$

Clearing of fractions, etc., $\qquad 10x^2-x+38=0$, (4)

or $\qquad x^2-5x-6=0$. (5)

Solving (4) by the formula, § 390, $$x=\frac{1\pm\sqrt{1-4\cdot 10\cdot 38}}{2\cdot 10}$$
$$=\tfrac{1}{20}(1\pm\sqrt{-1519}).$$

Factoring (5), $\qquad (x-6)(x+1)=0$.

$$\therefore x=6 \text{ or } -1.$$

Hence, $\qquad x=6 \text{ or } -1 \text{ or } \tfrac{1}{20}(1\pm\sqrt{-1519})$.

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46. $$\frac{x^2+1}{x-1}-\frac{4(x-1)}{x^2+1}=\frac{21}{5}. \quad (1)$$

If $\dfrac{x^2+1}{x-1}=p$, $$p-\frac{4}{p}=\frac{21}{5}. \quad (2)$$

Clearing of fractions, etc., $5p^2-21p-20=0.$

Factoring, $(p-5)(5p+4)=0.$

$$\therefore p=\frac{x^2+1}{x-1}=5 \text{ or } -\frac{4}{5}. \quad (3)$$

Clearing of fractions, etc., $x^2-5x+6=0, \quad (4)$

or $5x^2+4x+1=0. \quad (5)$

Factoring (4), $(x-2)(x-3)=0.$

$\therefore x=2$ or $3.$

Solving (5) by the formula, § 390, $$x=\frac{-4\pm\sqrt{16-4\cdot5\cdot1}}{2\cdot5}$$

$$=\tfrac{1}{5}(-2\pm\sqrt{-1}).$$

Hence, $x=2$ or 3 or $\frac{1}{5}(-2\pm\sqrt{-1}).$

47. $x^{\frac{3}{2}}-6x+8x^{\frac{1}{2}}=0.$

$x^{\frac{1}{2}}(x-6x^{\frac{1}{2}}+8)=0.$

$x^{\frac{1}{2}}(x^{\frac{1}{2}}-4)(x^{\frac{1}{2}}-2)=0.$

$x^{\frac{1}{2}}=0. \quad \therefore x=0.$

$x^{\frac{1}{2}}-4=0. \quad \therefore x=16.$

$x^{\frac{1}{2}}-2=0. \quad \therefore x=4.$

Hence, $x=0$ or 4 or $16.$

48. $2x-3x^{\frac{3}{4}}+x^{\frac{1}{2}}=0.$

$x^{\frac{1}{2}}(2x^{\frac{1}{2}}-3x^{\frac{1}{4}}+1)=0.$

$x^{\frac{1}{2}}(x^{\frac{1}{4}}-1)(2x^{\frac{1}{4}}-1)=0.$

$x^{\frac{1}{2}}=0. \quad \therefore x=0.$

$x^{\frac{1}{4}}-1=0. \quad \therefore x=1.$

$2x^{\frac{1}{4}}-1=0. \quad \therefore x=\frac{1}{16}.$

Hence, $x=0$ or 1 or $\frac{1}{16}.$

49. $\sqrt[4]{x}+3\sqrt{x}=30.$

Let $\sqrt[4]{x}=p$, then, $\sqrt{x}=p^2,$

and $3p^2+p-30=0.$

$(p-3)(3p+10)=0.$

$\therefore p=3$ or $-\frac{10}{3}$;

that is, $\sqrt[4]{x}=3$ or $-\frac{10}{3}.$

$\therefore x=81$ or $\frac{10000}{81}.$

$\frac{10000}{81}$ does not verify and is rejected.

50. $ax^{2n}+bx^n+c=0.$

$4a^2x^{2n}+4abx^n+b^2=b^2-4ac.$

$2ax^n+b=\pm\sqrt{b^2-4ac}.$

$$x^n=\frac{-b\pm\sqrt{b^2-4ac}}{2a}.$$

$$\therefore x=\left(\frac{-b\pm\sqrt{b^2-4ac}}{2a}\right)^{\frac{1}{n}}.$$

51. $x-7x^{\frac{2}{3}}+10x^{\frac{1}{3}}=0.$

$x^{\frac{1}{3}}(x^{\frac{2}{3}}-7x^{\frac{1}{3}}+10)=0.$

$x^{\frac{1}{3}}(x^{\frac{1}{3}}-2)(x^{\frac{1}{3}}-5)=0.$

$x^{\frac{1}{3}}=0. \quad \therefore x=0.$

$x^{\frac{1}{3}}-2=0. \quad \therefore x=8.$

$x^{\frac{1}{3}}-5=0. \quad \therefore x=125.$

Hence, $x=0$ or 8 or $125.$

52. $x\sqrt{x}+20\sqrt{x}=9x.$

$\sqrt{x}(x+20-9\sqrt{x})=0.$

$\sqrt{x}(\sqrt{x}-4)(\sqrt{x}-5)=0.$

$\sqrt{x}=0. \quad \therefore x=0.$

$\sqrt{x}-4=0. \quad \therefore x=16.$

$\sqrt{x}-5=0. \quad \therefore x=25.$

Hence, $x=0$ or 16 or $25.$

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53. $$x-5+2\sqrt{x-5}=8.$$

Put p for $\sqrt{x-5}$ and p^2 for $x-5$.

Then, $$p^2+2p=8.$$

Completing the square, $$p^2+2p+1=9.$$

$$p+1=\pm 3.$$

$$\therefore p=\sqrt{x-5}=2 \text{ or } -4.$$

$$x-5=4 \text{ or } 16.$$

$$\therefore x=9 \text{ or } 21.$$

21 does not verify and is rejected.

54. $$x+10=2\sqrt{x+10}+5.$$

Transposing, $$x+10-2\sqrt{x+10}=5.$$

Put p for $\sqrt{x+10}$ and p^2 for $x+10$.

Then, $$p^2-2p=5.$$

Completing the square, $$p^2-2p+1=6.$$

$$p-1=\pm\sqrt{6}.$$

$$\therefore p=\sqrt{x+10}=1\pm\sqrt{6}.$$

Squaring, $$x+10=7\pm 2\sqrt{6}.$$

$$\therefore x=-3\pm 2\sqrt{6}.$$

55. $$x-3=21-4\sqrt{x-3}.$$

Transposing, $$x-3+4\sqrt{x-3}-21=0.$$

Put p for $\sqrt{x-3}$ and p^2 for $x-3$.

Then, $$p^2+4p-21=0.$$

Factoring, $$(p-3)(p+7)=0.$$

$$\therefore p=\sqrt{x-3}=3 \text{ or } -7.$$

$$x-3=9 \text{ or } 49.$$

$$\therefore x=12 \text{ or } 52.$$

52 does not verify and is rejected.

56. $$2x-3\sqrt{2x+5}=-5.$$

Adding 5, $$2x+5-3\sqrt{2x+5}=0.$$

Put p for $\sqrt{2x+5}$ and p^2 for $2x+5$.

Then, $$p^2-3p=0.$$

Factoring, $$p(p-3)=0.$$

$$\therefore p=0 \text{ or } 3;$$

that is, $$\sqrt{2x+5}=0 \text{ or } 3.$$

$$2x+5=0 \text{ or } 9.$$

Solving these equations, $$x=-\tfrac{5}{2} \text{ or } 2.$$

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57. $$2x-6\sqrt{2x-1}=8.$$

Adding -1, $$2x-1-6\sqrt{2x-1}=7.$$

Put p for $\sqrt{2x-1}$ and p^2 for $2x-1$.

Then, transposing 7, $$p^2-6p-7=0.$$

Factoring, $$(p-7)(p+1)=0.$$

$$\therefore p=\sqrt{2x-1}=7 \text{ or } -1.$$

$$2x-1=49 \text{ or } 1.$$

$$\therefore x=25 \text{ or } 1.$$

1 does not verify and is rejected.

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58. $$x = 11 - 3\sqrt{x+7}.$$

Adding 7 and transposing,

$$x + 7 + 3\sqrt{x+7} - 18 = 0.$$

Put p for $\sqrt{x+7}$ and p^2 for $x + 7$.

Then, $$p^2 + 3p - 18 = 0.$$

Factoring, $$(p-3)(p+6) = 0.$$

$$\therefore p = \sqrt{x+7} = 3 \text{ or } -6.$$

$$x + 7 = 9 \text{ or } 36.$$

$$\therefore x = 2 \text{ or } 29.$$

29 does not verify and is rejected.

59. $$x^{-1} - 5x^{-\frac{1}{2}} + 6 = 0.$$

Factoring, $$(x^{-\frac{1}{2}} - 2)(x^{-\frac{1}{2}} - 3) = 0.$$

$$x^{-\frac{1}{2}} - 2 = 0. \qquad x^{-\frac{1}{2}} - 3 = 0.$$

$$\frac{1}{x^{\frac{1}{2}}} = 2. \qquad \frac{1}{x^{\frac{1}{2}}} = 3.$$

$$\frac{1}{x} = 4. \qquad \frac{1}{x} = 9.$$

$$\therefore x = \tfrac{1}{4}. \qquad \therefore x = \tfrac{1}{9}.$$

Hence, $x = \frac{1}{4}$ or $\frac{1}{9}$.

60. $$x^{-\frac{2}{3}} - 5x^{-\frac{1}{3}} + 4 = 0.$$

Multiplying by $x^{\frac{2}{3}}$, $$1 - 5x^{\frac{1}{3}} + 4x^{\frac{2}{3}} = 0.$$

Put p for $x^{\frac{1}{3}}$ and p^2 for $x^{\frac{2}{3}}$.

Then, $$1 - 5p + 4p^2 = 0.$$

Factoring, $$(1-p)(1-4p) = 0.$$

$$\therefore p = 1 \text{ or } \tfrac{1}{4};$$

that is, $$x^{\frac{1}{3}} = 1 \text{ or } \tfrac{1}{4}.$$

$$\therefore x = 1 \text{ or } \tfrac{1}{64}.$$

61. $$x^2 - 5x + 2\sqrt{x^2 - 5x - 2} = 10.$$

Adding -2, $x^2 - 5x - 2 + 2\sqrt{x^2 - 5x - 2} = 8.$

Put p for $\sqrt{x^2 - 5x - 2}$ and p^2 for $x^2 - 5x - 2$.

Then, $$p^2 + 2p = 8.$$

Completing the square, $$p^2 + 2p + 1 = 9.$$

$$p + 1 = \pm 3.$$

$$\therefore p = \sqrt{x^2 - 5x - 2} = 2 \text{ or } -4.$$

$$x^2 - 5x - 2 = 4 \text{ or } 16.$$

Solving these two equations, $x = 6$ or -1 or $\frac{5}{2} \pm \frac{1}{2}\sqrt{97}$.

$\frac{5}{2} \pm \frac{1}{2}\sqrt{97}$ does not verify and is rejected.

62. $$x^2 - x - \sqrt{x^2 - x + 4} - 8 = 0.$$

Separating -8 into $+4-12$,

$$x^2 - x + 4 - \sqrt{x^2 - x + 4} - 12 = 0.$$

Put p for $\sqrt{x^2 - x + 4}$ and p^2 for $x^2 - x + 4$.

Then, $$p^2 - p - 12 = 0.$$

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Factoring, $(p-4)(p+3)=0.$

$$\therefore p=\sqrt{x^2-x+4}=4 \text{ or } -3.$$

$$x^2-x+4=16 \text{ or } 9.$$

Solving these two equations, $x=4$ or -3 or $\frac{1}{2}(1\pm\sqrt{21})$.

$\frac{1}{2}(1\pm\sqrt{21})$ does not verify and is rejected.

63. $x^2-5x+5\sqrt{x^2-5x+1}=49.$

Adding 1, $x^2-5x+1+5\sqrt{x^2-5x+1}=50.$

Put p for $\sqrt{x^2-5x+1}$ and p^2 for x^2-5x+1.

Then, transposing 50, $p^2+5p-50=0.$

Factoring, $(p-5)(p+10)=0.$

$$\therefore p=\sqrt{x^2-5x+1}=5 \text{ or } -10.$$

$$x^2-5x+1=25 \text{ or } 100.$$

Solving these two equations, $x=8$ or -3 or $\frac{5}{2}\pm\frac{1}{2}\sqrt{421}$.

$\frac{5}{2}\pm\frac{1}{2}\sqrt{421}$ does not verify and is rejected.

64. $(x^2-2x)^2-2(x^2-2x)=3.$

Put p for x^2-2x and p^2 for $(x^2-2x)^2$.

Then, $p^2-2p=3.$

Completing the square, $p^2-2p+1=4.$

$$p-1=\pm 2.$$

$$\therefore p=3 \text{ or } -1;$$

that is, $x^2-2x=3$ or -1.

Completing the square, $x^2-2x+1=4$ or 0.

$$x-1=\pm 2 \text{ or } \pm 0.$$

$$\therefore x=3 \text{ or } -1 \text{ or } 1.$$

The roots of the given equation are 3, -1, 1, 1.

65. $(x^2-x)^2-(x^2-x)-132=0.$

Factoring, $(x^2-x-12)(x^2-x+11)=0.$

$$(x-4)(x+3)(x^2-x+11)=0.$$

$$\therefore x=4 \text{ or } -3 \text{ or } x^2-x+11=0.$$

Solving the last equation, $x=\frac{1}{2}(1\pm\sqrt{-43})$.

Hence, $x=4$ or -3 or $\frac{1}{2}(1\pm\sqrt{-43})$.

66.
$$\left(\frac{12}{x}-1\right)^2+8\left(\frac{12}{x}-1\right)=33.$$

Put p for $\dfrac{12}{x}-1$ and p^2 for $\left(\dfrac{12}{x}-1\right)^2$.

Then, transposing 33, $p^2+8p-33=0.$

Factoring, $(p-3)(p+11)=0.$

$$\therefore p=3 \text{ or } -11;$$

that is, $\dfrac{12}{x}-1=3$ or -11.

$$\frac{12}{x}=4 \text{ or } -10.$$

$$\therefore x=3 \text{ or } -\tfrac{6}{5}.$$

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67. $$\left(x+\frac{1}{x}\right)^2-2\left(x+\frac{1}{x}\right)=\frac{5}{4}. \quad (1)$$

Put p for $x+\frac{1}{x}$ and p^2 for $\left(x+\frac{1}{x}\right)^2$.

Then, $$p^2-2p=\tfrac{5}{4}. \quad (2)$$

Completing the square, $$p^2-2p+1=\tfrac{9}{4}.$$

$$p-1=\pm\tfrac{3}{2}.$$

$$\therefore p=\tfrac{5}{2} \text{ or } -\tfrac{1}{2}; \quad (3)$$

that is, $$x+\frac{1}{x}=\frac{5}{2} \text{ or } -\frac{1}{2}. \quad (4)$$

Clearing equations (4) of fractions and transposing,

$$2x^2-5x+2=0, \quad (5)$$

or $$2x^2+x+2=0. \quad (6)$$

Factoring (5), $$(x-2)(2x-1)=0.$$

$$\therefore x=2 \text{ or } \tfrac{1}{2}.$$

Solving (6) by the formula, § 390, $$x=\frac{-1\pm\sqrt{1-4\cdot 2\cdot 2}}{2\cdot 2}$$

$$=\tfrac{1}{4}(-1\pm\sqrt{-15}).$$

Hence, $$x=2 \text{ or } \tfrac{1}{2} \text{ or } \tfrac{1}{4}(-1\pm\sqrt{-15}).$$

68. $$\left(\frac{1+x^2}{x}\right)^2+2\left(\frac{1+x^2}{x}\right)=8. \quad (1)$$

Put p for $\frac{1+x^2}{x}$ and p^2 for $\left(\frac{1+x^2}{x}\right)^2$.

Then, $$p^2+2p=8. \quad (2)$$

Completing the square, $$p^2+2p+1=9.$$

$$p+1=\pm 3.$$

$$\therefore p=2 \text{ or } -4; \quad (3)$$

that is, $$\frac{1+x^2}{x}=2 \text{ or } -4. \quad (4)$$

Clearing equations (4) of fractions and transposing,

$$x^2-2x+1=0, \quad (5)$$

or $$x^2+4x+1=0. \quad (6)$$

Factoring (5), $$(x-1)(x-1)=0.$$

$$\therefore x=1 \text{ or } 1.$$

Solving (6), $$x=-2\pm\sqrt{3}.$$

Hence, the roots of (1) are 1, 1, and $-2\pm\sqrt{3}$.

69. $$\left(x-\frac{1}{x}\right)^2+\frac{5}{6}\left(x-\frac{1}{x}\right)=1. \quad (1)$$

Put p for $x-\frac{1}{x}$ and p^2 for $\left(x-\frac{1}{x}\right)^2$.

Then, $$p^2+\tfrac{5}{6}p=1. \quad (2)$$

Clearing of fractions, etc., $$6p^2+5p-6=0.$$

Factoring, $$(2p+3)(3p-2)=0.$$

$$\therefore p=x-\frac{1}{x}=-\frac{3}{2} \text{ or } \frac{2}{3}. \quad (3)$$

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Clearing equations (3) of fractions, and transposing,

$$2x^2 + 3x - 2 = 0, \quad (4)$$

or

$$3x^2 - 2x - 3 = 0. \quad (5)$$

Factoring (4), $(2x - 1)(x + 2) = 0.$

$\therefore\ x = \frac{1}{2}$ or $-2.$

Solving (5) by the formula, § 390, $x = \dfrac{2 \pm \sqrt{4 - 4 \cdot 3(-3)}}{2 \cdot 3}$

$= \frac{1}{3}(1 \pm \sqrt{10}).$

Hence, $x = \frac{1}{2}$ or -2 or $\frac{1}{3}(1 \pm \sqrt{10}).$

70. $x^4 - 10x^3 + 35x^2 - 50x + 24 = 0.$

Extracting the square root of the first member as far as possible, it is found that the first member lacks 1 of being the square of $x^2 - 5x + 5.$

Adding 1 to each member,

$$x^4 - 10x^3 + 35x^2 - 50x + 25 = 1.$$

Extracting the square root, $x^2 - 5x + 5 = \pm 1.$

$x^2 - 5x = -4$ or $-6.$

Solving these two equations, $x = 1$ or 4 or 2 or $3.$

71. $16x^4 - 8x^3 - 31x^2 + 8x + 15 = 0.$

Extracting the square root of the first member as far as possible, it is found that the first member lacks 1 of being the square of $4x^2 - x - 4.$

Adding 1 to each member,

$$16x^4 - 8x^3 - 31x^2 + 8x + 16 = 1.$$

Extracting the square root, $4x^2 - x - 4 = \pm 1.$

$4x^2 - x = 5$ or $3.$

Solving these two equations, $x = -1$ or $\frac{5}{4}$ or 1 or $-\frac{3}{4}.$

72. $4x^4 - 4x^3 - 7x^2 + 4x + 3 = 0.$

Extracting the square root of the first member as far as possible, it is found that the first member lacks 1 of being the square of $2x^2 - x - 2.$

Adding 1 to each member,

$$4x^4 - 4x^3 - 7x^2 + 4x + 4 = 1.$$

Extracting the square root, $2x^2 - x - 2 = \pm 1.$

$2x^2 - x = 3$ or $1.$

Solving these two equations, $x = -1$ or $\frac{3}{2}$ or 1 or $-\frac{1}{2}.$

73. $x^2 + x + 1 - \dfrac{1}{x^2 + x + 1} = \dfrac{8}{3}. \quad (1)$

If $x^2 + x + 1 = p,$ $\quad p - \dfrac{1}{p} = \dfrac{8}{3}. \quad (2)$

Clearing of fractions, etc., $3p^2 - 8p - 3 = 0.$

Factoring, $(p - 3)(3p + 1) = 0.$

$\therefore\ p = x^2 + x + 1 = 3$ or $-\frac{1}{3}. \quad (3)$

Transposing, etc., $x^2 + x - 2 = 0, \quad (4)$

or $3x^2 + 3x + 4 = 0. \quad (5)$

Factoring (4), $(x - 1)(x + 2) = 0.$

$\therefore\ x = 1$ or $-2.$

Solving (5) by the formula, § 390, $x = \dfrac{-3 \pm \sqrt{9 - 4 \cdot 3 \cdot 4}}{2 \cdot 3}$

$= \frac{1}{2}(-1 \pm \frac{1}{3}\sqrt{-39}).$

Hence, $x = 1$ or -2 or $\frac{1}{2}(-1 \pm \frac{1}{3}\sqrt{-39}).$

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74. $$x^2 - 2x + \frac{4}{x^2 - 2x + 1} = 4.$$

Adding 1 to each member and substituting p for $x^2 - 2x + 1$,

$$p + \frac{4}{p} = 5.$$

Clearing of fractions, etc., $p^2 - 5p + 4 = 0.$

Factoring, $(p-1)(p-4) = 0.$

$$\therefore p = x^2 - 2x + 1 = 1 \text{ or } 4.$$

Extracting the square root, $x - 1 = \pm 1$ or ± 2.

$$\therefore x = 2 \text{ or } 0 \text{ or } 3 \text{ or } -1.$$

75. $$x^2 - 3x + \frac{2}{x^2 - 3x + 2} = 1. \quad (1)$$

Adding 2 to each member and substituting p for $x^2 - 3x + 2$,

$$p + \frac{2}{p} = 3. \quad (2)$$

Clearing of fractions, etc., $p^2 - 3p + 2 = 0.$

Factoring, $(p-1)(p-2) = 0.$

$$\therefore p = x^2 - 3x + 2 = 1 \text{ or } 2. \quad (3)$$

Transposing, etc., $x^2 - 3x + 1 = 0,$ (4)

or $x^2 - 3x = 0.$ (5)

Solving (4) by the formula, § 390, $$x = \frac{3 \pm \sqrt{9 - 4 \cdot 1 \cdot 1}}{2 \cdot 1}$$

$$= \tfrac{1}{2}(3 \pm \sqrt{5}).$$

Factoring (5), $x(x-3) = 0.$

$$\therefore x = 0 \text{ or } 3.$$

Hence, $x = 0$ or 3 or $\frac{1}{2}(3 \pm \sqrt{5})$.

76. $$x^2 - x + \frac{2}{x^2 - x - 4} = 7. \quad (1)$$

Adding -4 to each member and substituting p for $x^2 - x - 4$,

$$p + \frac{2}{p} = 3. \quad (2)$$

Clearing of fractions, etc., $p^2 - 3p + 2 = 0.$

Factoring, $(p-1)(p-2) = 0.$

$$\therefore p = x^2 - x - 4 = 1 \text{ or } 2. \quad (3)$$

Transposing, etc., $x^2 - x - 5 = 0,$ (4)

or $x^2 - x - 6 = 0.$ (5)

Solving (4) by the formula, § 390, $$x = \frac{1 \pm \sqrt{1 - 4 \cdot 1(-5)}}{2}$$

$$= \tfrac{1}{2}(1 \pm \sqrt{21}).$$

Factoring (5), $(x-3)(x+2) = 0.$

$$\therefore x = 3 \text{ or } -2.$$

Hence, $x = 3$ or -2 or $\frac{1}{2}(1 \pm \sqrt{21})$.

77. $$\frac{x}{x^2 - 1} + \frac{x^2 - 1}{x} = -\frac{13}{6}. \quad (1)$$

If $\frac{x^2 - 1}{x} = p$, $$\frac{1}{p} + p = -\frac{13}{6}. \quad (2)$$

Clearing of fractions, etc., $6p^2 + 13p + 6 = 0.$

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Factoring, $(2p+3)(3p+2)=0.$

$$\therefore p=\frac{x^2-1}{x}=-\frac{3}{2} \text{ or } -\frac{2}{3}. \quad (3)$$

Clearing of fractions, etc., $2x^2+3x-2=0,$ (4)

or $3x^2+2x-3=0.$ (5)

Factoring (4), $(2x-1)(x+2)=0.$

$$\therefore x=\tfrac{1}{2} \text{ or } -2.$$

Solving (5) by the formula, § 390, $x=\dfrac{-2\pm\sqrt{4-4\cdot 3(-3)}}{2\cdot 3}$

$$=\tfrac{1}{3}(-1\pm\sqrt{10}).$$

Hence, $x=\frac{1}{2}$ or -2 or $\frac{1}{3}(-1\pm\sqrt{10})$.

78. $$\frac{1}{1+x+x^2}+\frac{2}{\sqrt{1+x+x^2}}-3=0. \quad (1)$$

Let $\sqrt{1+x+x^2}=p.$

Then, $$\frac{1}{p^2}+\frac{2}{p}-3=0. \quad (2)$$

Factoring, $$\left(\frac{1}{p}-1\right)\left(\frac{1}{p}+3\right)=0.$$

$$\therefore \frac{1}{p}=1 \text{ or } -3;$$

whence, $p=\sqrt{1+x+x^2}=1$ or $-\frac{1}{3}$. (3)

Squaring, $1+x+x^2=1$ or $\frac{1}{9}$. (4)

From (4), $x^2+x=0,$ (5)

or $x^2+x=-\frac{8}{9}.$ (6)

Factoring (5), $x(x+1)=0.$

$$\therefore x=0 \text{ or } -1.$$

Completing the square in (6), $x^2+x+\frac{1}{4}=-\frac{23}{36}.$

$$\therefore x=-\tfrac{1}{2}\pm\tfrac{1}{6}\sqrt{-23}.$$

Hence, $x=0$ or -1 or $-\frac{1}{2}\pm\frac{1}{6}\sqrt{-23}$.

$-\frac{1}{2}\pm\frac{1}{6}\sqrt{-23}$ does not verify and is rejected.

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2. $$\begin{cases} x^2+y^2=20, & (1)\\ x=2y. & (2)\end{cases}$$

Substituting (2) in (1), $4y^2+y^2=20.$ (3)

Solving (3), $y=\pm 2.$

Substituting $y=2$ in (2), $x=4.$

Substituting $y=-2$ in (2), $x=-4.$

Hence, when $x=4$, $y=2$; when $x=-4$, $y=-2$.

3. $$\begin{cases} 10x+y=3xy, & (1)\\ y-x=2. & (2)\end{cases}$$

From (2), $y=x+2.$ (3)

Substituting (3) in (1), $10x+x+2=3x(x+2).$ (4)

Solving (4), $x=2$ or $-\frac{1}{3}$. (5)

Substituting (5) in (3), $y=4$ or $\frac{5}{3}$.

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4. $\begin{cases} x^2 + xy = 12, & (1) \\ x - y = 2. & (2) \end{cases}$

From (2), $y = x - 2.$ (3)

Substituting (3) in (1), $x^2 + x(x - 2) = 12.$ (4)

Solving (4), $x = 3$ or $-2.$ (5)

Substituting (5) in (3), $y = 1$ or $-4.$

5. $\begin{cases} m^2 - 3n^2 = 13, & (1) \\ m - 2n = 1. & (2) \end{cases}$

From (2), $m = 1 + 2n.$ (3)

Substituting (3) in (1),

$1 + 4n + 4n^2 - 3n^2 = 13.$ (4)

Solving, $n = 2$ or $-6.$ (5)

Substituting (5) in (2), $m = 5$ or $-11.$

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6. $\begin{cases} x = 6 - y, & (1) \\ x^3 + y^3 = 72. & (2) \end{cases}$

Substituting (1) in (2),

$216 - 108y + 18y^2 - y^3 + y^3 = 72.$

$y^2 - 6y + 8 = 0.$ (3)

Solving (3), $y = 4$ or $2.$ (4)

Substituting (4) in (1), $x = 2$ or $4.$

7. $\begin{cases} xy(x - 2y) = 10, & (1) \\ xy = 10. & (2) \end{cases}$

Substituting(2) in (1), etc., $x - 2y = 1.$ (3)

From (3), $x = 2y + 1.$ (4)

Substituting (4) in (2), $(2y + 1)y = 10.$ (5)

Solving (5), $y = 2$ or $-\frac{5}{2}.$ (6)

Substituting (6) in (4), $x = 5$ or $-4.$

8. $\begin{cases} 3x(y + 1) = 12, & (1) \\ 3x = 2y. & (2) \end{cases}$

Substituting (2) in (1), $2y(y + 1) = 12.$ (3)

Solving (3), $y = 2$ or $-3.$ (4)

Substituting (4) in (2) and solving, $x = \frac{4}{3}$ or $-2.$

9. $\begin{cases} 3rs - 10r = s, & (1) \\ 2 - s = -r. & (2) \end{cases}$

From (2), $s = r + 2.$ (3)

Substituting (3) in (1), $3r(r + 2) - 10r = r + 2.$ (4)

Solving, $r = 2$ or $-\frac{1}{3}.$ (5)

Substituting (5) in (3), $s = 4$ or $\frac{5}{3}.$

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2. $\begin{cases} x^2 + y^2 = 25, & (1) \\ x + y = 7. & (2) \end{cases}$

Squaring (2), $x^2 + 2xy + y^2 = 49.$ (3)

Subtracting (1) from (3), $2xy = 24.$ (4)

Subtracting (4) from (1), $x^2 - 2xy + y^2 = 1.$ (5)

Extracting the square root of (5), $x - y = \pm 1.$ (6)

Adding (2) and (6), $x = 4$ or $3.$

Subtracting (6) from (2), $y = 3$ or $4.$

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4. $\begin{cases} x^2+y^2=50, & (1)\\ xy=7. & (2)\end{cases}$

Multiplying (2) by 2, $2xy=14.$ (3)

Adding (3) to (1), $x^2+2xy+y^2=64.$ (4)

Subtracting (3) from (1), $x^2-2xy+y^2=36.$ (5)

From (4), $x+y=\pm 8.$ (6)

From (5), $x-y=\pm 6.$ (7)

Since (4) and (5) are derived separately, neither from the other,

from (6) and (7), $\begin{cases} x+y=8,\\ x-y=6;\end{cases}$ or $\begin{cases} x+y=8,\\ x-y=-6;\end{cases}$ or $\begin{cases} x+y=-8,\\ x-y=6;\end{cases}$ or $\begin{cases} x+y=-8,\\ x-y=-6.\end{cases}$

From these equations, $x=7, 1, -1, -7;$

and $y=1, 7, -7, -1.$

5. $\begin{cases} x+y=8, & (1)\\ x^2+y^2=34. & (2)\end{cases}$

Squaring (1), $x^2+2xy+y^2=64.$ (3)

Subtracting (3) from (2), $-2xy=-30.$ (4)

Adding (4) to (2), $x^2-2xy+y^2=4.$ (5)

$\therefore x-y=\pm 2.$ (6)

From (1) and (6), $x=5$ or 3,

and $y=3$ or 5.

6. $\begin{cases} x+y=9, & (1)\\ x^3+y^3=243. & (2)\end{cases}$

Cubing (1), $x^3+3x^2y+3xy^2+y^3=729.$ (3)

Subtracting (2) from (3), $3x^2y+3xy^2=486.$

$xy(x+y)=162.$ (4)

Substituting (1) in (4), $9xy=162.$

$\therefore xy=18.$ (5)

Squaring (1), $x^2+2xy+y^2=81.$ (6)

Subtracting (5) $\times 4$ from (6), $x^2-2xy+y^2=9.$

$\therefore x-y=\pm 3.$ (7)

From (1) and (7), $x=6$ or 3,

and $y=3$ or 6.

7. $\begin{cases} x+y=8, & (1)\\ x^2+xy+y^2=49. & (2)\end{cases}$

Squaring (1), $x^2+2xy+y^2=64.$ (3)

Subtracting (2) from (3), $xy=15.$ (4)

Multiplying (4) by 4, $4xy=60.$ (5)

Subtracting (5) from (3), $x^2-2xy+y^2=4.$ (6)

Extracting the square root, $x-y=\pm 2.$ (7)

Adding (1) and (7), $x=5$ or 3.

Subtracting (7) from (1), $y=3$ or 5.

8. $\begin{cases} x^2+xy+y^2=31, & (1)\\ x^2+y^2=26. & (2)\end{cases}$

Subtracting (2) from (1), $xy=5.$ (3)

Adding (1) and (3), $x^2+2xy+y^2=36.$ (4)

Extracting the square root, $x+y=\pm 6.$ (5)

Multiplying (3) by 3, $3xy=15.$ (6)

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Subtracting (6) from (1), $x^2 - 2xy + y^2 = 16.$ (7)
Extracting the square root, $x - y = \pm 4.$ (8)
Since (4) and (7) are derived separately, neither from the other,
from (5) and (8), $\begin{cases} x+y=6, \\ x-y=4; \end{cases} \begin{cases} x+y=6, \\ x-y=-4; \end{cases} \begin{cases} x+y=-6, \\ x-y=4; \end{cases} \begin{cases} x+y=-6, \\ x-y=-4. \end{cases}$
From these equations, $x = 5, 1, -1, -5;$
and $y = 1, 5, -5, -1.$

9. $\begin{cases} x^2 + y^2 = 8, & (1) \\ x^2 - xy + y^2 = 4. & (2) \end{cases}$
Subtracting (2) from (1), $xy = 4.$ (3)
Subtracting (3) from (2), $x^2 - 2xy + y^2 = 0.$ (4)
Multiplying (3) by 3, $3xy = 12.$ (5)
Adding (2) and (5), $x^2 + 2xy + y^2 = 16.$ (6)
Extracting the square root of (4), $x - y = 0.$ (7)
Extracting the square root of (6), $x + y = \pm 4.$ (8)
From (7) and (8), $x = 2 \text{ or } -2,$
and $y = 2 \text{ or } -2.$

10. $\begin{cases} x^2 + y^2 = 13, & (1) \\ x + y + xy = 11. & (2) \end{cases}$
From (2), $x + y = 11 - xy.$ (3)
Squaring (3), $x^2 + 2xy + y^2 = 121 - 22xy + x^2y^2.$ (4)
Subtracting (1) from (4), $2xy = 108 - 22xy + x^2y^2.$
$x^2y^2 - 24xy + 108 = 0.$ (5)
$(xy - 6)(xy - 18) = 0.$
$\therefore xy = 6 \text{ or } 18.$ (6)
Subtracting (6) from (2), $x + y = 5 \text{ or } -7.$ (7)
Multiplying (6) by 2, $2xy = 12 \text{ or } 36.$ (8)
Subtracting $2xy = 12$ from (1),
$x^2 - 2xy + y^2 = 1;$
whence, $x - y = \pm 1.$ (9)
Subtracting $2xy = 36$ from (1),
$x^2 - 2xy + y^2 = -23;$
whence, $x - y = \pm\sqrt{-23}.$ (10)
When $xy = 6$, $\begin{cases} x + y = 5, \\ x - y = \pm 1. \end{cases}$ When $xy = 18$, $\begin{cases} x + y = -7, \\ x - y = \pm\sqrt{-23}. \end{cases}$
From these equations, $x = 3, 2, \frac{1}{2}(-7 + \sqrt{-23}), \frac{1}{2}(-7 - \sqrt{-23});$
and $y = 2, 3, \frac{1}{2}(-7 - \sqrt{-23}), \frac{1}{2}(-7 + \sqrt{-23}).$

11. $\begin{cases} x^2 + 3xy + y^2 = 31, & (1) \\ xy = 6. & (2) \end{cases}$
Subtracting (2) from (1), $x^2 + 2xy + y^2 = 25.$ (3)
Extracting the square root, $x + y = \pm 5.$ (4)
Multiplying (2) by 4, $4xy = 24.$ (5)
Subtracting (5) from (3), $x^2 - 2xy + y^2 = 1.$ (6)
Extracting the square root, $x - y = \pm 1.$ (7)
Since (3) and (6) are derived separately, neither from the other,
from (4)and (7), $\begin{cases} x+y=5, \\ x-y=1; \end{cases} \begin{cases} x+y=5, \\ x-y=-1; \end{cases} \begin{cases} x+y=-5, \\ x-y=1; \end{cases} \begin{cases} x+y=-5, \\ x-y=-1 \end{cases}$
From these equations, $x = 3, 2, -2, -3;$
and $y = 2, 3, -3, -2.$

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12. $$\begin{cases} x^2 + y^2 = 100, & (1) \\ (x+y)^2 = 196. & (2) \end{cases}$$

From (2), $x^2 + 2xy + y^2 = 196.$ (3)

Subtracting (1) from (3), $2xy = 96.$ (4)

Multiplying (4) by 2, $4xy = 192.$ (5)

Subtracting (5) from (3), $x^2 - 2xy + y^2 = 4.$ (6)

Extracting the square root, $x - y = \pm 2.$ (7)

From (2), $x + y = \pm 14.$ (8)

Since (3) and (6) are derived separately, neither from the other,

from (7) and (8), $\begin{cases} x+y=14, \\ x-y=2; \end{cases}$ $\begin{cases} x+y=14, \\ x-y=-2; \end{cases}$ $\begin{cases} x+y=-14, \\ x-y=2; \end{cases}$ $\begin{cases} x+y=-14, \\ x-y=-2. \end{cases}$

From these equations, $x = 8, 6, -6, -8;$

and $y = 6, 8, -8, -6.$

13. $$\begin{cases} x + xy + y = 19, & (1) \\ x^2y^2 = 144. & (2) \end{cases}$$

From (2), $xy = \pm 12.$ (3)

Subtracting (3) from (1), $x + y = 7 \text{ or } 31.$ (4)

From (3) and (4), $\begin{cases} x+y=7, \\ xy=12; \end{cases}$ $\begin{cases} x+y=7, \\ xy=-12; \end{cases}$ $\begin{cases} x+y=31, \\ xy=12; \end{cases}$ $\begin{cases} x+y=31, \\ xy=-12. \end{cases}$

Solving these sets of equations as in Ex. 1, p. 310,

$x = 4, 3, \frac{1}{2}(7+\sqrt{97}), \frac{1}{2}(7-\sqrt{97}), \frac{1}{2}(31+\sqrt{913}), \frac{1}{2}(31-\sqrt{913}),$
$\frac{1}{2}(31+\sqrt{1009}), \frac{1}{2}(31-\sqrt{1009});$

$y = 3, 4, \frac{1}{2}(7-\sqrt{97}), \frac{1}{2}(7+\sqrt{97}), \frac{1}{2}(31-\sqrt{913}), \frac{1}{2}(31+\sqrt{913}),$
$\frac{1}{2}(31-\sqrt{1009}), \frac{1}{2}(31+\sqrt{1009}).$

But, $x = \frac{1}{2}(7+\sqrt{97}), \frac{1}{2}(7-\sqrt{97}), \frac{1}{2}(31+\sqrt{913}), \frac{1}{2}(31-\sqrt{913});$

and $y = \frac{1}{2}(7-\sqrt{97}), \frac{1}{2}(7+\sqrt{97}), \frac{1}{2}(31-\sqrt{913}), \frac{1}{2}(31+\sqrt{913}),$

do not verify and are rejected.

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14. $$\begin{cases} x^4 + y^4 = 17, & (1) \\ x + y = 3. & (2) \end{cases}$$

Raising (2) to the fourth power,

$$x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4 = 81. \quad (3)$$

Subtracting (1) from (3), $4x^3y + 6x^2y^2 + 4xy^3 = 64.$ (4)

Dividing (4) by 2, $2x^3y + 3x^2y^2 + 2xy^3 = 32.$ (5)

$2xy \times$ square of (2), $2x^3y + 4x^2y^2 + 2xy^3 = 18xy.$ (6)

Subtracting (5) from (6), $x^2y^2 = 18xy - 32.$ (7)

Solving (7) for xy, $xy = 2 \text{ or } 16.$ (8)

Squaring (2), $x^2 + 2xy + y^2 = 9.$ (9)

Subtracting $4xy = 8$ from (9), $x^2 - 2xy + y^2 = 1;$

whence, $x - y = \pm 1.$ (10)

Subtracting $4xy = 64$ from (9), $x^2 - 2xy + y^2 = -55;$

whence, $x - y = \pm\sqrt{-55}.$ (11)

When $xy = 2$, $\begin{cases} x+y=3, \\ x-y=\pm 1. \end{cases}$ When $xy = 16$, $\begin{cases} x+y=3, \\ x-y=\pm\sqrt{-55}. \end{cases}$

From these equations, $x = 2, 1, \frac{1}{2}(3+\sqrt{-55}), \frac{1}{2}(3-\sqrt{-55});$

and $y = 1, 2, \frac{1}{2}(3-\sqrt{-55}), \frac{1}{2}(3+\sqrt{-55}).$

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15.
$$\begin{cases} x^4 + x^2y^2 + y^4 = 21, & (1) \\ x^2 + xy + y^2 = 7. & (2) \end{cases}$$

From (2), $x^2 + y^2 = 7 - xy.$ (3)

Squaring (3), $x^4 + 2x^2y^2 + y^4 = 49 - 14xy + x^2y^2.$ (4)

Subtracting (1) from (4), $x^2y^2 = 28 - 14xy + x^2y^2.$ (5)

Solving (5) for xy, $xy = 2.$ (6)

Adding (6) to (2), $x^2 + 2xy + y^2 = 9;$

whence, $x + y = \pm 3.$ (7)

Subtracting (6) × 3 from (2), $x^2 - 2xy + y^2 = 1;$

whence, $x - y = \pm 1.$ (8)

Since (7) and (8) have been derived separately, we have

$$\begin{cases} x+y=3, \\ x-y=1; \end{cases} \text{ or } \begin{cases} x+y=3, \\ x-y=-1; \end{cases} \text{ or } \begin{cases} x+y=-3, \\ x-y=1; \end{cases} \text{ or } \begin{cases} x+y=-3, \\ x-y=-1. \end{cases}$$

From these equations, $x = 2, 1, -1, -2;$

and $y = 1, 2, -2, -1.$

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2.
$$\begin{cases} 2x^2 - 3y - y^2 = 8, & (1) \\ 6x^2 - 5xy - 6y^2 = 0. & (2) \end{cases}$$

Factoring (2), $(3x + 2y)(2x - 3y) = 0.$

$$y = -\frac{3x}{2}, \quad (3)$$

or
$$y = \frac{2x}{3}. \quad (4)$$

Substituting (3) in (1), simplifying, etc., $x^2 - 18x + 32 = 0.$ (5)

Factoring (5), $(x - 16)(x - 2) = 0.$ (6)

From (6), $x = 16 \text{ or } 2.$ (7)

Substituting (7) in (3), $y = -24 \text{ or } -3.$ (8)

Substituting (4) in (1), simplifying, etc., $7x^2 - 9x - 36 = 0.$ (9)

Solving, $x = 3 \text{ or } -\frac{12}{7}.$ (10)

Substituting (10) in (4), $y = 2 \text{ or } -\frac{8}{7}.$ (11)

Hence, from (7), (8), (10), and (11), the roots of the given equations are,

$$\begin{cases} x = 16, 2, 3, -\frac{12}{7}; \\ y = -24, -3, 2, -\frac{8}{7}. \end{cases}$$

3.
$$\begin{cases} 5x^2 + 8xy - 4y^2 = 0, & (1) \\ xy + 2y^2 = 60. & (2) \end{cases}$$

Factoring (1), $(5x - 2y)(x + 2y) = 0,$

whence,
$$y = \frac{5x}{2}, \quad (3)$$

or,
$$y = -\frac{x}{2}. \quad (4)$$

Substituting (3) in (2), simplifying, etc., $30x^2 = 120.$

$\therefore x = 2 \text{ or } -2.$ (5)

Substituting (5) in (3), $y = 5 \text{ or } -5.$

From (2), $y(x + 2y) = 60.$ (6)

Substituting (4) in (6),
$$-\frac{x}{2}(x - x) = 60.$$

$x \cdot 0 = -120.$

$\therefore x = -\frac{120}{0}.$ (7)

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Substituting (7) in (4), $y = \dfrac{120}{2 \cdot 0} = \dfrac{60}{0}.$

NOTE. — The interpretation of such expressions will be found in § 543. Such roots are called *infinite roots*. The symbol for an infinite number is ∞, read "*infinity*." Hence, from $x + 2y = 0$, $x = \infty$, $y = \infty$.

4. $$\begin{cases} 2x^2 - xy - y^2 = 0, & (1) \\ 4x^2 + 4xy + y^2 = 36. & (2) \end{cases}$$

Factoring (1), $(2x + y)(x - y) = 0.$ (3)

From (3), $y = x,$ (4)

and $y = -2x.$ (5)

Substituting (4) in (2), solving, etc., $x = 2$ or $-2.$ (6)

Substituting (6) in (4), $y = 2$ or $-2.$

The value $y = -2x$ in (5) gives rise to infinite roots.

See Note, Ex. 3.

5. $$\begin{cases} 6x^2 + xy - 12y^2 = 0, & (1) \\ x^2 + xy - y = 1. & (2) \end{cases}$$

Factoring (1), $(3x - 4y)(2x + 3y) = 0.$ (3)

From (3), $y = \dfrac{3x}{4},$ (4)

and $y = -\dfrac{2x}{3}.$ (5)

Substituting (4) in (2), simplifying, etc., $7x^2 - 3x = 4.$

Solving, $x = 1$ or $-\frac{4}{7}.$ (6)

Substituting (6) in (4), $y = \frac{3}{4}$ or $-\frac{3}{7}.$ (7)

Substituting (5) in (2), $x^2 + 2x - 3 = 0.$

Solving, $x = 1$, or $-3.$ (8)

Substituting (8) in (5), $y = -\frac{2}{3}$ or $2.$ (9)

Hence, from (6), (7), (8), and (9), the roots are $\begin{cases} x = 1, -\frac{4}{7}, 1, -3; \\ y = \frac{3}{4}, -\frac{3}{7}, -\frac{2}{3}, 2. \end{cases}$

6. $$\begin{cases} 3x^2 - 7xy - 40y^2 = 0, & (1) \\ x^2 - xy - 12y^2 = 8. & (2) \end{cases}$$

Factoring (1), $(3x + 8y)(x - 5y) = 0.$ (3)

From (3), $y = \dfrac{x}{5},$ (4)

and $y = -\dfrac{3x}{8}.$ (5)

Substituting (4) in (2), simplifying, etc., $x = 5$ or $-5.$ (6)

Substituting (6) in (4), $y = 1$ or $-1.$ (7)

Substituting (5) in (2), $5x^2 = -128.$

$x = \frac{8}{5}\sqrt{-10}$ or $-\frac{8}{5}\sqrt{-10}.$ (8)

Substituting (8) in (5), $y = -\frac{3}{5}\sqrt{-10}$ or $\frac{3}{5}\sqrt{-10}.$ (9)

Hence, from (6), (7), (8), and (9), the roots are,

$$\begin{cases} x = 5, -5, \frac{8}{5}\sqrt{-10}, -\frac{8}{5}\sqrt{-10}; \\ y = 1, -1, -\frac{3}{5}\sqrt{-10}, \frac{3}{5}\sqrt{-10}. \end{cases}$$

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7. $$\begin{cases} x^2 - xy - y^2 = 20, & (1) \\ 3x^2 - 13xy + 12y^2 = 0. & (2) \end{cases}$$

Factoring (2), $(3x - 4y)(x - 3y) = 0.$ (3)

From (3), $y = \frac{x}{3},$ (4)

and $y = \frac{3x}{4}.$ (5)

Substituting (4) in (1), $5x^2 = 180.$

$\therefore x = 6 \text{ or } -6.$ (6)

Substituting (6) in (4), $y = 2 \text{ or } -2.$ (7)

Substituting (5) in (1), $x^2 = -64.$

$\therefore x = \pm 8\sqrt{-1}.$ (8)

Substituting (8) in (5), $y = \pm 6\sqrt{-1}.$ (9)

Hence, from (6), (7), (8), and (9), the roots are,

$$\begin{cases} x = 6, \ -6, \ 8\sqrt{-1}, \ -8\sqrt{-1}; \\ y = 2, \ -2, \ 6\sqrt{-1}, \ -6\sqrt{-1}. \end{cases}$$

8. $$\begin{cases} 3x^2 - 7xy + 4y^2 = 0, & (1) \\ 5x^2 - 7xy + 3y^2 = 4. & (2) \end{cases}$$

Factoring (1), $(x - y)(3x - 4y) = 0.$ (3)

From (3), $y = x,$ (4)

and $y = \frac{3x}{4}.$ (5)

Substituting (4) in (2), and solving, $x = 2 \text{ or } -2.$ (6)

Substituting (6) in (4), $y = 2 \text{ or } -2.$ (7)

Substituting (5) in (2), $23x^2 = 64.$

$\therefore x = \pm \frac{8}{23}\sqrt{23}.$ (8)

Substituting (8) in (5), $y = \pm \frac{6}{23}\sqrt{23}.$ (9)

Hence, from (6), (7), (8), and (9), the roots are,

$$\begin{cases} x = 2, \ -2, \ \frac{8}{23}\sqrt{23}, \ -\frac{8}{23}\sqrt{23}; \\ y = 2, \ -2, \ \frac{6}{23}\sqrt{23}, \ -\frac{6}{23}\sqrt{23}. \end{cases}$$

9. $$\begin{cases} x^2 + y^2 + x - y = 12, & (1) \\ 3x^2 + 2xy - y^2 = 0. & (2) \end{cases}$$

Factoring (2), $(3x - y)(x + y) = 0.$ (3)

From (3), $y = 3x,$ (4)

and $y = -x.$ (5)

Substituting (4) in (1), simplifying, etc.,

$$5x^2 - x = 6.$$

Solving, $x = \frac{6}{5} \text{ or } -1.$ (6)

Substituting (6) in (4), $y = \frac{18}{5} \text{ or } -3.$ (7)

Substituting (5) in (1), simplifying, etc.,

$$x^2 + x = 6.$$

Solving, $x = 2 \text{ or } -3.$ (8)

Substituting (8) in (5), $y = -2 \text{ or } 3.$ (9)

Hence, from (6), (7), (8), and (9), the roots are,

$$\begin{cases} x = \frac{6}{5}, \ -1, \ 2, \ -3; \\ y = \frac{18}{5}, \ -3, \ -2, \ 3. \end{cases}$$

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2. $\begin{cases} xy + 3y^2 = 20, & (1) \\ x^2 - 3xy = -8. & (2) \end{cases}$

Assume $x = vy.$ (3)

Substituting (3) in (1), $vy^2 + 3y^2 = 20.$ (4)

Substituting (3) in (2), $v^2y^2 - 3vy^2 = -8.$ (5)

From (4) and (5), $y^2 = \dfrac{20}{v+3} = \dfrac{-8}{v^2 - 3v}.$ (6)

Clearing of fractions, etc., $5v^2 - 13v + 6 = 0.$

Factoring, $(v-2)(5v-3) = 0.$

$\therefore v = 2 \text{ or } \frac{3}{5}.$

Substituting 2 for v in (6), $y = 2 \text{ or } -2,$
whence, by (3), $x = 4 \text{ or } -4,$ when $v = 2.$

Substituting, $\frac{3}{5}$ for v in (6), $y = \frac{5}{3}\sqrt{2} \text{ or } -\frac{5}{3}\sqrt{2},$
whence, by (3), $x = \sqrt{2} \text{ or } -\sqrt{2},$ when $v = \frac{3}{5}.$

Hence, $\begin{cases} x = 4, -4, \sqrt{2}, -\sqrt{2}; \\ y = 2, -2, \frac{5}{3}\sqrt{2}, -\frac{5}{3}\sqrt{2}. \end{cases}$

3. $\begin{cases} x^2 + xy = 12, & (1) \\ xy + 2y^2 = 5. & (2) \end{cases}$

Assume $x = vy.$ (3)

Substituting (3) in (1), $v^2y^2 + vy^2 = 12.$ (4)

Substituting (3) in (2), $vy^2 + 2y^2 = 5.$ (5)

From (4) and (5), $y^2 = \dfrac{12}{v^2 + v} = \dfrac{5}{v+2}.$ (6)

Clearing of fractions, etc., $5v^2 - 7v - 24 = 0.$

Factoring, $(v-3)(5v+8) = 0.$

$\therefore v = 3 \text{ or } -\frac{8}{5}.$

Substituting 3 for v in (6), $y = 1 \text{ or } -1,$
whence, by (3), $x = 3 \text{ or } -3,$ when $v = 3.$

Substituting $-\frac{8}{5}$ for v in (6), $y = \frac{5}{2}\sqrt{2} \text{ or } -\frac{5}{2}\sqrt{2},$
whence, by (3), $x = -4\sqrt{2} \text{ or } 4\sqrt{2},$ when $v = -\frac{8}{5}.$

Hence, $\begin{cases} x = 3, -3, 4\sqrt{2}, -4\sqrt{2}, \\ y = 1, -1, -\frac{5}{2}\sqrt{2}, \frac{5}{2}\sqrt{2}. \end{cases}$

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4. $\begin{cases} x^2 + 2y^2 = 44, & (1) \\ xy - y^2 = 8. & (2) \end{cases}$

Assume $x = vy.$ (3)

Substituting (3) in (1), $v^2y^2 + 2y^2 = 44.$ (4)

Substituting (3) in (2), $vy^2 - y^2 = 8.$ (5)

From (4) and (5), $y^2 = \dfrac{44}{v^2 + 2} = \dfrac{8}{v - 1}.$ (6)

Clearing of fractions, etc., $2v^2 - 11v + 15 = 0.$

$(v-3)(2v-5) = 0.$

$\therefore v = 3 \text{ or } \frac{5}{2}.$

Substituting 3 for v in (6), $y = 2 \text{ or } -2,$
whence, by (3), $x = 6 \text{ or } -6,$ when $v = 3.$

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Substituting $\frac{5}{2}$ for v in (6), whence, by (3),
$$\left.\begin{array}{l} y = \frac{4}{3}\sqrt{3} \text{ or } -\frac{4}{3}\sqrt{3}, \\ x = \frac{10}{3}\sqrt{3} \text{ or } -\frac{10}{3}\sqrt{3}, \end{array}\right\} \text{ when } v = \tfrac{5}{2}.$$

Hence,
$$\begin{cases} x = 6,\ -6,\ \frac{10}{3}\sqrt{3},\ -\frac{10}{3}\sqrt{3}; \\ y = 2,\ -2,\ \frac{4}{3}\sqrt{3},\ -\frac{4}{3}\sqrt{3}. \end{cases}$$

5.
$$\begin{cases} x(x-y) = 6, & (1) \\ x^2 - y^2 = 3. & (2) \end{cases}$$

Assume $\quad x = vy. \quad (3)$

Substituting (3) in (1), $\quad v^2y^2 - vy^2 = 6. \quad (4)$

Substituting (3) in (2), $\quad v^2y^2 - y^2 = 3. \quad (5)$

From (4) and (5),
$$y^2 = \frac{6}{v^2 - v} = \frac{3}{v^2 - 1}. \quad (6)$$

Clearing of fractions, etc., $\quad v^2 + v = 2.$

Solving, $\quad v = -2 \text{ or } 1.$

Substituting -2 for v in (6), whence, by (3),
$$\left.\begin{array}{l} y = 1 \text{ or } -1, \\ x = -2 \text{ or } 2, \end{array}\right\} \text{ when } v = -2.$$

Substituting 1 for v in (6) gives rise to infinite roots. See Note, Ex. 3, p. 295.

Hence,
$$\begin{cases} x = 2,\ -2; \\ y = -1,\ 1. \end{cases}$$

6.
$$\begin{cases} x^2 - xy - y^2 = 20, & (1) \\ x^2 - 3xy + 2y^2 = 8. & (2) \end{cases}$$

Assume $\quad x = vy. \quad (3)$

Substituting (3) in (1), $\quad v^2y^2 - vy^2 - y^2 = 20. \quad (4)$

Substituting (3) in (2), $\quad v^2y^2 - 3vy^2 + 2y^2 = 8. \quad (5)$

From (4) and (5),
$$y^2 = \frac{20}{v^2 - v - 1} = \frac{8}{v^2 - 3v + 2}. \quad (6)$$

Clearing of fractions, etc., $\quad 3v^2 - 13v + 12 = 0.$

Factoring, $\quad (v-3)(3v-4) = 0.$

$$\therefore v = 3 \text{ or } \tfrac{4}{3}.$$

Substituting 3 for v in (6), whence, by (3),
$$\left.\begin{array}{l} y = 2 \text{ or } -2, \\ x = 6 \text{ or } -6, \end{array}\right\} \text{ when } v = 3.$$

Substituting $\frac{4}{3}$ for v in (6), whence, by (3),
$$\left.\begin{array}{l} y = 6\sqrt{-1} \text{ or } -6\sqrt{-1}, \\ x = 8\sqrt{-1} \text{ or } -8\sqrt{-1}, \end{array}\right\} \text{ when } v = \tfrac{4}{3}.$$

Hence,
$$\begin{cases} x = 6,\ -6,\ 8\sqrt{-1},\ -8\sqrt{-1}; \\ y = 2,\ -2,\ 6\sqrt{-1},\ -6\sqrt{-1}. \end{cases}$$

7.
$$\begin{cases} x^2 - xy + y^2 = 21, & (1) \\ x^2 + 2y^2 = 27. & (2) \end{cases}$$

Assume $\quad x = vy. \quad (3)$

Substituting (3) in (1), $\quad v^2y^2 - vy^2 + y^2 = 21. \quad (4)$

Substituting (3) in (2), $\quad v^2y^2 + 2y^2 = 27. \quad (5)$

From (4) and (5),
$$y^2 = \frac{21}{v^2 - v + 1} = \frac{27}{v^2 + 2}. \quad (6)$$

Clearing of fractions, etc., $\quad 2v^2 - 9v - 5 = 0.$

Factoring, $\quad (v-5)(2v+1) = 0.$

$$\therefore v = 5 \text{ or } -\tfrac{1}{2}.$$

Substituting 5 for v in (6), whence, by (3),
$$\left.\begin{array}{l} y = 1 \text{ or } -1, \\ x = 5 \text{ or } -5, \end{array}\right\} \text{ when } v = 5.$$

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Substituting $-\frac{1}{2}$ for v in (6), $y = 2\sqrt{3}$ or $-2\sqrt{3}$, whence, by (3), $x = -\sqrt{3}$ or $\sqrt{3}$, when $v = -\frac{1}{2}$.

Hence, $\begin{cases} x = 5, -5, \sqrt{3}, -\sqrt{3}; \\ y = 1, -1, -2\sqrt{3}, 2\sqrt{3}. \end{cases}$

8.
$$\begin{cases} 2x^2 - 3xy + 2y^2 = 100, & (1) \\ x^2 - y^2 = 75. & (2) \end{cases}$$

Assume $x = vy$. (3)

Substituting (3) in (1), $2v^2y^2 - 3vy^2 + 2y^2 = 100$. (4)

Substituting (3) in (2), $v^2y^2 - y^2 = 75$. (5)

From (4) and (5), $y^2 = \dfrac{100}{2v^2 - 3v + 2} = \dfrac{75}{v^2 - 1}$. (6)

Reducing (6), $2v^2 - 9v + 10 = 0$.

Factoring, $(v-2)(2v-5) = 0$.

$\therefore v = 2$ or $\frac{5}{2}$.

Substituting 2 for v in (6), $y = 5$ or -5, whence, by (3), $x = 10$ or -10, when $v = 2$.

Substituting $\frac{5}{2}$ for v in (6), $y = \frac{10}{7}\sqrt{7}$ or $-\frac{10}{7}\sqrt{7}$, whence, by (3), $x = \frac{25}{7}\sqrt{7}$ or $-\frac{25}{7}\sqrt{7}$, when $v = \frac{5}{2}$.

9.
$$\begin{cases} x^2 - 5xy + 3y^2 = 8, & (1) \\ 3x^2 + xy + y^2 = 24. & (2) \end{cases}$$

Multiplying (1) by 3, $3x^2 - 15xy + 9y^2 = 24$. (3)

Subtracting (3) from (2), $16xy - 8y^2 = 0$.

$8y(2x - y) = 0$.

$y = 0$ or $2x$.

Substituting 0 for y in (1), $x = \pm 2\sqrt{2}$.

Substituting $2x$ for y in (1), $x = \pm \frac{2}{3}\sqrt{6}$.

Then, from $y = 2x$, $y = \pm \frac{4}{3}\sqrt{6}$.

Hence, the roots are $\begin{cases} x = 2\sqrt{2}, -2\sqrt{2}, \frac{2}{3}\sqrt{6}, -\frac{2}{3}\sqrt{6}; \\ y = 0, \quad 0, \frac{4}{3}\sqrt{6}, -\frac{4}{3}\sqrt{6}. \end{cases}$

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10.
$$\begin{cases} m^2 + mn = 2, & (1) \\ mn + n^2 = -1. & (2) \end{cases}$$

Adding (1) and (2), $m^2 + 2mn + n^2 = 1$. (3)

$\therefore m + n = 1$ or -1. (4)

Subtracting (2) from (1), $m^2 - n^2 = 3$. (5)

Dividing (5) by (4), $m - n = 3$ or -3. (6)

Combining (4) and (6), $m = 2, -2$,

and $n = -1, 1$.

11.
$$\begin{cases} p^2 + q^2 + p + q = 36, & (1) \\ pq = -15. & (2) \end{cases}$$

Multiplying (2) by 2, and adding (1), $p^2 + 2pq + q^2 + p + q = 6$.

Completing the square, $(p+q)^2 + (p+q) + (\frac{1}{2})^2 = \frac{25}{4}$.

Extracting the square root, $p + q + \frac{1}{2} = \pm \frac{5}{2}$.

$\therefore p + q = 2$ or -3. (3)

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Equations (2) and (3) give two pairs of simultaneous equations,

$$\begin{cases} p+q=2, \\ pq=-15; \end{cases} \text{ and } \begin{cases} p+q=-3, \\ pq=-15. \end{cases}$$

Solving, according to § 407, the corresponding values of p and q are,

$$\begin{cases} p=5,\ -3,\ \frac{1}{2}(-3+\sqrt{69}),\ \frac{1}{2}(-3-\sqrt{69}); \\ q=-3,\ 5,\ \frac{1}{2}(-3-\sqrt{69}),\ \frac{1}{2}(-3+\sqrt{69}). \end{cases}$$

12.
$$\begin{cases} a^2+b^2=130, & (1) \\ a-b=2. & (2) \end{cases}$$

Squaring (2), $a^2-2ab+b^2=4.$ (3)
Subtracting (3) from (1), $2ab=126.$ (4)
Adding (4) and (1), $a^2+2ab+b^2=256.$ (5)
Extracting the square root, $a+b=\pm 16.$ (6)
Adding (2) and (6), $a=9$ or $-7.$
Subtracting (2) from (6), $b=7$ or $-9.$

13.
$$\begin{cases} \frac{1}{x^2}+\frac{1}{y^2}=13, & (1) \\ \frac{1}{x}-\frac{1}{y}=1. & (2) \end{cases}$$

Squaring (2), $\frac{1}{x^2}-\frac{2}{xy}+\frac{1}{y^2}=1.$ (3)

Subtracting (3) from (1), $\frac{2}{xy}=12.$ (4)

Adding (4) and (1), $\frac{1}{x^2}+\frac{2}{xy}+\frac{1}{y^2}=25.$ (5)

Extracting the square root, $\frac{1}{x}+\frac{1}{y}=\pm 5.$ (6)

Adding (2) and (6), $\frac{1}{x}=3$ or $-2.$ (7)

Subtracting (2) from (6), $\frac{1}{y}=2$ or $-3.$ (8)

Hence, from (7) and (8), $\begin{cases} x=\frac{1}{3},\ -\frac{1}{2}; \\ y=\frac{1}{2},\ -\frac{1}{3}. \end{cases}$

14.
$$\begin{cases} x^4+y^4=17, & (1) \\ x-y=1. & (2) \end{cases}$$

Assume, $x=u+v,$ (3)
and $y=u-v.$ (4)
Substituting these values in (1),
$u^4+4u^3v+6u^2v^2+4uv^3+v^4+u^4-4u^3v+6u^2v^2-4uv^3+v^4=17,$ (5)
and in (2), $2v=1.$ (6)
Dividing (5) by 2, $u^4+6u^2v^2+v^4=\frac{17}{2}.$ (7)
Dividing (6) by 2, $v=\frac{1}{2}.$ (8)
Substituting $\frac{1}{2}$ for v in (7) and solving, $u=\pm\frac{3}{2}$ or $\pm\frac{1}{2}\sqrt{-15}.$ (9)
Substituting (8) and (9) in (3) and (4), the following values are found,

$$\begin{cases} x=2,\ -1,\ \frac{1}{2}(1+\sqrt{-15}),\ \frac{1}{2}(1-\sqrt{-15}); \\ y=1,\ -2,\ \frac{1}{2}(-1+\sqrt{-15}),\ \frac{1}{2}(-1-\sqrt{-15}). \end{cases}$$

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15. $\begin{cases} c^4 + c^2d^2 + d^4 = 3, & (1) \\ c^2 - cd + d^2 = 3. & (2) \end{cases}$

Dividing (1) by (2), $c^2 + cd + d^2 = 1.$ (3)

Subtracting (2) from (3), $2\,cd = -2;$

whence, $cd = -1.$ (4)

Adding (4) and (3), $c^2 + 2\,cd + d^2 = 0.$ (5)

Subtracting (4) from (2), $c^2 - 2\,cd + d^2 = 4.$ (6)

Extracting the square root of (5), $c + d = \pm 0$; that is, 0 or 0. (7)

Extracting the square root of (6), $c - d = 2$ or $-2.$ (8)

Since (7) and (8) were derived independently, with *each* value of $c + d$, may be associated *each* value of $c - d$. This gives the following equations,

$$\begin{cases} c + d = 0, \\ c - d = 2; \end{cases} \quad \begin{cases} c + d = 0, \\ c - d = -2; \end{cases} \quad \begin{cases} c + d = 0, \\ c - d = 2; \end{cases} \quad \begin{cases} c + d = 0, \\ c - d = -2. \end{cases}$$

Solving, $\begin{cases} c = 1, \ -1, \ 1, \ -1; \\ d = -1, \ 1, \ -1, \ 1. \end{cases}$

16. $\begin{cases} r^3 - s^3 = 54, & (1) \\ r - s = 6. & (2) \end{cases}$

Dividing (1) by (2), $r^2 + rs + s^2 = 9.$ (3)

Squaring (2), $r^2 - 2\,rs + s^2 = 36.$ (4)

Subtracting (4) from (3), $3\,rs = -27.$

$rs = -9.$ (5)

Adding (3) and (5), $r^2 + 2\,rs + s^2 = 0.$

$r + s = \pm 0$; that is, 0 or 0. (6)

Adding (2) and (6), $r = 3$ or $3.$

Subtracting (2) from (6), $s = -3$ or $-3.$

17. $\begin{cases} x^2 + 2\,xy = 7\,y, & (1) \\ 2\,x^2 - xy + y^2 = 8\,y. & (2) \end{cases}$

Multiplying (1) by 8, $8\,x^2 + 16\,xy = 56\,y.$ (3)

Multiplying (2) by 7, $14\,x^2 - 7\,xy + 7\,y^2 = 56\,y.$ (4)

Subtracting (3) from (4), $6\,x^2 - 23\,xy + 7\,y^2 = 0.$ (5)

Dividing (5) by y^2, $\dfrac{6\,x^2}{y^2} - \dfrac{23\,x}{y} + 7 = 0.$

Solving, $\dfrac{x}{y} = \dfrac{7}{2}$ or $\dfrac{1}{3}.$ (6)

From (6), $y = 3\,x,$ (7)

and $y = \dfrac{2\,x}{7}.$ (8)

Substituting (7) in (2), $8\,x^2 - 24\,x = 0;$

whence, $x = 0$ or $3.$ (9)

Substituting (9) in (7), $y = 0$ or $9.$ (10)

Substituting (8) in (2), $x = 0$ or $\frac{14}{11}.$ (11)

Substituting (11) in (8), $y = 0$ or $\frac{4}{11}.$ (12)

Hence, $\begin{cases} x = 0, \ 3, \ 0, \ \frac{14}{11}; \\ y = 0, \ 9, \ 0, \ \frac{4}{11}. \end{cases}$

18. $\begin{cases} xy + x = 32, & (1) \\ xy + y = 27. & (2) \end{cases}$

Subtracting (2) from (1), $x - y = 5;$

whence, $y = x - 5.$ (3)

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Substituting (3) in (1), $x(x-5)+x=32,$

or $x^2-4x=32.$

Solving, $x=8 \text{ or } -4.$ (4)

Substituting (4) in (3), $y=3 \text{ or } -9.$

19. $\begin{cases} 2x^2-3y^2=5, & (1) \\ 3x^2-2y^2=30. & (2) \end{cases}$

Multiplying (1) by 2, $4x^2-6y^2=10.$ (3)

Multiplying (2) by 3, $9x^2-6y^2=90.$ (4)

Subtracting (3) from (4), $5x^2=80.$

$\therefore x=4 \text{ or } -4.$ (5)

Substituting 4 for x in (1), $y=3 \text{ or } -3.$

Substituting -4 for x in (1), $y=3 \text{ or } -3.$

Hence, $\begin{cases} x=4, 4, -4, -4; \\ y=3, -3, 3, -3. \end{cases}$

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1. $\begin{cases} x+y=3, & (1) \\ xy=2. & (2) \end{cases}$

Squaring (1), $x^2+2xy+y^2=9.$ (3)

Multiplying (2) by 4, $4xy=8.$ (4)

Subtracting (4) from (3), $x^2-2xy+y^2=1.$ (5)

Extracting the square root, $x-y=\pm 1.$ (6)

Adding (1) and (6), $x=2 \text{ or } 1.$ (7)

Subtracting (6) from (1), $y=1 \text{ or } 2.$

2. $\begin{cases} 5x^2-4y^2=44, & (1) \\ 4x^2-5y^2=19. & (2) \end{cases}$

Multiplying (1) by 5, $25x^2-20y^2=220.$ (3)

Multiplying (2) by 4, $16x^2-20y^2=76.$ (4)

Subtracting (4) from (3), $9x^2=144.$

$x=4 \text{ or } -4.$

Substituting 4 for x in (1), $y=3 \text{ or } -3.$

Substituting -4 for x in (1), $y=3 \text{ or } -3.$

Hence, $\begin{cases} x=4, 4, -4, -4; \\ y=3, -3, 3, -3. \end{cases}$

3. $\begin{cases} 1+x=y, & (1) \\ x^2+y^2=61. & (2) \end{cases}$

Substituting $1+x$ for y in (2), $x^2+1+2x+x^2=61.$

$x^2+x-30=0.$

Factoring, $(x-5)(x+6)=0.$

$\therefore x=5 \text{ or } -6.$ (3)

Substituting (3) in (1), $y=6 \text{ or } -5.$

4. $\begin{cases} x^2-xy=48, & (1) \\ xy-y^2=12. & (2) \end{cases}$

Subtracting (2) from (1), $x^2-2xy+y^2=36.$ (3)

$\therefore x-y=6 \text{ or } -6.$ (4)

Adding (1) and (2), $x^2-y^2=60.$ (5)

Dividing (5) by (4), $x+y=10 \text{ or } -10.$ (6)

Adding (4) and (6), $x=8 \text{ or } -8.$

Subtracting (4) from (6), $y=2 \text{ or } -2.$

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5. $\begin{cases} a + ab + 28 = 0, & (1) \\ b + ab + 40 = 0. & (2) \end{cases}$

Subtracting (2) from (1), $a - b = 12;$
whence, $b = a - 12.$ (3)
Substituting (3) in (1), $a + a(a - 12) + 28 = 0,$
or $a^2 - 11a + 28 = 0.$
Solving, $a = 7 \text{ or } 4.$ (4)
Substituting (4) in (3), $b = -5 \text{ or } -8.$

6. $\begin{cases} x^2 - 3xy = 8x, & (1) \\ 2x^2 - xy + y^2 = 8x. & (2) \end{cases}$

Subtracting (1) from (2), $x^2 + 2xy + y^2 = 0.$
Extracting the square root, $x + y = \pm 0$; that is, 0 or 0.
$\therefore y = -x,$ (3)
and $y = -x.$ (4)
Substituting (3) in (2), $x = 2 \text{ or } 0.$ (5)
Substituting (5) in (3), $y = -2 \text{ or } 0.$ (6)
Substituting (4) in (2), $x = 2 \text{ or } 0.$
Substituting (6) in (4), $y = -2 \text{ or } 0.$

Hence, $\begin{cases} x = 2, 0, 2, 0; \\ y = -2, 0, -2, 0. \end{cases}$

7. $\begin{cases} x^4 + x^2y^2 + y^4 = 21, & (1) \\ x^2 - xy + y^2 = 7. & (2) \end{cases}$

Dividing (1) by (2), $x^2 + xy + y^2 = 3.$ (3)
Subtracting (2) from (3), $2xy = -4;$
whence, $xy = -2.$ (4)
Adding (4) and (3), $x^2 + 2xy + y^2 = 1.$ (5)
Subtracting (4) from (2), $x^2 - 2xy + y^2 = 9.$ (6)
Extracting the square root of (5), $x + y = 1 \text{ or } -1.$ (7)
Extracting the square root of (6), $x - y = 3 \text{ or } -3.$ (8)
Since (7) and (8) were derived independently, with each value of $x + y$ may be associated each value of $x - y$.

Hence, $\begin{cases} x + y = 1, \\ x - y = 3; \end{cases}$ $\begin{cases} x + y = 1, \\ x - y = -3; \end{cases}$ $\begin{cases} x + y = -1, \\ x - y = 3; \end{cases}$ $\begin{cases} x + y = -1, \\ x - y = -3. \end{cases}$

Solving these, $\begin{cases} x = 2, \ -1, \ 1, \ -2; \\ y = -1, \ 2, \ -2, \ 1. \end{cases}$

8. $\begin{cases} x^2 + y^2 + x + y = 26, & (1) \\ xy = -12. & (2) \end{cases}$

(2) × 2, added to (1), $x^2 + 2xy + y^2 + x + y = 2.$
Completing the square, $(x + y)^2 + (x + y) + (\frac{1}{2})^2 = \frac{9}{4}.$
Extracting the square root, $x + y + \frac{1}{2} = \pm \frac{3}{2}.$
$\therefore x + y = 1 \text{ or } -2.$ (3)

From (2) and (3), $\begin{cases} x + y = 1, \\ xy = -12; \end{cases}$ and $\begin{cases} x + y = -2, \\ xy = -12. \end{cases}$

Solving, $\begin{cases} x = -3, \ 4, \ -1 - \sqrt{13}, \ -1 + \sqrt{13}; \\ y = 4, \ -3, \ -1 + \sqrt{13}, \ -1 - \sqrt{13}. \end{cases}$

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9. $\begin{cases} x^2 + y^2 = 40, & (1) \\ xy = 12. & (2) \end{cases}$

Adding (2) × 2 to (1), $x^2 + 2xy + y^2 = 64;$
whence, $x + y = \pm 8.$ (3)

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Subtracting (2) × 2 from (1), $x^2 - 2xy + y^2 = 16;$
whence, $x - y = \pm 4.$ (4)

Solving the four sets of equations given in (3) and (4),
$$x = 6,\ 2,\ -2,\ -6;$$
and $y = 2,\ 6,\ -6,\ -2.$

10. $\begin{cases} x^2 + xy = -6, & (1) \\ xy + y^2 = 15. & (2) \end{cases}$

From (1), $x(x + y) = -6.$ (3)
From (2), $y(x + y) = 15.$ (4)

Dividing (4) by (3), $\dfrac{y}{x} = -\dfrac{5}{2};$

whence, $y = -\frac{5}{2}x.$ (5)
Substituting (5) in (1), $x^2 - \frac{5}{2}x^2 = -6;$
whence, $x = 2 \text{ or } -2.$ (6)
Substituting (6) in (5), $y = -5 \text{ or } 5.$
If $x = vy$, $v = -1$ or $-\frac{2}{5}$. The value $v = -1$ gives rise to infinite roots. See Note, Ex. 3, p. 295.

11. $\begin{cases} x^3 + y^3 = 28, & (1) \\ x + y = 4. & (2) \end{cases}$

Dividing (1) by (2), $x^2 - xy + y^2 = 7.$ (3)
Squaring (2), $x^2 + 2xy + y^2 = 16.$ (4)
Subtracting (3) from (4), $3xy = 9;$
whence, $xy = 3.$ (5)
Subtracting (5) from (3), $x^2 - 2xy + y^2 = 4;$
whence, $x - y = \pm 2.$ (6)
From (2) and (6), $x = 3 \text{ or } 1,$
and $y = 1 \text{ or } 3.$

12. $\begin{cases} x^4 + y^4 = 82, & (1) \\ x + y = 4. & (2) \end{cases}$

Let $x = u + v,$ (3)
and $y = u - v.$ (4)

Substituting (3) and (4) in (1) and (2), and dividing by 2,
$$u^4 + 6u^2v^2 + v^4 = 41, \quad (5)$$
and $u = 2.$ (6)
Substituting (6) in (5), $v^4 + 24v^2 - 25 = 0.$ (7)
Factoring, $(v - 1)(v + 1)(v^2 + 25) = 0.$
$$\therefore v = 1 \text{ or } -1 \text{ or } 5\sqrt{-1} \text{ or } -5\sqrt{-1}. \quad (8)$$

By (3), (6) + (8) gives, $x = 3 \text{ or } 1 \text{ or } 2 + 5\sqrt{-1} \text{ or } 2 - 5\sqrt{-1}.$
By (4), (6) − (8) gives, $y = 1 \text{ or } 3 \text{ or } 2 - 5\sqrt{-1} \text{ or } 2 + 5\sqrt{-1}.$

13. $\begin{cases} x^4 + y^4 = 17, & (1) \\ x - y = -3. & (2) \end{cases}$

Let $x = u + v,$ (3)
and $y = u - v.$ (4)

Substituting (3) and (4) in (1) and (2), and dividing by 2,
$$u^4 + 6u^2v^2 + v^4 = \tfrac{17}{2}, \quad (5)$$
and $v = -\frac{3}{2}.$ (6)
Substituting (6) in (5),
$$u^4 + \tfrac{27}{2}u^2 - \tfrac{55}{16} = 0.$$

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Completing the square and solving,

$$u^2 = \tfrac{1}{4} \text{ or } -\tfrac{55}{4}.$$

$$\therefore\ u = \pm \tfrac{1}{2} \text{ or } \pm \tfrac{1}{2}\sqrt{-55}. \qquad (7)$$

By (3), (6) + (7) gives,

$$x = -1,\ -2,\ \tfrac{1}{2}(-3+\sqrt{-55}),\ \tfrac{1}{2}(-3-\sqrt{-55});$$

By (4), (7) − (6) gives,

$$y = 2,\ 1,\ \tfrac{1}{2}(3+\sqrt{-55}),\ \tfrac{1}{2}(3-\sqrt{-55}).$$

14.
$$\begin{cases} xy + x^2 = 44, & (1) \\ xy + y^2 = -28. & (2) \end{cases}$$

Subtracting (2) from (1), $x^2 - y^2 = 72.$ (3)

Adding (1) and (2), $x^2 + 2xy + y^2 = 16.$

Extracting square root, $x + y = 4 \text{ or } -4.$ (4)

Dividing (3) by (4), $x - y = 18 \text{ or } -18.$ (5)

From (4) and (5), $x = 11 \text{ or } -11,$

and $y = -7 \text{ or } 7.$

15.
$$\begin{cases} x^2 + 4x + 3y = -1, & (1) \\ 2x^2 + 5xy + 2y^2 = 0. & (2) \end{cases}$$

Factoring (2), $(2x + y)(x + 2y) = 0.$ (3)

From (3), $y = -2x,$ (4)

and $y = -\dfrac{x}{2}.$ (5)

Substituting (4) in (1), $x = 1 \text{ or } 1.$ (6)

Substituting (6) in (4), $y = -2 \text{ or } -2.$ (7)

Substituting (5) in (1), $x = -\tfrac{1}{2} \text{ or } -2.$ (8)

Substituting (8) in (5), $y = \tfrac{1}{4} \text{ or } 1.$

Hence,
$$\begin{cases} x = 1,\ 1,\ -\tfrac{1}{2},\ -2; \\ y = -2,\ -2,\ \tfrac{1}{4},\ 1. \end{cases}$$

16.
$$\begin{cases} \dfrac{1}{m} + \dfrac{1}{n} = \dfrac{1}{2}, & (1) \\[2ex] \dfrac{1}{mn} - \dfrac{1}{18} = 0. & (2) \end{cases}$$

Squaring (1), $\dfrac{1}{m^2} + \dfrac{2}{mn} + \dfrac{1}{n^2} = \dfrac{1}{4}.$ (3)

(2) × 4, subtracted from (3),

$$\frac{1}{m^2} - \frac{2}{mn} + \frac{1}{n^2} = \frac{1}{36}.$$

Extracting the square root, $\dfrac{1}{m} - \dfrac{1}{n} = +\dfrac{1}{6} \text{ or } -\dfrac{1}{6}.$ (4)

Adding (1) and (4), $\dfrac{2}{m} = \dfrac{2}{3} \text{ or } \dfrac{1}{3}.$

Subtracting (4) from (1), $\dfrac{2}{n} = \dfrac{1}{3} \text{ or } \dfrac{2}{3}.$

Hence,
$$\begin{cases} m = 3,\ 6; \\ n = 6,\ 3. \end{cases}$$

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17. $\begin{cases} x^2 - xy = 6, & (1) \\ x^2 + y^2 = 61. & (2) \end{cases}$

Assume $x = vy$. (3)

Substituting (3) in (1), $v^2y^2 - vy^2 = 6$. (4)

Substituting (3) in (2), $v^2y^2 + y^2 = 61$. (5)

From (4) and (5), $y^2 = \dfrac{6}{v^2 - v} = \dfrac{61}{v^2 + 1}$. (6)

Clearing of fractions, etc., $55v^2 - 61v - 6 = 0$.

Factoring, $(5v - 6)(11v + 1) = 0$.

$\therefore v = \frac{6}{5}$ or $-\frac{1}{11}$.

Substituting $\frac{6}{5}$ for v in (6), $y = 5$ or -5,
whence, $x = 6$ or -6, when $v = \frac{6}{5}$.

Substituting $-\frac{1}{11}$ for v in (6), $y = \frac{11}{2}\sqrt{2}$ or $-\frac{11}{2}\sqrt{2}$,
whence, $x = -\frac{1}{2}\sqrt{2}$ or $\frac{1}{2}\sqrt{2}$, when $v = -\frac{1}{11}$.

Hence, $\begin{cases} x = 6, -6, \frac{1}{2}\sqrt{2}, -\frac{1}{2}\sqrt{2}; \\ y = 5, -5, -\frac{11}{2}\sqrt{2}, \frac{11}{2}\sqrt{2}. \end{cases}$

18. $\begin{cases} x^2 + xy = 77, & (1) \\ xy - y^2 = 12. & (2) \end{cases}$

Assume $x = vy$. (3)

Substituting (3) in (1), $v^2y^2 + vy^2 = 77$. (4)

Substituting (3) in (2), $vy^2 - y^2 = 12$. (5)

From (4) and (5), $y^2 = \dfrac{77}{v^2 + v} = \dfrac{12}{v - 1}$. (6)

Clearing of fractions, etc., $12v^2 - 65v + 77 = 0$.

Factoring, $(4v - 7)(3v - 11) = 0$.

$\therefore v = \frac{7}{4}$ or $\frac{11}{3}$.

Substituting $\frac{7}{4}$ for v in (6), $y = 4$ or -4,
whence, $x = 7$ or -7, when $v = \frac{7}{4}$.

Substituting $\frac{11}{3}$ for v in (6), $y = \frac{3}{2}\sqrt{2}$ or $-\frac{3}{2}\sqrt{2}$
whence, $x = \frac{11}{2}\sqrt{2}$ or $-\frac{11}{2}\sqrt{2}$, when $v = \frac{11}{3}$.

Hence, $\begin{cases} x = 7, -7, \frac{11}{2}\sqrt{2}, -\frac{11}{2}\sqrt{2}; \\ y = 4, -4, \frac{3}{2}\sqrt{2}, -\frac{3}{2}\sqrt{2}. \end{cases}$

19. $\begin{cases} 2x - y = 2, & (1) \\ 2x^2 + y^2 = \frac{8}{9}. & (2) \end{cases}$

From (1), $y = 2x - 2$. (3)

Substituting (3) in (2),

$$2x^2 + 4x^2 - 8x + 4 = \tfrac{8}{9}.$$

$$x^2 - \tfrac{4}{3}x = -\tfrac{14}{27}.$$

Completing the square,

$$x^2 - \tfrac{4}{3}x + (\tfrac{2}{3})^2 = \tfrac{4}{9} - \tfrac{14}{27} = \tfrac{1}{36}.$$

$$x - \tfrac{2}{3} = \pm\tfrac{1}{6}.$$

$$\therefore x = \tfrac{5}{6} \text{ or } \tfrac{1}{2}. \quad (4)$$

Substituting (4) in (3), $y = -\frac{1}{3}$ or -1.

20. $\begin{cases} x^2 + xy + y^2 = 19, & (1) \\ x^3 - y^3 = 19. & (2) \end{cases}$

Dividing (2) by (1), $x - y = 1$. (3)

Squaring (3), $x^2 - 2xy + y^2 = 1$. (4)

Subtracting (4) from (1), $3xy = 18$;
whence, $xy = 6$. (5)

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Adding (5) to (1), $x^2 + 2xy + y^2 = 25$;
whence, $x + y = \pm 5$. (6)
From (6) and (3), $x = 3$ or -2,
and $y = 2$ or -3.

21. $\begin{cases} x^2 + 3xy = y^2 + 23, & (1) \\ x + 3y = 9. & (2) \end{cases}$

From (1), $x(x + 3y) = y^2 + 23$. (3)
From (2), $x = 9 - 3y$. (4)
Substituting (4) and (2) in (3), $(9 - 3y)9 = y^2 + 23$.
$y^2 + 27y - 58 = 0$.
Factoring, $(y - 2)(y + 29) = 0$.
$\therefore y = 2$ or -29. (5)
Substituting (5) in (4), $x = 3$ or 96.

22. $\begin{cases} 4xy = 96 - x^2y^2, & (1) \\ x + y = 6. & (2) \end{cases}$

Solving (1) for xy, $xy = 8$ or -12. (3)
Squaring (2), $x^2 + 2xy + y^2 = 36$. (4)
Subtracting $4xy = 32$ from (4), $x^2 - 2xy + y^2 = 4$;
whence, $x - y = \pm 2$. (5)
Subtracting $4xy = -48$ from (4), $x^2 - 2xy + y^2 = 84$;
whence, $x - y = \pm 2\sqrt{21}$. (6)
From (2) and (5), $x = 4$ or 2 and $y = 2$ or 4.
From (2) and (6), $x = 3 + \sqrt{21}$ or $3 - \sqrt{21}$ and $y = 3 - \sqrt{21}$ or $3 + \sqrt{21}$.

Hence, $\begin{cases} x = 4,\ 2,\ 3 + \sqrt{21},\ 3 - \sqrt{21}; \\ y = 2,\ 4,\ 3 - \sqrt{21},\ 3 + \sqrt{21}. \end{cases}$

23. $\begin{cases} x^2 - xy = 8, & (1) \\ xy + y^2 = 12. & (2) \end{cases}$

Assume $x = vy$. (3)
Substituting (3) in (1), $v^2y^2 - vy^2 = 8$. (4)
Substituting (3) in (2), $vy^2 + y^2 = 12$. (5)
From (4) and (5), $y^2 = \dfrac{8}{v^2 - v} = \dfrac{12}{v + 1}$. (6)
Clearing of fractions, etc., $3v^2 - 5v - 2 = 0$.
Factoring, $(v - 2)(3v + 1) = 0$.
$\therefore v = 2$ or $-\frac{1}{3}$.

Substituting 2 for v in (6), whence, by (3), $\left.\begin{array}{l} y = 2 \text{ or } -2, \\ x = 4 \text{ or } -4, \end{array}\right\}$ when $v = 2$.

Substituting $-\frac{1}{3}$ for v in (6), whence, $\left.\begin{array}{l} y = 3\sqrt{2} \text{ or } -3\sqrt{2}, \\ x = -\sqrt{2} \text{ or } \sqrt{2}, \end{array}\right\}$ when $v = -\frac{1}{3}$.

Hence, $\begin{cases} x = 4,\ -4,\ \sqrt{2},\ -\sqrt{2}; \\ y = 2,\ -2,\ -3\sqrt{2},\ 3\sqrt{2}. \end{cases}$

24. $\begin{cases} x(x + y) = x, & (1) \\ y(x - y) = -1. & (2) \end{cases}$

It is evident that $x = 0$ satisfies (1).
Substituting 0 for x in (2), $-y^2 = -1$.
$\therefore y = \pm 1$. (3)
Dividing (1) by x, $x + y = 1$. (4)
Multiplying (4) by y, $y(x + y) = y$. (5)

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Subtracting (2) from (5), $2y^2 = y + 1.$

$$2y^2 - y - 1 = 0.$$
$$(y-1)(2y+1) = 0.$$
$$\therefore y = 1 \text{ or } -\tfrac{1}{2}. \qquad (6)$$

Substituting (6) in (4), $x = 0 \text{ or } \tfrac{3}{2}.$

Hence, $\begin{cases} x = 0,\ 0,\ 0,\ \tfrac{3}{2}; \\ y = 1,\ -1,\ 1,\ -\tfrac{1}{2}. \end{cases}$

25. $\begin{cases} x^2 + 3xy - y^2 = 43, & (1) \\ x + 2y = 10. & (2) \end{cases}$

From (2), $x = 10 - 2y. \qquad (3)$

Substituting (3) in (1),

$$100 - 40y + 4y^2 + 30y - 6y^2 - y^2 = 43.$$
$$3y^2 + 10y - 57 = 0.$$
$$(y-3)(3y+19) = 0.$$
$$\therefore y = 3 \text{ or } -\tfrac{19}{3}. \qquad (4)$$

Substituting (4) in (3), $x = 4 \text{ or } \tfrac{68}{3}.$

26. $\begin{cases} 2x^2 + 3xy + y^2 = 20, & (1) \\ 5x^2 + 4y^2 = 41. & (2) \end{cases}$

Multiplying (1) by 41, $82x^2 + 123xy + 41y^2 = 41 \cdot 20. \qquad (3)$

Multiplying (2) by 20, $100x^2 + 80y^2 = 20 \cdot 41. \qquad (4)$

Subtracting (3) from (4), $18x^2 - 123xy + 39y^2 = 0.$

$$6x^2 - 41xy + 13y^2 = 0. \qquad (5)$$

Factoring, $(3x - y)(2x - 13y) = 0.$

$$\therefore x = \tfrac{1}{3}y \text{ or } \tfrac{13}{2}y. \qquad (6)$$

Substituting $\tfrac{1}{3}y$ for x in (2), $y = 3 \text{ or } -3;$

whence, $x = 1 \text{ or } -1.$

Substituting $\tfrac{13}{2}y$ for x in (2), $y = \tfrac{2}{21}\sqrt{21} \text{ or } -\tfrac{2}{21}\sqrt{21};$

whence, $x = \tfrac{13}{21}\sqrt{21} \text{ or } -\tfrac{13}{21}\sqrt{21}.$

Hence, $\begin{cases} x = 1,\ -1,\ \tfrac{13}{21}\sqrt{21},\ -\tfrac{13}{21}\sqrt{21}; \\ y = 3,\ -3,\ \tfrac{2}{21}\sqrt{21},\ -\tfrac{2}{21}\sqrt{21}. \end{cases}$

27. $\begin{cases} 2xy - y^2 = 12, & (1) \\ 3xy + 5x^2 = 104. & (2) \end{cases}$

Multiplying (1) by 26, $52xy - 26y^2 = 312. \qquad (3)$

Multiplying (2) by 3, $9xy + 15x^2 = 312. \qquad (4)$

Subtracting (3) from (4), $15x^2 - 43xy + 26y^2 = 0. \qquad (5)$

Factoring, $(x - 2y)(15x - 13y) = 0.$

$$\therefore x = 2y \text{ or } \tfrac{13}{15}y. \qquad (6)$$

Substituting $2y$ for x in (1), $y = 2 \text{ or } -2;$

whence, $x = 4 \text{ or } -4.$

Substituting $\tfrac{13}{15}y$ for x in (1), $y = \tfrac{6}{11}\sqrt{55} \text{ or } -\tfrac{6}{11}\sqrt{55};$

whence, $x = \tfrac{26}{55}\sqrt{55} \text{ or } -\tfrac{26}{55}\sqrt{55}.$

Hence, $\begin{cases} x = 4,\ -4,\ \tfrac{26}{55}\sqrt{55},\ -\tfrac{26}{55}\sqrt{55}; \\ y = 2,\ -2,\ \tfrac{6}{11}\sqrt{55},\ -\tfrac{6}{11}\sqrt{55}. \end{cases}$

28. $\begin{cases} x^2 + xy + y^2 = 151, & (1) \\ x^2 + y^2 = 106. & (2) \end{cases}$

Subtracting (2) from (1), $xy = 45. \qquad (3)$

Adding (3) to (1), $x^2 + 2xy + y^2 = 196;$

whence, $x + y = \pm 14. \qquad (4)$

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Subtracting (3) $\times$ 3 from (1), $x^2 - 2xy + y^2 = 16$;
whence, $x - y = \pm 4$. (5)

Since (4) and (5) are derived separately, neither from the other,

$$\begin{cases} x + y = 14, \\ x - y = 4; \end{cases} \text{or} \begin{cases} x + y = 14, \\ x - y = -4; \end{cases} \text{or} \begin{cases} x + y = -14, \\ x - y = 4; \end{cases} \text{or} \begin{cases} x + y = -14, \\ x - y = -4. \end{cases}$$

Solving these equations, $x = 9, 5, -5, -9$;
and $y = 5, 9, -9, -5$.

29.
$$\begin{cases} 1 + x = y, & (1) \\ 1 + x^3 = \dfrac{y^3}{4}. & (2) \end{cases}$$

Substituting (1) in (2), and simplifying,

$$x^3 - x^2 - x + 1 = 0.$$

Factoring, $x^2(x-1) - (x-1) = 0$.

$$(x-1)(x-1)(x+1) = 0.$$

$$\therefore x = 1, 1, -1. \quad (3)$$

Substituting these values for x in (1), $y = 2, 2, 0$.

30.
$$\begin{cases} x^4 - y^4 = 369, & (1) \\ x^2 - y^2 = 9. & (2) \end{cases}$$

Dividing (1) by (2), $x^2 + y^2 = 41$. (3)
From (2) and (3), $x^2 = 25$;
whence, $x = \pm 5$; (4)
also, $y^2 = 16$;
whence, $y = \pm 4$. (5)
Since (4) and (5) are derived separately, $x = 5, 5, -5, -5$,
when $y = 4, -4, 4, -4$.

31.
$$\begin{cases} x^2 + xy + y^2 = 84, & (1) \\ x - \sqrt{xy} + y = 6. & (2) \end{cases}$$

Dividing (1) by (2), $x + \sqrt{xy} + y = 14$. (3)
Adding (2) and (3) and dividing by 2, $x + y = 10$. (4)
Subtracting (4) from (3), $\sqrt{xy} = 4$;
whence, $xy = 16$. (5)
Subtracting (5) $\times$ 3 from (1), $x^2 - 2xy + y^2 = 36$;
whence, $x - y = \pm 6$. (6)
From (4) and (6), $x = 8$ or 2,
and $y = 2$ or 8.

32.
$$\begin{cases} 4x^2 - 2xy + y^2 = 13, & (1) \\ 8x^3 + y^3 = 65. & (2) \end{cases}$$

Dividing (2) by (1), $2x + y = 5$. (3)
Squaring (3), $4x^2 + 4xy + y^2 = 25$. (4)
Subtracting (1) from (4), $6xy = 12$;
whence, $8xy = 16$. (5)
Subtracting (5) from (4), $4x^2 - 4xy + y^2 = 9$;
whence, $2x - y = \pm 3$. (6)
From (3) and (6), $x = 2$ or $\frac{1}{2}$,
and $y = 1$ or 4.

33.
$$\begin{cases} 6x^2 + 6y^2 = 13xy, & (1) \\ x^2 - y^2 = 20. & (2) \end{cases}$$

From (1), $6x^2 - 13xy + 6y^2 = 0$.

Factoring, $(2x-3y)(3x-2y)=0.$

$\therefore y=\frac{2}{3}x$ or $\frac{3}{2}x.$ (3)

Substituting $\frac{2}{3}x$ for y in (2), $x=6$ or $-6;$

whence, $y=4$ or $-4.$

Substituting $\frac{3}{2}x$ for y in (2), $x=4\sqrt{-1}$ or $-4\sqrt{-1};$

whence, $y=6\sqrt{-1}$ or $-6\sqrt{-1}.$

34. $\begin{cases} x^2+y^2-3(x+y)=8, & (1) \\ x+y+xy=11. & (2) \end{cases}$

Adding (2) $\times$ 2 to (1), $x^2+2xy+y^2-(x+y)=30.$

Completing the square, $(x+y)^2-(x+y)+\frac{1}{4}=30\frac{1}{4}=\frac{121}{4}.$ (3)

Extracting the square root, $x+y-\frac{1}{2}=\pm\frac{11}{2};$

whence, $x+y=6$ or $-5.$ (4)

Subtracting (4) from (2), when $x+y=6$, $xy=5,$ (5)

and when $x+y=-5,$ $xy=16.$ (6)

To form $x^2-2xy+y^2$, it is necessary to subtract $2xy-3(x+y)$ from (1).

From (5), when $x+y=6$, $2xy-3(x+y)=-8.$ (7)

From (6), when $x+y=-5$, $2xy-3(x+y)=47.$ (8)

Subtracting (7) from (1), $x^2-2xy+y^2=16.$

Therefore, when $x+y=6$, $x-y=\pm 4.$ (9)

Subtracting (8) from (1), $x^2-2xy+y^2=-39.$

Therefore, when $x+y=-5$, $x-y=\pm\sqrt{-39}.$ (10)

From (9), $x=5$ or $1,$

and $y=1$ or $5.$

From (10), $x=\frac{1}{2}(-5+\sqrt{-39})$ or $\frac{1}{2}(-5-\sqrt{-39});$

and $y=\frac{1}{2}(-5-\sqrt{-39})$ or $\frac{1}{2}(-5+\sqrt{-39}).$

Hence, $\begin{cases} x=5,\ 1,\ \frac{1}{2}(-5+\sqrt{-39}),\ \frac{1}{2}(-5-\sqrt{-39}); \\ y=1,\ 5,\ \frac{1}{2}(-5-\sqrt{-39}),\ \frac{1}{2}(-5+\sqrt{-39}). \end{cases}$

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35. $\begin{cases} x^3-y^3=37, & (1) \\ xy(y-x)=-12. & (2) \end{cases}$

Changing signs in (2), $xy(x-y)=12.$ (3)

Dividing (1) by (3), $\dfrac{x^2+xy+y^2}{xy}=\dfrac{37}{12}.$

Clearing of fractions, etc., $12x^2-25xy+12y^2=0.$

Factoring, $(3x-4y)(4x-3y)=0.$

$\therefore y=\frac{3}{4}x$ or $\frac{4}{3}x.$ (4)

Substituting $\frac{3}{4}x$ for y in (3), $x^3=64.$

Transposing and factoring, $(x-4)(x^2+4x+16)=0.$ (5)

Equating each factor to zero and solving,

$$x=4 \text{ or } 2(-1+\sqrt{-3}) \text{ or } 2(-1-\sqrt{-3});$$

whence, since $y=\frac{3}{4}x$, $y=3$ or $\frac{3}{2}(-1+\sqrt{-3})$ or $\frac{3}{2}(-1-\sqrt{-3});$

Substituting $\frac{4}{3}x$ for y in (2), $x^3=-27.$

Transposing and factoring,

$$(x+3)(x^2-3x+9)=0.$$

Equating each factor to zero and solving,

$$x=-3 \text{ or } \tfrac{3}{2}(1+\sqrt{-3}) \text{ or } \tfrac{3}{2}(1-\sqrt{-3});$$

whence, since $y=\frac{4}{3}x$, $y=-4$ or $2(1+\sqrt{-3})$ or $2(1-\sqrt{-3}).$

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Hence, $\begin{cases} x = 4,\ -3,\ 2(-1+\sqrt{-3}),\ 2(-1-\sqrt{-3}),\ \frac{3}{2}(1+\sqrt{-3}), \\ \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \frac{3}{2}(1-\sqrt{-3}); \\ y = 3,\ -4,\ \frac{3}{2}(-1+\sqrt{-3}),\ \frac{3}{2}(-1-\sqrt{-3}),\ 2(1+\sqrt{-3}), \\ \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad 2(1-\sqrt{-3}). \end{cases}$

36. $\begin{cases} x + y = 25, & (1) \\ \sqrt{x} + \sqrt{y} = 7. & (2) \end{cases}$

Squaring (2), $x + 2\sqrt{xy} + y = 49.$ (3)
Multiplying (1) by 2, $2x + 2y = 50.$ (4)
Subtracting (3) from (4), $x - 2\sqrt{xy} + y = 1;$
whence, $\sqrt{x} - \sqrt{y} = \pm 1.$ (5)
From (2) and (5), $\sqrt{x} = 4$ or 3,
and $\sqrt{y} = 3$ or 4.
Hence, $x = 16$ or 9,
and $y = 9$ or 16.

37. $\begin{cases} x^3 + y^3 = 225y, & (1) \\ x^2 - y^2 = 75. & (2) \end{cases}$

Dividing (1) by (2), $\dfrac{x^3+y^3}{x^2-y^2} = \dfrac{x^2-xy+y^2}{x-y} = 3y.$ (3)
Clearing of fractions, etc., $x^2 - 4xy + 4y^2 = 0.$
Factoring, $(x-2y)(x-2y) = 0.$ (4)
$\therefore x = 2y.$ (5)
Substituting (5) in (2), $y = 5$ or -5; (6)
whence, $x = 10$ or -10. (7)
Since the factors of (4) are equal, each of the values in (6) and (7) is twice used as a root.

38. $\begin{cases} x^2 + y^2 = 3xy + 5, & (1) \\ x^4 + y^4 = 2. & (2) \end{cases}$

Let $x = u + v,$ (3)
and $y = u - v.$ (4)
Substituting (3) and (4) in (1), $u^2 = 5(v^2 - 1),$ (5)
and in (2), $u^4 + 6u^2v^2 + v^4 = 1.$ (6)
Substituting (5) in (6), $56v^4 - 80v^2 + 24 = 0.$

$$7v^4 - 10v^2 + 3 = 0.$$
$$(v-1)(v+1)(7v^2-3) = 0.$$
$$\therefore v = 1,\ -1,\ \tfrac{1}{7}\sqrt{21},\ -\tfrac{1}{7}\sqrt{21}. \quad (7)$$

Substituting (7) in (5), $u = 0,\ 0,\ \pm\frac{2}{7}\sqrt{-35},\ \pm\frac{2}{7}\sqrt{-35}.$ (8)

$$\therefore \begin{cases} x = u + v = 1,\ -1,\ \tfrac{1}{7}(\pm 2\sqrt{-35} + \sqrt{21}),\ \tfrac{1}{7}(\pm 2\sqrt{-35} - \sqrt{21}); \\ y = u - v = -1,\ 1,\ \tfrac{1}{7}(\pm 2\sqrt{-35} - \sqrt{21}),\ \tfrac{1}{7}(\pm 2\sqrt{-35} + \sqrt{21}). \end{cases}$$

39. $\begin{cases} 3xy + 2x + y = 25, & (1) \\ 9x^2 - 4y^2 = 0. & (2) \end{cases}$

From (2), $9x^2 = 4y^2;$
whence, $3x = 2y,$ (3)
or $3x = -2y.$ (4)
Substituting (3) in (1), $2y^2 + \frac{2}{3}(2y) + y = 25.$
Clearing of fraction, etc., $6y^2 + 7y - 75 = 0.$
Factoring, $(y-3)(6y+25) = 0.$

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Therefore, when $3x = 2y$, $\quad y = 3 \text{ or } -\frac{25}{6};$
whence, $\quad x = 2 \text{ or } -\frac{25}{9}.$

Substituting (4) in (1), $\quad -2y^2 - \frac{2}{3}(2y) + y = 25.$

Clearing of fractions, etc., $\quad 6y^2 + y + 75 = 0.$

Solving, $$y = \frac{-1 \pm \sqrt{1 - 4 \cdot 6 \cdot 75}}{12} = -\tfrac{1}{12}(1 \mp \sqrt{-1799});$$

whence, since $\quad 3x = -2y,\ x = \frac{1}{18}(1 \mp \sqrt{-1799}).$

Hence, $$\begin{cases} x = 2,\ -\frac{25}{9},\ \frac{1}{18}(1 + \sqrt{-1799}),\ \frac{1}{18}(1 - \sqrt{-1799}); \\ y = 3,\ -\frac{25}{6},\ -\frac{1}{12}(1 + \sqrt{-1799}),\ -\frac{1}{12}(1 - \sqrt{-1799}). \end{cases}$$

40. $$\begin{cases} x^2 - 7xy + 12y^2 = 0, & (1) \\ xy + 3y = 2x + 21. & (2) \end{cases}$$

Factoring (1), $\quad (x - 4y)(x - 3y) = 0.$

$\therefore x = 4y \text{ or } 3y. \quad (3)$

Substituting $4y$ for x in (2), $\quad 4y^2 - 5y - 21 = 0.$

Factoring, $\quad (y - 3)(4y + 7) = 0.$

$\therefore y = 3 \text{ or } -\frac{7}{4};$

whence, $\quad x = 12 \text{ or } -7.$

Substituting $3y$ for x in (2), $\quad y^2 - y = 7.$

Completing the square, $\quad y^2 - y + \frac{1}{4} = \frac{29}{4}.$

Solving, $\quad y = \frac{1}{2}(1 + \sqrt{29}) \text{ or } \frac{1}{2}(1 - \sqrt{29});$

whence, $\quad x = \frac{3}{2}(1 + \sqrt{29}) \text{ or } \frac{3}{2}(1 - \sqrt{29}).$

41. $$\begin{cases} (x + y)(x^2 + y^2) = 65, & (1) \\ (x - y)(x^2 - y^2) = 5. & (2) \end{cases}$$

Dividing (1) by (2), $$\frac{x^2 + y^2}{(x - y)^2} = 13.$$

Clearing of fractions, etc., $\quad 6x^2 - 13xy + 6y^2 = 0.$

Factoring, $\quad (2x - 3y)(3x - 2y) = 0.$

$\therefore x = \frac{3}{2}y \text{ or } \frac{2}{3}y.$

Substituting $\frac{3}{2}y$ for x in (2), $\quad y^3 = 8.$

Transposing and factoring, $\quad (y - 2)(y^2 + 2y + 4) = 0.$

$\therefore y = 2 \text{ or } -1 + \sqrt{-3} \text{ or } -1 - \sqrt{-3};$

whence, $\quad x = 3 \text{ or } \frac{3}{2}(-1 + \sqrt{-3}) \text{ or } \frac{3}{2}(-1 - \sqrt{-3}).$

Substituting $\frac{2}{3}y$ for x in (2), $\quad y^3 = 27.$

Transposing and factoring, $\quad (y - 3)(y^2 + 3y + 9) = 0.$

$\therefore y = 3 \text{ or } \frac{3}{2}(-1 + \sqrt{-3}) \text{ or } \frac{3}{2}(-1 - \sqrt{-3});$

whence, $\quad x = 2, \text{ or } -1 + \sqrt{-3} \text{ or } -1 - \sqrt{-3}.$

Hence, $$\begin{cases} x = 3,\ 2,\ \frac{3}{2}(-1 + \sqrt{-3}),\ \frac{3}{2}(-1 - \sqrt{-3}),\ -1 + \sqrt{-3}, \\ \qquad -1 - \sqrt{-3}; \\ y = 2,\ 3,\ -1 + \sqrt{-3},\ -1 - \sqrt{-3},\ \frac{3}{2}(-1 + \sqrt{-3}), \\ \qquad \frac{3}{2}(-1 - \sqrt{-3}). \end{cases}$$

42. $$\begin{cases} x^2 + y = x - y^2 + 42, & (1) \\ xy = 20. & (2) \end{cases}$$

From (1), transposing, $\quad x^2 + y^2 - (x - y) = 42. \quad (3)$

Subtracting (2) $\times$ 2 from (3), and completing the square,
$$x^2 - 2xy + y^2 - (x - y) + \tfrac{1}{4} = 2 + \tfrac{1}{4} = \tfrac{9}{4}. \quad (4)$$

Extracting the square root, $\quad x - y - \frac{1}{2} = \pm\frac{3}{2};$

whence, $\quad x - y = 2 \text{ or } -1. \quad (5)$

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Squaring (5), $\quad x^2 - 2xy + y^2 = 4 \text{ or } 1. \quad (6)$

Adding (2) $\times$ 4 to (6), $\quad x^2 + 2xy + y^2 = 84 \text{ or } 81;$

whence, $\quad x + y = \pm 2\sqrt{21} \text{ or } \pm 9. \quad (7)$

Hence, $\begin{cases} x + y = 9, \\ x - y = -1; \end{cases}$ or $\begin{cases} x + y = -9, \\ x - y = -1; \end{cases}$ or $\begin{cases} x + y = 2\sqrt{21}, \\ x - y = 2; \end{cases}$

or $\begin{cases} x + y = -2\sqrt{21}, \\ x - y = 2. \end{cases}$

Solving these equations, $x = 4, \ -5, \ 1 + \sqrt{21}, \ 1 - \sqrt{21};$

and $\quad y = 5, \ -4, \ -1 + \sqrt{21}, \ -1 - \sqrt{21}.$

43.
$$\begin{cases} x + y + 2\sqrt{x+y} = 24, & (1) \\ x - y + 3\sqrt{x-y} = 10. & (2) \end{cases}$$

Completing squares, $\quad x + y + 2\sqrt{x+y} + 1 = 25, \quad (3)$

and $\quad x - y + 3\sqrt{x-y} + \frac{9}{4} = \frac{49}{4}. \quad (4)$

Extracting square roots, $\quad \sqrt{x+y} + 1 = \pm 5;$

whence, $\quad \sqrt{x+y} = 4 \text{ or } -6, \quad (5)$

and $\quad \sqrt{x-y} + \frac{3}{2} = \pm \frac{7}{2};$

whence, $\quad \sqrt{x-y} = 2 \text{ or } -5. \quad (6)$

Squaring (5), $\quad x + y = 16 \text{ or } 36. \quad (7)$

Squaring (6), $\quad x - y = 4 \text{ or } 25. \quad (8)$

Since (7) and (8) have been derived separately, we have the equations,

$\begin{cases} x + y = 16, \\ x - y = 4; \end{cases}$ or $\begin{cases} x + y = 16, \\ x - y = 25; \end{cases}$ or $\begin{cases} x + y = 36, \\ x - y = 4; \end{cases}$ or $\begin{cases} x + y = 36, \\ x - y = 25. \end{cases}$

Solving these equations, $\quad x = 10, \ \frac{41}{2}, \ 20, \ \frac{61}{2};$

and $\quad y = 6, \ -\frac{9}{2}, \ 16, \ \frac{11}{2}.$

$\begin{cases} x = \frac{41}{2}, \ 20, \ \frac{61}{2}, \\ y = -\frac{9}{2}, \ 16, \ \frac{11}{2}, \end{cases}$ do not verify and are rejected.

44.
$$\begin{cases} x^2 + y^2 + 6\sqrt{x^2+y^2} = 55, & (1) \\ x^2 - y^2 = 7. & (2) \end{cases}$$

Completing the square in (1),

$$x^2 + y^2 + 6\sqrt{x^2+y^2} + 9 = 64. \quad (3)$$

Extracting the square root, $\quad \sqrt{x^2+y^2} + 3 = \pm 8;$

whence, $\quad \sqrt{x^2+y^2} = 5 \text{ or } -11. \quad (4)$

Squaring (4), $\quad x^2 + y^2 = 25 \text{ or } 121. \quad (5)$

From (5) and (2), $\quad x^2 = 16 \text{ or } 64,$

and $\quad y^2 = 9 \text{ or } 57;$

that is, $\quad \begin{cases} x^2 = 16, \\ y^2 = 9; \end{cases}$ or $\begin{cases} x^2 = 64, \\ y^2 = 57. \end{cases}$

Whence, $\quad x = 4, \ 4, \ -4, \ -4, \ 8, \ 8, \ -8, \ -8;$

and $\quad y = 3, \ -3, \ 3, \ -3, \ \sqrt{57}, \ -\sqrt{57}, \ \sqrt{57}, \ -\sqrt{57}.$

$\begin{cases} x = 8, \ 8, \ -8, \ -8, \\ y = \sqrt{57}, \ -\sqrt{57}, \ \sqrt{57}, \ -\sqrt{57}, \end{cases}$ do not verify and are rejected.

45.
$$\begin{cases} x^2 - 6xy + 9y^2 + 2x - 6y - 8 = 0, & (1) \\ x^2 + 4xy + 4y^2 - 4x - 8y - 21 = 0. & (2) \end{cases}$$

From suggestion, $\quad (x - 3y)^2 + 2(x - 3y) - 8 = 0, \quad (3)$

$$(x + 2y)^2 - 4(x + 2y) - 21 = 0. \quad (4)$$

Solving (3), $\quad x - 3y = 2 \text{ or } -4. \quad (5)$

Solving (4), $\quad x + 2y = 7 \text{ or } -3. \quad (6)$

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From (5) and (6),

$$\begin{cases} x-3y=2, \\ x+2y=7; \end{cases} \quad \begin{cases} x-3y=2, \\ x+2y=-3; \end{cases} \quad \begin{cases} x-3y=-4, \\ x+2y=7; \end{cases} \quad \begin{cases} x-3y=-4, \\ x+2y=-3. \end{cases}$$

Solving, $\begin{cases} x=5,\ -1,\ \frac{13}{5},\ -\frac{17}{5}; \\ y=1,\ -1,\ \frac{11}{5},\ \frac{1}{5}. \end{cases}$

46.
$$\begin{cases} x^2-xy=a^2+b^2, & (1) \\ xy-y^2=2ab. & (2) \end{cases}$$

Subtracting (2) from (1), $x^2-2xy+y^2=a^2-2ab+b^2.$

Extracting the square root, $x-y=\pm(a-b).$ (3)

Adding (1) and (2), $x^2-y^2=(a+b)^2.$ (4)

Dividing (4) by (3), $x+y=\dfrac{(a+b)^2}{\pm(a-b)}.$ (5)

Hence, from (3) and (5), $\begin{cases} x+y=\dfrac{(a+b)^2}{a-b}, \\ x-y=a-b; \end{cases} \quad \begin{cases} x+y=-\dfrac{(a+b)^2}{a-b}, \\ x-y=-(a-b). \end{cases}$

Solving, $\begin{cases} x=\dfrac{a^2+b^2}{a-b},\quad -\dfrac{a^2+b^2}{a-b}; \\ y=\dfrac{2ab}{a-b},\quad -\dfrac{2ab}{a-b}. \end{cases}$

47.
$$\begin{cases} x-2y=2(a+b), & (1) \\ xy+2y^2=2b(b-a). & (2) \end{cases}$$

From (1), $x=2(y+a+b).$ (3)

Substituting (3) in (2), simplifying, etc.,

$$y^2+\tfrac{1}{2}(a+b)y=\tfrac{1}{2}b(b-a).$$

Completing the square, $y^2+\frac{1}{2}(a+b)y+\dfrac{(a+b)^2}{16}=\dfrac{9b^2-6ab+a^2}{16}.$

Extracting the square root, $y+\dfrac{a+b}{4}=\pm\dfrac{a-3b}{4}.$

$\therefore y=-b$ or $\dfrac{b-a}{2}.$ (4)

Substituting (4) in (3), $x=2a$ or $a+3b.$

48.
$$\begin{cases} x^3+y^3=2a(a^2+3b^2), & (1) \\ x^2y+xy^2=2a(a^2-b^2). & (2) \end{cases}$$

Dividing (1) by (2), $\dfrac{x^2-xy+y^2}{xy}=\dfrac{a^2+3b^2}{a^2-b^2}.$

Simplifying, etc.,

$(a^2-b^2)x^2-2(a^2+b^2)xy+(a^2-b^2)y^2=0.$ (3)

Factoring (3),

$[(a-b)x-(a+b)y][(a+b)x-(a-b)y]=0.$ (4)

From (4), $y=\dfrac{(a-b)x}{a+b}$ or $\dfrac{(a+b)x}{a-b}.$

Substituting $\dfrac{(a-b)x}{a+b}$ for y in (2),

$$\frac{(a-b)x^3}{(a+b)}+\frac{(a-b)^2x^3}{(a+b)^2}=2a(a^2-b^2).$$

Dividing by $a-b$, simplifying, etc., $2ax^3=2a(a+b)^3.$

$\therefore x=a+b.$ (5)

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Substituting (5) in $y = \dfrac{(a-b)x}{a+b}$, $\quad y = a - b.$

Substituting $\dfrac{(a+b)x}{a-b}$ for y in (2),

$$\frac{(a+b)x^3}{a-b} + \frac{(a+b)^2x^3}{(a-b)^2} = 2\,a(a^2-b^2).$$

Dividing by $a+b$, simplifying, etc.,

$$2\,ax^3 = 2\,a(a-b)^3.$$

$$\therefore x = a - b. \qquad (6)$$

Substituting (6) in $y = \dfrac{(a+b)x}{a-b}$, $\quad y = a + b.$

49.
$$\begin{cases} s = \frac{1}{2}\,at^2, & (1)\\ v = at. & (2)\end{cases}$$

From (2), $\quad t = \dfrac{v}{a}. \qquad (3)$

From (1), $\quad a = \dfrac{2\,s}{t^2}. \qquad (4)$

Substituting (3) in (4), solving, $\quad a = \dfrac{v^2}{2\,s}. \qquad (5)$

Substituting (5) in (3), $\quad t = \dfrac{2\,s}{v}.$

50.
$$\begin{cases} s = 6\,t + \frac{1}{2}\,at^2, & (1)\\ v = at. & (2)\end{cases}$$

Clearing (1) of fractions, etc., $at^2 + 12\,t = 2\,s.$

Solving by formula, $\quad t = \dfrac{-6 \pm \sqrt{36 + 2\,as}}{a}. \qquad (3)$

Substituting (3) in (2), $\quad v = -6 \pm \sqrt{36 + 2\,as}.$

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1. Let $x =$ one number,
and $y =$ the other.
Then, $x + y = 12,$
and $xy = 32.$
Solving, $x = 8$ or $4,$
and $y = 4$ or $8.$
Hence, the numbers are 4 and 8.

2. Let $x =$ one number,
and $y =$ the other.
Then, $x + y = 17,$
and $x^2 + y^2 = 157.$
Solving, $x = 11$ or $6,$
and $y = 6$ or $11.$
Hence, the numbers are 6 and 11.

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3. Let $x =$ the greater number,
and $y =$ the lesser number.
Then, $x - y = 1$,
and $x^3 - y^3 = 91$.
Solving, $x = 6$ or -5,
and $y = 5$ or -6.
Hence, the numbers are 6 and 5, or -5 and -6.

4. Let $x =$ one number,
and $y =$ the other.
Then, $x + y = 82$,
and $\sqrt{x} + \sqrt{y} = 10$.
Solving, $x = 81$ or 1,
and $y = 1$ or 81.
Hence, the numbers are 81 and 1.

5. Let $x =$ number of rods in length of garden,
and $y =$ number of rods in width of garden.
Then, since the length and width together are one half of the distance around the garden, and their product is the area in square rods,

$$x + y = 26,$$

and $xy = 160$.
Solving, $x = 16$ or 10,
and $y = 10$ or 16.
Hence, the garden is 16 rods long and 10 rods wide.

6. Let $x =$ one number,
and $y =$ the other.
Then, $xy - (x + y) = 59$,
and $x^2 + y^2 = 170$.
Solving, $x = 11$ or 7 or $-8 + \sqrt{21}$ or $-8 - \sqrt{21}$,
and $y = 7$ or 11 or $-8 - \sqrt{21}$ or $-8 + \sqrt{21}$.
Hence, the numbers are 11 and 7 or $-8 + \sqrt{21}$ and $-8 - \sqrt{21}$.

7. Let $x =$ tens' digit,
and $y =$ units' digit.
Then, $10x + y - 63 = 10y + x$, (1)
and $(10x + y)(x + y) = 729$. (2)
From (1), $y = x - 7$. (3)
Substituting (3) in (2), $(11x - 7)(2x - 7) = 729$. (4)
Solving (4), $x = 8$ or $-\frac{5}{2}\frac{5}{2}$.
Rejecting the second value, from (3), $y = 1$.
Hence, the number is 81.

8. Let $x =$ number of yards,
and $y =$ number of cents per yard.
Then, $xy = 600$, (1)
and $(x + 5)(y - 4) = 600$. (2)
Subtracting (2) from (1), $4x - 5y + 20 = 0$; (3)
whence, $5y = 4x + 20$. (4)
From (1), $x \cdot 5y = 3000$. (5)
Substituting (4) in (5), $4x^2 + 20x = 3000$;
whence, $x = 25$ or -30. (6)
Substituting (6) in (1), $y = 24$ or -20.
Hence, the man bought 25 yards at 24 cents per yard.

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9. Let $x =$ number of gallons of Italian oil,
and $y =$ number of cents Italian oil cost per gallon.
Then, $xy = 225$, (1)
and $(x - \frac{1}{2})(y + 50) = 200$. (2)
From (2), $2xy + 100x - y = 450$. (3)
Substituting $\frac{225}{x}$ for y in (3), and solving,
$$x = \pm \tfrac{3}{2}. \quad (4)$$
Substituting (4) in (1), $y = 150$.
Then, $x - \frac{1}{2} = 1$,
and $y + 50 = 200$.
Hence, $1\frac{1}{2}$ gallons of Italian oil were bought at $1.50 per gallon, and 1 gallon of French oil at $2.00 per gallon.

10. Let $x =$ number of yards in length,
and $y =$ number of yards in width.
Then, $2x + 2y = 18$, (1)
and $xy = \frac{1}{2} \times 40$. (2)
From (2), $y = \dfrac{20}{x}$. (3)
Substituting (3) in (1), $x^2 - 9x = -20$.
Solving, $x = 5$ or 4. (4)
Substituting (4) in (3), $y = 4$ or 5.
Hence, the length is 5 yards or 15 feet and the width 4 yards or 12 feet.

11. Let $x =$ number of bales he pressed,
and $y =$ number of pounds in each bale.
Then, $xy = 6120$, (1)
and $(x - 1)(y + 20) = 6120$. (2)
From (2), $xy + 20x - y = 6140$. (3)
Substituting (1) in (3), etc., $x^2 - x = 306$.
Solving, $x = 18$ or -17. (4)
Substituting 18 for x in (1), $y = 340$.
Hence, there were 18 bales, each bale weighing 340 pounds.

12. Let $x =$ number of feet in width of screen.
Then, $x + \frac{1}{3} =$ number of feet in length of screen.
Also let $y =$ number of feet in width of frame.
Then, $x(x + \frac{1}{3}) = 10$, (1)
and $(x - 2y)(x + \frac{1}{3} - 2y) = 8$. (2)
From (1), $x = 3$ or $-\frac{10}{3}$.
Substituting 3 for x in (2), and solving,
$$y = 3 \text{ or } \tfrac{1}{6}.$$
Since the width of the frame could not be as great as the width of the screen, the value 3 is inadmissible. Hence, the width of the frame is $\frac{1}{6}$ of a foot or 2 inches.

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13. Let $x =$ number of inches in side of blotter after it is cut,
and $y =$ number of inches cut from width of blotter.
Then, $y + 4 =$ number of inches cut from length of blotter.
Then, $x^2 = 256$, (1)
and $(x + y)(x + y + 4) = 480$. (2)
From (1), $x = 16$.
Substituting 16 for x in (2), and simplifying,
$$y^2 + 36y = 160.$$
Solving, $y = 4$,
and $y + 4 = 8$.
Hence, 8 inches were cut from length and 4 inches from width.

14. Let $x =$ number of loads of gravel bought,
and $y =$ number of cents paid for 1 load of gravel.
Then, $x + 4 =$ number of loads of sand bought,
and $y - 50 =$ number of cents paid for 1 load of sand.
Then, $xy = 1000$, (1)
and $(x + 4)(y - 50) = 900$. (2)
Substituting (1) in (2) and simplifying,
$$x^2 + 2x = 80.$$
Solving, $x = 8$ or -10.
Substituting 8 for x in (1), $y = 125$.
Then, $x + 4 = 12$,
and $y - 50 = 75$.
Hence, 8 loads of gravel were bought at $\$1\frac{1}{4}$ per load and 12 loads of sand were bought at $\$\frac{3}{4}$ per load.

15. Let $x =$ number of miles in hypotenuse of track,
and $y =$ number of miles in 1 leg of track.
Then, $y + 3 =$ number of miles in other leg of track.
Then, $x + y + y + 3 = 36$, (1)
and $x^2 = y^2 + (y + 3)^2$. (2)
From (1), $x = 33 - 2y$. (3)
Substituting (3) in (2),
$$y^2 - 69y = -540.$$
Solving, $y = 60$ or 9.
The value 60 is inadmissible, since the whole track is only 36 miles long.
Then, considering $y = 9$, $y + 3 = 12$,
and from (1), $x = 15$.
Hence, the dimensions of the course are 9 miles, 12 miles, and 15 miles.

16. Let $x =$ number of feet in length of rink,
and $y =$ number of feet in width of rink.
Then, $(x + 50)(y + 50) = 37{,}500$, (1)
and $xy + \frac{7}{8}xy = 37{,}500$. (2)
From (2), $xy = 20{,}000$. (3)
From (1) and (3), $x^2 - 300x = -20{,}000$.
Solving, $x = 200$ or 100. (4)
Substituting (4) in (3), $y = 100$ or 200.
Hence, the length is 200 feet and the width is 100 feet.

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17. Let $x =$ number of feet in width of produce car,
and $y =$ number of feet in height of produce car.
Then, $x + \frac{1}{2} =$ number of feet in width of furniture car,
and $y + 1 =$ number of feet in height of furniture car.

Then, $$33xy = 1848, \quad (1)$$
and $$(33 + 3)(x + \tfrac{1}{2})(y + 1) = 2448. \quad (2)$$

From (1), $$y = \frac{56}{x}. \quad (3)$$

Substituting (3) in (2), simplifying, etc.,
$$2x^2 - 23x = -56.$$
Solving, $$x = 8. \quad (4)$$
Substituting (4) in (3), $$y = 7.$$
Then, $$x + \tfrac{1}{2} = 8\tfrac{1}{2},$$
and $$y + 1 = 8.$$

Hence, the length of produce car is 33 feet, width is 8 feet, and the height is 7 feet. The length of the furniture car is 36 feet, the width $8\frac{1}{2}$ feet, and the height is 8 feet.

18. Let $x =$ number of days it will take the first man,
and $y =$ number of days it will take the second.

Then, $$\frac{1}{x} + \frac{1}{y} = \frac{1}{6\frac{2}{3}} = \frac{3}{20},$$
and $$x - y = 3.$$
Solving, $$x = 15 \text{ or } \tfrac{4}{3},$$
and $$y = 12 \text{ or } -\tfrac{5}{3}.$$

Hence, the first man can do the work alone in 15 days, and the second can do it alone in 12 days.

19. Let $x =$ number of rods in length of field,
and $y =$ number of rods in width of field.
Then, $xy = 1120$, area of field in square rods. (1)

Since, in making the circuit of the field 11 times, 11×6 ft., or 66 ft., is cut on each side, both the length and the width of the field is reduced by 2×66 ft., or 132 ft., or 8 rd.

Then, $(x - 8)(y - 8) = 640$, area of grass still standing. (2)

From (1), $$y = \frac{1120}{x}. \quad (3)$$

Substituting (3) in (2), simplifying, etc.,
$$x^2 - 68x + 1120 = 0.$$
Solving, $x = 40$ or 28. (4)
Substituting (4) in (3), $y = 28$ or 40.
Hence, the length of the field is 40 rods, and the width 28 rods.

20. Let $x =$ number of yards in circumference of fore wheel,
and $y =$ number of yards in circumference of hind wheel.

Then, $$\frac{240}{x} - \frac{240}{y} = 12, \quad (1)$$
and $$\frac{240}{x+1} - \frac{240}{y+1} = 8. \quad (2)$$

Dividing (1) by 12 and clearing of fractions,
$$-20x + 20y = xy. \quad (3)$$

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Dividing (2) by 8, clearing of fractions, transposing, etc.,

$$-31x + 29y = xy + 1. \qquad (4)$$

Subtracting (3) from (4), $\quad -11x + 9y = 1;$

whence, $$y = \frac{11x+1}{9}. \qquad (5)$$

Solving (5) and (3), or (5) and (4), $\quad x = 4$ or $-\frac{5}{11}$,
and $\quad y = 5$ or $-\frac{4}{9}$.

Hence, the fore wheel is 4 yards and the hind wheel 5 yards in circumference.

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21. Let $\quad x =$ number of dollars in larger loan,
and $\quad y\% =$ rate of interest on larger loan.

Then, $\quad \frac{xy}{100} =$ number of dollars interest yielded by each investment.

$1000 - x =$ number of dollars in smaller loan,

and $\frac{xy}{100} \div (1000 - x)$, or $\frac{xy}{1000-x}\% =$ rate of interest on smaller loan.

$$\therefore \frac{x \cdot xy}{1000 - x} = 3600, \qquad (1)$$

and $$(1000 - x)y = 1600. \qquad (2)$$

Multiplying (1) by (2), $\quad x^2y^2 = 36 \cdot 16 \cdot 100 \cdot 100;$

whence, $$xy = \pm 6 \cdot 4 \cdot 100 = \pm 2400. \qquad (3)$$

Since the product of principal and rate cannot be negative, the negative value in (3) is rejected.

Adding (3) and (2), $\quad 1000y = 4000.$

$$\therefore y = 4. \qquad (4)$$

Substituting (4) in (3), $\quad x = 600. \qquad (5)$

From (4) and (5), $\quad 1000 - x = 400,$

and $$\frac{xy}{1000 - x} = \frac{2400}{400} = 6.$$

Hence, the sums invested were \$600 at 4% and \$400 at 6%.

22. Let $\quad x =$ number of dollars in the principal,
and $\quad y =$ number of per cent in the rate.

Then, $$x + .01xy = 11{,}130, \qquad (1)$$

and $$(x + 100)(1 + .01y - .01) = 11{,}130. \qquad (2)$$

Simplifying (2), $\quad .99x + y + 99 + .01xy = 11{,}130. \qquad (3)$

Subtracting (3) from (1), $\quad .01x - y - 99 = 0;$

whence, $$x = 100y + 9900. \qquad (4)$$

Substituting (4) in (1) and simplifying,

$$y^2 + 199y - 1230 = 0. \qquad (5)$$

Factoring (5), $\quad (y - 6)(y + 205) = 0.$

$$\therefore y = 6 \text{ or } -205.$$

Rejecting the second value and substituting the first in (4),

$$x = 10{,}500.$$

Hence, the principal was \$10,500, and the rate was 6%.

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23. Let $x =$ number of miles per hour he walks,
and $y =$ number of miles per hour he rows.
Then, since he walks 4 miles and rows 8 miles,

$$\frac{4}{x}+\frac{8}{y}=3, \qquad (1)$$

and
$$\frac{4}{x}+\frac{8}{y-2}=5. \qquad (2)$$

Subtracting (1) from (2), $\frac{8}{y-2}-\frac{8}{y}=2.$ (3)

Simplifying, $y^2-2y-8=0.$ (4)

Solving (4), $y=4$ or $-2.$ (5)

Substituting (5) in (1), $x=4$ or $\frac{4}{7}$.

Hence, his rates of walking and rowing are each 4 miles an hour.

Since in rowing back, he rows 2 miles an hour less than before, his rate of rowing back is 2 miles per hour.

24. Let $x =$ number of miles each traveled,
and $y =$ number of miles per hour A traveled.

Then, $\frac{x}{y} =$ number of hours it took A,

and $\frac{x}{y-2} =$ number of hours it took C.

$$\therefore \frac{20}{y-2}+\frac{x-20}{y}=\frac{x}{y-2}-\frac{2}{3}, \qquad (1)$$

and
$$\frac{20}{y-2}+\frac{x-20}{y}=\frac{x}{y}+\frac{1}{3}. \qquad (2)$$

Simplifying (1), $y^2-2y=3x-60.$ (3)

Subtracting (1) from (2), and simplifying,
$$y^2-2y=2x. \qquad (4)$$

Subtracting (4) from (3), etc., $x=60.$ (5)

Substituting (5) in (4), $y=12$ or $-10.$

Rejecting the negative value, it is seen that the distance each traveled was 60 miles: A rode 12 miles an hour, C 10 miles an hour, and B 10 miles an hour for 20 miles, and 12 miles an hour for 40 miles.

25. Let $x =$ number of seconds torpedo was falling,
and $y =$ number of seconds sound was returning.

Then, $x+y=5,$ (1)

and $16.08x^2-1125.6y=0.$ (2)

From (1), $y=5-x.$ (3)

Substituting (3) in (2), simplifying, etc.,
$$x^2+70x=350.$$

Solving, $x=4.7.$

Hence, it took 4.7 seconds for the torpedo to fall.

26. Let $\frac{cp}{100} =$ number of pounds of clay in p pounds.

Then, $\frac{c+10}{100}\cdot 240 =$ number of pounds of clay in 240 pounds,

and $\frac{c-8}{100}\cdot 250 =$ number of pounds of clay in 250 pounds.

$240-p =$ number of pounds of clay added to make 240 pounds.

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Then, $$\frac{(c+10)240}{100} - \frac{cp}{100} = 240 - p, \qquad (1)$$

and $$\frac{cp}{100} - \frac{(c-8)250}{100} = 0. \qquad (2)$$

Simplifying (1), $240\,c - cp + 100\,p = 21{,}600.$ (3)
Simplifying (2), $250\,c - cp = 2000.$ (4)
Subtracting (3) from (4), $10\,c - 100\,p = -19{,}600.$ (5)
From (5), $c = 10\,p - 1960.$ (6)
Substituting (6) in (4),
$250(10\,p - 1960) - (10\,p - 1960)p = 2000.$
Simplifying, $p^2 - 446\,p = -49{,}200.$
Solving, $p = 246$ or $200.$ (7)
Substituting (7) in (6), $c = 500$ or $40.$
Evidently the first value of p is inadmissible, since after the addition of clay, the weight of the mixture was only 240 pounds. Hence, $p = 200$, and $c = 40$.

GRAPHIC SOLUTIONS

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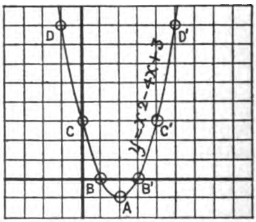
1. Since the coefficient of x is -4, § 418, first substitute 2 for x.

$$y = x^2 - 4\,x + 3$$

x	y	POINTS
2	-1	A
1, 3	0	B, B'
0, 4	3	C, C'
-1, 5	8	D, D'

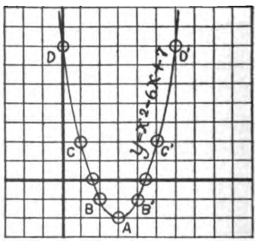


Plotting these points and drawing a smooth curve through them, we have the graph of $y = x^2 - 4\,x + 3$, which crosses the x-axis at 1 and 3. Hence, the roots of $x^2 - 4\,x + 3 = 0$ are 1 and 3.

2. Since the coefficient of x is -6, § 418, first substitute 3 for x.

$$y = x^2 - 6\,x + 7$$

x	y	POINTS
3	-2	A
2, 4	-1	B, B'
1, 5	2	C, C'
0, 6	7	D, D'



Plotting these points and drawing a smooth curve through them, we have the graph of $y = x^2 - 6\,x + 7$, which crosses the x-axis approximately at 1.6 and 4.4.
Hence, the roots of $x^2 - 6\,x + 7 = 0$ are 1.6 and 4.4.

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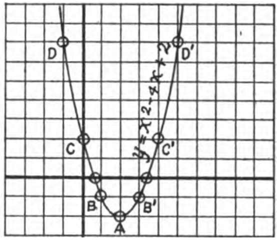
3. Putting $x^2 - 4x = -2$ in the form $x^2 - 4x + 2 = 0$, since the coefficient of x is -4, § 418, first substitute 2 for x.

$$y = x^2 - 4x + 2$$

x	y	POINTS
2	-2	A
1, 3	-1	B, B'
0, 4	2	C, C'
-1, 5	7	D, D'



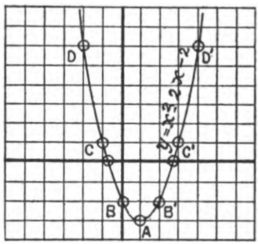
Plotting these points and drawing a smooth curve through them, we have the graph of $y = x^2 - 4x + 2$, which crosses the x-axis approximately at .6 and 3.4.

Hence, the roots of $x^2 - 4x + 2 = 0$, or of $x^2 - 4x = -2$, are .6 and 3.4.

4. Putting $x^2 = 2(x + 1)$ in the form $x^2 - 2x - 2 = 0$, since the coefficient of x is -2, § 418, first substitute 1 for x.

$$y = x^2 - 2x - 2$$

x	y	POINTS
1	-3	A
0, 2	-2	B, B'
-1, 3	1	C, C'
-2, 4	6	D, D'



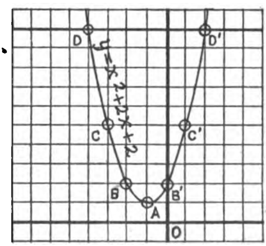
Plotting these points and drawing a smooth curve through them, we have the graph of $y = x^2 - 2x - 2$, which crosses the x-axis approximately at $-.7$ and 2.7.

Hence, the roots of $x^2 - 2x - 2 = 0$, or of $x^2 = 2(x + 1)$, are $-.7$ and 2.7.

5. Putting $x^2 + 2(x + 1) = 0$ in the form $x^2 + 2x + 2 = 0$, since the coefficient of x is 2, § 418, first substitute -1 for x.

$$y = x^2 + 2x + 2$$

x	y	POINTS
-1	1	A
-2, 0	2	B, B'
-3, 1	5	C, C'
-4, 2	10	D, D'



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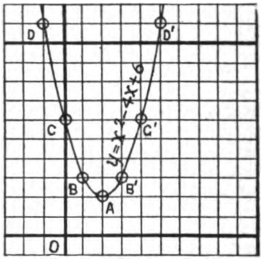
Plotting these points and drawing a smooth curve through them, we have the graph of $y = x^2 + 2x + 2$, whose minimum point lies above the x-axis.

Hence, § 420, 4, the roots of $x^2 + 2x + 2 = 0$, or of $x^2 + 2(x + 1) = 0$, are imaginary.

6. Since the coefficient of x is -4, § 418, first substitute 2 for x.

$y = x^2 - 4x + 6$

x	y	Points
2	2	A
1, 3	3	B, B'
0, 4	6	C, C'
-1, 5	11	D, D'



Plotting these points and drawing a smooth curve through them, we have the graph of $y = x^2 - 4x + 6$, whose minimum point lies above the x-axis.

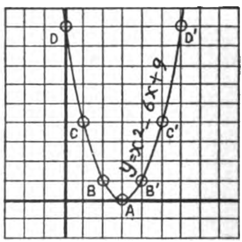
Hence, § 420, 4, the roots of $x^2 - 4x + 6 = 0$ are imaginary.

7. Since this equation is the same as the one in Ex. 4, though in different form, its graph and roots are the same. (For solution see Ex. 4.)

8. Putting $x^2 = 6x - 9$ in the form $x^2 - 6x + 9 = 0$, since the coefficient of x is -6, first substitute 3 for x.

$y = x^2 - 6x + 9$

x	y	Points
3	0	A
2, 4	1	B, B'
1, 5	4	C, C'
0, 6	9	D, D'



Plotting these points and drawing a smooth curve through them, we have the graph of $y = x^2 - 6x + 9$, whose minimum point lies on the x-axis at $x = 3$.

Hence, § 420, 2, the roots of $x^2 - 6x + 9 = 0$, or of $x^2 = 6x - 9$, are real and equal; that is, $x = 3$ or 3.

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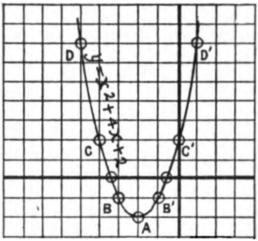
9. Since the coefficient of x is 4, § 418, first substitute -2 for x.

$y = x^2 + 4x + 2$

x	y	POINTS
-2	-2	A
$-3, -1$	-1	B, B'
$-4, 0$	2	C, C'
$-5, 1$	7	D, D'



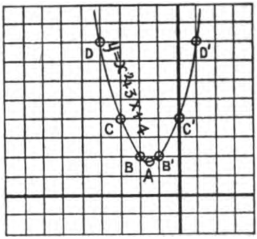
Plotting these points and drawing a smooth curve through them, we have the graph of $y = x^2 + 4x + 2$, which crosses the x-axis approximately at -3.4 and $-.6$.

Hence, the roots of $x^2 + 4x + 2 = 0$ are -3.4 and $-.6$.

10. Since the coefficient of x is 3, § 418, first substitute $-1\frac{1}{2}$ for x.

$y = x^2 + 3x + 4$

x	y	POINTS
$-1\frac{1}{2}$	$1\frac{3}{4}$	A
$-2, -1$	2	B, B'
$-3, 0$	4	C, C'
$-4, 1$	8	D, D'



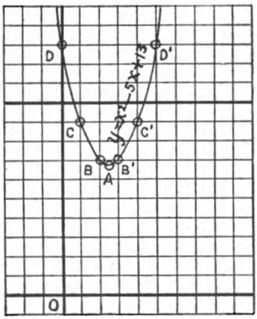
Plotting these points and drawing a smooth curve through them, we have the graph of $y = x^2 + 3x + 4$, whose minimum point lies above the x-axis.

Hence, § 420, 4, the roots of $x^2 + 3x + 4 = 0$ are imaginary.

11. Since the coefficient of x is -5, § 418, first substitute $2\frac{1}{2}$ for x.

$y = x^2 - 5x + 13$

x	y	POINTS
$2\frac{1}{2}$	$6\frac{3}{4}$	A
$2, 3$	7	B, B'
$1, 4$	9	C, C'
$0, 5$	13	D, D'



Plotting these points and drawing a smooth curve through them, we have the graph of $y = x^2 - 5x + 13$, whose minimum point lies above the x-axis.

Hence, § 420, 4, the roots of $x^2 - 5x + 13 = 0$ are imaginary.

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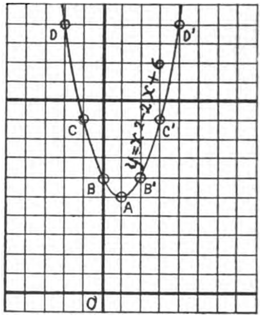
12. Since the coefficient of x is -2, § 418, first substitute 1 for x.

$$y = x^2 - 2x + 6$$

x	y	Points
1	5	A
0, 2	6	B, B'
$-1, 3$	9	C, C'
$-2, 4$	14	D, D'

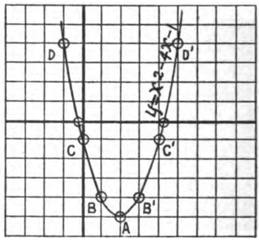


Plotting these points and drawing a smooth curve through them, we have the graph of $y = x^2 - 2x + 6$, whose minimum point lies above the x-axis.
Hence, § 420, 4, the roots of $x^2 - 2x + 6 = 0$ are imaginary.

13. Since the coefficient of x is -4, § 418, first substitute 2 for x.

$$y = x^2 - 4x - 1$$

x	y	Points
2	-5	A
1, 3	-4	B, B'
0, 4	-1	C, C'
$-1, 5$	4	D, D'

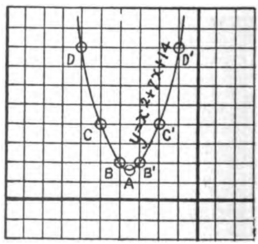


Plotting these points and drawing a smooth curve through them, we have the graph of $y = x^2 - 4x - 1$, which crosses the x-axis approximately at $-.2$ and 4.2.
Hence, the roots of $x^2 - 4x - 1 = 0$ are $-.2$ and 4.2.

14. Since the coefficient of x is 7, § 418, first substitute $-3\frac{1}{2}$ for x.

$$y = x^2 + 7x + 14$$

x	y	Points
$-3\frac{1}{2}$	$1\frac{3}{4}$	A
$-4, -3$	2	B, B'
$-5, -2$	4	C, C'
$-6, -1$	8	D, D'



Plotting these points and drawing a smooth curve through them, we have the graph of $y = x^2 + 7x + 14$, whose minimum point lies above the x-axis.
Hence, § 420, 4, the roots of $x^2 + 7x + 14 = 0$ are imaginary.

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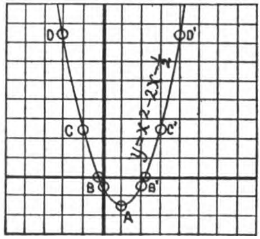
15. Since in $y = x^2 - 2x - \frac{1}{2}$, the coefficient of x is -2, § 418, first substitute 1 for x.

$$y = x^2 - 2x - \tfrac{1}{2}$$

x	y	POINTS
1	$-1\frac{1}{2}$	A
0, 2	$-\frac{1}{2}$	B, B'
-1, 3	$2\frac{1}{2}$	C, C'
-2, 4	$7\frac{1}{2}$	D, D'



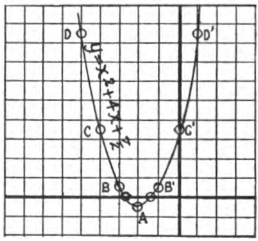
Plotting these points and drawing a smooth curve through them, we have the graph of $y = x^2 - 2x - \frac{1}{2}$, which crosses the x-axis approximately at $-.2$ and 2.2.

Hence, the roots of $x^2 - 2x - \frac{1}{2} = 0$, or of $4x - 2x^2 + 1 = 0$, are $-.2$ and 2.2.

16. Putting $2x^2 + 8x + 7 = 0$ in the form $x^2 + 4x + \frac{7}{2} = 0$, since the coefficient of x is 4, § 418, first substitute -2 for x.

$$y = x^2 + 4x + \tfrac{7}{2}$$

x	y	POINTS
-2	$-\frac{1}{2}$	A
-3, -1	$\frac{1}{2}$	B, B'
-4, 0	$3\frac{1}{2}$	C, C'
-5, 1	$8\frac{1}{2}$	D, D'



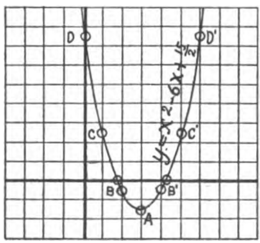
Plotting these points and drawing a smooth curve through them, we have the graph of $y = x^2 + 4x + \frac{7}{2}$, which crosses the x-axis approximately at -2.7 and -1.3.

Hence, the roots of $x^2 + 4x + \frac{7}{2} = 0$, or of $2x^2 + 8x + 7 = 0$, are -2.7 and -1.3.

17. Putting $2x^2 - 12x + 15 = 0$ in the form $x^2 - 6x + \frac{15}{2} = 0$, since the coefficient of x is -6, § 418, first substitute 3 for x.

$$y = x^2 - 6x + \tfrac{15}{2}$$

x	y	POINTS
3,	$-1\frac{1}{2}$	A
2, 4	$-\frac{1}{2}$	B, B'
1, 5	$2\frac{1}{2}$	C, C'
0, 6	$7\frac{1}{2}$	D, D'



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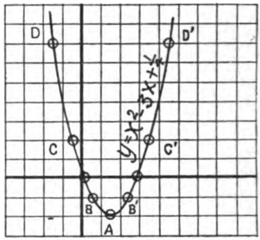
Plotting these points and drawing a smooth curve through them, we have the graph of $y = x^2 - 6x + \frac{15}{2}$, which crosses the x-axis approximately at 1.8 and 4.2.

Hence, the roots of $x^2 - 6x + \frac{15}{2} = 0$, or of $2x^2 - 12x + 15 = 0$, are 1.8 and 4.2.

18. Putting $12x - 4x^2 - 1 = 0$ in the form $x^2 - 3x + \frac{1}{4} = 0$, since the coefficient of x is -3, § 418, first substitute $1\frac{1}{2}$ for x.

$$y = x^2 - 3x + \tfrac{1}{4}$$

x	y	POINTS
$1\frac{1}{2}$	-2	A
$\frac{1}{2}, 2\frac{1}{2}$	-1	B, B'
$-\frac{1}{2}, 3\frac{1}{2}$	2	C, C'
$-1\frac{1}{2}, 4\frac{1}{2}$	7	D, D'



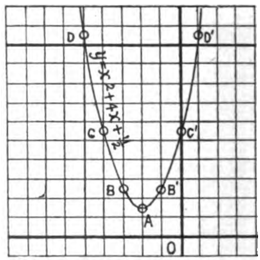
Plotting these points and drawing a smooth curve through them, we have the graph of $y = x^2 - 3x + \frac{1}{4}$, which crosses the x-axis approximately at .1 and 2.9.

Hence, the roots of $x^2 - 3x + \frac{1}{4} = 0$, or of $12x - 4x^2 - 1 = 0$, are .1 and 2.9.

19. Putting $11 + 8x + 2x^2 = 0$ in the form $x^2 + 4x + \frac{11}{2} = 0$, since the coefficient of x is 4, § 418, first substitute -2 for x.

$$y = x^2 + 4x + \tfrac{11}{2}$$

x	y	POINTS
-2	$1\frac{1}{2}$	A
$-3, -1$	$2\frac{1}{2}$	B, B'
$-4, 0$	$5\frac{1}{2}$	C, C'
$-5, 1$	$10\frac{1}{2}$	D, D'



Plotting these points and drawing a smooth curve through them, we have the graph of $y = x^2 + 4x + \frac{11}{2}$, whose minimum point lies above the x-axis.

Hence, § 420, 4, the roots of $x^2 + 4x + \frac{11}{2} = 0$, or of $11 + 8x + 2x^2 = 0$, are imaginary.

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2. Solving $(x-2)^2 + (y-3)^2 = 9$ for y, $y = 3 \pm \sqrt{9 - (x-2)^2}$.

Since any value less than -1 or greater than $+5$ substituted for x will make the value of y imaginary, the graph lies between $x = -1$ and $+5$;

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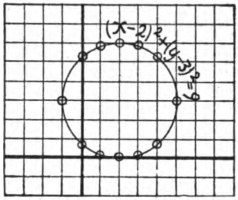
consequently, we substitute for x only -1, $+5$, and intermediate values.

x	y
-1	3
0	5.2, .8
1	5.8, .2
2	6, 0
3	5.8, .2
4	5.2, .8
5	3



Plotting these twelve points and drawing a smooth curve through them, we have the graph of $(x-2)^2+(y-3)^2=9$, which is a circle whose radius is 3 and whose center is at the point (2, 3).

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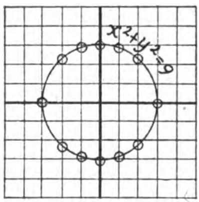
7. Solving $x^2+y^2=9$ for y, $y=\pm\sqrt{9-x^2}$.

Since any value numerically greater than 3 substituted for x will make the value of y imaginary, we substitute only values of x between and including -3 and $+3$. The corresponding values of y, or of $\pm\sqrt{9-x^2}$, are given in the table.

x	y
0	± 3
± 1	± 2.8
± 2	± 2.2
± 3	0



Plotting these twelve points and drawing a smooth curve through them, we have the graph of $x^2+y^2=9$, which is a circle whose radius is 3 and whose center is at the origin.

8. Solving $y^2=5x+8$ for y, $y=\pm\sqrt{5x+8}$.

It will be observed that any value smaller than -1.6 substituted for x will make y imaginary; consequently, no point of the graph lies to the left of $x=-1.6$. Then, beginning with $x=-1.6$, we determine corresponding values of y from succeeding values of x as shown in the table.

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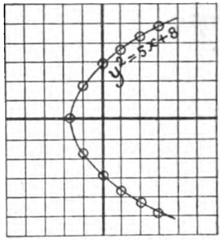
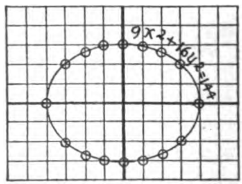
x	y
-1.6	0
-1	± 1.7
0	± 2.8
1	± 3.6
2	± 4.2
3	± 4.8



Plotting these ten points and drawing a smooth curve through them, we have the graph of $y^2 = 5x + 8$, which is a parabola.

9. Solving $9x^2 + 16y^2 = 144$ for y, $y = \pm \frac{3}{4}\sqrt{16 - x^2}$.
Since any value numerically greater than 4 substituted for x will make the value of y imaginary, no point of the graph lies farther to the right or to the left of the origin than 4 units; consequently, we substitute for x only values between and including -4 and $+4$.

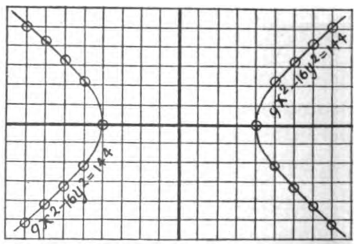
x	y
0	± 3
± 1	± 2.9
± 2	± 2.6
± 3	± 2
± 4	0



Plotting these sixteen points and drawing a smooth curve through them, we have the graph of $9x^2 + 16y^2 = 144$, which is an ellipse.

10. Solving $9x^2 - 16y^2 = 144$ for y, $y = \pm \frac{3}{4}\sqrt{x^2 - 16}$.
Since any value numerically less than 4 substituted for x will make the value of y imaginary, no point of the graph lies between $x = -4$ and $+4$; consequently, we substitute for x only ± 4 and values numerically greater than 4.

x	y
± 4	0
± 5	$\pm 2\frac{1}{4}$
± 6	± 3.3
± 7	± 4.3
± 8	± 5.2



Plotting these eighteen points and drawing a smooth curve through each group of points, the two branches of the curve found constitute the graph of the equation $9x^2 - 16y^2 = 144$, which is an hyperbola.

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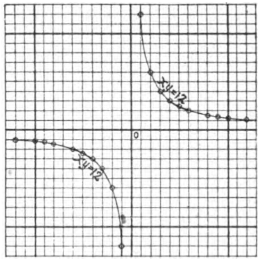
11. Substituting values for x and solving $xy = 12$ for y, the corresponding values found are as given in the table.

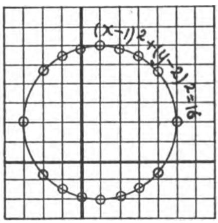
x	y	x	y
1	12	-1	-12
2	6	-2	-6
3	4	-3	-4
4	3	-4	-3
5	2.4	-5	-2.4
6	2	-6	-2
8	$1\frac{1}{2}$	-8	$-1\frac{1}{2}$
9	$1\frac{1}{3}$	-9	$-1\frac{1}{3}$
10	1.2	-10	-1.2
12	1	-12	-1



Plotting these points and drawing a smooth curve through each group of points, the two branches of the curve found constitute the graph of the equation $xy = 12$, which is an hyperbola.

12. Solving $(x-1)^2 + (y-2)^2 = 16$ for y, $y = 2 \pm \sqrt{16-(x-1)^2}$. Since any value less than -3 or greater than $+5$ substituted for x will make the value of y imaginary, the graph lies between $x = -3$ and $+5$; consequently, only -3, $+5$, and intermediate values are substituted for x.

x	y
-3	2
-2	$4.6, -.6$
-1	$5.5, -1.5$
0	$5.9, -1.9$
1	$6, -2$
2	$5.9, -1.9$
3	$5.5, -1.5$
4	$4.6, -.6$
5	2



Plotting these sixteen points and drawing a smooth curve through them, we have the graph of $(x-1)^2 + (y-2)^2 = 16$, which is a circle whose radius is 4 and whose center is at the point (1, 2).

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6. Imagine the circle $x^2 + y^2 = 25$ in exercise 5 to become smaller and smaller until it coincides with the circle $x^2 + y^2 = 9$ (see dotted circle in the cut). The four real unequal roots represented by the coördinates of the points of intersection of the graphs come together in pairs at the points (3, 0) and $(-3, 0)$ where the circle $x^2 + y^2 = 9$ is tangent to the

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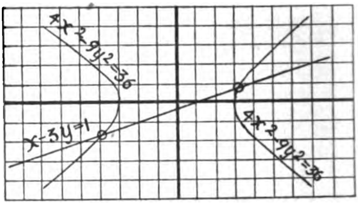
hyperbola $4x^2 - 9y^2 = 36$; consequently, the equations $4x^2 - 9y^2 = 36$ and $x^2 + y^2 = 9$ have two pairs of equal real roots, namely:

$$\begin{cases} x = 3, & 3, & -3, & -3; \\ y = 0, & 0, & 0, & 0. \end{cases}$$

7. The graph of $4x^2 - 9y^2 = 36$, the same as that constructed in § 422, exercise 5, is an hyperbola; and that of $x - 3y = 1$ (§ 267) is a straight line.

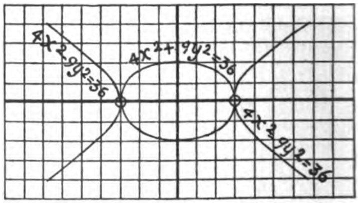


The straight line intersects the hyperbola approximately at the points (3.2, .7) and (– 3.8, – 1.6).

Hence, there are two real roots, namely:

$$\begin{cases} x = 3.2, & -3.8; \\ y = .7, & -1.6. \end{cases}$$

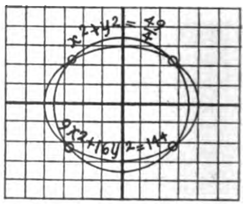
8. The graph of $4x^2 - 9y^2 = 36$, the same as that constructed in § 422, exercise 5, is an hyperbola; and that of $4x^2 + 9y^2 = 36$, constructed by the method of § 422, exercise 4, is an ellipse.



The ellipse is tangent to the hyperbola at the points (3, 0) and (– 3, 0); consequently, the equations have two pairs of equal real roots, namely:

$$\begin{cases} x = 3, & 3, & -3, & -3; \\ y = 0, & 0, & 0, & 0. \end{cases}$$

9. The graph of $9x^2 + 16y^2 = 144$, the same as that constructed for exercise 9, page 333, is an ellipse; and that for $x^2 + y^2 = \frac{49}{4}$, constructed by the method of § 422, exercise 1, is a circle.

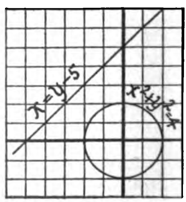


These graphs intersect approximately at the points (2.7, 2.2), (2.7, – 2.2), (– 2.7, 2.2), and (– 2.7, – 2.2).

Hence, the equations have four real roots, namely:

$$\begin{cases} x = 2.7, & 2.7, & -2.7, & -2.7; \\ y = 2.2, & -2.2, & 2.2, & -2.2. \end{cases}$$

10. The graph of $x^2 + y^2 = 4$, constructed by the method of § 422, exercise 1, is a circle; and that of $x = y - 5$ (§ 267) is a straight line.



Since one equation is simple and the other quadratic, they have two roots, and since their graphs have no points in common, the roots are imaginary.

11. The graph of $x^2 - 4y^2 = 4$, constructed by method of § 422, ex. 5, is an hyperbola; and that of $x^2 + y^2 = 4$, same as first equation of ex. 10, is a circle.

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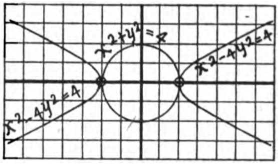
The hyperbola and circle are tangent to each other at the points (2, 0) and (−2, 0).

Hence, the equations have two pairs of equal real roots, namely:

$$\begin{cases} x = 2, \ \ 2, \ \ -2, \ \ -2; \\ y = 0, \ \ 0, \ \ \ \ 0, \ \ \ \ 0. \end{cases}$$

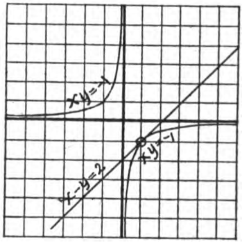


12. The graph of $x - y = 2$ (§ 267) is a straight line; and that of $xy = -1$, constructed by the method of § 422, exercise 6, is an hyperbola.

The straight line is tangent to the hyperbola at the point (1, − 1).

Hence, the equations have a pair of equal real roots, namely:

$$\begin{cases} x = \ \ \ 1, \ \ \ 1; \\ y = -1, \ -1. \end{cases}$$

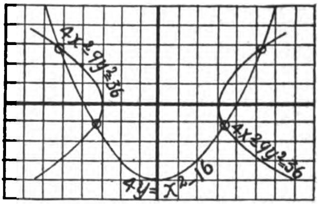


13. The graph of $4x^2 - 9y^2 = 36$, the same as that constructed in § 422, exercise 5, is an hyperbola; and that of $4y = x^2 - 16$, constructed by the method of § 422, exercise 3, is a parabola.

These graphs intersect approximately at the points (5.2, 2.8) (− 5.2, 2.8), (3.4, − 1.1), and (− 3.4, − 1.1).

Hence, the equations have four real roots, namely:

$$\begin{cases} x = 5.2, \ -5.2, \ \ \ 3.4, \ -3.4; \\ y = 2.8, \ \ \ \ 2.8, \ -1.1, \ -1.1. \end{cases}$$

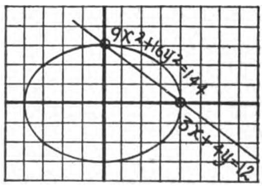


14. The graph of $9x^2 + 16y^2 = 144$, the same as that constructed for exercise 9, page 333, is an ellipse; and that for $3x + 4y = 12$ (§ 267) is a straight line.

The straight line intersects the ellipse at the points (4, 0) and (0, 3).

Hence, the equations have two real roots, namely:

$$\begin{cases} x = 4, \ 0; \\ y = 0, \ 3. \end{cases}$$



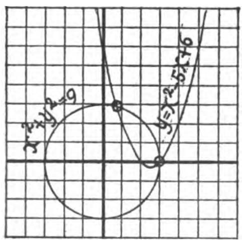
15. The graph of $x^2 + y^2 = 9$, constructed by the method of § 422, exercise 1, is a circle; and that of $y = x^2 - 5x + 6$, constructed by the method of § 418, is a parabola.

These two graphs intersect approximately at the points (3, 0) and (.7, 2.9).

Hence, the equations have two real roots, namely:

$$\begin{cases} x = 3, \ \ .7; \\ y = 0, \ 2.9. \end{cases}$$

Since both equations are quadratic, they have four roots; hence, the other two roots are imaginary.



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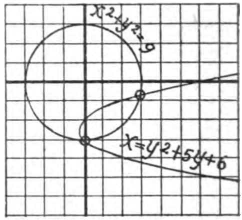
16. The graph of $x^2 + y^2 = 9$, constructed by the method of § 422, exercise 1, is a circle; and that of $x = y^2 + 5y + 6$, constructed by the method of § 418 by substituting values of y and solving for x, is a parabola.

These graphs intersect approximately at the points $(0, -3)$ and $(2.9, -.7)$.

Hence, the equations have two real roots,

namely: $\begin{cases} x = 0, & 2.9; \\ y = -3, & -.7. \end{cases}$

Since both equations are quadratic they have four roots; hence, the other two roots are imaginary.

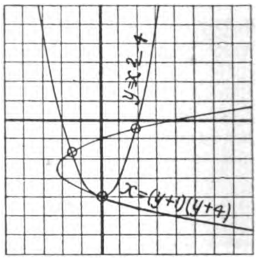


17. The graph of $y = x^2 - 4$, constructed by the method of § 418, is a parabola; and that of $x = (y+1)(y+4)$, constructed by the same method by substituting values of y and solving for x, is also a parabola.

These graphs are tangent to each other at the point $(0, -4)$ and intersect approximately at the points $(1.8, -.5)$ and $(-1.6, -1.7)$.

Hence, the equations have a pair of equal real roots and two other real roots, namely:

$$\begin{cases} x = 0, & 0, & 1.8, & -1.6; \\ y = -4, & -4, & -.5, & -1.7. \end{cases}$$

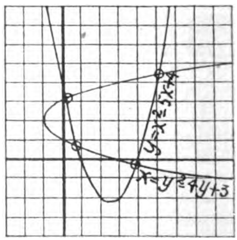


18. The graph of $y = x^2 - 5x + 4$, constructed by the method of § 418, is a parabola; and that of $x = y^2 - 4y + 3$, constructed by the same method by substituting values of y and solving for x, is also a parabola.

These graphs intersect approximately at the points $(.2, 3.1)$, $(.7, .7)$, $(3.9, -.2)$, and $(5.1, 4.5)$.

Hence, the equations have four real roots,

namely: $\begin{cases} x = .2, & .7, & 3.9, & 5.1; \\ y = 3.1, & .7, & -.2, & 4.5. \end{cases}$



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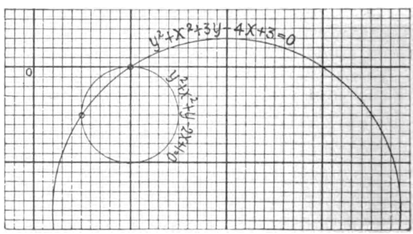
19. Solving $y^2 + x^2 + y - 2x + 1 = 0$ for y, as in § 422, Ex. 1, we find that any value of x less than .5 or greater than 1.5 will make the value of y imaginary; then, the graph lies between $x = .5$ and 1.5; hence, we substitute for x only .5, 1.5, and intermediate values.

In plotting the graph, we plot to a larger scale than in the preceding exercises, letting one division of the squared paper represent .1 of a unit instead of 1 unit.



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The graph is a circle whose radius is .5 and whose center is at the point $(1, -\frac{1}{2})$.

Proceeding in a similar manner with the equation $y^2 + x^2 + 3y - 4x + 3 = 0$, it is found that its graph lies between $x = .2$ and $x = 3.8$.

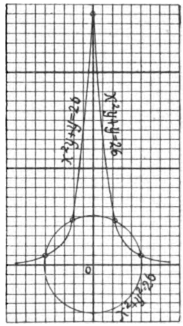
Plotting to the same scale, it is found that the graph is a circle whose radius is 1.8 and whose center is at the point $(2, -1.5)$.

These graphs intersect at the points $(1, 0)$ and $(\frac{1}{2}, -\frac{1}{2})$.

Hence, the equations have two real roots, namely: $\begin{cases} x = 1, & \frac{1}{2}; \\ y = 0, & -\frac{1}{2}. \end{cases}$

Since both equations are quadratic, they have four roots; hence, the other two roots are imaginary.

20. Both from the form of the equation and from plotting points as in § 422, exercise 1, the graph of $x^2 + y^2 = 26$ is found to be a circle whose center is at the origin and whose radius is equal to $\sqrt{26}$.



By solving $x^2y + y = 26$ for y, substituting positive and negative values for x, solving for y, plotting these points, and proceeding as in previous exercises, we find the graph of $x^2y + y = 26$, as shown in the margin.

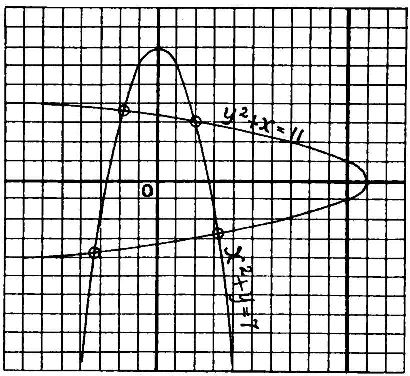
By substituting negative values for y and solving for x, it is seen that x is imaginary when y is negative, showing that the graph does not extend below the x-axis. In fact, the curve never quite reaches the x-axis though it approaches indefinitely near to it.

The graphs of the given equations intersect approximately at the points $(5, 1)$, $(-5, 1)$, $(2.2, 4.6)$, and $(-2.2, 4.6)$.

Hence, the equations have four real roots, namely:

$$\begin{cases} x = 5, & -5, & 2.2, & -2.2; \\ y = 1, & 1, & 4.6, & 4.6. \end{cases}$$

21. The graph of $x^2+y=7$, constructed as in § 422, exercise 3, is a parabola; and that of $y^2+x=11$, constructed by the same method by substituting values of y and solving for x, is also a parabola.



These parabolas intersect approximately at the points $(3.2, -2.8)$, $(2, 3)$, $(-1.8, 3.6)$, and $(-3.3, -3.8)$.

Hence, the equations have four real roots, namely:

$$\begin{cases} x = 3.2, & 2, & -1.8, & -3.3; \\ y = -2.8, & 3, & 3.6, & -3.8. \end{cases}$$

NOTE. — A special solution by quadratic methods for one of the roots is here given:

$$\begin{cases} x^2 + y = 7, & (1) \\ y^2 + x = 11. & (2) \end{cases}$$

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From (1), $y - 3 = 4 - x^2$. (3)

From (2), $y^2 - 9 = 2 - x$. (4)

Dividing (4) by $y + 3$, $y - 3 = \dfrac{2}{y+3} - \dfrac{x}{y+3}$. (5)

From (3) and (5), $4 - x^2 = \dfrac{2}{y+3} - \dfrac{x}{y+3}$.

Transposing, etc., $x^2 - \dfrac{x}{y+3} = 4 - \dfrac{2}{y+3}$.

Completing squares, $x^2 - \dfrac{x}{y+3} + \dfrac{1}{4(y+3)^2} = 4 - \dfrac{2}{y+3} + \dfrac{1}{4(y+3)^2}$.

Extracting square root, $x - \dfrac{1}{2(y+3)} = 2 - \dfrac{1}{2(y+3)}$.

Whence, $x = 2$, and substituting in (1), $y = 3$.

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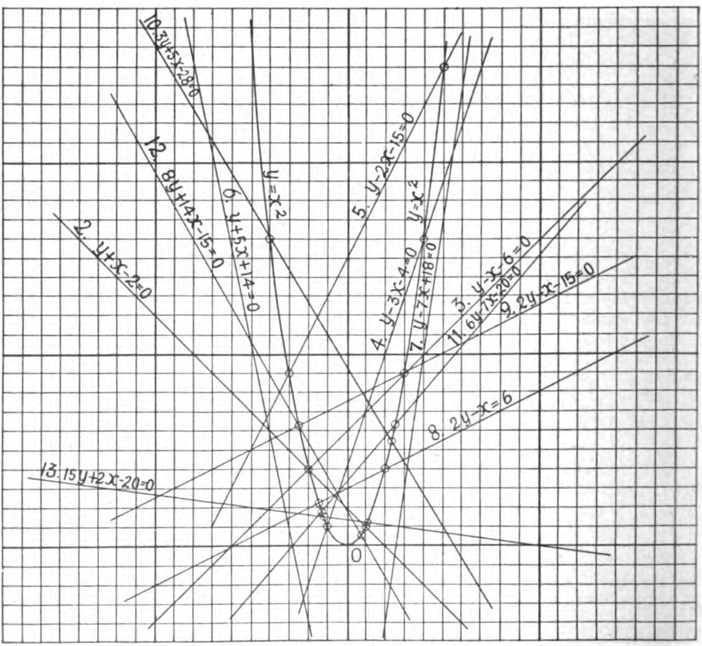
2. Substituting y for x^2 in $x^2 + x - 2 = 0$, we have $y + x - 2 = 0$ whose graph, shown in the figure, is a straight line that intersects the parabola $y = x^2$ at $x = 1$ and $x = -2$. Hence, the roots of the equation $x^2 + x - 2 = 0$ are 1 and -2.



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3. Substituting y for x^2 in $x^2 - x - 6 = 0$, we have $y - x - 6 = 0$ whose graph, shown in the figure, is a straight line that intersects the parabola $y = x^2$ at $x = 3$ and $x = -2$. Hence, the roots of the equation $x^2 - x - 6 = 0$ are 3 and -2.

4. Substituting y for x^2 in $x^2 - 3x - 4 = 0$, we have $y - 3x - 4 = 0$ whose graph, shown in the figure, is a straight line that intersects the parabola $y = x^2$ at $x = 4$ and $x = -1$. Hence, the roots of the equation $x^2 - 3x - 4 = 0$ are 4 and -1.

5. Substituting y for x^2 in $x^2 - 2x - 15 = 0$, we have $y - 2x - 15 = 0$ whose graph, shown in the figure, is a straight line that intersects the parabola $y = x^2$ at $x = 5$ and $x = -3$. Hence, the roots of the equation $x^2 - 2x - 15 = 0$ are 5 and -3.

6. Substituting y for x^2 in $x^2 + 5x + 14 = 0$, we have $y + 5x + 14 = 0$ whose graph, shown in the figure, is a straight line that has no point in common with the parabola $y = x^2$. Hence, the roots of the equation $x^2 + 5x + 14 = 0$ are imaginary.

7. Substituting y for x^2 in $x^2 - 7x + 18 = 0$, we have $y - 7x + 18 = 0$ whose graph, shown in the figure, is a straight line that has no point in common with the parabola $y = x^2$. Hence, the roots of the equation $x^2 - 7x + 18 = 0$ are imaginary.

8. Substituting y for x^2 in $2x^2 - x = 6$, we have $2y - x = 6$ whose graph, shown in the figure, is a straight line that intersects the parabola $y = x^2$ at $x = 2$ and $x = -1.5$. Hence, the roots of the equation $2x^2 - x = 6$ are 2 and -1.5.

9. Substituting y for x^2 in $2x^2 - x - 15 = 0$, we have $2y - x - 15 = 0$ whose graph, shown in the figure, is a straight line that intersects the parabola $y = x^2$ at $x = 3$ and $x = -2.5$. Hence, the roots of the equation $2x^2 - x - 15 = 0$ are 3 and -2.5.

10. Substituting y for x^2 in $3x^2 + 5x - 28 = 0$, we have, $3y + 5x - 28 = 0$ whose graph, shown in the figure, is a straight line that intersects the parabola $y = x^2$ approximately at $x = 2.3$ and $x = -4$. Hence, the roots of the equation $3x^2 + 5x - 28 = 0$ are 2.3 and -4.

11. Substituting y for x^2 in $6x^2 - 7x - 20 = 0$, we have $6y - 7x - 20 = 0$ whose graph, shown in the figure, is a straight line that intersects the parabola $y = x^2$ approximately at $x = 2.5$ and $x = -1.3$. Hence, the roots of the equation $6x^2 - 7x - 20 = 0$ are 2.5 and -1.3.

12. Substituting y for x^2 in $8x^2 + 14x - 15 = 0$, we have $8y + 14x - 15 = 0$ whose graph, shown in the figure, is a straight line that intersects the parabola $y = x^2$ approximately at $x = .75$ and $x = -2.5$. Hence, the roots of the equation $8x^2 + 14x - 15 = 0$ are .75 and -2.5.

13. Substituting y for x^2 in $15x^2 + 2x - 20 = 0$, we have $15y + 2x - 20 = 0$ whose graph, shown in the figure, is a straight line that intersects the parabola $y = x^2$ approximately at $x = 1.1$ and $x = -1.2$. Hence, the roots of the equation $15x^2 + 2x - 20 = 0$ are 1.1 and -1.2.

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PROPERTIES OF QUADRATIC EQUATIONS

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3. $$ax^2 + bx + c = x^2 - 5x - 75 = 0.$$

Since $b^2 - 4ac = 25 + 300 = 325$, the roots are real and unequal (§ 429, Prin. 1), and irrational (§ 429, Prin. 4). Since c is negative, the roots have opposite signs and, b being negative, the positive root is the greater numerically (§ 431, Prin.).

4. $$ax^2 + bx + c = x^2 + 5x + 6 = 0.$$

Since $b^2 - 4ac = 25 - 24 = 1 = 1^2$, the roots are real and unequal (§ 429, Prin. 1), and rational (§ 429, Prin. 4). Since c is positive and b is positive, both roots are negative (§ 431, Prin.).

5. $$ax^2 + bx + c = x^2 + 7x - 30 = 0.$$

Since $b^2 - 4ac = 49 + 120 = 169 = 13^2$, the roots are real and unequal (Prin. 1), and rational (§ 429, Prin. 4). Since c is negative, the roots have opposite signs and, b being positive, the negative root is the greater numerically (§ 431, Prin.).

6. $$ax^2 + bx + c = x^2 - 3x + 5 = 0.$$

Since $b^2 - 4ac = 9 - 20 = -11$, both roots are imaginary (§ 429, Prin. 3).

7. $$ax^2 + bx + c = x^2 + 3x - 5 = 0.$$

Since $b^2 - 4ac = 9 + 20 = 29$, the roots are real and unequal (§ 429, Prin. 1), and irrational (§ 429, Prin. 4). Since c is negative, the roots have opposite signs and, b being positive, the negative root is the greater numerically (§ 431, Prin.).

8. $$ax^2 + bx + c = 4x^2 - 4x + 1 = 0.$$

Since $b^2 - 4ac = 16 - 16 = 0$, the roots are real and equal (§ 429, Prin. 2), and rational (§ 429, Prin. 4). Since c is positive and b is negative, both roots are positive (§ 431, Prin.).

9. $$ax^2 + bx + c = 4x^2 + 6x - 4 = 0.$$

Since $b^2 - 4ac = 36 + 64 = 100 = 10^2$, the roots are real and unequal (§ 429, Prin. 1), and rational (§ 429, Prin. 4). Since c is negative, the roots have opposite signs and, b being positive, the negative root is the greater numerically (§ 431, Prin.).

10. $$ax^2 + bx + c = x^2 + x + 2 = 0.$$

Since $b^2 - 4ac = 1 - 8 = -7$, both roots are imaginary (§ 429, Prin. 3).

11. $$ax^2 + bx + c = 4x^2 + 16x + 7 = 0.$$

Since $b^2 - 4ac = 256 - 112 = 144 = 12^2$, the roots are real and unequal (§ 429, Prin. 1), and rational (§ 429, Prin. 4). Since c is positive and b is positive, both roots are negative (§ 431, Prin.).

12. $$ax^2 + bx + c = 9x^2 + 12x + 4 = 0.$$

Since $b^2 - 4ac = 144 - 144 = 0$, the roots are real and equal (§ 429, Prin. 2), and rational (§ 429, Prin. 4). Since c is positive and b is positive, both roots are negative (§ 431, Prin.).

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14. The roots will be equal, if the discriminant equals zero (§ 429, Prin. 2); that is, if $$25m^2 - 4 \cdot 9 \cdot 25 = 0,$$
or, solving, if $$m = \pm 6.$$
The roots will be real, if the discriminant is positive (§ 429, Prin. 1); that is, if $$25m^2 > 4 \cdot 9 \cdot 25 \text{ numerically},$$
or, solving, if $$m > 6 \text{ numerically}.$$
The roots are also real, if the discriminant equals zero (§ 429, Prin. 2); that is, if, as above, $$m = \pm 6.$$
The roots will be imaginary, if the discriminant is negative (§ 429, Prin. 3); that is, if $$25m^2 - 4 \cdot 9 \cdot 25 \text{ is negative},$$
which will be true when m is numerically less than 6.

15. The roots will be real and equal, when the discriminant equals zero (§ 429, Prin. 2); that is, if $$[2(a-3)]^2 - 4 \cdot 4 \cdot 1 = 0,$$
or, solving, if $$a = 5 \text{ or } 1.$$
The roots will be real and unequal, if the discriminant is positive, (§ 429, Prin. 1); that is, if $$[2(a-3)]^2 > 4 \cdot 4 \cdot 1 \text{ numerically},$$
or, solving, if $$a > 5 \text{ numerically}.$$
The roots will be imaginary, if the discriminant is negative (§ 429, Prin. 3); that is, if $$[2(a-3)]^2 - 4 \cdot 4 \cdot 1 \text{ is negative},$$
which will be true when $a < 5$, in all cases except $a = 1$, when the roots are real and equal.

16. The roots will be equal, when the discriminant equals zero (§ 429, Prin. 2); that is, when $$(m+1)^2 - 4 \cdot 4 \cdot 1 = 0,$$
or, solving, when $$m = 3 \text{ or } -5.$$
Substituting 3 for m in the given equation, $4x^2 + 3x + x + 1 = 0.$
Solving, $$x = -\tfrac{1}{2}.$$
Substituting -5 for m in the given equation, $4x^2 - 5x + x + 1 = 0.$
Solving, $$x = \tfrac{1}{2}.$$
Hence, the corresponding values of x are $-\frac{1}{2}$ and $\frac{1}{2}$.

17. Expressing the equation in the form of $ax^2 + bx + c = 0$,
$$(3+2n)x^2 - 4nx + (n+1) = 0.$$
The roots will be real and equal, if the discriminant equals zero, that is, if $$(4n)^2 - 4 \cdot (3+2n)(n+1) = 0,$$
or, solving, if $$n = 3 \text{ or } -\tfrac{1}{2}.$$

18. The roots will be numerically equal but opposite in sign, if $b = 0$ (§ 431, Note); that is, if $$a - 1 = 0,$$
or, solving, if $$a = 1.$$
Substituting 1 for a in $$ax^2 - (a-1)x + 1 = 0.$$
$$x^2 - (1-1)x + 1 = 0.$$
Solving, $$x = \pm\sqrt{-1}.$$

19. From § 429 it is seen, that if $c = 0$, $r_1 = \dfrac{-b+\sqrt{b^2}}{2a}$ and $r_2 = \dfrac{-b-\sqrt{b^2}}{2a}$, which reduces to $r_1 = \dfrac{-b+b}{2a} = 0$ and $r_2 = \dfrac{-b-b}{2a} = -\dfrac{b}{a}$.
Hence, it is evident, that if the absolute term equals zero, the equation will have a zero root. Applying this principle to the given equation,

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$$x^2 + (2-d)x = 3d^2 - 27$$

will have a zero root, if $3d^2 - 27 = 0$,

or, solving, if $d = \pm 3$.

Substituting 3 for d in the given equation,

$$x^2 + (2-3)x = 3(3)^2 - 27.$$

Solving, $x = 0$ or 1.

Substituting -3 for d, $x^2 + (2+3)x = 3(-3)^2 - 27$.

Solving, $x = 0$ or -5.

Hence, when $d = 3$, the roots are 0 and 1, and when $d = -3$, the roots are 0 and -5.

20. The roots of the equation $(m + \frac{5}{2})x^2 - 2(m+1)x + 2 = 0$ will be equal, if the discriminant equals zero, that is, if

$$[2(m+1)]^2 - 4 \cdot (m + \tfrac{5}{2}) \cdot 2 = 0,$$

or, solving, if $m = \pm 2$.

21.
$$\begin{cases} 3x^2 - 4y^2 = 8, & (1) \\ 5(x-k) - 4y = 0. & (2) \end{cases}$$

From (2), $y = \dfrac{5x - 5k}{4}$. (3)

Substituting (3) in (1), simplifying, etc.,

$$13x^2 - 50kx + 25k^2 + 32 = 0.$$

Solving, $x = \frac{1}{13}(25k \pm 2\sqrt{75k^2 - 104})$. (4)

Substituting (4) in (3), $y = \dfrac{\frac{5}{13}(25k \pm 2\sqrt{75k^2 - 104}) - 5k}{4}$.

Simplifying, $y = \frac{5}{26}(6k \pm \sqrt{75k^2 - 104})$.

The roots will be real, if the discriminant is positive, that is, if

$$2500k^2 > 4(25k^2 + 32) \cdot 13, \text{ numerically.}$$

$$k^2 > \tfrac{104}{75} \text{ numerically,}$$

or, solving, if $k > \frac{2}{15}\sqrt{78}$ numerically.

The roots will be imaginary, if $k < \frac{2}{15}\sqrt{78}$ numerically (§ 429, Prin. 3).

The roots will be equal, if the discriminant equals zero (Prin. 2); that is, if $2500k^2 - 4 \cdot 13(25k^2 + 32) = 0$,

or, solving, if $k = \frac{2}{15}\sqrt{78}$.

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2. The equation is $(x-6)(x-4) = 0$, or $x^2 - 10x + 24 = 0$.

3. The equation is $(x-5)(x+3) = 0$, or $x^2 - 2x - 15 = 0$.

4. The equation is $(x-3)(x+\frac{1}{3}) = 0$, or $x^2 - \frac{8}{3}x - 1 = 0$, or, multiplying by 3, $3x^2 - 8x - 3 = 0$.

5. The equation is $(x - \frac{2}{3})(x - \frac{5}{3}) = 0$, or $x^2 - \frac{7}{3}x + \frac{10}{9} = 0$, or, multiplying by 9, $9x^2 - 21x + 10 = 0$.

6. The equation is $(x+2)(x+\frac{1}{2}) = 0$, or $x^2 + \frac{5}{2}x + 1 = 0$, or, multiplying by 2, $2x^2 + 5x + 2 = 0$.

7. The equation is $(x+\frac{1}{2})(x+\frac{3}{2}) = 0$, or $x^2 + 2x + \frac{3}{4} = 0$, or, multiplying by 4, $4x^2 + 8x + 3 = 0$.

8. The equation is $(x-a)(x+3a) = 0$, or $x^2 + 2ax - 3a^2 = 0$.

9. The equation is $(x-a-2)(x-a+2) = 0$, or $x^2 - 2ax + a^2 - 4 = 0$.

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10. The equation is $(x-b-1)(x-b+1)=0$, or $x^2-2bx+b^2-1=0$.

11. The equation is $(x-a-b)(x-a+b)=0$, or $x^2-2ax+a^2-b^2=0$.

12. The equation is

$$(x-\sqrt{a}+\sqrt{b})(x-\sqrt{b})=0, \text{ or } x^2-x\sqrt{a}+\sqrt{ab}-b=0.$$

13. The equation is $(x-\frac{1}{2}a-\frac{1}{2}\sqrt{b})(x-\frac{1}{2}a+\frac{1}{2}\sqrt{b})=0$, or multiplying each factor by 2,

$$(2x-a-\sqrt{b})(2x-a+\sqrt{b})=0, \text{ or } 4x^2-4ax+a^2-b=0.$$

14. The sum of the roots is 6 and their product is $9-2$, or 7. Hence, the equation is $x^2-6x+7=0$.

15. The sum of the roots is 4 and their product is $4-5$ or -1. Hence, the equation is $x^2-4x-1=0$.

16. The sum of the roots is 4 and their product is $4-3$ or 1. Hence, the equation is $x^2-4x+1=0$.

17. The sum of the roots is -3 and their product is $\frac{9}{4}-\frac{6}{4}$, or $\frac{3}{4}$. Hence, the equation is $x^2+3x+\frac{3}{4}=0$, or $4x^2+12x+3=0$.

18. The sum of the roots is -1 and their product is $\frac{1}{4}-\frac{2}{4}$, or $-\frac{1}{4}$. Hence, the equation is $x^2+x-\frac{1}{4}=0$, or $4x^2+4x-1=0$.

19. The sum of the roots is $4a$ and their product is $4a^2-a^2\cdot 4\cdot 5$, or $4a^2-20a^2$, or $-16a^2$. Hence, the equation is $x^2-4ax-16a^2=0$.

20. The sum of the roots of $2m^2x^2-(5m-1)x=6$ is found by reducing the equation to the form $x^2+px+q=0$, that is,

$$x^2-\frac{5m-1}{2m^2}x-\frac{6}{2m^2}=0.$$

Then, the sum of the roots equals $+\left(\frac{5m-1}{2m^2}\right)$ by § 433, Prin.

The sum of the roots will equal 2, if $\frac{5m-1}{2m^2}=2$,

or, solving, if $$m=1 \text{ or } \tfrac{1}{4}.$$

22. Writing $4x^2-3ax+b=3$ in the form $x^2-\frac{3a}{4}x+\frac{b-3}{4}=0$, and representing the roots by r and $2r$, it is evident that

$$r+2r=3r=\frac{3a}{4}, \qquad (1)$$

and $$r\cdot 2r=2r^2=\frac{b-3}{4}. \qquad (2)$$

From (1), $$r=\frac{a}{4}. \qquad (3)$$

Substituting $\frac{a}{4}$ for r in (2), $$2\left(\frac{a}{4}\right)^2=\frac{b-3}{4}.$$

Solving, $$a^2=2(b-3).$$

Hence, $a^2=2(b-3)$ expresses the condition that one root of the given equation is twice the other.

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23. Let r and $r+3$ represent the two roots.

Then, $$r+r+3=\frac{-b}{a}, \quad (1)$$

and $$r(r+3)=\frac{c}{a}. \quad (2)$$

From (1), $$r=\frac{-b-3a}{2a}, \quad (3)$$

and from (2), $$r=\frac{-3a\pm\sqrt{9a^2+4ac}}{2a}. \quad (4)$$

Equating the values of r in (3) and (4),

$$\frac{-3a\pm\sqrt{9a^2+4ac}}{2a}=\frac{-b-3a}{2a}.$$

Simplifying, $$\pm\sqrt{9a^2+4ac}=-b.$$
Solving, $$4ac=b^2-9a^2.$$

24. Let r and $\frac{1}{r}$ represent the two roots.

Then, by § 433, Prin., $$r\left(\frac{1}{r}\right)=\frac{c}{a},$$

or, solving, $$a=c.$$

25. By § 433, Prin., $$r_1+r_2=-\frac{b}{a}.$$

Then, $$-r_1+(-r_2)=-\left(-\frac{b}{a}\right)=\frac{b}{a}.$$

By § 433, Prin., $$r_1r_2=\frac{c}{a}.$$

Then, $$-r_1\cdot -r_2=(r_1\cdot r_2)=\frac{c}{a}.$$

Hence, the equation is $ax^2-bx+c=0.$

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27. Let r_1 and r_2 represent the roots of the equation.
Then, sum of roots $= r_1+r_2=5.$ (1)
Product of roots $= r_1r_2=-6.$ (2)
Squaring (1), $r_1^2+2r_1r_2+r_2^2=25.$ (3)
Multiplying (2) by 2, $2r_1r_2=-12.$ (4)
Subtracting (4) from (3), $r_1^2+r_2^2=37.$
Hence, the sum of the squares of the roots is 37.

28. Let r_1 and r_2 represent the roots of the equation.
Then, by § 433, Prin., $r_1+r_2=\frac{3}{2},$ (1)
and $r_1r_2=\frac{1}{2}.$ (2)
Cubing (1), $r_1^3+3r_1^2r_2+3r_1r_2^2+r_2^3=\frac{27}{8}.$ (3)
Multiplying (1) by (2), $r_1^2r_2+r_1r_2^2=\frac{3}{4}.$ (4)
Multiplying (4) by 3, $3r_1^2r_2+3r_1r_2^2=\frac{9}{4}.$ (5)
Subtracting (5) from (3), $r_1^3+r_2^3=\frac{9}{8}.$
Hence, the sum of the cubes of the roots is $\frac{9}{8}$.

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29. Let r_1 and r_2 represent the roots of the equation.

Then, by § 433, Prin., $r_1 + r_2 = -\frac{1}{12}$, (1)

and $r_1 r_2 = -\frac{1}{12}$. (2)

Squaring (1), $r_1^2 + 2r_1r_2 + r_2^2 = \frac{1}{144}$. (3)

Multiplying (2) by 4, $4r_1r_2 = -\frac{1}{3}$. (4)

Subtracting (4) from (3), $r_1^2 - 2r_1r_2 + r_2^2 = \frac{49}{144}$.

Extracting the square root, $r_1 - r_2 = \pm\frac{7}{12}$.

Hence, the difference between the roots is $\pm\frac{7}{12}$.

30. Let r_1 and r_2 represent the roots of the equation.

Then, by § 433, Prin., $r_1 + r_2 = 7$, (1)

and $r_1 r_2 = 12$. (2)

Squaring (1), $r_1^2 + 2r_1r_2 + r_2^2 = 49$. (3)

Multiplying (2) by 2, $2r_1r_2 = 24$. (4)

Subtracting (4) from (3), $r_1^2 + r_2^2 = 25$. (5)

Extracting the square root, $\sqrt{r_1^2 + r_2^2} = \pm 5$.

Hence, the square root of the sum of the squares of the roots is ± 5.

32. Let r_1 and r_2 represent the roots of the equation.

Then, $\frac{1}{r_1} - \frac{1}{r_2} = \frac{r_2 - r_1}{r_1 r_2}$. (1)

By the method in Ex. 29, $r_2 - r_1 = \pm\frac{1}{4}$. (2)

By § 433, Prin., $r_1 r_2 = \frac{3}{8}$. (3)

Substituting (2) and (3) in (1), $\frac{1}{r_1} - \frac{1}{r_2} = \frac{\pm\frac{1}{4}}{\frac{3}{8}} = \pm\frac{2}{3}$.

Hence, the difference between the reciprocals of the roots is $\pm\frac{2}{3}$.

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2. Let $4x^2 - 4x - 3 = 0$.

Dividing by 4, transposing, completing the square, and solving,

$$x = \tfrac{3}{2} \text{ or } -\tfrac{1}{2}.$$

Forming an equation having these roots, § 434,

$$(x - \tfrac{3}{2})(x + \tfrac{1}{2}) = 0,$$

or, $$(2x - 3)(2x + 1) = 4x^2 - 4x - 3 = 0.$$

3. Let $5x^2 + 3x - 2 = 0$.

Dividing by 5, transposing, completing the square, and solving,

$$x = \tfrac{2}{5} \text{ or } -1.$$

Forming an equation having these roots, § 434,

$$(x + 1)(x - \tfrac{2}{5}) = 0,$$

or, $$(x + 1)(5x - 2) = 5x^2 + 3x - 2 = 0.$$

4. Let $3x^2 + 14x - 5 = 0$.

Dividing by 3, transposing, completing the square, and solving,

$$x = \tfrac{1}{3} \text{ or } -5.$$

Forming an equation having these roots, § 434,

$$(x - \tfrac{1}{3})(x + 5) = 0,$$

or, $$(3x - 1)(x + 5) = 3x^2 + 14x - 5 = 0.$$

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5. Let $8x^2 - 14x + 3 = 0.$

Dividing by 8, transposing, completing the square, and solving,

$$x = \tfrac{3}{2} \text{ or } \tfrac{1}{4}.$$

Forming an equation having these roots, § 434,

$$(x - \tfrac{3}{2})(x - \tfrac{1}{4}) = 0,$$

or, $$(2x - 3)(4x - 1) = 8x^2 - 14x + 3 = 0.$$

6. Let $7x^2 + 13x - 2 = 0.$

Dividing by 7, transposing, completing the square, and solving,

$$x = \tfrac{1}{7} \text{ or } -2.$$

Forming an equation having these roots, § 434,

$$(x + 2)(x - \tfrac{1}{7}) = 0,$$

or, $$(x + 2)(7x - 1) = 7x^2 + 13x - 2 = 0.$$

7. Let $24x^2 - 10x - 25 = 0.$

Dividing by 24, transposing, completing the square, and solving,

$$x = \tfrac{5}{4} \text{ or } -\tfrac{5}{6}.$$

Forming an equation having these roots, § 434,

$$(x - \tfrac{5}{4})(x + \tfrac{5}{6}) = 0,$$

or, $$(4x - 5)(6x + 5) = 24x^2 - 10x - 25 = 0.$$

8. Let $10x^2 + 21x - 10 = 0.$

Dividing by 10, transposing, completing the square, and solving,

$$x = \tfrac{2}{5} \text{ or } -\tfrac{5}{2}.$$

Forming an equation having these roots, § 434,

$$(x + \tfrac{5}{2})(x - \tfrac{2}{5}) = 0,$$

or, $$(2x + 5)(5x - 2) = 10x^2 + 21x - 10 = 0.$$

9. Let $15x^2 - 5.5x - 1 = 0.$

Dividing by 15, transposing, completing the square, and solving,

$$x = \tfrac{1}{2} \text{ or } -\tfrac{2}{15}.$$

Forming an equation having these roots, § 434,

$$(x + \tfrac{2}{15})(x - \tfrac{1}{2}) = 0,$$

or, $$\tfrac{1}{2}(15x + 2)(2x - 1) = 15x^2 - 5.5x - 1 = 0.$$

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11. Let $x^2 + 4x - 6 = 0.$

Solving, $x = -2 \pm \sqrt{10}.$

Hence, § 434, $(x + 2 + \sqrt{10})(x + 2 - \sqrt{10}) = x^2 + 4x - 6 = 0.$

12. Let $y^2 - 6y + 3 = 0.$

Solving, $y = 3 \pm \sqrt{6}.$

Hence, § 434, $(y - 3 + \sqrt{6})(y - 3 - \sqrt{6}) = y^2 - 6y + 3 = 0.$

13. Let $z^2 - 5z - 1 = 0.$

Solving, $z = \tfrac{5}{2} \pm \tfrac{1}{2}\sqrt{29}.$

Hence, § 434, $(z - \tfrac{5}{2} + \tfrac{1}{2}\sqrt{29})(z - \tfrac{5}{2} - \tfrac{1}{2}\sqrt{29}) = z^2 - 5z - 1 = 0,$

or, the factors are $\tfrac{1}{4}(2z - 5 + \sqrt{29})(2z - 5 - \sqrt{29}).$

14. Let $x^2 + x + 1 = 0.$

Solving, $x = -\tfrac{1}{2} \pm \tfrac{1}{2}\sqrt{-3}.$

Hence, § 434, $(x + \tfrac{1}{2} + \tfrac{1}{2}\sqrt{-3})(x + \tfrac{1}{2} - \tfrac{1}{2}\sqrt{-3}) = x^2 + x + 1 = 0,$

or, the factors are $\tfrac{1}{4}(2x + 1 + \sqrt{-3})(2x + 1 - \sqrt{-3}).$

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15. Let $a^2 + 3a - 5 = 0.$

Solving, $a = -\frac{3}{2} \pm \frac{1}{2}\sqrt{29}.$

Hence, § 434, $(a + \frac{3}{2} + \frac{1}{2}\sqrt{29})(a + \frac{3}{2} - \frac{1}{2}\sqrt{29}) = a^2 + 3a - 5 = 0,$

or, the factors are $\frac{1}{4}(2a + 3 + \sqrt{29})(2a + 3 - \sqrt{29}).$

16. Let $t^2 + 3t + 7 = 0.$

Solving, $t = -\frac{3}{2} \pm \frac{1}{2}\sqrt{-19}.$

Hence, § 434, $(t + \frac{3}{2} + \frac{1}{2}\sqrt{-19})(t + \frac{3}{2} - \frac{1}{2}\sqrt{-19}) = t^2 + 3t + 7 = 0,$

or, the factors are $\frac{1}{4}(2t + 3 + \sqrt{-19})(2t + 3 - \sqrt{-19}).$

17. $2 - 3x - 2x^2 = -2(x^2 + \frac{3}{2}x - 1).$

Let $(x^2 + \frac{3}{2}x - 1) = 0.$

Solving, $x = \frac{1}{2}$ or $-2.$

Hence, § 434, $(x + 2)(x - \frac{1}{2}) = 0.$

Then, $-2(x + 2)(x - \frac{1}{2}) = (x + 2)(1 - 2x) = 2 - 3x - 2x^2 = 0.$

18. Let $2x^2 + 2x - 1 = 0.$

Dividing by 2, transposing, completing the square, and solving,

$$x = -\frac{1}{2} \pm \frac{1}{2}\sqrt{3}.$$

Hence, § 434, $(x + \frac{1}{2} + \frac{1}{2}\sqrt{3})(x + \frac{1}{2} - \frac{1}{2}\sqrt{3}) = 0.$

$\frac{1}{4}(2x + 1 + \sqrt{3})(2x + 1 - \sqrt{3}) = 0.$

Multiplying by 2, $\frac{1}{2}(2x + 1 + \sqrt{3})(2x + 1 - \sqrt{3}) = 2x^2 + 2x - 1 = 0.$

19. Let $9x^2 - 4x + 1 = 0.$

Dividing by 9, transposing, completing the square, and solving,

$$x = \frac{2}{9} \pm \frac{1}{9}\sqrt{-5}.$$

Hence, § 434, $(x - \frac{2}{9} + \frac{1}{9}\sqrt{-5})(x - \frac{2}{9} - \frac{1}{9}\sqrt{-5}) = 0.$

$\frac{1}{81}(9x - 2 + \sqrt{-5})(9x - 2 - \sqrt{-5}) = 0.$

Multiplying by 9, $\frac{1}{9}(9x - 2 + \sqrt{-5})(9x - 2 - \sqrt{-5}) = 9x^2 - 4x + 1 = 0.$

20. Let $24x - 16x^2 - 3 = 0.$

Dividing by -16, transposing, completing the square, and solving,

$$x = \frac{3}{4} \pm \frac{1}{4}\sqrt{6}.$$

Hence, § 434, $(x - \frac{3}{4} + \frac{1}{4}\sqrt{6})(x - \frac{3}{4} - \frac{1}{4}\sqrt{6}) = 0.$

$\therefore \frac{1}{16}(4x - 3 + \sqrt{6})(4x - 3 - \sqrt{6}) = 0.$

Multiplying by -16, $-(4x - 3 + \sqrt{6})(4x - 3 - \sqrt{6}) = 24x - 16x^2 - 3 = 0.$

21. Let $9a^2 - 12a + 5 = 0.$

Dividing by 9, transposing, completing the square, and solving,

$$a = \frac{2}{3} \pm \frac{1}{3}\sqrt{-1}.$$

Hence, § 434,

$(a - \frac{2}{3} + \frac{1}{3}\sqrt{-1})(a - \frac{2}{3} - \frac{1}{3}\sqrt{-1}) = 0.$

$\therefore \frac{1}{9}(3a - 2 + \sqrt{-1})(3a - 2 - \sqrt{-1}) = 0.$

Multiplying by 9,

$(3a - 2 + \sqrt{-1})(3a - 2 - \sqrt{-1}) = 9a^2 - 12a + 5 = 0.$

22. Let $16v(1 - v) - 9 = 0.$

$\therefore 16v - 16v^2 - 9 = 0.$

Dividing by -16, transposing, completing the square, and solving.

$$v = \frac{1}{2} \pm \frac{1}{4}\sqrt{-5}.$$

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Hence, § 434,

$$(v - \tfrac{1}{2} + \tfrac{1}{4}\sqrt{-5})(v - \tfrac{1}{2} - \tfrac{1}{4}\sqrt{-5}) = 0.$$

$$\therefore \tfrac{1}{16}(4v - 2 + \sqrt{-5})(4v - 2 - \sqrt{-5}) = 0.$$

Multiplying by -16,

$$-(4v - 2 + \sqrt{-5})(4v - 2 - \sqrt{-5}) = 16v(1 - v) - 9 = 0.$$

23. Let $$16(3 + n) + 3n^2 = 0.$$

Dividing by 3, transposing, completing the square, and solving,

$$n = -\tfrac{8}{3} \pm \tfrac{4}{3}\sqrt{-5}.$$

Hence, § 434,

$$(n + \tfrac{8}{3} + \tfrac{4}{3}\sqrt{-5})(n + \tfrac{8}{3} - \tfrac{4}{3}\sqrt{-5}) = 0.$$

$$\therefore \tfrac{1}{9}(3n + 8 + 4\sqrt{-5})(3n + 8 - 4\sqrt{-5}) = 0.$$

Multiplying by 3,

$$\tfrac{1}{3}(3n + 8 + 4\sqrt{-5})(3n + 8 - 4\sqrt{-5}) = 16(3 + n) + 3n^2 = 0.$$

24. Let $$100x^2 + 70xy - 119y^2 = 0.$$

Completing the square and solving, $x = -\tfrac{7}{20}y \pm \tfrac{5}{20}y\sqrt{21}.$

Hence, § 434,

$$(x + \tfrac{7}{20}y + \tfrac{5}{20}y\sqrt{21})(x + \tfrac{7}{20}y - \tfrac{5}{20}y\sqrt{21}) = 0.$$

$$\therefore \tfrac{1}{400}(20x + 7y + 5y\sqrt{21})(20x + 7y - 5y\sqrt{21}) = 0.$$

Multiplying by 100,

$$\tfrac{1}{4}(20x + 7y + 5y\sqrt{21})(20x + 7y - 5y\sqrt{21}) = 100x^2 + 70xy - 119y^2 = 0.$$

25. Let $$4b^2 - 48b + 143 = 0.$$

Completing the square and solving, $b = \tfrac{11}{2}$ or $\tfrac{13}{2}$.

Hence, § 434, $$(b - \tfrac{11}{2})(b - \tfrac{13}{2}) = 0.$$

$$\therefore (2b - 11)(2b - 13) = 4b^2 - 48b + 143 = 0.$$

26. Let $$9r^2 - 12r + 437 = 0.$$

Completing the square and solving, $r = \tfrac{2}{3} \pm \tfrac{1}{3}\sqrt{-433}.$

Hence, § 434,

$$(r - \tfrac{2}{3} + \tfrac{1}{3}\sqrt{-433})(r - \tfrac{2}{3} - \tfrac{1}{3}\sqrt{-433}) = 0.$$

$$\therefore \tfrac{1}{9}(3r - 2 + \sqrt{-433})(3r - 2 - \sqrt{-433}) = 0.$$

Multiplying by 9,

$$(3r - 2 + \sqrt{-433})(3r - 2 - \sqrt{-433}) = 9r^2 - 12r + 437 = 0.$$

27. Let $$4a^2 + 12a - 135 = 0.$$

Completing the square and solving, $a = -\tfrac{15}{2}$ or $\tfrac{9}{2}$.

Hence, § 434, $$(a + \tfrac{15}{2})(a - \tfrac{9}{2}) = 0.$$

$$\therefore (2a + 15)(2a - 9) = 4a^2 + 12a - 135 = 0.$$

28. Let $$16p(p + 1) - 1517 = 0.$$

Completing the square and solving, $p = -\tfrac{41}{4}$ or $\tfrac{37}{4}$.

Hence, § 434, $$(p + \tfrac{41}{4})(p - \tfrac{37}{4}) = 0.$$

$$\therefore (4p + 41)(4p - 37) = 16p(p + 1) - 1517 = 0.$$

29. Let $$25e^2 - 2h(5e - 2h) = 0.$$

$$\therefore 25e^2 - 10he + 4h^2 = 0.$$

Completing the square and solving, $e = \tfrac{1}{5}h \pm \tfrac{1}{5}h\sqrt{-3}.$

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Hence, § 434,

$$(e-\tfrac{1}{5}h+\tfrac{1}{5}h\sqrt{-3})(e-\tfrac{1}{5}h-\tfrac{1}{5}h\sqrt{-3})=0.$$

$$\therefore \tfrac{1}{25}(5e-h+h\sqrt{-3})(5e-h-h\sqrt{-3})=0.$$

Multiplying by 25,

$$(5e-h+h\sqrt{-3})(5e-h-h\sqrt{-3})=25e^2-2h(5e-2h)=0.$$

30. $$3h(4k-3h)-7k^2=-(9h^2-12hk+7k^2).$$

Let $$9h^2-12hk+7k^2=0.$$

Completing the square and solving, $h=\tfrac{2}{3}k\pm\tfrac{1}{3}k\sqrt{-3}.$

Hence, § 434,

$$(h-\tfrac{2}{3}k+\tfrac{1}{3}k\sqrt{-3})(h-\tfrac{2}{3}k-\tfrac{1}{3}k\sqrt{-3})=0.$$

$$\therefore \tfrac{1}{9}(3h-2k+k\sqrt{-3})(3h-2k-k\sqrt{-3})=0.$$

Multiplying by -9,

$$-(3h-2k+k\sqrt{-3})(3h-2k-k\sqrt{-3})=3h(4k-3h)-7k^2=0$$

32. Let $$x^4+6x^3+11x^2+6x-8=0.$$

Completing the square, $(x^4+6x^3+9x^2)+2(x^2+3x)+1=9.$

Extracting the square root, $x^2+3x+1=3$ or $-3.$

$$\therefore x^4+6x^3+11x^2+6x-8=(x^2+3x+1-3)(x^2+3x+1+3)$$
$$=(x^2+3x-2)(x^2+3x+4).$$

33. Let $$x^6+2x^5+5x^4+8x^3+8x^2+8x+3=0.$$

Completing the square,

$$(x^6+2x^5+5x^4+4x^3+4x^2)+4(x^3+x^2+2x)+4=1.$$

Extracting the square root, $x^3+x^2+2x+2=1$ or $-1.$

$$\therefore x^6+2x^5+5x^4+8x^3+8x^2+8x+3$$
$$=(x^3+x^2+2x+2-1)(x^3+x^2+2x+2+1)$$
$$=(x^3+x^2+2x+1)(x^3+x^2+2x+3).$$

34. Let $x^6-4x^5+6x^4+6x^3-19x^2+10x+9=0.$

Completing the square,

$$(x^6-4x^5+6x^4-4x^3+x^2)+10(x^3-2x^2+x)+25=16.$$

Extracting the square root, $x^3-2x^2+x+5=4$ or $-4.$

$$\therefore x^6-4x^5+6x^4+6x^3-19x^2+10x+9$$
$$=(x^3-2x^2+x+5-4)(x^3-2x^2+x+5+4)$$
$$=(x^3-2x^2+x+1)(x^3-2x^2+x+9).$$

35. Let $4x^6+12x^5+25x^4+40x^3+40x^2+32x+15=0.$

Completing the square,

$$(4x^6+12x^5+25x^4+24x^3+16x^2)+8(2x^3+3x^2+4x)+16=1.$$

Extracting the square root, $2x^3+3x^2+4x+4=1$ or $-1.$

$$\therefore 4x^6+12x^5+25x^4+40x^3+40x^2+32x+15$$
$$=(2x^3+3x^2+4x+4-1)(2x^3+3x^2+4x+4+1)$$
$$=(2x^3+3x^2+4x+3)(2x^3+3x^2+4x+5).$$

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37. $$x^4+16=x^4+8x^2+16-8x^2=(x^2+4)^2-(2\sqrt{2}\,x)^2$$
$$=(x^2+2x\sqrt{2}+4)(x^2-2x\sqrt{2}+4).$$

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38. $$a^4+b^4=a^4+2a^2b^2+b^4-2a^2b^2=(a^2+b^2)^2-(\sqrt{2}\,ab)^2$$
$$=(a^2+ab\sqrt{2}+b^2)(a^2-ab\sqrt{2}+b^2).$$

39. $$x^4+2a^2x^2+4a^4=x^4+4a^2x^2+4a^4-2a^2x^2=(x^2+2a^2)^2-(ax\sqrt{2})^2$$
$$=(x^2+ax\sqrt{2}+2a^2)(x^2-ax\sqrt{2}+2a^2).$$

40. $$v^4-4n^2v^2-2n^4=v^4-4n^2v^2+4n^4-6n^4=(v^2-2n^2)^2-(n^2\sqrt{6})^2$$
$$=(v^2-2n^2+n^2\sqrt{6})(v^2-2n^2-n^2\sqrt{6}).$$

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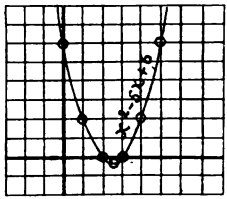
3. Since x^2 is always positive, whether x is positive or negative, for very large numerical values of x, x^2-5x+6 is very large numerically, and positive; that is, when $x=\pm\infty$, $x^2-5x+6=+\infty$.

Draw the graph of x^2-5x+6, plotting values of x as abscissas and values of x^2-5x+6 as ordinates (§ 418).



Referring to the graph, and observing the form of the function itself, we see that:

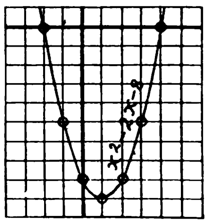
As x increases continuously from $-\infty$ to $+2\frac{1}{2}$, x^2-5x+6 decreases continuously from $+\infty$ to its minimum value, $-\frac{1}{4}$, crossing the x-axis at $x=2$, which is therefore a root of the equation $x^2-5x+6=0$.

As x increases continuously from $+2\frac{1}{2}$ to $+\infty$, x^2-5x+6 increases continuously from its minimum value, $-\frac{1}{4}$, to $+\infty$, crossing the x-axis at $x=3$, which is therefore the other root of the equation $x^2-5x+6=0$.

The function is positive for all values of x outside the limits $x=2$ and $x=3$, and negative for all values of x within these limits.

4. Since x^2 is always positive, whether x is positive or negative, for very large numerical values of x, x^2-2x-8 is very large numerically, and positive; that is, when $x=\pm\infty$, $x^2-2x-8=+\infty$.

Draw the graph of x^2-2x-8, plotting values of x as abscissas and values of x^2-2x-8 as ordinates (§ 418).



Referring to the graph, and observing the form of the function itself, we see that:

As x increases continuously from $-\infty$ to $+1$, x^2-2x-8 decreases continuously from $+\infty$ to its minimum value, -9, crossing the x-axis at $x=-2$, which is therefore a root of the equation $x^2-2x-8=0$.

As x increases continuously from $+1$ to $+\infty$, x^2-2x-8 increases continuously from its minimum value, -9, to $+\infty$, crossing the x-axis at $x=4$, which is therefore the other root of the equation $x^2-2x-8=0$.

The function is positive for all values of x outside the limits $x=-2$ and $x=4$, and negative for all values of x within these limits.

5. Since x^2 is always positive, whether x is positive or negative, for very large numerical values of x, $x^2+2x-15$ is very large numerically, and positive; that is, when $x=\pm\infty$, $x^2+2x-15=+\infty$.

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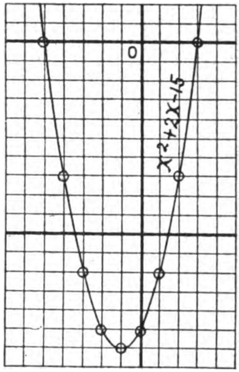
Draw the graph of $x^2 + 2x - 15$, plotting values of x as abscissas and values of $x^2 + 2x - 15$ as ordinates (§ 418).



Referring to the graph, and observing the form of the function itself, we see that:

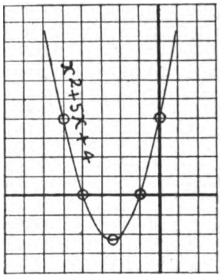
As x increases continuously from $-\infty$ to -1, $x^2 + 2x - 15$ decreases continuously from $+\infty$ to its minimum value, -16, crossing the x-axis at $x = -5$, which is therefore a root of the equation $x^2 + 2x - 15 = 0$.

As x increases continuously from -1 to $+\infty$, $x^2 + 2x - 15$ increases continuously from its minimum value, -16, to $+\infty$, crossing the x-axis at $x = 3$, which is therefore the other root of the equation $x^2 + 2x - 15 = 0$.

The function is positive for all values of x outside the limits $x = -5$ and $x = 3$, and negative for all values of x within these limits.

6. Since x^2 is always positive, whether x is positive or negative, for very large numerical values of x, $x^2 + 5x + 4$ is very large numerically, and positive; that is, when $x = \pm\infty$, $x^2 + 5x + 4 = +\infty$.

Draw the graph of $x^2 + 5x + 4$, plotting values of x as abscissas and values of $x^2 + 5x + 4$ as ordinates (§ 418).



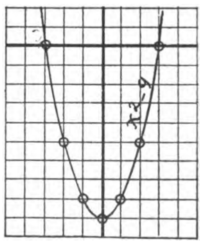
Referring to the graph, and observing the form of the function itself, we see that:

As x increases continuously from $-\infty$ to $-2\frac{1}{2}$, $x^2 + 5x + 4$ decreases continuously from $+\infty$ to its minimum value, $-2\frac{1}{4}$, crossing the x-axis at $x = -4$, which is therefore a root of the equation $x^2 + 5x + 4 = 0$.

As x increases continuously from $-2\frac{1}{2}$ to $+\infty$, $x^2 + 5x + 4$ increases continuously from its minimum value, $-2\frac{1}{4}$, to $+\infty$, crossing the x-axis at $x = -1$, which is therefore the other root of the equation $x^2 + 5x + 4 = 0$.

The function is positive for all values of x outside of the limits $x = -4$ and $x = -1$, and negative for all values of x within these limits.

7. Since x^2 is always positive, whether x is positive or negative, for very large numerical values of x, $x^2 - 9$ is very large numerically, and positive; that is, when $x = \pm\infty$, $x^2 - 9 = +\infty$.



Draw the graph of $x^2 - 9$, plotting values of x as abscissas and values of $x^2 - 9$ as ordinates (§ 418).

Referring to the graph and observing the form of the function itself, we see that:

As x increases continuously from $-\infty$ to 0, $x^2 - 9$ decreases continuously from $+\infty$ to its minimum value, -9, crossing the x-axis at $x = -3$, which is therefore a root of the equation $x^2 - 9 = 0$.

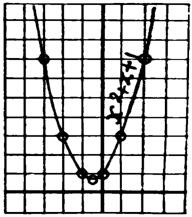
As x increases continuously from 0 to $+\infty$, $x^2 - 9$ increases continuously from its minimum

value, -9, to $+\infty$, crossing the x-axis at $x=3$, which is therefore the other root of the equation $x^2-9=0$.

The function is positive for all values of x outside the limits $x=-3$ and $x=3$, and negative for all values of x within these limits.

8. Since x^2 is always positive, whether x is positive or negative, for very large numerical values of x, x^2+x+1 is very large numerically, and positive; that is, when $x=\pm\infty$, $x^2+x+1=+\infty$.

Draw the graph of x^2+x+1, plotting values of x as abscissas and values of x^2+x+1 as ordinates (§ 418).



Referring to the graph and observing the form of the function itself, we see that:

As x increases continuously from $-\infty$ to $-\frac{1}{2}$, x^2+x+1 decreases continuously from $+\infty$ to its minimum value, $\frac{3}{4}$.

As x increases continuously from $-\frac{1}{2}$ to $+\infty$, x^2+x+1 increases continuously from its minimum value, $\frac{3}{4}$, to $+\infty$.

Since the graph has no point in common with the x-axis, the roots of the equation $x^2+x+1=0$ are imaginary; and since the graph lies entirely *above* the x-axis, the function is *positive* for all values of x.

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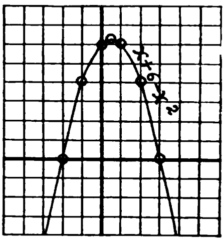
10. Referring to the graph of $3+2x-x^2$ and observing the form of the function itself, the discussion may be completed as follows:

As x increases from $-\infty$ to $+1$, $3+2x-x^2$ increases continuously from $-\infty$ to its maximum value, 4, crossing the x-axis at $x=-1$, which is therefore a root of the equation $3+2x-x^2=0$.

As x increases continuously from $+1$ to $+\infty$, $3+2x-x^2$ decreases continuously from its maximum value, 4, to $-\infty$, crossing the x-axis at $x=3$, which is therefore the other root of the equation $3+2x-x^2=0$.

The function is negative for all values of x outside the limits $x=-1$ and $x=3$, and positive for all values of x within these limits.

11. Since $-x^2$ is always negative, whether x is positive or negative, for very large numerical values of x, $x+6-x^2$ is very large numerically, and negative; that is, when $x=\pm\infty$, $x+6-x^2=-\infty$.



Since $x+6-x^2=-(x^2-x-6)$, and x^2-x-6 has a minimum value at $x=\frac{1}{2}$, the given function has a maximum value at $x=\frac{1}{2}$; that is, when $x=\frac{1}{2}$, $x+6-x^2=6\frac{1}{4}$, the maximum value.

Draw the graph of $x+6-x^2$, plotting values of x as abscissas and values of $x+6-x^2$ as ordinates (§ 418).

Referring to the graph and observing the form of the function itself, we see that:

As x increases continuously from $-\infty$ to $+\frac{1}{2}$, $x+6-x^2$ increases continuously from $-\infty$ to its maximum value, $6\frac{1}{4}$, crossing the x-axis at $x=-2$, which is therefore a root of the equation $x+6-x^2=0$.

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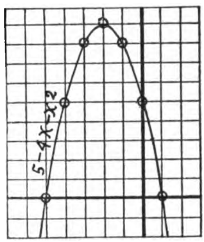
As x increases continuously from $+\frac{1}{2}$ to $+\infty$, $x+6-x^2$ decreases continuously from its maximum value, $6\frac{1}{4}$, to $-\infty$, crossing the x-axis at $x=3$, which is therefore the other root of the equation $x+6-x^2=0$.

The function is negative for all values of x outside the limits $x=-2$ and $x=3$, and positive for all values of x within these limits.

12. Since $-x^2$ is always negative, whether x is positive or negative, for very large numerical values of x, $5-4x-x^2$ is very large numerically, and negative; that is, when $x=\pm\infty$, $5-4x-x^2=-\infty$.



Since $5-4x-x^2=-(x^2+4x-5)$, and x^2+4x-5 has a minimum value at $x=-2$, the given function has a maximum value at $x=-2$; that is, when $x=-2$, $5-4x-x^2=9$, the maximum value.

Draw the graph of $5-4x-x^2$, plotting values of x as abscissas, and values of $5-4x-x^2$ as ordinates (§ 418).

Referring to the graph and observing the form of the function itself, we see that:

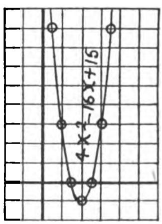
As x increases continuously from $-\infty$ to -2, $5-4x-x^2$ increases continuously from $-\infty$ to its maximum value, 9, crossing the x-axis at $x=-5$, which is therefore a root of the equation $5-4x-x^2=0$.

As x increases continuously from -2 to $+\infty$, $5-4x-x^2$ decreases continuously from its maximum value, 9, to $-\infty$, crossing the x-axis at $x=1$, which is therefore the other root of the equation $5-4x-x^2=0$.

The function is negative for all values of x outside the limits $x=-5$ and $x=1$, and positive for all values of x within these limits.

13. Since $4x^2$ is always positive, whether x is positive or negative, for very large numerical values of x, $4x^2-16x+15$ is very large numerically, and positive; that is, when $x=\pm\infty$, $4x^2-16x+15=+\infty$.

Since $4x^2-16x+15=4(x^2-4x+\frac{15}{4})$, and $x^2-4x+\frac{15}{4}$ has a minimum value at $x=2$, the given function has a minimum value at $x=2$; that is, when $x=2$, $4x^2-16x+15=-1$, the minimum value.



Draw the graph of $4x^2-16x+15$, plotting values of x as abscissas and values of $4x^2-16x+15$ as ordinates (§ 418).

Referring to the graph and observing the form of the function itself, we see that:

As x increases continuously from $-\infty$ to $+2$, $4x^2-16x+15$ decreases continuously from $+\infty$ to its minimum value, -1, crossing the x-axis at $x=1\frac{1}{2}$, which is a root of the equation $4x^2-16x+15=0$.

As x increases continuously from $+2$ to $+\infty$, $4x^2-16x+15$ increases continuously from its minimum value, -1, to $+\infty$, crossing the x-axis at $x=2\frac{1}{2}$, which is therefore the other root of the equation $4x^2-16x+15=0$.

The function is positive for all values of x outside the limits $x=1\frac{1}{2}$ and and $x=2\frac{1}{2}$, and negative for all values of x within these limits.

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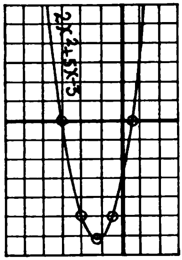
14. Since $2x^2$ is always positive, whether x is positive or negative, for very large numerical values of x, $2x^2 + 5x - 3$ is very large numerically, and positive; that is, when $x = \pm\infty$, $2x^2 + 5x - 3 = +\infty$.

Since $2x^2 + 5x - 3 = 2(x^2 + \frac{5}{2}x - \frac{3}{2})$, and $x^2 + \frac{5}{2}x - \frac{3}{2}$ has a minimum value at $x = -1\frac{1}{4}$, the given function has a minimum value at $x = -1\frac{1}{4}$; that is, when $x = -1\frac{1}{4}$, $2x^2 + 5x - 3 = -6\frac{1}{8}$, the minimum value.



Draw the graph of $2x^2 + 5x - 3$, plotting values of x as abscissas and values of $2x^2 + 5x - 3$ as ordinates (§ 418).

Referring to the graph and observing the form of the function itself, we see that:

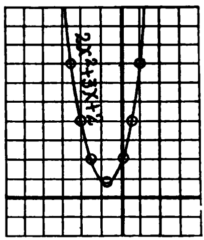
As x increases continuously from $-\infty$ to $-1\frac{1}{4}$, $2x^2 + 5x - 3$ decreases continuously from $+\infty$ to its minimum value, $-6\frac{1}{8}$, crossing the x-axis at $x = -3$, which is therefore a root of the equation $2x^2 + 5x - 3 = 0$.

As x increases continuously from $-1\frac{1}{4}$ to $+\infty$, $2x^2 + 5x - 3$ increases continuously from its minimum value, $-6\frac{1}{8}$, to $+\infty$, crossing the x-axis at $x = \frac{1}{2}$, which is therefore the other root of the equation $2x^2 + 5x - 3 = 0$.

The function is positive for all values of x outside the limits $x = -3$ and $x = \frac{1}{2}$, and negative for all values of x within these limits.

15. Since $2x^2$ is always positive, whether x is positive or negative, for very large numerical values of x, $2x^2 + 3x + 2$ is very large numerically, and positive; that is, when $x = \pm\infty$, $2x^2 + 3x + 2 = +\infty$.

Since $2x^2 + 3x + 2 = 2(x^2 + \frac{3}{2}x + 1)$, and $x^2 + \frac{3}{2}x + 1$ has a minimum value at $x = -\frac{3}{4}$, the given function has a minimum value at $x = -\frac{3}{4}$; that is, when $x = -\frac{3}{4}$, $2x^2 + 3x + 2 = \frac{7}{8}$, the minimum value.



Draw the graph of $2x^2 + 3x + 2$, plotting values of x as abscissas and values of $2x^2 + 3x + 2$ as ordinates (§ 418).

Referring to the graph and observing the form of the function itself, we see that:

As x increases continuously from $-\infty$ to $-\frac{3}{4}$, $2x^2 + 3x + 2$ decreases continuously from $+\infty$ to its minimum value, $\frac{7}{8}$.

As x increases continuously from $-\frac{3}{4}$ to $+\infty$, $2x^2 + 3x + 2$ increases continuously from its minimum value, $\frac{7}{8}$, to $+\infty$.

Since the graph has no point in common with the x-axis, the roots of $2x^2 + 3x + 2 = 0$ are imaginary; and since the graph lies entirely *above* the x-axis, the function is *positive* for all values of x.

16. Since $-x^2$ is always negative, whether x is positive or negative, for very large numerical values of x, $4x - 6 - x^2$ is very large numerically, and negative; that is, when $x = \pm\infty$, $4x - 6 - x^2 = -\infty$.

Since $4x - 6 - x^2 = -(x^2 - 4x + 6)$, and $x^2 - 4x + 6$ has a minimum value at $x = 2$, the given function has a maximum value at $x = 2$; that is, when $x = 2$, $4x - 6 - x^2 = -2$, the maximum value.

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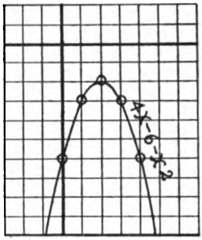

Draw the graph of $4x - 6 - x^2$, plotting values of x as abscissas and values of $4x - 6 - x^2$ as ordinates (§ 418).

Referring to the graph and observing the form of the function itself, we see that:

As x increases continuously from $-\infty$ to 2, $4x - 6 - x^2$ increases continuously from $-\infty$ to its maximum value, -2.

As x increases continuously from 2 to $+\infty$, $4x - 6 - x^2$ decreases continuously from its maximum value, -2, to $-\infty$.

Since the graph has no point in common with the x-axis, the roots of the equation $4x - 6 - x^2 = 0$ are imaginary; and since the graph lies entirely below the x-axis, the function is negative for all values of x.

18. $$x^2 - 3x - 28 = (x - 7)(x + 4).$$

$x^2 - 3x - 28$ is positive when both factors are positive or when both are negative; that is, when x is greater than 7 or less than -4, these values being the roots of the equation $x^2 - 3x - 28 = 0$.

$x^2 - 3x - 28$ is negative when one factor is positive and the other negative; that is, when x is less than 7 or greater than -4.

19. Let $$ax^2 + bx + c = x^2 - 6x + 12 = 0.$$

Since $b^2 - 4ac = 36 - 48 = -12$, both roots of the equation $x^2 - 6x + 12 = 0$ are *imaginary* (§ 429, Prin. 3). This means that the graph of the function $x^2 - 6x + 12$ has no point in common with the x-axis; consequently, it must lie entirely above or entirely below the x-axis.

By § 418, $x^2 - 6x + 12$ has a minimum value at $x = 3$; that is, when $x = 3$, $x^2 - 6x + 12 = 3$, the minimum value; and since the minimum value is positive, the graph lies entirely above the x-axis and *the function $x^2 - 6x + 12$ is positive for all real values of x.*

20. Since $x - x^2 - 1 = 0$ is equivalent to the equation $x^2 - x + 1 = 0$, let $$ax^2 + bx + c = x^2 - x + 1 = 0.$$

Since $b^2 - 4ac = 1 - 4 = -3$, the roots of the equation $x^2 - x + 1 = 0$, or of the equivalent equation $x - x^2 - 1 = 0$, are imaginary (§ 429, Prin. 3). This means that the graph of $x - x^2 - 1$ has no point in common with the x-axis; consequently, it must lie entirely above or entirely below the x-axis.

Since $x - x^2 - 1 = -(x^2 - x + 1)$, and $x^2 - x + 1$ has a minimum value at $x = \frac{1}{2}$, $x - x^2 - 1$ has a maximum value at $x = \frac{1}{2}$; that is, when $x = \frac{1}{2}$, $x - x^2 - 1 = -\frac{3}{4}$, the maximum value; and since the maximum value is negative, the graph lies entirely below the x-axis and the function $x - x^2 - 1$ is negative for all real values of x.

21. If the function $ax^2 + bx + c$ has the same sign for all real values of x, its graph lies entirely above or entirely below the x-axis; that is, it has no point in common with the x-axis; consequently, the roots of the equation $ax^2 + bx + c = 0$ are imaginary; but the condition that the roots of $ax^2 + bx + c = 0$ shall be imaginary is that $b^2 - 4ac$ shall be negative (§ 429, Prin. 3).

Hence, $ax^2 + bx + c$ will have the same sign for all real values of x, if $b^2 - 4ac$ is negative, when a is positive and b and c are either positive or negative.

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GENERAL REVIEW

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1.
$$\begin{array}{rl}
& x\sqrt{y} + y\sqrt{x} + \sqrt{xy} \\
& -x\sqrt{y} - y\sqrt{x} + x^{\frac{1}{2}}y^{\frac{1}{2}} \\
& x\sqrt{y} - y\sqrt{x} - \sqrt{xy} \\
& -2x\sqrt{y} + y\sqrt{x} - 3\sqrt{xy} \\
\hline
\text{Sum} = & -x\sqrt{y} \qquad\qquad -2\sqrt{xy}
\end{array}$$

2. The number is $(a-b)-(b-a+c) = a-b-b+a-c = 2a-2b-c.$

3.
$$\begin{aligned}
& a-\{b-a-[a-b-(2a+b)+(2a-b)-a]-b\} \\
&= a-\{-a-[-b-(2a+b)+(2a-b)]\} \\
&= a-\{-a-[-b-2a-b+2a-b]\} \\
&= a-\{-a-[-3b]\} \\
&= a-\{-a+3b\} \\
&= a+a-3b = 2a-3b.
\end{aligned}$$

4.
$$\begin{array}{l}
x\sqrt{x} + x\sqrt{y} + y\sqrt{x} + y\sqrt{y} \\
\sqrt{x} - \sqrt{y} \\
\hline
x^2 + x\sqrt{xy} + xy + y\sqrt{xy} \\
\quad - x\sqrt{xy} - xy - y\sqrt{xy} - y^2 \\
\hline
x^2 \qquad\qquad\qquad\qquad\qquad - y^2
\end{array}$$

5.
$$\begin{aligned}
\left(2x^{\frac{a}{2b}} - 5y^{\frac{a+b}{2}}\right)\left(2x^{\frac{a}{2b}} + 5y^{\frac{a+b}{2}}\right) &= 4\left(x^{\frac{a}{2b}}\right)^2 - 25\left(y^{\frac{a+b}{2}}\right)^2 \\
&= 4x^{\frac{a}{b}} - 25y^{a+b}.
\end{aligned}$$

6.
$$\begin{aligned}
(x^n - y^n)(x^n + y^n)(x^{2n} + y^{2n}) &= (x^{2n} - y^{2n})(x^{2n} + y^{2n}) \\
&= x^{4n} - y^{4n}.
\end{aligned}$$

8.
$$\begin{array}{l|l}
1+0+0-3+0-20 & 1-2 \\
\cline{2-2}
\underline{1-2} & 1+2+4+5+10 \\
\quad\; 2 & = x^4 + 2x^3 + 4x^2 + 5x + 10. \\
\quad\; \underline{2-4} & \\
\qquad\;\; 4-3 & \\
\qquad\;\; \underline{4-8} & \\
\qquad\qquad 5 & \\
\qquad\qquad \underline{5-10} & \\
\qquad\qquad\quad 10-20 & \\
\qquad\qquad\quad \underline{10-20} &
\end{array}$$

9. In $x^{50} - b^{50}$ substitute $-b$ for x.
Then, $x^{50} - b^{50} = (-b)^{50} - b^{50} = b^{50} - b^{50} = 0.$
Therefore, by the factor theorem, $x^{50} - b^{50}$ is divisible by $x - (-b)$, or $x + b$.

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10. $(a+b)+x \;\big|\; (a+b)^{\frac{1}{3}}+x^{\frac{1}{3}}$

$$\begin{array}{l} (a+b)+x \\ \underline{(a+b)+(a+b)^{\frac{2}{3}}x^{\frac{1}{3}}} \\ \qquad -(a+b)^{\frac{2}{3}}x^{\frac{1}{3}} \\ \qquad \underline{-(a+b)^{\frac{2}{3}}x^{\frac{1}{3}}-(a+b)^{\frac{1}{3}}x^{\frac{2}{3}}} \\ \qquad\qquad (a+b)^{\frac{1}{3}}x^{\frac{2}{3}}+x \\ \qquad\qquad \underline{(a+b)^{\frac{1}{3}}x^{\frac{2}{3}}+x} \end{array} \qquad \begin{array}{|l} (a+b)^{\frac{1}{3}}+x^{\frac{1}{3}} \\ \hline (a+b)^{\frac{2}{3}}-(a+b)^{\frac{1}{3}}x^{\frac{1}{3}}+x^{\frac{2}{3}} \end{array}$$

11. $9x^2-12x+4=(3x)^2-4(3x)+4=(3x-2)(3x-2).$
$9x^2+\ 9x+2=(3x)^2+3(3x)+2=(3x+1)(3x+2).$
If 1 is substituted for x, $x^3-3x+2=1-3+2=0.$
Therefore, $x-1$ is a factor of x^3-3x+2.
By dividing by $x-1$, $x^3-3x+2=(x-1)(x^2+x-2)$
$=(x-1)(x-1)(x+2).$

$$\begin{aligned} a^4+1 &= a^4+2a^2+1-2a^2 \\ &= (a^2+1)^2-(\sqrt{2}\,a)^2 \\ &= (a^2+a\sqrt{2}+1)(a^2-a\sqrt{2}+1) \\ &= (a+\tfrac{1}{2}\sqrt{2}+\tfrac{1}{2}\sqrt{-2})(a+\tfrac{1}{2}\sqrt{2}-\tfrac{1}{2}\sqrt{-2})(a-\tfrac{1}{2}\sqrt{2}+\tfrac{1}{2}\sqrt{-2}) \\ &\qquad (a-\tfrac{1}{2}\sqrt{2}-\tfrac{1}{2}\sqrt{-2}). \end{aligned}$$

12. In $x^n+3ax^{n-1}-4a^n$ substitute a for x.
Then, $x^n+3ax^{n-1}-4a^n=a^n+3a^n-4a^n=0.$
Hence, by the factor theorem, $x^n+3ax^{n-1}-4a^n$ is divisible by $x-a$.

13. $$\begin{aligned} a^{12}-1 &= (a^6+1)(a^6-1) \\ &= (a^2+1)(a^4-a^2+1)(a^3+1)(a^3-1) \\ &= (a^2+1)(a^4-a^2+1)(a+1)(a^2-a+1)(a-1)(a^2+a+1). \end{aligned}$$

14. $$\begin{aligned} &4(ad+bc)^2-(a^2-b^2-c^2+d^2)^2 \\ &= (2ad+2bc+a^2-b^2-c^2+d^2)(2ad+2bc-a^2+b^2+c^2-d^2) \\ &= [a^2+2ad+d^2-(b^2-2bc+c^2)][b^2+2bc+c^2-(a^2-2ad+d^2)] \\ &= [(a+d)^2-(b-c)^2][(b+c)^2-(a-d)^2] \\ &= (a+d+b-c)(a+d-b+c)(b+c+a-d)(b+c-a+d) \\ &= (a+b-c+d)(a-b+c+d)(a+b+c-d)(-a+b+c+d). \end{aligned}$$

16. $$2x^4-7x^3+4x^2+7x-6. \qquad (1)$$
Substituting -1 for x in (1), $2+7+4-7-6=0.$
Substituting 1 for x in (1), $2-7+4+7-6=0.$
Substituting 2 for x in (1), $32-56+16+14-6=0.$
Therefore, by the factor theorem, $x+1$, $x-1$, and $x-2$ are factors of (1).

Then, dividing out each one of these factors in succession, the remaining factor of (1) is found to be $2x-3$;
that is, $2x^4-7x^3+4x^2+7x-6=(x+1)(x-1)(x-2)(2x-3).$

By trial of these factors in each of the other given expressions, it is found that the only one contained in each expression is $2x-3$;
that is, $2x^4+x^3-4x^2+7x-15=(2x-3)(x^3+2x^2+x+5),$
and $2x^4+x^3-x-12=(2x-3)(x^3+2x^2+3x+4).$

Hence, $\qquad$ H. C. F. $=2x-3.$

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19. $\dfrac{a^2-b^2-c^2-2bc}{a^2-b^2+c^2+2ac}=\dfrac{(a+b+c)(a-b-c)}{(a+c+b)(a+c-b)}=\dfrac{a-b-c}{a-b+c}.$

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20. $$\frac{x}{x+1}-\frac{x}{1-x}+\frac{x^2}{x^2-1}=\frac{x}{x+1}+\frac{x}{x-1}+\frac{x^2}{x^2-1}$$
$$=\frac{x(x-1)+x(x+1)+x^2}{x^2-1}=\frac{3x^2}{x^2-1}.$$

21. $$\frac{x+y}{2x-2y}+\frac{x-y}{2x+2y}+\frac{4x^2y^2}{y^4-x^4}=\frac{(x+y)^2+(x-y)^2}{2(x^2-y^2)}-\frac{4x^2y^2}{x^4-y^4}$$
$$=\frac{x^2+y^2}{x^2-y^2}-\frac{4x^2y^2}{x^4-y^4}$$
$$=\frac{x^4+2x^2y^2+y^4-4x^2y^2}{x^4-y^4}$$
$$=\frac{(x^2-y^2)(x^2-y^2)}{(x^2+y^2)(x^2-y^2)}=\frac{x^2-y^2}{x^2+y^2}.$$

22. $$\frac{1}{(a-b)(b-c)}-\frac{1}{(c-b)(c-a)}+\frac{1}{(c-a)(a-b)}$$
$$=\frac{1}{(a-b)(b-c)}+\frac{1}{(b-c)(c-a)}+\frac{1}{(c-a)(a-b)}$$
$$=\frac{(c-a)+(a-b)+(b-c)}{(a-b)(b-c)(c-a)}=\frac{0}{(a-b)(b-c)(c-a)}=0.$$

23. $$\left[\frac{x}{1+\frac{1}{x}}+1-\frac{1}{x+1}\right]\div\left[\frac{x}{1-\frac{1}{x}}-x-\frac{1}{x-1}\right]$$
$$=\left(\frac{x^2}{x+1}+1-\frac{1}{x+1}\right)\div\left(\frac{x^2}{x-1}-x-\frac{1}{x-1}\right)$$
$$=\left(\frac{x^2-1}{x+1}+1\right)\div\left(\frac{x^2-1}{x-1}-x\right)=(x-1+1)\div(x+1-x)=x.$$

24. $$\frac{1}{x-\dfrac{1}{x+\frac{1}{x}}}-\frac{1}{x+\dfrac{1}{x-\frac{1}{x}}}=\frac{1}{x-\dfrac{x}{x^2+1}}-\frac{1}{x+\dfrac{x}{x^2-1}}$$
$$=\frac{x^2+1}{(x^3+x)-x}-\frac{x^2-1}{(x^3-x)+x}$$
$$=\frac{(x^2+1)-(x^2-1)}{x^3}=\frac{2}{x^3}.$$

26. $(2a+3b)^4$
$$=(2a)^4+4(2a)^3(3b)+6(2a)^2(3b)^2+4(2a)(3b)^3+(3b)^4$$
$$=16a^4+96a^3b+216a^2b^2+216ab^3+81b^4.$$

$(\sqrt{x}+\sqrt[3]{y})^6$
$$=(\sqrt{x})^6+6(\sqrt{x})^5(\sqrt[3]{y})+15(\sqrt{x})^4(\sqrt[3]{y})^2+20(\sqrt{x})^3(\sqrt[3]{y})^3$$
$$+15(\sqrt{x})^2(\sqrt[3]{y})^4+6(\sqrt{x})(\sqrt[3]{y})^5+(\sqrt[3]{y})^6$$
$$=x^3+6x^2\sqrt{x}\sqrt[3]{y}+15x^2\sqrt[3]{y^2}+20xy\sqrt{x}+15xy\sqrt[3]{y}+6y\sqrt{x}\sqrt[3]{y^2}+y^2.$$

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$$(-1-\sqrt{-3})^3$$
$$=(-1)^3-3(-1)^2\sqrt{-3}+3(-1)(\sqrt{-3})^2-(\sqrt{-3})^3$$
$$=-1-3\sqrt{-3}+9+3\sqrt{-3}=8.$$

27.

$$\begin{array}{r|l}
\multicolumn{2}{l}{4'82'68'09 \ \underline{|\,2197}} \\
4 & \\
\hline
 & 82 \\
41 & 41 \\
\hline
 & 41\ 68 \\
429 & 38\ 61 \\
\hline
 & 3\ 07\ 09 \\
4387 & 3\ 07\ 09 \\
\hline
\end{array}
\qquad\qquad
\begin{array}{r|l}
\multicolumn{2}{l}{2'197 \ \underline{|\,13}} \\
1 & \\
\hline
300 & 1\ 197 \\
90 & \\
9 & \\
\hline
399 & 1\ 197 \\
\hline
\end{array}$$

Therefore, the sixth root of 4826809 is 13.

28. $\sqrt{\tfrac{2}{3}}=\sqrt{\tfrac{6}{9}}=\sqrt{\tfrac{1}{9}}\times\sqrt{6}=\tfrac{1}{3}\sqrt{6}.$

29. $\sqrt[4]{25\,a^4}=a\sqrt[4]{25}=a\sqrt[4]{5^2}=a(5)^{\frac{2}{4}}=a(5)^{\frac{1}{2}}=a\sqrt{5}.$

30. $\dfrac{1}{\sqrt{2}}=\dfrac{1\times\sqrt{2}}{\sqrt{2}\times\sqrt{2}}=\tfrac{1}{2}\sqrt{2}=\tfrac{1}{2}$ of $1.414+=.707+$.

31. $(2+\sqrt{8})(1-\sqrt{2})=(2+2\sqrt{2})(1-\sqrt{2})=2(1+\sqrt{2})(1-\sqrt{2})$
$$=2(1-2)=-2.$$
$(2+\sqrt{-8})(1-\sqrt{-2})=(2+2\sqrt{-2})(1-\sqrt{-2})$
$$=2(1+\sqrt{-2})(1-\sqrt{-2})=2(1+2)=6.$$

32. $\dfrac{\sqrt{3}+3\sqrt{2}}{\sqrt{6}+2}=\dfrac{(\sqrt{3}+3\sqrt{2})(\sqrt{6}-2)}{(\sqrt{6}+2)(\sqrt{6}-2)}=\dfrac{3\sqrt{2}-2\sqrt{3}+6\sqrt{3}-6\sqrt{2}}{6-4}$
$$=2\sqrt{3}-\tfrac{3}{2}\sqrt{2}.$$

33. Since $(ax)^m\div(ax)^m=(ax)^{m-m}$, or $(ax)^0$, and since $(ax)^m\div(ax)^m=\dfrac{(ax)^m}{(ax)^m}=1$, by Ax. 1, $(ax)^0=1$.

34. Prove that $ax^{-5}=\dfrac{a}{x^5}$.

$$ax^{-5}=\frac{ax^{-5}\cdot x^5}{x^5}=\frac{ax^{-5+5}}{x^5}=\frac{ax^0}{x^5}=\frac{a}{x^5}.$$

35. Prove that $x^{\frac{2}{3}}=\sqrt[3]{x^2}$; also that $x^{\frac{2}{3}}=(\sqrt[3]{x})^2$.

$$x^{\frac{2}{3}}\cdot x^{\frac{2}{3}}\cdot x^{\frac{2}{3}}=x^{\frac{2}{3}+\frac{2}{3}+\frac{2}{3}}=x^2;\ \therefore\ x^{\frac{2}{3}}=\sqrt[3]{x^2}.$$

$x^{\frac{1}{3}}\cdot x^{\frac{1}{3}}=x^{\frac{1}{3}+\frac{1}{3}}=x^{\frac{2}{3}}$; $\therefore\ x^{\frac{2}{3}}=(x^{\frac{1}{3}})^2=(\sqrt[3]{x})^2$ by the first part of the proof.

36. $125^{\frac{2}{3}}=(\sqrt[3]{125})^2=5^2=25.$

$$\left(\frac{x^5}{32}\right)^{-\frac{3}{5}}=\left[\left(\frac{x}{2}\right)^5\right]^{-\frac{3}{5}}=\left(\frac{x}{2}\right)^{-3}=\left[\left(\frac{x}{2}\right)^3\right]^{-1}=\left(\frac{x^3}{8}\right)^{-1}=\frac{8}{x^3}.$$

Substituting .5 for x, $\dfrac{8}{x^3}=\dfrac{8}{.125}=64.\quad\therefore\ \left(\dfrac{x^5}{32}\right)^{-\frac{3}{5}}=64.$

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37.
$$\frac{1}{a-b}+\frac{a-b}{x}=\frac{1}{a+b}+\frac{a+b}{x}.$$

Transposing,
$$\frac{1}{a-b}-\frac{1}{a+b}=\frac{a+b}{x}-\frac{a-b}{x}.$$

Uniting terms,
$$\frac{2b}{a^2-b^2}=\frac{2b}{x}.$$
$$\therefore x=a^2-b^2.$$

38.
$$\frac{x-\frac{1}{a}}{c}+\frac{x-\frac{1}{b}}{a}+\frac{x-\frac{1}{c}}{b}=0.$$

Clearing of fractions,
$$abx-b+bcx-c+cax-a=0.$$
$$\therefore x=\frac{a+b+c}{ab+bc+ca}.$$

39.
$$mx^2-nx=mn.$$

Completing the square,
$$4m^2x^2-4mnx+n^2=4m^2n+n^2.$$

Extracting the square root, $2mx-n=\pm\sqrt{4m^2n+n^2}.$
$$\therefore x=\frac{1}{2m}(n\pm\sqrt{4m^2n+n^2}).$$

40.
$$x^4+\frac{1}{2}=\frac{3x^2}{2}.$$

Clearing of fractions and transposing, $2x^4-3x^2+1=0.$

Factoring, $(x^2-1)(2x^2-1)=0.$

Equating each factor to zero and solving, $x=\pm 1$ or $\pm\frac{1}{2}\sqrt{2}.$

41.
$$x^6+8=9x^3.$$

Transposing, $x^6-9x^3+8=0.$

Factoring, $(x-1)(x^2+x+1)(x-2)(x^2+2x+4)=0.$

Equating each factor to zero and solving,
$$x=1 \text{ or } \tfrac{1}{2}(-1\pm\sqrt{-3}) \text{ or } 2 \text{ or } -1\pm\sqrt{-3}.$$

42.
$$\sqrt{x-9}=\sqrt{x}-1.$$

Squaring, $x-9=x-2\sqrt{x}+1.$

Canceling, etc., $5=\sqrt{x}.$

Squaring, $x=25.$

43.
$$x^2+\sqrt{x^2+16}=14.$$

Adding 16 to each member and transposing,
$$(x^2+16)+\sqrt{x^2+16}-30=0.$$

Factoring, $(\sqrt{x^2+16}-5)(\sqrt{x^2+16}+6)=0.$
$$\therefore \sqrt{x^2+16}=5 \text{ or } -6.$$

Squaring, $x^2+16=25$ or $36.$
$$x^2=9 \text{ or } 20.$$
$$\therefore x=\pm 3 \text{ or } \pm 2\sqrt{5}.$$

$\pm 2\sqrt{5}$ does not verify and is rejected.

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44. $$\left(\frac{4}{x}+x\right)^2-\left(\frac{4}{x}+x\right)=20$$

Transposing and factoring,

$$\left(\frac{4}{x}+x-5\right)\left(\frac{4}{x}+x+4\right)=0.$$

$$\therefore \frac{4}{x}+x=5 \text{ or } -4.$$

Clearing of fractions and transposing, we have the equations

$$x^2-5x+4=0,$$

whence, $$x=1 \text{ or } 4,$$

and $$x^2+4x+4=0,$$

whence, $$x=-2 \text{ or } -2.$$

Hence, the roots are 1, 4, -2, -2.

45. $$(1+x)^5+(1-x)^5=242.$$

Expanding, uniting terms, etc., $x^4+2x^2-24=0.$

Factoring, $(x^2-4)(x^2+6)=0.$

Equating each factor to zero and solving, $x=\pm 2$ or $\pm\sqrt{-6}$.

46. $$x+x^2+(1+x+x^2)^2=55.$$

Adding 1 to each member and transposing,

$$(1+x+x^2)^2+(1+x+x^2)-56=0.$$

Factoring, $(1+x+x^2-7)(1+x+x^2+8)=0.$

Equating each factor to zero and solving,

$$x=2 \text{ or } -3 \text{ or } \tfrac{1}{2}(-1\pm\sqrt{-35}).$$

47. $$\frac{1+x}{1+x+\sqrt{1+x^2}}=a-\frac{1+x}{1-x+\sqrt{1+x^2}}.$$

Rationalizing, $$\frac{(1+x-\sqrt{1+x^2})(1+x)}{2x}=a-\frac{(1-x-\sqrt{1+x^2})(1+x)}{-2x},$$

or $$\frac{(1+x-\sqrt{1+x^2})(1+x)}{2x}=a+\frac{(1-x-\sqrt{1+x^2})(1+x)}{2x}.$$

Canceling, etc., $$\frac{2x(1+x)}{2x}=a,$$

whence, $$1+x=a.$$

$$\therefore x=a-1.$$

48. $$\begin{cases}\dfrac{1}{x}+\dfrac{1}{y}=10, & (1)\\[2mm] \dfrac{3}{x}+\dfrac{2}{y}=10. & (2)\end{cases}$$

Subtracting (1) $\times$ 2 from (2), $$\frac{1}{x}=-10.$$

$$\therefore x=-\tfrac{1}{10}.$$

Subtracting (2) from (1) $\times$ 3, $$\frac{1}{y}=20.$$

$$\therefore y=\tfrac{1}{20}.$$

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49.
$$\begin{cases} 2x+3y+z=9, & (1)\\ x+2y+3z=13, & (2)\\ 3x+y+2z=11. & (3)\end{cases}$$

Adding (1), (2), and (3), $6x+6y+6z=33.$ (4)

Dividing (4) by 3, $2x+2y+2z=11.$ (5)

Subtracting (5) from (3), $x-y=0,$

whence, $x=y.$ (6)

Subtracting (5) from (1), $y-z=-2,$

whence, $z=y+2$ or $x+2.$ (7)

Substituting (6) and (7) in (2), $x+2x+3x+6=13.$

$\therefore x=\frac{7}{6},$

whence, by (6), $y=\frac{7}{6},$

and, by (7), $z=\frac{19}{6}.$

50.
$$\begin{cases} ax+y+z=2(a+1), & (1)\\ x+ay+z=3a+1, & (2)\\ x+y+az=a^2+3. & (3)\end{cases}$$

Adding (1), (2), and (3), $(a+2)(x+y+z)=a^2+5a+6.$ (4)

Dividing (4) by $a+2$, $x+y+z=a+3.$ (5)

Subtracting (5) from (1), $(a-1)x=a-1.$

$\therefore x=1.$

Subtracting (5) from (2), $(a-1)y=2a-2.$

$\therefore y=2.$

Subtracting (5) from (3), $(a-1)z=a^2-a.$

$\therefore z=a.$

51.
$$\begin{cases} x^2+xy=24, & (1)\\ y^2+xy=12. & (2)\end{cases}$$

Adding, $x^2+2xy+y^2=36,$

whence, $x+y=6$ or $-6.$ (3)

Subtracting (2) from (1), $x^2-y^2=12.$ (4)

Dividing (4) by (3), $x-y=2$ or $-2;$ (5)

that is, the corresponding values of $x+y$ and $x-y$ are 6 and 2, or -6 and -2.

From (3) and (5), $x=4$ or $-4,$

and $y=2$ or $-2.$

52.
$$\begin{cases} x^2+3xy=7, & (1)\\ xy+4y^2=18. & (2)\end{cases}$$

Adding, $x^2+4xy+4y^2=25,$

whence, extracting the square root, $x+2y=5$ or $-5.$ (3)

From (3), $x=5-2y$ or $-5-2y.$ (4)

Substituting $5-2y$ for x in (2), $5y-2y^2+4y^2=18.$ (5)

Solving (5), $y=2$ or $-\frac{9}{2},$

whence, since $x=5-2y,$ $x=1$ or $14.$

Substituting $-5-2y$ for x in (2), $-5y-2y^2+4y^2=18.$ (6)

Solving (6), $y=\frac{9}{2}$ or $-2,$

whence, since $x=-5-2y,$ $x=-14$ or $-1.$

Hence, $x=1,\ 14,\ -14,\ -1;$

and $y=2,\ -\frac{9}{2},\ \frac{9}{2},\ -2.$

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53. $$\begin{cases} x^2 + x = 26 - y^2 - y, & (1) \\ xy = 8. & (2) \end{cases}$$

From (1), $x^2 + y^2 + (x + y) - 26 = 0.$ (3)

From (2), $2xy - 16 = 0.$ (4)

Adding (4) to (3), $(x + y)^2 + (x + y) - 42 = 0.$ (5)

Factoring (5), $(x + y - 6)(x + y + 7) = 0.$

$\therefore x + y = 6 \text{ or } -7.$ (6)

Squaring (6), $x^2 + 2xy + y^2 = 36 \text{ or } 49.$ (7)

Subtracting (2) $\times$ 4 from (7),

$x^2 - 2xy + y^2 = 4 \text{ or } 17,$

whence, $x - y = \pm 2 \text{ or } \pm\sqrt{17}.$ (8)

From (6) and (8), $x = 4 \text{ or } 2 \text{ or } \frac{1}{2}(-7 + \sqrt{17}) \text{ or } \frac{1}{2}(-7 - \sqrt{17}),$

and $y = 2 \text{ or } 4 \text{ or } \frac{1}{2}(-7 - \sqrt{17}) \text{ or } \frac{1}{2}(-7 + \sqrt{17}).$

54. $$\begin{cases} \sqrt{xy} = 12, & (1) \\ x + y - \sqrt{x + y} = 20. & (2) \end{cases}$$

Transposing in (2), $x + y - \sqrt{x + y} - 20 = 0.$ (3)

Factoring (3), $(\sqrt{x + y} - 5)(\sqrt{x + y} + 4) = 0.$

$\therefore \sqrt{x + y} = 5 \text{ or } -4.$ (4)

Squaring (4), $x + y = 25 \text{ or } 16.$ (5)

Squaring (1), $xy = 144.$ (6)

Squaring (5), $x^2 + 2xy + y^2 = 625 \text{ or } 256.$ (7)

Subtracting (6) $\times$ 4 from (7), $x^2 - 2xy + y^2 = 49 \text{ or } -320,$

whence, $x - y = \pm 7 \text{ or } \pm 8\sqrt{-5}.$ (8)

From (5) and (8) $x = 16 \text{ or } 9 \text{ or } 8 + 4\sqrt{-5} \text{ or } 8 - 4\sqrt{-5},$

and $y = 9 \text{ or } 16 \text{ or } 8 - 4\sqrt{-5} \text{ or } 8 + 4\sqrt{-5}.$

$\left.\begin{matrix} x = 8 + 4\sqrt{-5}, 8 - 4\sqrt{-5}, \\ y = 8 - 4\sqrt{-5}, 8 + 4\sqrt{-5}, \end{matrix}\right\}$ do not verify and are rejected.

55. $$\begin{cases} xy - xy^2 = -6, & (1) \\ x - xy^3 = 9. & (2) \end{cases}$$

Multiplying (1) by 3, etc. $3xy(1 - y) = -18.$ (3)

Multiplying (2) by 2, etc.,

$2x(1 + y + y^2)(1 - y) = 18.$ (4)

Adding (3) and (4), $x(2 + 5y + 2y^2)(1 - y) = 0.$ (5)

When, from (5), $2y^2 + 5y + 2 = 0,$

factoring, $(y + 2)(2y + 1) = 0,$

whence, $y = -2 \text{ or } -\frac{1}{2}.$ (6)

Substituting these values in (1), $x = 1 \text{ or } 8.$ (7)

When, from (5), $1 - y = 0,$ $y = 1,$ (8)

and, in (2), $x = \frac{9}{0} = \infty.$ (9)

When, from (5), $x = 0$, substituting in (2), $y = -\frac{9}{0} = -\infty.$

Hence, $$\begin{cases} x = 1, \; 8, \; \infty, \; 0; \\ y = -2, \; -\frac{1}{2}, \; 1, \; -\infty. \end{cases}$$

$\left.\begin{matrix} x = \infty, 0, \\ y = 1, -\infty \end{matrix}\right\}$ do not verify and are rejected.

56. $$\begin{cases} xy = x + y, & (1) \\ x^2 + y^2 = 8. & (2) \end{cases}$$

From (1), $2xy - 2(x + y) = 0.$ (3)

Adding (3) to (2), $x^2 + 2xy + y^2 - 2(x + y) = 8.$ (4)

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Completing the square,

$$(x+y)^2-2(x+y)+1=9.$$

Extracting the square root, $x+y-1=\pm 3.$

$$\therefore x+y=4 \text{ or } -2. \quad (5)$$

From (1) and (5), $2xy=8 \text{ or } -4.$ (6)

Subtracting (6) from (2), $x^2-2xy+y^2=0 \text{ or } 12,$

whence, $x-y=\pm 0 \text{ or } \pm 2\sqrt{3}.$ (7)

From (5) and (7), $x=2 \text{ or } 2 \text{ or } -1+\sqrt{3} \text{ or } -1-\sqrt{3},$

and $y=2 \text{ or } 2 \text{ or } -1-\sqrt{3} \text{ or } -1+\sqrt{3}.$

57.
$$\begin{cases} x^2y^2-4xy=5, & (1) \\ x^2+4y^2=29. & (2) \end{cases}$$

Solving (1) for xy, $xy=5 \text{ or } -1,$ (3)

whence, $4xy=20 \text{ or } -4.$ (4)

Adding (4) to (2), $x^2+4xy+4y^2=49 \text{ or } 25,$

whence, $x+2y=\pm 7 \text{ or } \pm 5.$ (5)

Subtracting (4) from (2), $x^2-4xy+4y^2=9 \text{ or } 33,$

whence, $x-2y=\pm 3 \text{ or } \pm\sqrt{33}.$ (6)

From (5) and (6), since they are derived separately,

$x=5, 2, -2, -5, \frac{1}{2}(5+\sqrt{33}), \frac{1}{2}(5-\sqrt{33}), \frac{1}{2}(-5+\sqrt{33}), \frac{1}{2}(-5-\sqrt{33});$

$y=1, \frac{5}{2}, -\frac{5}{2}, -1, \frac{1}{4}(5-\sqrt{33}), \frac{1}{4}(5+\sqrt{33}), \frac{1}{4}(-5-\sqrt{33}), \frac{1}{4}(-5+\sqrt{33}).$

58.
$$\begin{cases} 2x^3+2y^3=9xy, & (1) \\ x+y=3. & (2) \end{cases}$$

From (1), $2(x+y)(x^2-xy+y^2)=9xy.$ (3)

Substituting (2) in (3), $6(x^2-xy+y^2)=9xy.$

Transposing, $6x^2-15xy+6y^2=0.$ (4)

Factoring (4), $(x-2y)(6x-3y)=0.$

$$\therefore x=2y \text{ or } \tfrac{1}{2}y. \quad (5)$$

Substituting $2y$ for x in (2), $y=1,$

whence, $x=2.$

Substituting $\frac{1}{2}y$ for x in (2), $y=2,$

whence, $x=1.$

59.
$$\begin{cases} x^{\frac{1}{2}}+y^{\frac{1}{3}}=4, & (1) \\ x^{\frac{3}{2}}+y=16. & (2) \end{cases}$$

Dividing (2) by (1), $x-x^{\frac{1}{2}}y^{\frac{1}{3}}+y^{\frac{2}{3}}=4.$ (3)

Squaring (1), $x+2x^{\frac{1}{2}}y^{\frac{1}{3}}+y^{\frac{2}{3}}=16.$ (4)

Subtracting (3) from (4), $3x^{\frac{1}{2}}y^{\frac{1}{3}}=12.$

$$\therefore x^{\frac{1}{2}}y^{\frac{1}{3}}=4. \quad (5)$$

Subtracting (5) from (3), $x-2x^{\frac{1}{2}}y^{\frac{1}{3}}+y^{\frac{2}{3}}=0,$

whence, $x^{\frac{1}{2}}-y^{\frac{1}{3}}=\pm 0.$ (6)

From (1) and (6), $x^{\frac{1}{2}}=2,$

and $y^{\frac{1}{3}}=2.$

Hence, $x=4 \text{ and } y=8.$

It is seen from (6) that each of the roots occurs twice.

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1. Let $x =$ number of miles down the river.

$12 + 3 = 15$, number of miles per hour downstream,

and $12 - 3 = 9$, number of miles per hour upstream;

$$\therefore \frac{x}{15} + \frac{x}{9} = 8.$$

Solving, $x = 45$.

Hence, the steamboat can go 45 miles down the river and return in 8 hours.

2. Let $x =$ number of cents paid per dozen.

Then, $$3x = 64 \div \frac{x}{12}.$$

Solving, $x = \pm 16$.

Hence, the price was 16 cents per dozen.

3. Let $x =$ number of horses he had.

Then, $\frac{1}{x-8} =$ part of the stable room occupied by 1 horse,

and $\frac{1}{x+8} =$ part of the stable room occupied by 1 horse after the new stable was built.

Since the stable room was increased one half,

$$\frac{1}{x-8} = \frac{\frac{3}{2}}{x+8}.$$

Solving, $x = 40$, the number of horses.

4. Let $x =$ number of pounds the whole weighed.

Then, $\frac{1}{2}x - 5 =$ number of pounds of copper,

$\frac{1}{3}(\frac{1}{2}x + 5) + 5 =$ number of pounds of lead and of tin;

$$\therefore \tfrac{1}{2}x - 5 + \tfrac{2}{3}(\tfrac{1}{2}x + 5) + 10 = x.$$

Solving, $x = 50$,

whence, $\frac{1}{2}x - 5 = 20$ and $\frac{1}{3}(\frac{1}{2}x + 5) + 5 = 15$.

Hence, there were 20 pounds of copper and 15 pounds of lead and of tin.

5. Let $x =$ number of miles he may ride.

Then, $\frac{x}{m} =$ number of hours he rides,

and $\frac{x}{n} =$ number of hours he walks;

$$\therefore \frac{x}{m} + \frac{x}{n} = a.$$

Solving, $$x = \frac{amn}{m+n}.$$

6. Let $x =$ number of car loads he had at first.

Then, $\frac{1}{2}x - \frac{1}{2} =$ number of car loads left after 1st sale,

$\frac{1}{2}(\frac{1}{2}x - \frac{1}{2}) - \frac{1}{2} = \frac{1}{4}x - \frac{3}{4} =$ number of car loads left after 2d sale,

and $\frac{1}{2}(\frac{1}{4}x - \frac{3}{4}) - \frac{1}{2} = \frac{1}{8}x - \frac{7}{8} =$ number of car loads left after 3d sale, had he made the sale;

$$\therefore \tfrac{1}{8}x - \tfrac{7}{8} = 0.$$

Solving, $x = 7$.

Hence, he had 7 car loads of grain at first.

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7. Let $3x =$ number of days he worked.

Then, $x =$ number of days he was idle;

$$\therefore \frac{5}{2}\cdot 3x - \frac{3}{2}x = 24.$$

Multiplying by $\frac{2}{3}$, $5x - x = 16.$

$$\therefore x = 4,$$

whence, $3x = 12$, the number of days he worked.

8. Let $x =$ number of dollars 1st cup is worth,

and $y =$ number of dollars 2d cup is worth.

Then, $x + \frac{3}{2} = \frac{15}{8}y,$

and $y + \frac{3}{2} = \frac{11}{12}x.$

Solving, $x = 6,$

and $y = 4.$

Hence, the first cup is worth \$6, and the second, \$4.

9. Let $x =$ greater number,

and $y =$ less number.

Then, $x + y = xy = x^2 - y^2.$

Since these equations stand for three simultaneous equations, any two of which are independent, while the third is derived from the two independent equations, we may select the equations

$$\begin{cases} x + y = xy, & (1) \\ x^2 - y^2 = xy. & (2) \end{cases}$$

Dividing (2) by (1), $x - y = 1.$ (3)

Squaring (3), $x^2 - 2xy + y^2 = 1.$ (4)

From (1), $4xy - 4(x + y) = 0.$ (5)

Adding (5) to (4), $x^2 + 2xy + y^2 - 4(x + y) = 1.$

Completing the square, $(x + y)^2 - 4(x + y) + 4 = 5.$

Extracting the square root, $x + y - 2 = \pm\sqrt{5}.$

$$\therefore x + y = 2 \pm \sqrt{5}. \quad (6)$$

From (6) and (3), $x = \frac{3}{2} + \frac{1}{2}\sqrt{5}$ or $\frac{3}{2} - \frac{1}{2}\sqrt{5},$

and $y = \frac{1}{2} + \frac{1}{2}\sqrt{5}$ or $\frac{1}{2} - \frac{1}{2}\sqrt{5}.$

Hence, the numbers are $\frac{1}{2}(3 + \sqrt{5})$ and $\frac{1}{2}(1 + \sqrt{5})$ or $\frac{1}{2}(3 - \sqrt{5})$ and $\frac{1}{2}(1 - \sqrt{5})$.

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10. Let $x =$ number of feet in length before the increase,

and $y =$ number of feet in width before the increase.

Then, $$\left(x + \frac{x}{8}\right)\left(y + \frac{y}{4}\right) = xy + 13, \quad (1)$$

and $$2\cdot\frac{x}{8} + 2\cdot\frac{y}{4} = 4. \quad (2)$$

From (1), $xy = 32,$ (3)

and from (2), $x + 2y = 16.$ (4)

From (3) and (4), $x = 8,$

and $y = 4.$

Then, $$x + \frac{x}{8} = 9,$$

and $$y + \frac{y}{4} = 5.$$

Hence, the finished rug is 9 feet long and 5 feet wide.

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11. Let $x =$ number of tons each cart can carry,
and $y =$ number of tons each wagon can carry.

Then, $15x + 12y = 28,$
and $24x + 8y = 28.$
Solving, $x = \frac{2}{3},$
and $y = \frac{3}{2}.$

Hence, each cart can carry $\frac{2}{3}$ of a ton, and each wagon $1\frac{1}{2}$ tons.

12. Let x represent the units' digit.

Then, $100x + 10x + x =$ the number;

$$\therefore 100x + 10x + x - 7 \cdot 3x = 180.$$

Solving, $x = 2.$

Hence, the number is 222.

13. Let $x =$ number of days in which A can do it,
$y =$ number of days in which B can do it,
and $z =$ number of days in which C can do it.

Then,
$$\frac{1}{x} + \frac{1}{y} = \frac{1}{m}, \quad (1)$$
$$\frac{1}{y} + \frac{1}{z} = \frac{1}{n}, \quad (2)$$
and
$$\frac{1}{x} + \frac{1}{z} = \frac{1}{p}. \quad (3)$$

Adding and dividing by 2, $$\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = \frac{np + mp + mn}{2mnp}. \quad (4)$$

Since (4) represents the part of the work all together can do in one day, it will take them all $1 \div \frac{np + mp + mn}{2mnp}$ days, or $\frac{2mnp}{mp + np + mn}$ days, to complete the work.

Subtracting (2), (3), and (1) in succession from (4), and solving,

$x = \frac{2mnp}{-mp + np + mn}$, number of days in which A can do it,

$y = \frac{2mnp}{mp + np - mn}$, number of days in which B can do it,

and $z = \frac{2mnp}{mp - np + mn}$, number of days in which C can do it.

14. Let $x =$ number of pounds of baggage allowed each passenger without charge.

Under the first condition $(400 - 2x)$ pounds are subject to charge at $\frac{100}{400 - 2x}$ cents per pound, and under the second condition $(400 - x)$ pounds at $\frac{150}{400 - x}$ cents per pound.

Therefore, $$\frac{100}{400 - 2x} = \frac{150}{400 - x}.$$

Solving, $x = 100$, number of pounds allowed.

15. Let $x =$ number of men.

Then, $x^2 = 4(x + 3).$
Solving, $x = 6$ or $-2.$

Hence, there were 6 men.

16. Let $x =$ number of pounds in the first lot.
Then, $x - 20 =$ number of pounds in the second lot;
$$\therefore x^2 + (x - 20)^2 = 3400.$$
Solving, $x = 50$ or -30,
whence, rejecting the negative value, $x - 20 = 30$.
Hence, there were 50 pounds in the first lot and 30 pounds in the second.

17. Let $x =$ number of shillings paid per week.
Under the first condition A had to pay $(x - 18)$ shillings for the pasturage of 4 horses. Hence, $\frac{x-18}{4}$ shillings per horse were paid by both A and B, and the number of horses in the pasture was $x \div \frac{x-18}{4}$, or $\frac{4x}{x-18}$.

Similarly, under the second condition, the number of horses was $\frac{4x}{x-20}$.

Since the second condition was due to increasing the number of horses by 2,
$$\frac{4x}{x-20} = \frac{4x}{x-18} + 2.$$
Reducing, $x^2 - 42x + 360 = 0$.
Solving this equation, $x = 30$ or 12.
The second value is evidently inadmissible.
Hence, the cost of hiring the pasture was 30 shillings per week.

18. Let $x =$ number of cents per dozen in first price,
and $x - 1 =$ number of cents per dozen in second price.
Then,
$$\frac{60}{x-1} = \frac{60}{x} + 5.$$
Solving, $x = 4$ or -3.
Hence, he sold apples for 4 cents per dozen at first.

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19. Let $x =$ number of miles per hour traveled before accident,
and $y =$ number of hours required to make the trip at the rate of x miles per hour.
Then, $xy =$ number of miles in the whole distance,
$2x =$ number of miles traveled before accident,
and $\frac{xy - 2x}{\frac{3}{5}x} =$ number of hours occupied in completing trip.
$$\therefore 2 + 1 + \frac{xy - 2x}{\frac{3}{5}x} = y + 7\tfrac{2}{3}. \qquad (1)$$
Under the second condition,
$2x + 50 =$ number of miles traveled before accident,
$2 + \frac{50}{x} =$ number of hours before accident,
and $\frac{xy - 2x - 50}{\frac{3}{5}x} =$ number of hours occupied in completing trip.
$$\therefore 2 + \frac{50}{x} + 1 + \frac{xy - 2x - 50}{\frac{3}{5}x} = y + 6\tfrac{1}{3}. \qquad (2)$$

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From (1), $y = 12.$ (3)

Subtracting (2) from (1), $-\frac{50}{x} + \frac{50}{\frac{5}{6}x} = \frac{4}{3}.$ (4)

Solving (4), $x = 25.$ (5)

From (3) and (5), $xy = 300.$

Hence, the whole distance was 300 miles.

20. Let $x =$ number of miles per hour A walked,
and $y =$ number of miles per hour B walked.

Then, $$\frac{30}{x} = \frac{30}{y} + 2, \quad (1)$$

and $$\frac{42}{x + \frac{1}{2}} = \frac{42}{y + \frac{1}{2}} + 2. \quad (2)$$

From (1), $15y = 15x + xy.$ (3)

Multiplying each member of (2) by $2(x + \frac{1}{2})(y + \frac{1}{2})$,

$$84y + 42 = 84x + 42 + 4xy + 2x + 2y + 1. \quad (4)$$

Canceling $42 = 42$, subtracting (3) $\times$ 4, and reducing,

$$22y = 26x + 1. \quad (5)$$

Multiplying (3) by 22, $15 \cdot 22y = 330x + x \cdot 22y.$ (6)

Substituting (5) in (6),

$$15(26x + 1) = 330x + x(26x + 1).$$

$$26x^2 - 59x - 15 = 0.$$

$$(2x - 5)(13x + 3) = 0.$$

$$\therefore x = \tfrac{5}{2} \text{ or } -\tfrac{3}{13}.$$

Rejecting the negative value and substituting $\frac{5}{2}$ for x in (5),

$$y = 3.$$

Hence, A traveled $2\frac{1}{2}$ miles per hour and B 3 miles per hour.

21. From § 237, Ex. 20, $pd = WD.$

Substituting for d and D, the values given in the figure, and for W, 360,000, the weight on the wall column. Then p equals the weight on the interior column.

$$\therefore 12p = 4 \times 360{,}000.$$

Solving, $p = 120{,}000.$

But, the weight on the fulcrum equals the sum of the weights on the wall column and on the interior column.

Hence, the weight on the fulcrum equals 360,000 pounds plus 120,000 pounds, or 480,000 pounds.

22. Let $x =$ number of feet per second sound travels,
and $y =$ number of feet per second projectile travels.

Then, $$\frac{3360}{x} + \frac{3360}{y} = 4\tfrac{1}{3}, \quad (1)$$

and $$\frac{5600}{x} - \left(\frac{3360}{y} + \frac{2240}{x}\right) = 1\tfrac{2}{3}. \quad (2)$$

From (1), $10{,}080y + 10{,}080x = 13xy,$ (3)

From (2), $10{,}080y - 10{,}080x = 5xy.$ (4)

(3) + (4), $18xy = 20{,}160y.$

$$\therefore x = 1120.$$

(3) − (4), $8xy = 20{,}160x.$

$$\therefore y = 2520.$$

Hence, sound travels 1120 feet per second, and the average velocity of the projectile is 2520 feet per second.

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23. Substituting in $v = gt$, the given values,

$$t = \frac{2010}{32.16}.$$

Solving, $t = 62.5.$

Hence, if it takes 62.5 seconds for the bullet to rise, it will take twice that time for it to rise and return to earth, or 125 seconds.

Substituting in $s = \frac{1}{2} gt^2$ the value for g and the value for t just found, $s = 62{,}812.5.$

Hence, the bullet will rise 62,812.5 feet.

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1. Solving $4x + 5y = 24$ for y, $y = \frac{4}{5}(6 - x).$

When $x = -4,\ y = 8;$

when $x = 6,\quad y = 0.$

Locate $A = (-4, 8),\ B = (6, 0).$

A straight line drawn through A and B is the graph of $4x + 5y = 24.$

Solving $3x - 2y = -5$ for y,

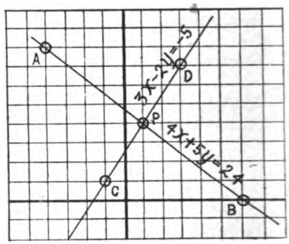
$$y = \tfrac{1}{2}(3x + 5).$$

When $x = -1,\ y = 1;$

when $x = 3,\quad y = 7.$

Locate $C = (-1, 1),\ D = (3, 7).$

A straight line drawn through C and D is the graph of $3x - 2y = -5.$



These two graphs intersect at $P = (1, 4)$. Hence, $x = 1$ and $y = 4$.

2.
$$\frac{3}{2x+3} + \frac{1}{x-5} - \frac{8}{2x^2 - 7x - 15} = 0.$$

Clearing of fractions, $3x - 15 + 2x + 3 - 8 = 0.$

Solving, $x = 4.$

3.
$$\begin{cases} (x-y)^2 = c^2, & (1) \\ (y-a)(x-b) = 0. & (2) \end{cases}$$

From (2), $y = a$ and $x = b.$

Substituting a for y in (1), $x = a + c$ or $a - c$, when $y = a.$

Substituting b for x in (1), $y = b - c$ or $b + c$, when $x = b.$

Hence, the values are
$$\begin{cases} x = a + c, & a - c, & b, & b; \\ y = a, & a, & b - c, & b + c. \end{cases}$$

4.
$$\begin{cases} x^2 + xy + z = 2, & (1) \\ x + 2y + z = 3, & (2) \\ x - y + z = 0. & (3) \end{cases}$$

Subtracting (3) from (2), $3y = 3. \quad \therefore y = 1.$

Subtracting (2) from (1), $x^2 + xy - x - 2y = -1.$ (4)

Substituting 1 for y in (4), $x^2 = 1. \quad \therefore x = \pm 1.$

5.
$$\begin{cases} 2(x+y)^2 - (x+y)(x-2y) = 70, & (1) \\ 2(x+y) - 3(x-2y) = 2. & (2) \end{cases}$$

From (2), $x = 8y - 2.$ (3)

Substituting (3) in (1),

$$2(9y-2)^2 - 2(9y-2)(3y-1) = 70.$$

Simplifying, etc., $18y^2 - 7y = 11.$

Solving, $y = 1$ or $-\frac{11}{18}.$

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Substituting 1 for y in (3), $x = 6.$

Substituting $-\frac{11}{18}$ for y in (3), $x = -\frac{62}{9}.$

Hence, the values are $\begin{cases} x = 6, -\frac{62}{9}; \\ y = 1, -\frac{11}{18}. \end{cases}$

6. $$\begin{cases} x + y = xy, & (1) \\ 2x + 2z = xz, & (2) \\ 3z + 3y = yz. & (3) \end{cases}$$

From (1), $y = \dfrac{x}{x-1}.$ (4)

From (2), $z = \dfrac{2x}{x-2}.$ (5)

Substituting (4) and (5) in (3),

$$\frac{6x}{x-2} + \frac{3x}{x-1} = \frac{2x^2}{(x-1)(x-2)}. \qquad (6)$$

Simplifying, etc., $7x^2 - 12x = 0.$

$\therefore x = 0$ or $\frac{12}{7}.$ (7)

Substituting (7) in (4), $y = 0$ or $\frac{12}{5}.$

Substituting (7) in (5), $z = 0$ or $-12.$

7. $$\begin{cases} x - y - \sqrt{x-y} = 2; & (1) \\ x^3 - y^3 = 2044. & (2) \end{cases}$$

Solving (1), $x - y = 4$ or $1.$ (3)

$\therefore x = y + 4$ or $y + 1.$

Substituting $y + 4$ for x in (2), $(y+4)^3 - y^3 = 2044.$

Simplifying, $y^2 + 4y = 165.$

Solving, $y = 11$ or $-15.$ (4)

Substituting (4) in $x = y + 4$, $x = 15$ or $-11.$

Substituting $y + 1$ for x in (2), $(y+1)^3 - y^3 = 2044.$

Simplifying, $y^2 + y = 681.$

Solving, $y = \frac{1}{2}(-1 \pm 5\sqrt{109}).$ (5)

Substituting (5) in $x = y + 1$, $x = \frac{1}{2}(1 \pm 5\sqrt{109}).$

$x = \frac{1}{2}(1 \pm 5\sqrt{109})$ and $y = \frac{1}{2}(-1 \pm 5\sqrt{109})$ do not verify and are rejected.

8. $$\begin{cases} (a+c)x - (a-c)y = 2ab, & (1) \\ (a+b)x - (a-b)y = 2ac. & (2) \end{cases}$$

From (1), $ax + cx - ay + cy = 2ab,$ (3)

and from (2), $ax + bx - ay + by = 2ac.$ (4)

Subtracting (3) from (4), $(b-c)x + (b-c)y = -2a(b-c).$

$\therefore y = -2a - x.$ (5)

Substituting (5) in (3), $ax + cx + 2a^2 + ax - 2ac - cx = 2ab.$

Solving, $x = -(a - b - c).$ (6)

Substituting (6) in (5), $y = -(a + b + c).$

9.
$$\begin{aligned} 4x^4 + 1 &= 4x^4 + 4x^2 + 1 - 4x^2. \\ &= (2x^2+1)^2 - (2x)^2 \\ &= (2x^2 + 2x + 1)(2x^2 - 2x + 1) \\ &= \tfrac{1}{4}(2x + 1 + \sqrt{-1})(2x + 1 - \sqrt{-1})(2x - 1 + \sqrt{-1}) \\ &\qquad (2x - 1 - \sqrt{-1}). \end{aligned}$$

$$27x^2 + 3x - 2 = (3x+1)(9x-2).$$

$$\begin{aligned} 4x^4 + y^4 - 5x^2y^2 &= (4x^2 - y^2)(x^2 - y^2) \\ &= (2x+y)(2x-y)(x+y)(x-y). \end{aligned}$$

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10. $$3(a-1)^3-(1+a)=3a^3-9a^2+8a-4$$

If 2 is substituted for a, $\quad = 24-36+16-4=0.$

Therefore, $a-2$ is a factor of $3a^3-9a^2+8a-4$.

By dividing by $a-2$,

$$3(a-1)^3-(1+a)=(a-2)(3a^2-3a+2).$$

$$\begin{aligned} a^4-a^2b^2-b^2-1 &= (a^4-1)-b^2(a^2+1) \\ &= (a^2+1)(a^2-1)-b^2(a^2+1) \\ &= (a^2+1)(a^2-b^2-1). \end{aligned}$$

$$3x^{-\frac{1}{2}}+7x^{-\frac{1}{4}}-6=(x^{-\frac{1}{4}}+3)(3x^{-\frac{1}{4}}-2).$$

11. $$4x^2+mx+5=0.$$

Completing the square, $\quad 4x^2+mx+\dfrac{m^2}{16}=\dfrac{m^2-80}{16}.$

Extracting the square root, $\quad 2x+\dfrac{m}{4}=\pm\dfrac{1}{4}\sqrt{m^2-80}.$

$$\therefore x=\tfrac{1}{8}(-m\pm\sqrt{m^2-80}).$$

The roots will be imaginary if the discriminant is negative, that is, if

$$m^2<80,$$

or, solving, if $\quad m<4\sqrt{5}$ numerically.

12. Let $\quad x =$ number of pounds of water to be added.

Then, $\quad x+80 =$ number of pounds in whole solution.

$$\therefore x+80=\frac{4}{\frac{4}{100}}.$$

$$x+80=100.$$

$$x=20.$$

Hence, 20 pounds of water must be added.

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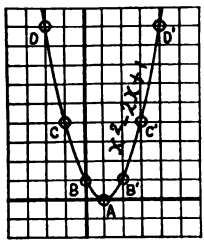
13. Let $y=x^2-2x+1$. By § 418, the function has a minimum value at $x=1$; that is, when $x=1$, $y=0$, the minimum value. The table gives the coördinates of other points.

x	y	Points
1	0	A
0, 2	1	B, B'
-1, 3	4	C, C'
-2, 4	9	D, D'



Plotting these points and drawing a smooth curve through them, we have the graph of the function x^2-2x+1, which is a parabola.

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15.

$$\begin{array}{l|l}
 & x^2 - 3x + 1 \;\Big|\; x - \frac{3}{2} - \frac{5}{8x} - \frac{15}{16x^2} \\
 & x^2 \\ \hline
2x & -3x + 1 \\
2x - \frac{3}{2} & -3x + \frac{9}{4} \\ \hline
2x - 3 & -\frac{5}{4} \\
2x - 3 - \frac{5}{8x} & -\frac{5}{4} + \frac{15}{8x} + \frac{25}{64x^2} \\ \hline
2x - 3 - \frac{5}{4x} & -\frac{15}{8x} - \frac{25}{64x^2} \\
2x - 3 - \frac{5}{4x} - \frac{15}{16x^2} & -\frac{15}{8x} + \frac{45}{16x^2} + \frac{75}{64x^3} + \frac{225}{256x^4} \\ \hline
\end{array}$$

16.

$$\begin{array}{l|l}
 & \frac{9a^2c^{2m}}{4b^{12}} - \frac{3ac^{m+n}}{b^8} + b^6c^{2n} - \frac{2^8ac^m}{b^6} + \frac{2^9b^3c^n}{3} + \frac{2^{16}}{9} \;\Big|\; \frac{3ac^m}{2b^6} - b^3c^n - \frac{2^8}{3} \\
 & \frac{9a^2c^{2m}}{4b^{12}} \\ \hline
\frac{3ac^m}{b^6} & -\frac{3ac^{m+n}}{b^8} + b^6c^{2n} \\
\frac{3ac^m}{b^6} - b^3c^n & -\frac{3ac^{m+n}}{b^8} + b^6c^{2n} \\ \hline
\frac{3ac^m}{b^6} - 2b^3c^n & -\frac{2^8ac^m}{b^6} + \frac{2^9b^3c^n}{3} + \frac{2^{16}}{9} \\
\frac{3ac^m}{b^6} - 2b^3c^n - \frac{2^8}{3} & -\frac{2^8ac^m}{b^6} + \frac{2^9b^3c^n}{3} + \frac{2^{16}}{9} \\ \hline
\end{array}$$

17. $$x^2 + 7x - 3 = \sqrt{2x^2 + 14x + 2}.$$

Multiplying by 2, $$2x^2 + 14x - 6 = 2\sqrt{2x^2 + 14x + 2}.$$

Adding 8, $$2x^2 + 14x + 2 = 2\sqrt{2x^2 + 14x + 2} + 8.$$

Putting p for $\sqrt{2x^2 + 14x + 2}$ and p^2 for $2x^2 + 14x + 2$,

$$p^2 - 2p = 8.$$

Solving, $$p = 4 \text{ or } -2.$$

Substituting 4 for p, $$\sqrt{2x^2 + 14x + 2} = 4.$$

Squaring, $$2x^2 + 14x + 2 = 16.$$

Solving, $$x = \tfrac{1}{2}(-7 \pm \sqrt{77}).$$

Substituting -2 for p, $$\sqrt{2x^2 + 14x + 2} = -2.$$

Squaring, $$2x^2 + 14x + 2 = 4.$$

Solving, $$x = \tfrac{1}{2}(-7 \pm \sqrt{53}).$$

$\frac{1}{2}(-7 \pm \sqrt{53})$ does not verify and is rejected.

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18. $$\frac{bc(b-c)+ac(c-a)+ab(a-b)}{a^2+bc-ac-ab}=\frac{b^2c-bc^2+ac^2-a^2c+a^2b-ab^2}{a^2+bc-ac-ab}.$$
$$=\frac{-c(a^2-b^2)+c^2(a-b)+ab(a-b)}{a(a-b)-c(a-b)}.$$
$$=\frac{[-c(a+b)+c^2+ab][a-b]}{(a-b)(a-c)}.$$
$$=\frac{-ac-bc+c^2+ab}{a-c}.$$
$$=\frac{b(a-c)-c(a-c)}{a-c}=b-c.$$

19. $$\begin{cases}\dfrac{2x}{3}-\dfrac{5y}{12}-\left(\dfrac{3x}{2}-\dfrac{4y}{3}\right)=-\dfrac{2}{3}, & (1)\\[2ex] \dfrac{x-y}{x+y}=\dfrac{1}{5}. & (2)\end{cases}$$

From (2), $y=\dfrac{2x}{3}$. (3)

Substituting $\dfrac{2x}{3}$ for y in (1),
$$\frac{2x}{3}-\frac{5x}{18}-\left(\frac{3x}{2}-\frac{8x}{9}\right)=-\frac{2}{3}.$$
Solving, $x=3$.
Substituting 3 for x in (3), $y=2$.

20. $$\begin{cases}\sqrt{x}-\sqrt{y}=2, & (1)\\ (\sqrt{x}-\sqrt{y})\sqrt{xy}=30. & (2)\end{cases}$$
Substituting (1) in (2), $2\sqrt{xy}=30$.
Dividing by 2 and squaring, $xy=225$.
$$\therefore y=\frac{225}{x}. \quad (3)$$

Substituting $\dfrac{225}{x}$ for y in (1),
$$\sqrt{x}-\frac{15}{\sqrt{x}}=2.$$
$$x-2\sqrt{x}=15.$$
Solving, $x=25$ or 9. (4)
Substituting (4) in (3), $y=9$ or 25.
$x=9$ and $y=25$ do not verify and are rejected.

21. Substituting $-b$ for a, $a^n+b^n=(-b)^n+b^n$
If n is odd, $=-b^n+b^n=0$.
$\therefore a+b$ is a factor of a^n+b^n for all positive odd integral values of n.

22. Let $x=$ number of feet in length of room,
$y=$ number of feet in width of room,
and $z=$ number of feet in height of room.
Then, $xy=120$, (1)
$yz=80$, (2)
and $xz=96$. (3)

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From (1), $$y = \frac{120}{x}, \tag{4}$$

and from (3), $$z = \frac{96}{x}. \tag{5}$$

Substituting (4) and (5) in (2), $x^2 = 144,$
$$x = \pm 12. \tag{6}$$

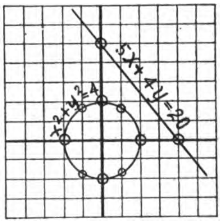
Substituting (6) in (4), $y = 10.$
Substituting (6) in (5), $z = 8.$
Hence, the length of the room is 12 feet, the width 10 feet, and the height 8 feet.

23. $$\sqrt[3]{3} = (3)^{\frac{1}{3}} = (3)^{\frac{5}{15}} = \sqrt[15]{243}.$$
$$\sqrt[5]{6} = (6)^{\frac{1}{5}} = (6)^{\frac{3}{15}} = \sqrt[15]{216}.$$

Hence, $\sqrt[3]{3}$ is greater than $\sqrt[5]{6}$.

24. Solving $x^2 + y^2 = 4$ for y, $y = \pm\sqrt{4 - x^2}$.
Since any value numerically greater than 2 substituted for x will make the value of y imaginary, we substitute only values of x between and including -2 and $+2$.
Corresponding values of x and y are recorded in the table.

x	y
0	± 2
± 1	± 1.7
± 2	0



Plotting these eight points and drawing a smooth curve through them, we have the graph of $x^2 + y^2 = 4$, which is a circle.
Solving $5x + 4y = 20$ for y, $y = \frac{5}{4}(4 - x)$.
When $x = 0$, $y = 5$; when $x = 4$, $y = 0$.
Plotting the points (0, 5) and (4, 0), and drawing a straight line through them, we have the graph of $5x + 4y = 20$.
These two graphs have no point in common. This means that the roots of this system of equations are imaginary.

25. Let x = number of inches in length of box,
y = number of inches in width of box,
and z = number of inches in height of box.
Then, $x + 12$ = number of inches in length of tin,
and $y + 12$ = number of inches in width of tin.
Then, $x + 12 = y + 12 + 4.$
$$\therefore y = x - 4. \tag{1}$$
$$xyz = 840. \tag{2}$$

Since a 6-inch square is cut from each corner, and the ends and sides are then turned up, the height of the box is 6 inches.
Substituting (1) in (2), $x(x - 4)6 = 840.$
$$x^2 - 4x = 140.$$

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Solving, $x = 14.$ (3)
Substituting (3) in (1), $y = 10.$

Hence, the length of the box is 14 inches, the width is 10 inches, and the height is 6 inches.

INEQUALITIES

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3. $6x - 5 > 13.$
Transposing, Prin. 3, $6x > 18.$
Dividing by 6, Prin. 2, $x > 3.$

4. $5x - 1 < 6x + 4.$
Transposing, Prin. 3, $-x < 5.$
Changing signs, Prin. 4, $x > -5.$

5. $3x - \frac{1}{2}x < 30.$
Uniting terms, $\frac{5}{2}x < 30.$
Multiplying by $\frac{2}{5}$, Prin. 2, $x < 12.$

6. $4x + 1 < 6x - 11.$
Transposing, Prin. 3, $-2x < -12.$
Dividing by -2, Prin. 2, $x > 6.$

7. $\begin{cases} 4x - 11 > \frac{1}{3}x, & (1) \\ 20 - 2x > 10. & (2) \end{cases}$
Multiplying (1) by 3, Prin. 2, $12x - 33 > x.$
Transposing, Prin. 3, $11x > 33.$
Dividing by 11, Prin. 2, $x > 3.$
Transposing, in (2), Prin. 3, $-2x > -10.$
Dividing by -2, Prin. 2, $x < 5.$

Hence, x is greater than 3 and less than 5; that is, x can have all values that lie between 3 and 5.

8. $\begin{cases} 3 - 4x < 7, & (1) \\ 5x + 10 < 20. & (2) \end{cases}$
Transposing in (1), Prin. 3, $-4x < 4.$
Dividing by -4, Prin. 2, $x > -1.$
Dividing (2) by 5, Prin. 2, $x + 2 < 4.$
Transposing, Prin. 3, $x < 2.$
Hence, x is greater than -1 and less than 2.

9. $x + \frac{2x}{3} + \frac{5x}{6} > 25$ and $< 30.$
Multiplying by 6, Prin. 2, $6x + 4x + 5x > 150$ and $< 180.$
$15x > 150$ and $< 180.$
Dividing by 15, Prin. 2, $x > 10$ and $< 12.$

11. $\begin{cases} 2x - 3y < 2, & (1) \\ 2x + 5y = 25. & (2) \end{cases}$
Subtracting (2) from (1), Prin. 1, $-8y < -23.$ (3)
Dividing (3) by -8, Prin. 2, $y > 2\frac{7}{8}.$ (4)
Multiplying (1) by 5, Prin. 2, $10x - 15y < 10.$ (5)

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Multiplying (2) by 3, $6x + 15y = 75.$ (6)
Adding (6) to (5), Prin. 1, $16x < 85.$ (7)
Dividing (7) by 16, Prin. 2, $x < 5\frac{5}{16}.$
Hence, the positive integral values are $x = 5$, $y = 3$.

12. $\begin{cases} 3x + 2y = 42, & (1) \\ 3x - \dfrac{y}{7} > 16. & (2) \end{cases}$

Subtracting (2) from (1), Prin. 6, $\frac{15}{7}y < 26.$ (3)
Multiplying (3) by $\frac{7}{15}$, Prin. 2, $y < 12\frac{2}{15}.$ (4)
Multiplying (2) by 14, Prin. 2, $42x - 2y > 224.$ (5)
Adding (1) to (5), Prin. 1, $45x > 266.$ (6)
Dividing (6) by 45, Prin. 2, $x > 5\frac{41}{45}.$
Hence, the positive integral values are $x = 6$, $y = 12$; $x = 8$, $y = 9$; $x = 10$, $y = 6$; $x = 12$, $y = 3$.

13. $\begin{cases} x + y = 10, & (1) \\ 4x < 3y. & (2) \end{cases}$

From (1), $y = 10 - x.$ (3)
Substituting (3) in (2), $4x < 30 - 3x.$ (4)
Transposing in (4), Prin. 3, $7x < 30.$ (5)
Dividing (5) by 7, Prin. 2, $x < 4\frac{2}{7}.$ (6)
From (1), $x = 10 - y.$ (7)
Substituting (7) in (2), $40 - 4y < 3y.$ (8)
Transposing in (8), Prin. 3, $-7y < -40.$ (9)
Dividing (9) by -7, Prin. 2, $y > 5\frac{5}{7}.$
Hence, the positive integral values are $x = 4$, $y = 6$; $x = 3$, $y = 7$; $x = 2$, $y = 8$; $x = 1$, $y = 9$.

14. $\begin{cases} y = 3x + 4, & (1) \\ 25 < 2y + 3x. & (2) \end{cases}$

Substituting (1) in (2), $25 < 6x + 8 + 3x.$ (3)
Transposing in (3), Prin. 3, $-9x < -17.$ (4)
Dividing (4) by -9, Prin. 2, $x > 1\frac{8}{9}.$ (5)
Subtracting (1) from (2), Prin. 1, $25 - y < 2y - 4.$ (6)
Transposing in (6), Prin. 3, $-3y < -29.$ (7)
Dividing (7) by -3, Prin. 2, $y > 9\frac{2}{3}.$
Hence, the positive integral values are $x = 4$, $y = 16$; $x = 5$, $y = 19$; $x = 6$, $y = 22$; etc.

15. $\begin{cases} y - x > 9, & (1) \\ \dfrac{7x}{20} + \dfrac{y}{15} = 9. & (2) \end{cases}$

Multiplying (2) by 60, $21x + 4y = 540.$ (3)
Multiplying (1) by 4, Prin. 2, $-4x + 4y > 36.$ (4)
Subtracting (4) from (3), Prin. 6, $25x < 504.$ (5)
Dividing (5) by 25, Prin. 2, $x < 20\frac{4}{25}.$ (6)
Multiplying (1) by 21, Prin. 2, $-21x + 21y > 189.$ (7)
Adding (3) to (7), Prin. 1, $25y > 729.$ (8)
Dividing (8) by 25, Prin. 2, $y > 29\frac{4}{25}.$
Hence, the positive integral values are $x = 20$, $y = 30$; $x = 16$, $y = 51$; $x = 12$, $y = 72$; $x = 8$, $y = 93$; $x = 4$, $y = 114$.

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16. $\begin{cases} x > y + 4, & (1) \\ x - 2y = 8. & (2) \end{cases}$

Subtracting (2) from (1), Prin. 1, $2y > y - 4.$ (3)
Transposing in (3), Prin. 3, $y > -4.$ (4)
Multiplying (1) by 2, Prin. 2, $2x > 2y + 8.$ (5)
Subtracting (2) from (5), Prin. 1, $x + 2y > 2y.$
Canceling $2y = 2y$, Prin. 1, $x > 0.$

Hence, the positive integral values are $y = 1, x = 10$; $y = 2, x = 12$; $y = 3, x = 14$; etc.

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18. $x^2 + 3x > 10.$
Transposing, Prin. 3, $x^2 + 3x - 10 > 0.$
Factoring, $(x - 2)(x + 5) > 0.$

Since $(x - 2)(x + 5)$ is positive, both factors are positive or both factors are negative. If both factors are positive, $x > 2$. If both factors are negative, $x < -5$.

Hence, x can have all values except 2 and -5 and those which lie between 2 and -5.

19. $x^2 + 8x > 20.$
Transposing, Prin. 3, $x^2 + 8x - 20 > 0.$
Factoring, $(x - 2)(x + 10) > 0.$

Since $(x - 2)(x + 10)$ is positive, both factors are positive or both factors are negative. If both factors are positive, $x > 2$. If both factors are negative, $x < -10$.

Hence, x can have all values except 2 and -10 and those which lie between 2 and -10.

20. $x^2 + 5x > 24.$
Transposing, Prin. 3, $x^2 + 5x - 24 > 0.$
Factoring, $(x - 3)(x + 8) > 0.$

Since $(x - 3)(x + 8)$ is positive, both factors are positive or both factors are negative. If both factors are positive, $x > 3$. If both factors are negative, $x < -8$.

Hence, x can have all values except 3 and -8 and those which lie between 3 and -8.

21. $(x - 2)(3 - x) > 0.$
Changing signs, Prin. 4, $(x - 2)(x - 3) < 0.$

Since $(x - 2)(x - 3)$ is negative, the factors have unlike signs. Hence, $(x - 2)$, the greater factor, is positive, and $x - 3$, the less factor, is negative; that is, x can have all values between 2 and 3, but no others.

22. $x^2 > 9x - 18.$
Transposing, Prin. 3, $x^2 - 9x + 18 > 0.$
Factoring, $(x - 3)(x - 6) > 0.$

Since $(x - 3)(x - 6)$ is positive, both factors are positive or both factors are negative. If both factors are positive, $x > 6$. If both factors are negative, $x < 3$.

Hence, x can have all values except 6 and 3 and those which lie between 6 and 3.

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23. $$x^2 + 40x > 3(4x - 25).$$
$$x^2 + 40x > 12x - 75.$$

Transposing, Prin. 3, $$x^2 + 28x + 75 > 0.$$

Factoring, $$(x+3)(x+25) > 0.$$

Since $(x+3)(x+25)$ is positive, both factors are positive or both factors are negative. If both factors are positive, $x > -3$. If both factors are negative, $x < -25$.

Hence, x can have all values except -3 and -25 and intermediate values.

24. $$x^2 + bx > ax + ab.$$

Transposing, Prin. 3, $$x^2 - ax + bx - ab > 0.$$

Factoring, $$(x-a)(x+b) > 0.$$

Since $(x-a)(x+b)$ is positive, both factors are positive or both factors are negative. If both factors are positive, $x > a$. If both factors are negative, $x < -b$.

Hence, x can have all values except a and $-b$ and those which lie between a and $-b$.

25. $$(x-3)(5-x) > 0.$$

Changing signs, Prin. 4, $$(x-3)(x-5) < 0.$$

Since $(x-3)(x-5)$ is negative, the factors have unlike signs. Hence, $x-3$, the greater factor, is positive, and $x-5$, the less factor, is negative; that is, x can have all values between 3 and 5, but no others.

27. $$\frac{a+b}{a+2b} - \frac{a+2b}{a+3b} = 1 - \frac{b}{a+2b} - \left(1 - \frac{b}{a+3b}\right)$$
$$= \frac{b}{a+3b} - \frac{b}{a+2b} = \frac{-b^2}{(a+3b)(a+2b)}.$$

Hence, $\dfrac{a+2b}{a+3b} > \dfrac{a+b}{a+2b}$, a and b being positive.

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28. If a, b, and c are unequal, since $(a-b)^2$, $(a-c)^2$, and $(b-c)^2$ are positive,
$$a^2 - 2ab + b^2 > 0,$$
$$a^2 - 2ac + c^2 > 0,$$
and $$b^2 - 2bc + c^2 > 0.$$

Adding, Prin. 5, and dividing by 2, Prin. 2,
$$a^2 + b^2 + c^2 - ab - ac - bc > 0.$$

Transposing, Prin. 3, $$a^2 + b^2 + c^2 > ab + ac + bc.$$

29. If a and b are unequal, $(a-b)^2$ is positive.
$$\therefore a^2 - 2ab + b^2 > 0.$$

Transposing, Prin. 3, $$a^2 - ab + b^2 > ab.$$

If a and b are positive, multiplying by $a+b$, Prin. 2,
$$a^3 + b^3 > a^2b + ab^2.$$

30. $$\frac{a^3+b^3}{a^2+b^2} - \frac{a^2+b^2}{a+b} = \frac{ab(a^2+b^2-2ab)}{(a^2+b^2)(a+b)}. \quad (1)$$

If a and b are unequal and positive, $a^2 + b^2 - 2ab$, or $(a-b)^2$, is positive, ab is positive, and $(a^2+b^2)(a+b)$ is positive.

Hence, the second member of (1) is positive, and
$$\frac{a^3+b^3}{a^2+b^2} - \frac{a^2+b^2}{a+b} > 0; \qquad \therefore \frac{a^3+b^3}{a^2+b^2} > \frac{a^2+b^2}{a+b}.$$

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31. Except when $2a = 3b$, $(2a - 3b)^2$ is positive; that is,
$$4a^2 - 12ab + 9b^2 > 0.$$
Transposing $-12ab$, Prin. 3, $$4a^2 + 9b^2 > 12ab.$$
Dividing by $12ab$, if a and b are positive, Prin. 2,
$$\frac{a}{3b} + \frac{3b}{4a} > 1.$$

32. Except when $a = 3b$, $(a - 3b)^2$ is positive; that is,
$$a^2 - 6ab + 9b^2 > 0.$$
Subtracting b^2 from each member, Prin. 1,
$$a^2 - 6ab + 8b^2 > -b^2.$$
Factoring, $$(a - 2b)(a - 4b) > -b^2.$$
Changing signs, Prin. 4, $$(a - 2b)(4b - a) < b^2.$$

33. It has been proved in Ex. 28 that $a^2 + b^2 + c^2 > ab + ac + bc$.
Transposing, Prin. 3, $a^2 + b^2 + c^2 - ab - ac - bc > 0$.
Multiplying by $a + b + c$, if a, b, and c are positive, Prin. 2,
$$a^3 + b^3 + c^3 - 3abc > 0.$$
Transposing $-3abc$, Prin. 3, $$a^3 + b^3 + c^3 > 3abc.$$

34. Let a represent any positive real number, except 1.

It is to be proved that $$a + \frac{1}{a} > 2.$$

Except when $a = 1$, $(a - 1)^2$ is positive; that is,
$$a^2 - 2a + 1 > 0.$$
Transposing $-2a$, Prin. 3, $$a^2 + 1 > 2a.$$
Dividing by a, Prin. 2, $$a + \frac{1}{a} > 2.$$

35. Let $\frac{a}{b}$ represent the positive proper fraction.

Then, $a < b$, and it is to be proved that $\frac{a+c}{b+c} > \frac{a}{b}$, c being a positive number.

Prin. 3, $$\frac{a+c}{b+c} - \frac{a}{b} = \frac{c(b-a)}{b(b+c)}.$$
Since $b - a$ is positive, because $a < b$,
$$\frac{c(b-a)}{b(b+c)} \text{ is positive.}$$
$$\therefore \frac{a+c}{b+c} - \frac{a}{b} \text{ is positive.}$$
Hence, § 446, $$\frac{a+c}{b+c} > \frac{a}{b}.$$

36. Let $x =$ the number.

Then, $$\tfrac{1}{2}x - 1 > \tfrac{1}{3}x + 3.$$
By Prin. 3, $$\frac{x}{2} - \frac{x}{3} > 4.$$
$$\frac{x}{6} > 4.$$
By Prin. 2, $$x > 24.$$
Hence, the number must be 25, since it must be greater than 24.

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37. Let $x =$ number of pupils in the department.

Then, $5x + 25 < 6x - 74,$ (1)

and $2x + 50 > 3x - 51.$ (2)

By Prin. 3, from (1), $x > 99,$

and from (2), $x < 101.$

Hence, since the number of pupils must be greater than 99 and less than 101, there are 100 pupils in the department.

38. By the first condition, $5 \text{ A's} + \text{B's} > 51.$ (1)

By the second condition, $3 \text{ A's} - \text{B's} = 21.$ (2)

From (2), $\text{B's} = 3 \text{ A's} - 21.$ (3)

Substituting (3) in (1), $5 \text{ A's} + 3 \text{ A's} - 21 > 51.$

By Prin. 3, $8 \text{ A's} > 72.$

By Prin. 2, $\text{A's} > 9.$

Substituting in (1), $\text{B's} > 6.$

Hence, A must have more than \$9 and B more than \$6.

39. Let $x =$ number of passenger trains entering city daily.

Then, by first condition, $4x - 136 < 3x + 24,$ (1)

and by second condition, $4x + 63 < 5x - 95.$ (2)

From (1), by Prin. 3, $x < 160.$

From (2), by Prin. 3, $x > 158.$

Hence, the number of passenger trains is 159.

40. Let $x =$ the number of soldiers in a full regiment.

By first condition, $3x - 593 < 2x + 608,$ (1)

and by second condition, $8x - 577 < 9x - 1776.$ (2)

From (1), by Prin. 3, $x < 1201.$

From (2), by Prin. 3, $x > 1199.$

Hence, the number of soldiers in a full regiment is 1200.

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33. It is to be proved that, $\frac{2+x}{3+x} > \frac{2}{3}$, where $\frac{2}{3}$ is a given ratio, and x is a positive number.

$$\frac{2+x}{3+x} - \frac{2}{3} = \frac{x}{3(3+x)}.$$

$$\frac{x}{3(3+x)} \text{ is positive.}$$

$$\therefore \frac{2+x}{3+x} - \frac{2}{3} \text{ is positive.}$$

Hence, $$\frac{2+x}{3+x} > \frac{2}{3}.$$

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34. The ratio of gross earnings to capital stock $= \dfrac{\$1,500,000}{\$7,500,000} = \dfrac{1}{5} = 1:5.$

The ratio of net earnings to gross earnings $= \dfrac{\$600,000}{\$1,500,000} = \dfrac{2}{5} = 2:5.$

The ratio of net earnings to capital stock $= \dfrac{\$600,000}{\$7,500,000} = \dfrac{2}{25} = 2:25.$

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18. Given, $a:b=c:d.$

By alternation, $a:c=b:d.$

By inversion, $c:a=d:b,$

that is, $d:b=c:a.$

19. Given, $a:b=c:d.$

By alternation, $a:c=b:d.$

By inversion, $c:a=d:b.$

Writing as a fractional equation, $\dfrac{c}{a}=\dfrac{d}{b},$

which may be written, $c\times\dfrac{1}{a}=d\times\dfrac{1}{b}.$

Writing as a proportion, $c:d=\dfrac{1}{b}:\dfrac{1}{a}.$

20. Given, $a:b=c:d.$

By alternation, $a:c=b:d,$

or $b:d=a:c.$

Cubing each proportional, $b^3:d^3=a^3:c^3.$

21. Given, $a:b=c:d.$

Squaring each proportional, $a^2:b^2=c^2:d^2.$

Expressing as a fractional equation, $\dfrac{a^2}{b^2}=\dfrac{c^2}{d^2}.$

Dividing each member by c^2, $\dfrac{a^2}{b^2c^2}=\dfrac{1}{d^2}.$

Expressing as a proportion, $a^2:b^2c^2=1:d^2.$

22. Given, $a:b=c:d.$

Expressing as a fractional equation, $\dfrac{a}{b}=\dfrac{c}{d}.$

Multiplying each member by $\dfrac{m}{\frac{1}{2}}$, $\dfrac{ma}{\frac{b}{2}}=\dfrac{mc}{\frac{d}{2}}.$

Expressing as a proportion, $ma:\dfrac{b}{2}=mc:\dfrac{d}{2}.$

23. Given, $a:b=c:d.$

Expressing as a fractional equation, $\frac{a}{b}=\frac{c}{d}.$

Multiplying each member by $\frac{c}{d}$, $\frac{ac}{bd}=\frac{c^2}{d^2}.$

Expressing as a proportion, $ac:bd=c^2:d^2.$

24. Given, $a:b=c:d.$
Expressing as an integral equation, $ad=bc.$
Extracting the square root of each member,

$$\sqrt{ad}=\sqrt{bc},$$

which may be written, $\sqrt{ad}\times 1=\sqrt{b}\times\sqrt{c}.$

Expressing as a proportion, $\sqrt{ad}:\sqrt{b}=\sqrt{c}:1.$

25. Given, $a:b=c:d.$
By composition and division, $a+b:a-b=c+d:c-d.$
By alternation, $a+b:c+d=a-b:c-d.$

26. Given, $a:b=c:d.$
By composition, $a+b:a=c+d:c.$
By inversion, $a:a+b=c:c+d.$
By alternation, $a:c=a+b:c+d.$
By composition, $a+c:a=a+b+c+d:a+b.$
By inversion, $a:a+c=a+b:a+b+c+d.$
By alternation, $a:a+b=a+c:a+b+c+d.$

27. Given, $a:b=c:d.$ (1)
By Prin. 9, $a^2:b^2=c^2:d^2.$
By composition, $a^2+b^2:a^2=c^2+d^2:c^2.$
By inversion, $a^2:a^2+b^2=c^2:c^2+d^2.$
By alternation, $a^2:c^2=a^2+b^2:c^2+d^2.$
By Prin. 9, $a:c=\sqrt{a^2+b^2}:\sqrt{c^2+d^2}.$ (2)
By composition in (1), $a+b:a=c+d:c.$
By alternation, $a+b:c+d=a:c.$ (3)
Substituting (3) in (2), $a+b:c+d=\sqrt{a^2+b^2}:\sqrt{c^2+d^2}.$

28. Given, $a:b=c:d.$ (1)
Raising each proportional to the fourth power,

$$a^4:b^4=c^4:d^4. \quad (2)$$

By division, $a^4-b^4:a^4=c^4-d^4:c^4.$ (3)
Taking (1) by division, $a-b:a=c-d:c.$ (4)
Dividing (3) by (4),

$$a^3+a^2b+ab^2+b^3:a^3=c^3+c^2d+cd^2+d^3:c^3.$$

29. Given, $a:b=c:d.$ (1)

Expressing as a fractional equation, $\frac{a}{b}=\frac{c}{d}.$ (2)

Multiplying by $\frac{2}{3}$, $\frac{2a}{3b}=\frac{2c}{3d}.$

Expressing as a proportion, $2a:3b=2c:3d.$
By composition, $2a+3b:2a=2c+3d:2c.$
By alternation, $2a+3b:2c+3d=2a:2c.$

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Multiplying by 3, $6a + 9b : 6c + 9d = 6a : 6c.$ (3)

Multiplying (2) by $\frac{3}{4}$, $\frac{3a}{4b} = \frac{3c}{4d}.$

Expressing as a proportion, $3a : 4b = 3c : 4d.$

By composition, $3a + 4b : 3a = 3c + 4d : 3c.$

By alternation, $3a + 4b : 3c + 4d = 3a : 3c.$

Multiplying by 2, $6a + 8b : 6c + 8d = 6a : 6c.$ (4)

Substituting (4) in (3), $6a + 9b : 6c + 9d = 6a + 8b : 6c + 8d.$

By alternation, $6a + 9b : 6a + 8b = 6c + 9d : 6c + 8d.$

Expressing as a fractional equation, $\frac{6a+9b}{6a+8b} = \frac{6c+9d}{6c+8d}.$

Dividing by $\frac{3}{2}$, $\frac{2a + 3b}{3a + 4b} = \frac{2c + 3d}{3c + 4d}.$

Expressing as a proportion,

$$2a + 3b : 3a + 4b = 2c + 3d : 3c + 4d.$$

30. Given, $a : b = c : d.$

By alternation, $a : c = b : d.$

Expressing as a fractional equation, $\frac{a}{c} = \frac{b}{d}.$

Multiplying the first member by $\frac{2}{3}$ and the second by $\frac{8}{12}$,

$$\frac{2a}{3c} = \frac{8b}{12d}.$$

Expressing as a proportion, $2a : 3c = 8b : 12d.$

By composition and division,

$$2a + 3c : 2a - 3c = 8b + 12d : 8b - 12d.$$

31. Given, $a : b = c : d.$

By composition and division,

$$a + b : a - b = c + d : c - d.$$

By alternation, $a + b : c + d = a - b : c - d.$

By composition and division,

$$a + b + c + d : a + b - c - d = a - b + c - d : a - b - c + d.$$

By alternation,

$$a + b + c + d : a - b + c - d = a + b - c - d : a - b - c + d.$$

32. Given, $a : b = c : d,$ (1)

$a : b = b : x,$ (2)

and $b : c = c : y.$ (3)

Solving (2) for x, $x = \frac{b^2}{a}.$ (4)

Solving (3) for y, $y = \frac{c^2}{b}.$ (5)

Solving (1) for d, $d = \frac{bc}{a}.$ (6)

Represent the mean proportional between x and y by z, and the mean proportional between c and d by v.

Then, $x : z = z : y,$ (7)

and $c : v = v : d.$ (8)

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Substituting (4) and (5) in (7),

$$\frac{b^2}{a} : z = z : \frac{c^2}{b}.$$

Solving, $$z = c\sqrt{\frac{b}{a}}.$$

Substituting (6) in (8), $$c : v = v : \frac{bc}{a}.$$

Solving, $$v = c\sqrt{\frac{b}{a}}.$$

Hence, $z = v$, or the mean proportional between x and y equals the mean proportional between c and d.

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35. $$\frac{\sqrt{x} + \sqrt{m}}{\sqrt{x} - \sqrt{m}} = \frac{m}{n}.$$

By composition and division, $$\frac{2\sqrt{x}}{2\sqrt{m}} = \frac{m+n}{m-n}.$$

Dividing both terms of the first ratio by 2,

$$\frac{\sqrt{x}}{\sqrt{m}} = \frac{m+n}{m-n}.$$

Squaring, $$\frac{x}{m} = \frac{(m+n)^2}{(m-n)^2}.$$

Solving, $$x = \frac{m(m+n)^2}{(m-n)^2}.$$

36. $$\frac{\sqrt{x} + \sqrt{2a}}{\sqrt{x} - \sqrt{2a}} = \frac{2}{1}.$$

By composition and division, $$\frac{2\sqrt{x}}{2\sqrt{2a}} = \frac{3}{1}.$$

Dividing both terms of the first ratio by 2,

$$\frac{\sqrt{x}}{\sqrt{2a}} = \frac{3}{1}.$$

Squaring, $$\frac{x}{2a} = \frac{9}{1}.$$

Solving, $$x = 18a.$$

37. $$\frac{x + \sqrt{x-1}}{x - \sqrt{x-1}} = \frac{13}{7}.$$

By composition and division, $$\frac{2x}{2\sqrt{x-1}} = \frac{20}{6}.$$

Dividing both terms of each ratio by 2,

$$\frac{x}{\sqrt{x-1}} = \frac{10}{3}.$$

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Squaring, $$\frac{x^2}{x-1}=\frac{100}{9}.$$

Solving, $$x = 10 \text{ or } \tfrac{10}{9}.$$

38. $$\frac{\sqrt{x+b}+\sqrt{x-b}}{\sqrt{x+b}-\sqrt{x-b}}=a.$$

Writing $\frac{a}{1}$ for a and taking the proportion by composition and division,

$$\frac{2\sqrt{x+b}}{2\sqrt{x-b}}=\frac{a+1}{a-1}.$$

Dividing both terms of the first ratio by 2 and squaring,

$$\frac{x+b}{x-b}=\frac{a^2+2a+1}{a^2-2a+1}.$$

By composition and division, $$\frac{2x}{2b}=\frac{2(a^2+1)}{4a}.$$

Dividing both terms of each ratio by 2, $$\frac{x}{b}=\frac{a^2+1}{2a}.$$

Solving, $$x=\frac{b(a^2+1)}{2a}.$$

39. $$\frac{\sqrt{a}+\sqrt{a-x}}{\sqrt{a}-\sqrt{a-x}}=\frac{1}{a}.$$

By composition and division, $$\frac{2\sqrt{a}}{2\sqrt{a-x}}=\frac{1+a}{1-a}.$$

Dividing both terms of the first ratio by 2 and squaring,

$$\frac{a}{a-x}=\frac{1+2a+a^2}{1-2a+a^2}.$$

By division, $$\frac{x}{a}=\frac{4a}{(1+a)^2}.$$

Solving, $$x=\frac{4a^2}{(1+a)^2}.$$

40. $$\frac{\sqrt{ax}-b}{\sqrt{ax}+b}=\frac{3\sqrt{ax}-2b}{3\sqrt{ax}+5b}.$$

By composition and division, $$\frac{2\sqrt{ax}}{2b}=\frac{6\sqrt{ax}+3b}{7b}.$$

Dividing both consequents by b, and both terms of the first ratio by 2,

$$\frac{\sqrt{ax}}{1}=\frac{6\sqrt{ax}+3b}{7}.$$

Solving for $\sqrt{ax}$, $$\sqrt{ax}=3b.$$

Squaring, $$ax=9b^2.$$

$$\therefore x=\frac{9b^2}{a}.$$

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41. $$\frac{\sqrt{a}+\sqrt{a+x}}{\sqrt{a}-\sqrt{a+x}}=\frac{\sqrt{b}+\sqrt{x-b}}{\sqrt{b}-\sqrt{x-b}}.$$

By composition and division, dividing both terms of each ratio by 2,

$$\frac{\sqrt{a}}{\sqrt{a+x}}=\frac{\sqrt{b}}{\sqrt{x-b}}.$$

Squaring, $$\frac{a}{a+x}=\frac{b}{x-b}.$$

Solving, $$x=\frac{2ab}{a-b}.$$

42. $$\frac{\sqrt{x+1}+\sqrt{x-2}}{\sqrt{x+1}-\sqrt{x-2}}=\frac{\sqrt{x-3}+\sqrt{x-4}}{\sqrt{x-3}-\sqrt{x-4}}.$$

By composition and division, dividing both terms of each ratio by 2,

$$\frac{\sqrt{x+1}}{\sqrt{x-2}}=\frac{\sqrt{x-3}}{\sqrt{x-4}}.$$

Squaring, $$\frac{x+1}{x-2}=\frac{x-3}{x-4}.$$

By composition and division, $$\frac{2x-1}{3}=\frac{2x-7}{1}.$$

Solving, $$x=5.$$

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1. Let $x=$ number of dollars in smaller share.
Then, $35-x=$ number of dollars in larger share.

$$\therefore \frac{x}{35-x}=\frac{3}{4}.$$

By composition, $$\frac{35}{x}=\frac{7}{3}.$$

$$\therefore x=15,$$

whence, $$35-x=20.$$

Hence, the smaller share is \$15 and the larger is \$20.

2. Let $x=$ greater number,
and $y=$ less number.

Then, $$\frac{x}{y}=\frac{3}{2}, \quad (1)$$

and $$\frac{x+4}{y+4}=\frac{4}{3}. \quad (2)$$

From (1), $$3y=2x. \quad (3)$$

Multiplying both terms of the first ratio in (2) by 3, and both terms of the second ratio by x, $$\frac{3x+12}{3y+12}=\frac{4x}{3x}. \quad (4)$$

Substituting (3) in (4), $$\frac{3x+12}{2x+12}=\frac{4x}{3x}. \quad (5)$$

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Taking (5) by composition and division,

$$\frac{5x+24}{x}=\frac{7x}{x}.$$

$$\therefore 5x+24=7x.$$

$$x=12, \text{ greater number.} \quad (6)$$

Substituting (6) in (3), $y=8$, less number.

3. Let $x=$ greater part,
and $y=$ less part.
Then, $x+y=16, \quad (1)$

and $$\frac{xy}{x^2+y^2}=\frac{3}{10}. \quad (2)$$

Multiplying each ratio in (2) by 2 and taking the proportion by inversion,

$$\frac{x^2+y^2}{2xy}=\frac{5}{3}.$$

By composition and division, $\dfrac{x^2+2xy+y^2}{x^2-2xy+y^2}=\dfrac{8}{2}=\dfrac{4}{1}.$

Substituting 16 for $x+y$, etc., $x-y=\sqrt{\dfrac{16^2}{4}}=8. \quad (3)$

From (1) and (3), $x=12$, greater part,
and $y=4$, less part.

4. Let $x=$ smaller part.
Then, $25-x=$ larger part;

$$\therefore \frac{26-x}{x-1}=\frac{4}{1}.$$

By composition, $\dfrac{25}{x-1}=\dfrac{5}{1}.$

$$\therefore x-1=5,$$

whence, $x=6$, smaller part,
and $25-x=19$, larger part.

5. Let $x=$ one number,
and $y=$ the other.
Then, $x+y=4, \quad (1)$

and $$\frac{x^2+2xy+y^2}{x^2+y^2}=\frac{8}{5}. \quad (2)$$

Substituting 16 for $x^2+2xy+y^2$ in (2) and changing $\frac{8}{5}$ to $\frac{16}{10}$,

$$\frac{16}{x^2+y^2}=\frac{16}{10}.$$

$$\therefore x^2+y^2=10. \quad (3)$$

Taking (2) by division, $\dfrac{2xy}{x^2+y^2}=\dfrac{3}{5}. \quad (4)$

Substituting (3) in (4), $\dfrac{2xy}{10}=\dfrac{3}{5}.$

$$\therefore 2xy=6. \quad (5)$$

Subtracting (5) from (3), $x^2-2xy+y^2=4,$
whence, $x-y=\pm 2. \quad (6)$
From (1) and (6), $x=3$ or 1,
and $y=1$ or 3.
Hence, the numbers are 3 and 1.

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6. Let $x =$ the number.
Then, $x + 1 : x + 2 = x + 4 : x + 7.$
By Prin. 1, $x^2 + 8x + 7 = x^2 + 6x + 8.$
Solving, $x = \frac{1}{2}.$

7. Let $x =$ the number of native-born inhabitants,
and $y =$ the number of foreign-born inhabitants.
Then, $x : y = 5 : 2,$ (1)
and $x + y = 1{,}750{,}000.$ (2)
From (2), $y = 1{,}750{,}000 - x.$ (3)
Substituting (3) in (1),
$x : 1{,}750{,}000 - x = 5 : 2.$
By Prin. 1, $7x = 8{,}750{,}000.$
$\therefore x = 1{,}250{,}000.$ (4)
Substituting (4) in (3), $y = 500{,}000.$
Hence, there were 1,250,000 native-born inhabitants, and 500,000 foreign-born inhabitants.

8. Let $x =$ number of dollars in first partner's share,
$y =$ number of dollars in second partner's share,
and $z =$ number of dollars in third partner's share.
Then, $x + y + z = 19{,}000.$ (1)
$\therefore x + z = 13{,}000.$ (2)
$x : y = y : z.$ (3)
Substituting 6000 for y, by Prin. 1,
$xz = 36{,}000{,}000.$ (4)
From (2), $z = 13{,}000 - x.$ (5)
Substituting (5) in (4), and solving,
$x = 4000$ or $9000.$ (6)
Substituting (6) in (2), $z = 9000$ or $4000.$
Hence, the shares of the three partners are \$4000, \$6000, and \$9000.

9. Let $x =$ the number to be added.
Then, $x + 11 : x + 17 = x + 2 : x + 5.$
By Prin. 1, $x^2 + 16x + 55 = x^2 + 19x + 34.$
Solving, $x = 7.$

10. Let w, x, y, and z represent the four numbers in proportion.
Then, $w : x = y : z.$ (1)
$w - y = \frac{8}{3}.$ (2)
$x + y = \frac{19}{3}.$ (3)
$y : z = 4 : 5.$ (4)
From (4), by Prin. 2, $y = \dfrac{4z}{5}.$ (5)
Adding (2) and (3), $w + x = 9.$
$\therefore x = 9 - w.$ (6)
Substituting (5) and (6) in (1),
$$w : 9 - w = \frac{4z}{5} : z.$$
Solving, $w = 4.$ (7)
Substituting (7) in (6), $x = 5.$ (8)
Substituting (8) in (3), $y = \frac{4}{3}.$ (9)
Substituting (9) in (5), $z = \frac{5}{3}.$

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11. Given, n, $n+1$, $n+2$, $n+3$, four consecutive integers.

If $\qquad n : n+1 = n+2 : n+3,$

then, by Prin. 1, $\qquad n^2 + 3n = n^2 + 3n + 2.$

But this is seen to be untrue. Hence, since these were any four consecutive integers, no four consecutive integers can form a proportion.

12. If $\qquad 2m+1 : 2n = 2x : 2y+1, \qquad (1)$

then, by Prin. 1, $\qquad 4nx = 4my + 2y + 2m + 1. \qquad (2)$

But since $2n$ and $2x$ are even numbers, $2n \cdot 2x$, or $4nx$, is an even number; similarly, $4my$ is even; then, $4my + 2y + 2m$ is even; and $4my + 2y + 2m + 1$ is odd; hence, (2) is not true, for an even number cannot be equal to an odd number. Therefore, (1) is not a true proportion.

That is, the ratio of an odd number to an even number cannot equal the ratio of another even number to another odd number.

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13. Given, $\qquad \text{Area} = \frac{1}{2}ab$ or $\text{Area} = \frac{1}{2}ch.$

$$\therefore \tfrac{1}{2}ab = \tfrac{1}{2}ch.$$

Dividing by $\frac{1}{2}$, $\qquad ab = ch.$

By Prin. 3, $\qquad a : c = h : b.$

14. Given, $\qquad a : p = p : b. \qquad (1)$

By Prin. 1, $\qquad p^2 = ab. \qquad (2)$

Substituting the values, $\qquad (20)^2 = a(50 - a).$

Solving, $\qquad a = 40$ or $10. \qquad (3)$

Substituting (3) in (2), $\qquad b = 10$ or $40.$

Since, from the figure a is the shorter part of the diameter, the small value is to be taken for a. Hence, $\qquad a = 10$ and $b = 40.$

15. Given, $\qquad c + e : t = t : e.$

By Prin. 1, $\qquad t^2 = ce + e^2.$

Substituting the given values, $\qquad t^2 = \frac{252}{5} \cdot \frac{48}{5} + \left(\frac{48}{5}\right)^2.$

Solving, $\qquad t = 24.$

16. Given, $\qquad N : N' = l' : l.$

Substituting the given values, $\qquad 256 : 384 = l' : 42.$

By Prin. 1, $\qquad 384\,l' = 10{,}752.$

Solving, $\qquad l' = 28.$

Hence, the length of the g string is 28 inches.

17. Given, $\qquad T^2 : t^2 = L : l.$

Substituting given values, $\qquad T^2 : 1 = 156.4 : 39.1.$

By Prin. 1, $\qquad 39.1\,T^2 = 156.4.$

$$T^2 = 4.$$

$$\therefore T = 2.$$

Hence, a pendulum 156.4 inches long will oscillate once in 2 seconds.

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18. Given, $T^2 : t^2 = L : l.$

Then, $(60)^2 : 1 = L : 39.1.$

By Prin. 1, $L = 140{,}760.$

Hence, a pendulum would have to be 140,760 inches, or 11,730 feet, in length to oscillate once a minute.

VARIATION

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2. Given, $x \propto \frac{y}{z}$, or $x = \frac{ky}{z}$, (1)

and $x = 2$ when $y = 12$ and $z = 2$.

Substituting these values in (1) and solving for k,

$k = \frac{1}{3}.$ (2)

From (1) and (2), $x = \frac{y}{3z}.$ (3)

Substituting 84 for y and 7 for z in (3), $x = \frac{84}{21} = 4.$

3. Given, $x \propto \frac{y}{z}$, or $x = \frac{ky}{z}$, (1)

and $x = 60$ when $y = 24$ and $z = 2$.

Substituting these values in (1) and solving for k,

$k = 5.$ (2)

From (1) and (2), $x = \frac{5y}{z}$,

whence, the value of y is $y = \frac{xz}{5}.$ (3)

Substituting 20 for x and 7 for z in (3), $y = 28.$

4. Given, $x \propto \frac{yz}{w^2}$, or $x = \frac{kyz}{w^2}$, (1)

and $x = 30$ when $y = 3$, $z = 5$, and $w = 4$.

Substituting these values in (1) and solving for k,

$k = 32.$ (2)

From (1) and (2), $x = \frac{32\,yz}{w^2}.$

6. Given $x \propto \frac{1}{y}$ and $y \propto \frac{1}{z}$, to prove that $x \propto z$.

Let m be the constant ratio of x to $\frac{1}{y}$, and n the constant ratio of y to $\frac{1}{z}$.

Then, $x = \frac{m}{y}$, (1)

and $y = \frac{n}{z}.$ (2)

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From (2), $$\frac{n}{z} = y. \qquad (3)$$

Multiplying (1) by (3), $$\frac{nx}{z} = m.$$

$$\therefore x = \frac{m}{n}z. \qquad (4)$$

Since m and n are constants, $\frac{m}{n}$ is a constant.

Hence, (4) may be written $$x \propto z.$$

7. Given $x \propto y$ and $z \propto y$, to prove that $(x \pm z) \propto y$.

Let m be the constant ratio of x to y, and n the constant ratio of z to y.

Then, $$x = my, \qquad (1)$$

and $$z = ny. \qquad (2)$$

Adding (2) to (1), $$x + z = (m + n)y. \qquad (3)$$

Subtracting (2) from (1), $$x - z = (m - n)y. \qquad (4)$$

Since m and n are constants, $m + n$ and $m - n$ are constants.

Hence, $$(x \pm z) \propto y.$$

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2. Let c stand for the circumference and d for the diameter of any circle, and let k be the constant ratio c to d.

Then, $$c \propto d, \text{ or } c = kd. \qquad (1)$$

Substituting 3.1416 for c and 1 for d, (1) becomes
$$3.1416 = k.$$

Substituting 3.1416 for k and 100 for d in (1),
$$c = 3.1416 \times 100 = 314.16.$$

Hence, the circumference is 314.16 feet.

3. Let a represent the area and d the diameter of any circle, and let k represent the constant ratio of a to d^2.

Then, $$a \propto d^2, \text{ or } a = kd^2. \qquad (1)$$

Substituting 78.54 for a and 10 for d, (1) becomes
$$78.54 = k \times 100.$$
$$\therefore k = .7854.$$

Substituting .7854 for k and 20 for d in (1),
$$a = .7854 \times 20^2 = 314.16.$$

Hence, the area is 314.16 square feet.

4. Let d represent the distance in feet and t the time in seconds of any falling body, and let k represent the constant ratio of d to t^2.

Then, $$d \propto t^2, \text{ or } d = kt^2. \qquad (1)$$

Substituting 64.32 for d and 2 for t, (1) becomes
$$64.32 = k \cdot 4.$$
$$\therefore k = 16.08.$$

Substituting 16.08 for k and 5 for t in (1),
$$d = 16.08 \times 25 = 402.$$

Hence, the stone will fall 402 feet in 5 seconds.

5. Let V and D denote the volume and diameter of the earth, and V' and D', the volume and diameter of the sun.

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Then, $V \propto D^3$ or $V = kD^3$, (1)

and $V' \propto D'^3$ or $V' = kD'^3$. (2)

From problem, $\frac{D'}{D} = 109.3.$ (3)

Cubing (3), $\frac{D'^3}{D^3} = 1{,}305{,}751.357.$ (4)

From (1), $D^3 = \frac{V}{k}.$ (5)

From (2) $D'^3 = \frac{V'}{k}.$ (6)

Substituting (5) and (6) in (4), $V' = 1{,}305{,}751.357\ V.$

Hence, the volume of the sun is 1,305,751.357 times that of the earth.

6. Let a represent the number of men, t the number of days, and w the number of days' work in the piece of work.

Then, $w \propto at$, or $w = at$, k being 1. (1)

Substituting 10 for a and 20 for t, (1) becomes

$$w = 10 \times 20 = 200.$$

Substituting 200 for w and 25 for a in (1),

$$200 = 25\ t.$$

$$\therefore t = 8.$$

Hence, 25 men can do the work in 8 days.

7. Adopting the notation of Ex. 6, and substituting ab for w, and c for t in (1), if $x =$ the number of men required,

$$ab = xc.$$

$$\therefore x = \frac{ab}{c}.$$

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8. Let a represent the area, b the base, and h the altitude of any triangle, and let k represent the constant ratio of a to $b \times h$.

Then, $a \propto bh$, or $a = kbh.$ (1)

Substituting 36 for a, 12 for b, and 6 for h, (1) becomes

$$36 = k \times 12 \times 6.$$

$$\therefore k = \tfrac{1}{2}, \text{ the constant ratio.}$$

Substituting $\frac{1}{2}$ for k, 8 for b, and 10 for h in (1),

$$a = \tfrac{1}{2} \times 8 \times 10 = 40.$$

Hence, the area of the triangle is 40 square inches.

9. Let w represent the weight in pounds, l the length in yards, and a the area in square inches of the cross section of a wrought iron bar, and let k represent the constant ratio of w to $l \times a$.

Then, $w \propto la$, or $w = kla.$ (1)

Substituting 10 for w, 1 for l, and 1 for a, (1) becomes

$$10 = k \times 1 \times 1.$$

$$\therefore k = 10.$$

Substituting 10 for k, 12 for l, and 16 for a in (1),

$$w = 10 \times 12 \times 16 = 1920.$$

Hence, the wrought iron bar weighs 1920 pounds.

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10. Adopting the notation of Ex. 9 and substituting $\frac{10}{12}$ for k, 8 for l, and 144 for a in (1),

$$w = \tfrac{10}{12} \times 8 \times 144 = 960.$$

Hence, the wooden beam weighs 960 pounds.

11. Adopting the notation of Ex. 9, and substituting $\frac{10}{4}$ for k, $\frac{2}{9}$ for l, and 8 for a in (1),

$$w = \tfrac{10}{4} \times \tfrac{2}{9} \times 8 = 4\tfrac{4}{9}.$$

Hence, the brick weighs $4\frac{4}{9}$ pounds.

12. Let W represent the larger weight in pounds and D its distance in feet from the fulcrum; also let w represent the smaller weight and d its distance from the fulcrum, in the same units.

Then, $$D : d = \frac{1}{W} : \frac{1}{w},$$

or $$D : d = w : W. \tag{1}$$

§ 475, Prin. 1, $$WD = wd. \tag{2}$$

Let $x =$ number of feet the heavier boy has.

Then, $\frac{17}{2} - x =$ number of feet the lighter boy has.

Substituting x for D, $\frac{17}{2} - x$ for d, 90 for W, and 80 for w in (2),

$$90x = 80(\tfrac{17}{2} - x). \tag{3}$$

Solving (3), $$x = 4,$$

whence, $$\tfrac{17}{2} - x = 4\tfrac{1}{2}.$$

Hence, the heavier boy has 4 feet of the board and the lighter boy has $4\frac{1}{2}$ feet.

13. Let $x =$ number of feet the greater weight has.

Then, $4 - x =$ number of feet the less weight has.

By (2), Ex. 12, $$100x = 60(4 - x).$$

Solving, $$x = 1\tfrac{1}{2},$$

whence, $$4 - x = 2\tfrac{1}{2}.$$

Hence, the point of the stick resting on his shoulder is $1\frac{1}{2}$ feet from the 100-pound end, or $2\frac{1}{2}$ feet from the 60-pound end.

14. From problem, $$H \propto ND^3 \text{ or } H = kND^3. \tag{1}$$

Substituting the given values in (1), $$585 = k \cdot 150 \cdot (5)^3.$$

$$\therefore k = \frac{585}{18{,}750} = \frac{39}{1250}.$$

Substituting the second set of values in (1), $$H = \frac{39}{1250} \cdot 75 \cdot (6)^3.$$

$$\therefore H = 505.44.$$

Hence, the shaft could transmit 505.44 horse power.

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15. Let W represent the weight in pounds of a body near the earth's surface and d its distance in miles from the earth's center.

Then, $$W \propto \frac{1}{d^2} \text{ or } W = \frac{k}{d^2}, \tag{1}$$

k being the constant ratio between W and $\frac{1}{d^2}$.

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Since a brick 4000 miles from the earth's center weighs 4 pounds, substituting 4 for W and 4000 for d, (1) becomes

$$4 = \frac{k}{(4000)^2}.$$

$$\therefore k = 4 \times (4000)^2.$$

Substituting $4 \times (4000)^2$ for k and 8000 for d in (1), the weight of the brick 8000 miles from the earth's center is

$$W = \frac{4 \times (4000)^2}{(8000)^2} = 1.$$

Hence, the brick would weigh 1 pound.

16. Let $x =$ the number of pounds the wire weighs,
$y =$ the number of miles in length of wire,
and $d =$ the number of inches in diameter of wire.

Then, $$x \propto d^2y \text{ or } x = kd^2y. \qquad (1)$$

Since 3 miles of wire .08 of an inch in diameter weighs 288 pounds, substituting 288 for x, 3 for y, and .08 for d, (1) becomes

$$288 = 3 \times .0064 \times k.$$

$$\therefore k = 15{,}000.$$

Substituting 15,000 for k, $\frac{1}{2}$ for y, and .16 for d, (1) becomes

$$x = \tfrac{1}{2} \times (.16)^2 \times 15{,}000.$$

$$\therefore x = 192.$$

17. Let d represent the distance in feet and u the illumination from a source of light upon an object d feet away.

Then, $$u \propto \frac{1}{d^2}, \text{ or } u = \frac{k}{d^2}. \qquad (1)$$

It is evident from (1) that if the first member, u, is divided by 4, the second member must be divided by 4 by changing d^2 to $4d^2$, that is, to $(2d)^2$, since k cannot be changed. That is, the distance of the screen from the lantern must be doubled.

Hence, the screen must be moved 10 feet farther away.

18. Let l represent the length in inches of a pendulum, and n the number of times the pendulum oscillates in any given time, as 1 second.

Then, $$n \propto \frac{1}{\sqrt{l}}, \text{ or } n = \frac{k}{\sqrt{l}}. \qquad (1)$$

Substituting 1 for n and 39.1 for l in (1),

$$1 = \frac{k}{\sqrt{39.1}}.$$

$$\therefore k = \sqrt{39.1}. \qquad (2)$$

Substituting $\sqrt{39.1}$ for k and 2 for n in (1),

$$2 = \frac{\sqrt{39.1}}{\sqrt{l}}.$$

Solving, $$l = \frac{39.1}{4} = 9.775.$$

Hence, a pendulum that oscillates twice a second must be 9.775 inches long.

Since the pendulum oscillates once in three seconds, in one second it would go $\frac{1}{3}$ of the distance. Hence, $n = \frac{1}{3}$.

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Substituting $\frac{1}{3}$ for n and $\sqrt{39.1}$ for k in (1),

$$l = \frac{\sqrt{39.1}}{\frac{1}{3}} = 351.9.$$

Hence, a pendulum that oscillates once in three seconds must be 351.9 inches long.

19. Let V represent the volume in cubic inches and R the radius, in inches, of a sphere.

Then, $V \propto R^3$, or $V = kR^3$. (1)

Let a, b, and c represent the volumes, respectively, of the spheres whose radii are 6 inches, 8 inches, and 10 inches.

Then, by (1), $a = k \times 6^3 = 216\,k$, (2)

$b = k \times 8^3 = 512\,k$, (3)

and $c = k \times 10^3 = 1000\,k$. (4)

Adding, the volume of all is $a + b + c = 1728\,k$. (5)

But (5) may be written $a + b + c = k \times 12^3$.

Hence, by (1), the radius of the resulting sphere is 12 inches.

20. Let V represent the volume in cubic feet, H the altitude in feet, and D the diameter of the base, in feet, of any cone.

Then, $V \propto D^2H$, or $V = kD^2H$. (1)

By (1), $P = k \times 25 \times 10 = 250\,k$, (2)

and $R = k \times 100 \times 5 = 500\,k$. (3)

Adding, $P + R = S = 750\,k$. (4)

Let d represent the diameter of the base, in feet, of S.

Then, by (1), $S = k \times d^2 \times 30 = 30\,d^2k$. (5)

From (5) and (4), $30\,d^2k = 750\,k$. (6)

Solving (6) for d, $d = \pm 5$.

Hence, the diameter of the base of S is 5 feet.

21. Adopting the notation of Ex. 4, $d = kt^2$. (1)

Substituting 31.5 for d and 1.4 for t, (1) becomes

$$31.5 = k \times 1.96.$$

$$\therefore k = \frac{31.5}{1.96}.$$

Substituting $\frac{31.5}{1.96}$ for k and 3 for t in (1), $d = \frac{31.5 \times 9}{1.96} = 144\frac{9}{14}$.

Hence, the height of the tower is $144\frac{9}{14}$ feet.

PROGRESSIONS

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11. $s = 1 + 2 + 3 + \cdots + 12$

$$= \frac{n}{2}(a + l) = 6 \times 13 = 78.$$

12. $l = a + (n - 1)\,d = 16\frac{1}{12} + 9 \times 32\frac{1}{6} = 305\frac{7}{12}$.

$$s = \frac{n}{2}(a + l) = 5 \times 321\frac{2}{3} = 1608\frac{1}{3}.$$

Hence, the body will fall $1608\frac{1}{3}$ feet in 10 seconds.

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13. The series is 8, 16, 24, etc.

$$l = a + (n-1)d = 8 + (30-1)8 = 240.$$

$$s = \frac{n}{2}(a+l) = 15 \times 248 = 3720.$$

Hence, she must walk 3720 feet.

14. $l = a + (n-1)d = 4 + (12-1)2 = 26.$

$$s = \frac{n}{2}(a+l) = \frac{12}{2}(4+26) = 180.$$

Hence, the toboggan slide is 180 feet long.

15. $l = a + (n-1)d = .18 + [(180+40) - 1].36 = 79.02.$

$$s = \frac{n}{2}(a+l) = \frac{220}{2}(.18 + 79.02) = 8712.$$

Hence, the train went 8712 feet before reaching top speed.

16. $l = a + (n-1)d = 12 + (8-1)\,12 = 96.$

$$s = \frac{n}{2}(a+l) = \frac{8}{2}(12+96) = 432.$$

Hence, each contestant must go 432 feet in order to finish the race.

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3. In the series 2, 6, 10, $\cdots$ 66, $a = 2$, $l = 66$, and $d = 4$; and n is to be found.

From $\quad l = a + (n-1)d,\ n = \dfrac{l-a}{d} + 1 = \dfrac{66-2}{4} + 1 = 17.$

4. In the series 1, 6, 11, $\cdots$ 61, $a = 1$, $l = 61$, and $d = 5$; and s is to be found.

From $\quad l = a + (n-1)d,\ n = \dfrac{l-a}{d} + 1 = \dfrac{61-1}{5} + 1 = 13.$

$$s = n\left(\frac{a+l}{2}\right) = 13 \times 31 = 403.$$

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5. In the series $-1, 2, 5, \cdots$, $a = -1$, $d = 3$, and $s = 221$ are given; and n is to be found.

From $\quad l = a + (n-1)d = -1 + 3(n-1)$ we obtain

$$l = 3n - 4. \tag{1}$$

From $\quad s = \dfrac{n}{2}(a+l)$, or $221 = \dfrac{n}{2}(-1+l)$ we obtain

$$l = \frac{442+n}{n}. \tag{2}$$

Eliminating l and solving (1) and (2) for n,

$$n = 13 \text{ or } -11\tfrac{1}{3}.$$

Hence, the given series has 13 terms.

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6. In the series 2, 9, 16, $\cdots$ 86, $a = 2$, $l = 86$, and $d = 7$; and n and s are to be found.

From $\quad l = a + (n-1)d,\ n = \dfrac{l-a}{d} + 1 = \dfrac{86-2}{7} + 1 = 13.$

$$s = n\left(\frac{a+l}{2}\right) = 13 \times 44 = 572.$$

7. In the series -10, $-8\frac{1}{2}$, -7, $\cdots$ to 10 terms, $a = -10$, $d = \frac{3}{2}$, and $n = 10$; and l and s are to be found.

$$l = a + (n-1)d = -10 + 9 \times \tfrac{3}{2} = 3\tfrac{1}{2}.$$

$$s = \frac{n}{2}(a+l) = 5(-10 + 3\tfrac{1}{2}) = -32\tfrac{1}{2}.$$

8. In the series $\cdots$ 22, 27, 32, $\cdots$, we have $d = 5$, $s = 714$, and $n = 17$; and a and l are to be found.

From $l = a + (n-1)d$, $\qquad l - a = 16 \times 5 = 80, \qquad (1)$

and from $s = \dfrac{n}{2}(a+l)$, $\qquad l + a = \dfrac{2s}{n} = \dfrac{1428}{17} = 84. \qquad (2)$

From (2) and (1), $\qquad a = 2,$

and $\qquad l = 82.$

9. Given $s = 113\frac{2}{3}$, $a = \frac{1}{3}$, and $d = 2$, to find n.

From $s = \dfrac{n}{2}(a+l)$, $\qquad n = \dfrac{2s}{a+l} = \dfrac{227\frac{1}{3}}{\frac{1}{3}+l} = \dfrac{682}{1+3l}. \qquad (1)$

Substituting $a + (n-1)d = \frac{1}{3} + 2n - 2$ for l in (1),

$$n = \frac{682}{1+1+6n-6} = \frac{682}{6n-4} = \frac{341}{3n-2}. \qquad (2)$$

Clearing (2) of fractions, $\qquad 3n^2 - 2n = 341. \qquad (3)$

Solving (3), $\qquad n = 11 \text{ or } -\frac{31}{3}.$

Rejecting the negative value, $\qquad n = 11.$

10. In the series -16, -11, -6, $\cdots$ 34, $a = -16$, $l = 34$, and $d = 5$; and s is required.

$$s = n\left(\frac{a+l}{2}\right) = n\left(\frac{-16+34}{2}\right) = 9n. \qquad (1)$$

From $\quad l = a + (n-1)d,\ n = \dfrac{l-a}{d} + 1 = \dfrac{34+16}{5} + 1 = 11. \qquad (2)$

Substituting 11 for n in (1), $\qquad s = 99.$

11. In the series $\cdots -1$, 3, 7, $\cdots$ 23, $d = 4$, $l = 23$, and $n = 16$; and s is required.

$$s = \frac{n}{2}(a+l) = 8a + 184. \qquad (1)$$

From $\quad l = a + (n-1)d,\ a = l - (n-1)d = 23 - 15 \times 4 = -37. \qquad (2)$

Substituting -37 for a in (1), $\qquad s = -296 + 184 = -112.$

12. Given $d = 2$, $s = 300$, and $n = 20$, to find a and l.

From $s = \dfrac{n}{2}(a+l)$, $\qquad a + l = \dfrac{2s}{n} = \dfrac{600}{20} = 30. \qquad (1)$

From $l = a + (n-1)d$, $a - l = -(n-1)d = -19 \times 2 = -38. \qquad (2)$

From (1) and (2), $\qquad a = -4,$

and $\qquad l = 34.$

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13. Since the 6th term is 10 and the 11th term is 0, in these five terms, the A.P. is decreased by 10. Hence, the common difference is -2.

$$l = a + (n-1)d = a + (14-1)(-2) = a - 26. \quad (1)$$

$$s = \frac{n}{2}(a+l).$$

$$98 = \frac{14}{2}(a+l).$$

$$7a + 7l = 98. \quad (2)$$

From (1), $$a - l = 26. \quad (3)$$

(3) × 7, $$7a - 7l = 182. \quad (4)$$

(2) + (4), $$14a = 280.$$

$$a = 20. \quad (5)$$

Substituting (5) in (1), $$l = -6.$$

Hence, the A.P. is 20, 18, 16 ⋯ 0, -2, -4, -6.

14. Substituting in $l = a + (n-1)d$, 15th term $= a + 14d$. (1)

14th term $= a + 13d$. (2)

13th term $= a + 12d$. (3)

Adding (1), (2), and (3),

$$a + 14d + a + 13d + a + 12d = 132.$$

$$3a + 39d = 132.$$

$$\therefore a + 13d = 44. \quad (4)$$

7th term $= a + 6d$. (5)

6th term $= a + 5d$. (6)

5th term $= a + 4d$. (7)

Adding (5), (6), and (7), $$3a + 15d = 60.$$

$$\therefore a + 5d = 20. \quad (8)$$

Subtracting (8) from (4), $$8d = 24.$$

$$d = 3. \quad (9)$$

Substituting (9) in (8), $$a = 5.$$

Hence, the series is 5, 8, 11, 14, ⋯, 41, 44, 47.

15. $$s = \frac{n}{2}(a+l).$$

$$2s = an + nl.$$

$$l = \frac{2s - an}{n}.$$

16. $$l = a + (n-1)d. \quad (1)$$

$$s = \frac{n}{2}(a+l). \quad (2)$$

From (1) $$n = \frac{l - a + d}{d}. \quad (3)$$

Substituting (3) in (2), $$s = \frac{l - a + d}{2d}(a+l).$$

$$s = \frac{l^2 + dl - a^2 + ad}{2d}.$$

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17. $$l = a + (n-1)d. \quad (1)$$

$$s = \frac{n}{2}(a + l). \quad (2)$$

Substituting (1) in (2), $$s = \frac{n}{2}(a + a + nd - d).$$

Simplifying, $$2s = 2an + n^2d - dn.$$

$$2an = 2s - dn^2 + dn.$$

$$\therefore a = \frac{2s - dn^2 + dn}{2n}.$$

18. $$l = a + (n-1)d. \quad (1)$$

$$s = \frac{n}{2}(a + l). \quad (2)$$

Substituting (1) in (2), $$s = \frac{n}{2}(a + a + dn - d).$$

$$2s = 2an + dn^2 - dn.$$

$$dn^2 - dn = 2s - 2an.$$

$$d = \frac{2s - 2an}{n^2 - n}.$$

19. $$l = a + (n-1)d. \quad (1)$$

From (1), $$a = l - nd + d. \quad (2)$$

$$s = \frac{n}{2}(a + l). \quad (3)$$

Substituting (2) in (3), $$s = \frac{n}{2}(l - dn + d + l).$$

$$2s = 2ln - dn^2 + dn.$$

$$dn^2 - dn = 2ln - 2s.$$

$$d = \frac{2ln - 2s}{n^2 - n}.$$

20. $$s = \frac{n}{2}(a + l).$$

$$2s = an + ln.$$

$$n = \frac{2s}{a + l}.$$

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2. From $l = a + (n-1)d$, $d = \dfrac{l-a}{n-1} = \dfrac{6-1}{11-1} = \dfrac{5}{10} = \dfrac{1}{2}$.

Hence, the A.P. is 1, $1\frac{1}{2}$, 2, $2\frac{1}{2}$, 3, $3\frac{1}{2}$, 4, $4\frac{1}{2}$, 5, $5\frac{1}{2}$, 6.

3. From $l = a + (n-1)d$, $d = \dfrac{l-a}{n-1} = \dfrac{2-24}{12-1} = -2$.

Hence, the A.P. is 24, 22, 20, 18, $\cdots$, 2.

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4. From $l = a + (n-1)d$, $d = \dfrac{l-a}{n-1} = \dfrac{-14-10}{9-1} = -3$.

Hence, the A.P. is 10, 7, 4, 1, -2, -5, -8, -11, -14.

5. From $l = a + (n-1)d$, $d = \dfrac{l-a}{n-1} = \dfrac{2-(-1)}{8-1} = \dfrac{3}{7}$.

Hence, the A.P. is -1, $-\frac{4}{7}$, $-\frac{1}{7}$, $\frac{2}{7}$, $\frac{5}{7}$, $1\frac{1}{7}$, $1\frac{4}{7}$, 2.

6. From $l = a + (n-1)d$, $d = \dfrac{l-a}{n-1} = \dfrac{20-15}{16-1} = \dfrac{1}{3}$.

Hence the A.P. is 15, $15\frac{1}{3}$, $15\frac{2}{3}$, 16, $16\frac{1}{3}$, $\cdots$, 20.

7. From $l = a + (n-1)d$, $d = \dfrac{l-a}{n-1} = \dfrac{(a+b)-(a-b)}{5-1} = \dfrac{b}{2}$.

Hence, the A.P. is $a-b,\ a-b+\dfrac{b}{2},\ a-b+b,\ a-b+\dfrac{3b}{2},\ a+b$;

or
$$a-b,\ \frac{2a-b}{2},\ a,\ \frac{2a+b}{2},\ a+b.$$

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2. Let the series be $x-y$, x, $x+y$.

Then, $3x = 18$, (1)

and $x(x-y)(x+y) = 120$. (2)

From (1), $x = 6$. (3)

Substituting (3) in (2), $6(6-y)(6+y) = 120$.

$36 - y^2 = 20$. (4)

Solving (4), $y = \pm 4$.

Forming the series from $x = 6$ and $y = \pm 4$, the terms are

2, 6, 10, or 10, 6, 2;

that is, the numbers are 2, 6, and 10.

3. Let the series be $x-y$, x, $x+y$.

Then, $3x = 21$, (1)

and $(x-y)^2 + x^2 + (x+y)^2 = 155$. (2)

From (1), $x = 7$. (3)

From (2), $3x^2 + 2y^2 = 155$. (4)

Substituting (3) in (4), $147 + 2y^2 = 155$. (5)

Solving (5), $y = \pm 2$.

Forming the series from $x = 7$ and $y = \pm 2$, the terms are

5, 7, 9, or 9, 7, 5;

that is, the numbers are 5, 7, and 9.

4. Let the series be $x-y$, x, $x+y$.

Then, $(x-y)^2 + x^2 + (x+y)^2 = 93$, (1)

and $x + y = 4(x-y)$. (2)

From (1), $3x^2 + 2y^2 = 93$. (3)

From (2), $y = \frac{3}{5}x$. (4)

Solving (3) and (4), $x = 5$ or -5,

and $y = 3$ or -3.

Forming the series from $x = 5$ and $y = 3$, and from $x = -5$ and $y = -3$, the terms are 2, 5, 8, or -2, -5, -8.

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5. In the series $1, 3, 5, \cdots 99$, $a = 1$, $d = 2$, and $l = 99$; and s is to be found.

From $$l = a + (n-1)d,\ n = \frac{l-a}{d} + 1 = 50.$$

$$s = \frac{n}{2}(a+l) = 25 \times 100 = 2500.$$

6. Given $$al = 70, \tag{1}$$

and in $s = \frac{n}{2}(a+l)$, since $n = 10$, and $s = 95$,

$$a + l = 19. \tag{2}$$

Solving (1) and (2), $a = 14$ or 5,

and $l = 5$ or 14.

Hence, the extremes are 5 and 14.

7. Let l = number of logs in the bottom layer.

Then, in the series, $1, 2, 3, \cdots l$, $a = 1$, $d = 1$, and $s = 55$; and l is to be found.

From $l = a + (n-1)d$, $$n = \frac{l-a}{d} + 1 = \frac{l-1}{1} + 1 = l. \tag{1}$$

From $s = \frac{n}{2}(a+l)$, $$n = \frac{2s}{a+l} = \frac{110}{1+l}. \tag{2}$$

Eliminating n, $$l = \frac{110}{1+l}. \tag{3}$$

Solving (3) for l, $l = 10$ or -11.

Hence, there were 10 logs in the bottom layer.

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8. Let n = number of yards in depth of well.

Then, in the series $1\frac{1}{2}, 1\frac{3}{4}, 2, \cdots$, $a = 1\frac{1}{2}$, $d = \frac{1}{4}$, and $s = 19$; and n is to be found.

From $s = \frac{n}{2}(a+l)$, $$l = \frac{2s}{n} - a = \frac{38}{n} - 1\frac{1}{2}. \tag{1}$$

Also, $$l = a + (n-1)d = 1\frac{1}{2} + \frac{n-1}{4}. \tag{2}$$

Eliminating l, $$1\frac{1}{2} + \frac{n-1}{4} = \frac{38}{n} - 1\frac{1}{2}. \tag{3}$$

Solving (3) for n, $n = 8$ or -19.

Hence, the well was 8 yards deep.

9. Let x = number of means required.

Then, from $s = \frac{n}{2}(a+l)$, $$116 = \frac{n}{2}(4 + 25),$$

$$\therefore n = 8.$$

Since the number of means is 2 less than the number of terms,

$$x = n - 2 = 6.$$

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10. Let $a, a + d, a + 2d, \cdots$ be an A.P., and let m be any common multiplier of its terms.

It is to be proved that ma, $m(a + d)$, and $m(a + 2d)$, $\cdots$ are in arithmetical progression.

$$m(a + d) - ma = m(a + d - a) = md.$$
$$m(a + 2d) - m(a + d) = m[a + 2d - (a + d)] = md.$$

Since the successive terms of the series ma, $m(a + d)$, $m(a + 2d)$, $\cdots$ have a common difference md, the series is an A.P.

11. Let x, $x + 1$, $x + 2$, $x + 3$, $\cdots$ be any consecutive integers.

It is to be proved that $(x+1)^2 - x^2$, $(x+2)^2 - (x+1)^2$, $(x+3)^2 - (x+2)^2$, $\cdots$ are in arithmetical progression, and that the common difference is 2.

$$(x + 1)^2 - x^2 = x^2 + 2x + 1 - x^2 = 2x + 1.$$
$$(x + 2)^2 - (x + 1)^2 = x^2 + 4x + 4 - (x^2 + 2x + 1) = 2x + 3.$$
$$(x + 3)^2 - (x + 2)^2 = x^2 + 6x + 9 - (x^2 + 4x + 4) = 2x + 5.$$

But $2x + 1$, $2x + 3$, $2x + 5$, are three terms of an A.P. whose common difference is 2,

Since to x may be assigned any integral value or any number of successive integral values, the proof given above is general.

12. In the series, 1, 3, 5, $\cdots$ to n terms, $a = 1$, $d = 2$, $l = a + (n - 1)d = 1 + 2(n - 1) = 2n - 1$; and s is to be found.

$$s = n\left(\frac{a + l}{2}\right) = n\left(\frac{1 + 2n - 1}{2}\right) = n \cdot \frac{2n}{2} = n^2.$$

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12. $l = ar^{n-1} = a^{19}b \times \left(\frac{b}{a}\right)^{10} = a^9b^{11}.$

13. $l = ar^{n-1} = 2 \times \left(\frac{1}{\sqrt{2}}\right)^{n-1} = 2^{1-\frac{n-1}{2}} = 2^{\frac{3-n}{2}}.$

14. $l = ar^{n-1} = 2000 \times 2^5 = 64{,}000.$
Hence, he will have \$64,000 at the end of the sixth year.

15. $l = ar^{n-1} = 76.3 \times 2^{5-1} = 1220.8.$
Hence, the population will be 1220.8 millions in the year 2000.

16. $l = ar^{n-1} = 512 \times (\frac{5}{4})^{6-1} = 1562\frac{1}{2}.$
Hence, his salary the sixth year was \$1562.50.

17. $l = ar^{n-1} = 20{,}736 \times (\frac{5}{4})^{5-1} = 50{,}625.$
Hence, the population at the end of 40 years was 50,625.

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18. $l = ar^{n-1} = 2 \times 2^9 = 1024.$
Hence, the last bushel cost him \$10.24.

19. $l = ar^{n-1} = 20{,}000(\frac{9}{10})^5 = 20{,}000 \times \frac{59049}{100000} = 11{,}809.80.$
Hence, the machinery at the end of 5 years will be worth \$11,809.80.

20. $l = ar^{n-1} = 1 \times 150^4$, number of grains, fourth year.

$\frac{150 \times 150 \times 150 \times 150}{150 \times 75} = 45{,}000$, number of bushels harvested the fourth year.

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2. $s=\dfrac{ar^n-a}{r-1}=\dfrac{2^8-1}{1}=255.$

3. $s=\dfrac{ar^n-a}{r-1}=\dfrac{a-ar^n}{1-r}=\dfrac{1-(\frac{1}{2})^8}{1-\frac{1}{2}}=\dfrac{255}{128}.$

4. $s=\dfrac{ar^n-a}{r-1}=\dfrac{(\frac{3}{2})^{10}-1}{\frac{3}{2}-1}=\dfrac{58025}{512}.$

5. $s=\dfrac{ar^n-a}{r-1}=\dfrac{a-ar^n}{1-r}=\dfrac{2-2(-\frac{1}{3})^7}{1+\frac{1}{3}}=\dfrac{2+\frac{2}{2187}}{\frac{4}{3}}=\dfrac{1094}{729}.$

6. $s=\dfrac{ar^n-a}{r-1}=\dfrac{a-ar^n}{1-r}=\dfrac{-\frac{1}{2}-(-\frac{1}{2})(-\frac{1}{2})^{12}}{1+\frac{1}{2}}=-\dfrac{1365}{4096}.$

7. $s=\dfrac{ar^n-a}{r-1}=\dfrac{(2x)^7-1}{2x-1}=\dfrac{128x^7-1}{2x-1}.$

8. $s=\dfrac{ar^n-a}{r-1}=\dfrac{(-2x)^7-1}{-2x-1}=\dfrac{128x^7+1}{2x+1}.$

9. $s=\dfrac{ar^n-a}{r-1}=\dfrac{(x^2)^n-1}{x^2-1}=\dfrac{x^{2n}-1}{x^2-1}.$

10. $s=\dfrac{ar^n-a}{r-1}=\dfrac{2^n-1}{2-1}=2^n-1.$

11. $s=\dfrac{ar^n-a}{r-1}=\dfrac{a-ar^n}{1-r}=\dfrac{1-(\frac{1}{3})^n}{1-\frac{1}{3}}=\dfrac{3^n-1}{3^n\times\frac{2}{3}}=\dfrac{3^n-1}{3^{n-1}\times 2}=\dfrac{1}{2}\left(\dfrac{3^n-1}{3^{n-1}}\right).$

12. $s=\dfrac{rl-a}{r-1}=\dfrac{3\times 729-1}{2}=1093.$

13. $s=\dfrac{rl-a}{r-1}=\dfrac{2\times 192-3}{1}=381.$

14. $s=\dfrac{a-rl}{1-r}=\dfrac{7-(-2)(-224)}{1-(-2)}=\dfrac{7-448}{3}=-147.$

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2. $s=\dfrac{a}{1-r}=\dfrac{1}{1-\frac{1}{2}}=\dfrac{1}{\frac{1}{2}}=2.$

3. $s=\dfrac{a}{1-r}=\dfrac{3}{1-\frac{1}{4}}=\dfrac{3}{\frac{3}{4}}=4.$

4. $s=\dfrac{a}{1-r}=\dfrac{1}{1-(-\frac{1}{3})}=\dfrac{1}{\frac{4}{3}}=\dfrac{3}{4}.$

5. $s=\dfrac{a}{1-r}=\dfrac{-4}{1-\frac{1}{4}}=\dfrac{-4}{\frac{3}{4}}=-\dfrac{16}{3}=-5\frac{1}{3}.$

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6. $s = \dfrac{a}{1-r} = \dfrac{-2}{1+\frac{1}{5}} = \dfrac{-2}{\frac{6}{5}} = -\dfrac{10}{6} = -1\frac{2}{3}.$

7. $s = \dfrac{a}{1-r} = \dfrac{100}{1+\frac{1}{10}} = \dfrac{100}{\frac{11}{10}} = \dfrac{1000}{11} = 90\frac{10}{11}.$

8. Substituting .9 for x in the given series, $1 + .9 + .81 + .729 + \cdots.$

$$s = \frac{a}{1-r} = \frac{1}{1-.9} = \frac{1}{1-\frac{9}{10}} = \frac{1}{\frac{1}{10}} = 10.$$

9. Substituting $\frac{3}{8}$ for x in the given series, $1 - \frac{3}{8} + \frac{9}{64} - \frac{27}{512} + \cdots.$

$$s = \frac{a}{1-r} = \frac{1}{1+\frac{3}{8}} = \frac{1}{\frac{11}{8}} = \frac{8}{11}.$$

11. $.407407 \cdots = .407 + .000407 + .000000407 + \cdots.$

$$s = \frac{a}{1-r} = \frac{.407}{1-\frac{1}{1000}} = \frac{407}{1000} \div \frac{999}{1000} = \frac{407}{999} = \frac{11}{27}.$$

12. $.363636 \cdots = .36 + .0036 + .000036 + \cdots.$

$$s = \frac{a}{1-r} = \frac{.36}{1-\frac{1}{100}} = \frac{36}{100} \div \frac{99}{100} = \frac{36}{99} = \frac{4}{11}.$$

13. $1.94444 = (1 + \frac{9}{10}) + (\frac{4}{100} + \frac{4}{1000} + \frac{4}{10000} + \cdots).$

$$s = \frac{19}{10} + \frac{a}{1-r} = \frac{19}{10} + \frac{.04}{1-\frac{1}{10}}$$
$$= \tfrac{19}{10} + \tfrac{4}{90} = \tfrac{175}{90} = 1\tfrac{17}{18}.$$

14. $.020303 \cdots = (.02) + (.0003 + .000003 + \cdots).$

$$s = .02 + \frac{a}{1-r} = .02 + \frac{.0003}{1-.01} = .02 + \frac{.0003}{.99} = \frac{.0201}{.99} = \frac{67}{3300}.$$

15. $.007007 = .007 + .000007 + \cdots.$

$$s = \frac{a}{1-r} = \frac{.007}{1-.001} = \frac{7}{1000} \times \frac{1000}{999} = \frac{7}{999}.$$

16. $5.032828 = (5 + \frac{3}{100}) + (\frac{28}{10000} + \frac{28}{1000000} + \cdots).$

$$s = \frac{503}{100} + \frac{a}{1-r} = \frac{503}{100} + \frac{\frac{28}{10000}}{1-\frac{1}{100}} = \frac{503}{100} + \frac{28}{9900} = 5\tfrac{13}{396}.$$

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2. Substituting 625 for l, 1 for a, and 5 for n in $l = ar^{n-1}$,

$$625 = r^4.$$
$$\therefore r = \pm 5.$$

Hence, the series is either 1, 5, 25, 125, 625, or 1, -5, 25, -125, 625.

3. Substituting $\frac{2048}{81}$ for l, $\frac{9}{2}$ for a, and 7 for n in $l = ar^{n-1}$,

$$\tfrac{2048}{81} = \tfrac{9}{2}r^6.$$
$$r^6 = \frac{4096}{729} = \frac{4^6}{3^6}.$$
$$\therefore r = \pm \tfrac{4}{3}.$$

Hence, the series is either $4\frac{1}{2}$, 6, 8, $\frac{32}{3}$, $\frac{128}{9}$, $\frac{512}{27}$, $\frac{2048}{81}$;

or $4\frac{1}{2}$, -6, 8, $-\frac{32}{3}$, $\frac{128}{9}$, $-\frac{512}{27}$, $\frac{2048}{81}$.

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4. Substituting $\frac{64}{49}$ for l, $\frac{343}{16}$ for a, and 6 for n in $l = ar^{n-1}$,

$$\tfrac{64}{49} = \tfrac{343}{16} r^5.$$

$$r = \sqrt[5]{\frac{64 \cdot 16}{49 \cdot 343}} = \sqrt[5]{\frac{4^3 \cdot 4^2}{7^2 \cdot 7^3}} = \sqrt[5]{\frac{4^5}{7^5}} = \frac{4}{7}.$$

Hence, the series is $\frac{343}{16}$, $\frac{49}{4}$, 7, 4, $\frac{16}{7}$, $\frac{64}{49}$.

5. Substituting 5120 for a, 5 for l, and 6 for n in $l = ar^{n-1}$,

$$5120\, r^5 = 5.$$

$$\therefore r = \tfrac{1}{4}.$$

Hence, the series is 5120, 1280, 320, 80, 20, 5.

6. Substituting 1 for l, $4\sqrt{2}$ for a, and 6 for n in $l = ar^{n-1}$,

$$1 = 4\sqrt{2}\, r^5 = (2)^{\frac{5}{2}} r^5.$$

$$\therefore r = \frac{1}{(2)^{\frac{1}{2}}} = \tfrac{1}{2}\sqrt{2}.$$

Hence, the series is $4\sqrt{2}$, 4, $2\sqrt{2}$, 2, $\sqrt{2}$, 1.

7. Substituting b^6 for l, a^6 for a, and 7 for n in $l = ar^{n-1}$,

$$b^6 = a^6 r^6.$$

$$\therefore r = \pm \frac{b}{a}.$$

Hence, the series is either $a^6, a^5b, a^4b^2, a^3b^3, a^2b^4, ab^5, b^6$;

or $a^6, -a^5b, a^4b^2, -a^3b^3, a^2b^4, -ab^5, b^6$.

8. Substituting $-y$ for l, x for a, and 6 for n in $l = ar^{n-1}$,

$$-y = xr^5.$$

$$\therefore r = \sqrt[5]{-\frac{y}{x}} = -\frac{y^{\frac{1}{5}}}{x^{\frac{1}{5}}}.$$

Hence, the series is x, $-x^{\frac{4}{5}}y^{\frac{1}{5}}$, $x^{\frac{3}{5}}y^{\frac{2}{5}}$, $-x^{\frac{2}{5}}y^{\frac{3}{5}}$, $x^{\frac{1}{5}}y^{\frac{4}{5}}$, $-y$.

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1. Given r, l, and s, to find a.

Formula III, $$s = \frac{rl - a}{r - 1}. \quad (1)$$

Solving for a, $$a = rl - s(r - 1). \quad (2)$$

2. By (2), Ex. 1, $a = rl - s(r - 1) = 5 \times 625 - 775 \times 4 = 25$.

3. Since the last term of an infinite decreasing geometrical series may be neglected, or counted as 0, by (2), Ex. 1,

$$a = rl - s(r - 1) = \tfrac{1}{10} \times 0 - \tfrac{1}{3}(\tfrac{1}{10} - 1) = \tfrac{3}{10}.$$

4. Find l in terms of a, r, and s.

Formula III, $$s = \frac{rl - a}{r - 1}. \quad (1)$$

Solving for l, $$l = \frac{a + s(r - 1)}{r}. \quad (2)$$

5. By (2), Ex. 4, $$l = \frac{5 + 155 \times 1}{2} = 80.$$

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6. By (2), Ex. 4, $l = \dfrac{\frac{1}{8} + \frac{15}{8}(1+\sqrt{2})(\sqrt{2}-1)}{\sqrt{2}} = \dfrac{\frac{1}{8}+\frac{15}{8}}{\sqrt{2}} = \sqrt{2}.$ (1)

Substituting $\sqrt{2}$ for l, $\frac{1}{8}$ for a, and $\sqrt{2}$ for r in $l = ar^{n-1}$,

$$\sqrt{2} = \tfrac{1}{8}(\sqrt{2})^{n-1}, \text{ or } 2^{\frac{1}{2}} = 2^{-3} \cdot 2^{\frac{n-1}{2}} = 2^{\frac{n-7}{2}} = 2^{\frac{1}{2}(n-7)}.$$

Now since $2^{\frac{1}{2}} = 2^{\frac{1}{2}(n-7)}$, the exponents must be equal.

$$\therefore \tfrac{1}{2} = \tfrac{1}{2}(n-7), \text{ whence, } n = 8.$$

7. Deduce the formula for r in terms of a, l, and s.

Formula III, $s = \dfrac{rl - a}{r - 1}.$ (1)

Solving for r, $r = \dfrac{s - a}{s - l}.$ (2)

8. By (2), Ex. 7, $r = \dfrac{s-a}{s-l} = \dfrac{665 - 32}{665 - 243} = \dfrac{3}{2}.$

Hence, the series is 32, 48, 72, 108, 162, 243.

9. By (2), Ex. 7, $r = \dfrac{s-a}{s-l} = \dfrac{700}{525} = \dfrac{4}{3}.$

Since the sum is 525 greater than the last term and 700 greater than the first term, the last term is 175 greater than the first term.

$$\therefore l = 81 + 175 = 256.$$

Hence, the progression is 81, 108, 144, 192, 256.

10. Deduce the formula for r in terms of a, n, and l.

Formula I, $l = ar^{n-1}.$ (1)

Solving for r, $r = \sqrt[n-1]{\dfrac{l}{a}}$ (2)

11. By (2), Ex. 10, $r = \sqrt[n-1]{\dfrac{l}{a}} = \sqrt[5]{\dfrac{729}{3}} = \sqrt[5]{243} = 3.$

Hence, the series is 3, 9, 27, 81, 243, 729.

12. Find l in terms of r, n, and s.

Formula I, $l = ar^{n-1}.$ (1)

Formula II, $s = \dfrac{a(r^n - 1)}{r - 1}.$ (2)

From (1), $\dfrac{l}{a} = r^{n-1}.$ (3)

From (2), $a = \dfrac{s(r-1)}{r^n - 1}.$ (4)

Multiplying (3) by (4), $l = \dfrac{r^{n-1}s(r-1)}{r^n - 1}.$ (5)

13. $s = 100[1 + \frac{4}{5} + (\frac{4}{5})^2 + \cdots].$

The series in brackets is an infinite decreasing geometrical progression whose ratio is $\frac{4}{5}$ and first term 1.

$$\therefore s = 100 \times \frac{1}{1 - \frac{4}{5}} = 100 \times 5 = 500.$$

Hence, the sled will go 500 feet.

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14. $$s=\frac{ar^n-a}{r-1}=\frac{2^{16}-1}{2-1}=2^{16}-1=65{,}535.$$

15. Whole distance passed through $= 200 + 80 + 32 + \cdots$.

In the series 200, 80, 32, $\cdots$, $a = 200$, $r = \frac{2}{5}$, and the number of terms is infinite. Let s represent the whole distance passed through, in feet.

Then, $$s=\frac{a}{1-r}=\frac{200}{1-\frac{2}{5}}=\frac{200}{\frac{3}{5}}=333\tfrac{1}{3}.$$

16. The amount is $1\,(1+r)$, or $1+r$, at the end of 1 year; $(1+r)(1+r)$, or $(1+r)^2$, at the end of 2 years; $(1+r)^2(1+r)$, or $(1+r)^3$, at the end of 3 years; etc.

Hence, the amounts $1+r$, $(1+r)^2$, $(1+r)^3$, $\cdots$ for 1, 2, 3, $\cdots$ years, respectively, form a geometrical progression whose ratio is $1+r$.

17. Let $$a,\ ar,\ ar^2,\ \cdots$$ be a geometrical progression, and m any common multiplier of its terms.

Then, $$ma,\ mar,\ mar^2,\ \cdots$$ also is a geometrical progression having the same ratio as the given geometrical progression, since $mr \div m = r$, $mr^2 \div mr = r$, etc.

18. Let the series be x^2, xy, y^2.

Then, $x^2 + xy + y^2 = 19$, (1)

and $x^4 + x^2y^2 + y^4 = 133$. (2)

Dividing (2) by (1), $x^2 - xy + y^2 = 7$. (3)

Subtracting (3) from (1), $2\,xy = 12$. (4)

Adding (4) $\div$ 2 to (1), $x^2 + 2\,xy + y^2 = 25$,

whence, $x + y = \pm 5$. (5)

Subtracting (4) $\div$ 2 from (3), $x^2 - 2\,xy + y^2 = 1$,

whence, $x - y = \pm 1$. (6)

From (5) and (6), $x = 3$ or 2 or -2 or -3, (7)

and $y = 2$ or 3 or -3 or -2. (8)

From (7) and (8), $x^2 = 9$ or 4, $xy = 6$, and $y^2 = 4$ or 9.

Hence, the numbers are 4, 6, and 9.

19. Let the series be x^2, xy, y^2.

Then, $x^3y^3 = 8$, (1)

and $x^4 + x^2y^2 + y^4 = 21$. (2)

From (1), $xy = 2$, or $y = \frac{2}{x}$. (3)

Substituting (3) in (2), $x^4 + 4 + \frac{16}{x^4} = 21$.

Clearing of fractions, etc., $x^8 - 17\,x^4 + 16 = 0$.

Factoring, $(x-1)(x+1)(x-2)(x+2)(x^2+1)(x^2+4) = 0$. (4)

From (4), $x = 1$, $x = -1$, $x = 2$, $x = -2$, $x^2 = -1$, $x^2 = -4$.

$\therefore x^2 = 1$ or 4 or -1 or -4. (5)

Substituting (5) in the square of (3),

$y^2 = 4$ or 1 or -4 or -1. (6)

From (5), (3), and (6) the numbers are found to be

1, 2, and 4, or -1, 2, and -4.

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20. Let the series be $\frac{x^2}{y}, x, y, \frac{y^2}{x}$.

Then, $$\frac{x^2}{y} + x = 15, \quad (1)$$

and $$y + \frac{y^2}{x} = 60. \quad (2)$$

Dividing (2) by (1), $$\frac{y^2}{x^2} = 4.$$

$$\therefore y = 2x \text{ or } -2x. \quad (3)$$

Substituting $2x$ for y in (1), $x = 10$. (4)
Substituting $-2x$ for y in (1), $x = 30$. (5)
Substituting (4) and (5) in (3), $y = 20$ or -60.
Forming the series from $x = 10$ and $y = 20$, and from $x = 30$ and $y = -60$, the numbers are 5, 10, 20, and 40, or -15, 30, -60, and 120.

21. At first the contents of the cask is all vinegar; then $\frac{2}{3}$ vinegar; then $\frac{2}{3}$ of $\frac{2}{3}$, or $\frac{4}{9}$ vinegar; etc. Hence, the part of the contents of the cask which will be vinegar after the sixth time is equal to the seventh term of the geometrical progression 1, $\frac{2}{3}$, $\frac{4}{9}$, $\cdots$.

$$\text{7th term} = l = ar^{n-1} = 1(\tfrac{2}{3})^6 = \tfrac{64}{729}.$$

Then, $\frac{729}{729} - \frac{64}{729} = \frac{665}{729}$, the part of contents which is water.
Hence, the contents of the cask will be more than $\frac{9}{10}$ water.

22. $$l = ar^{n-1} = 14.7\left(\frac{9}{10}\right)^6 = 14.7 \times \frac{531,441}{1,000,000} = 7.8.$$

Hence, after 6 strokes, the pressure will be 7.8 pounds.

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24. By reasoning similar to that in Ex. 23, the portions of the principal paid form a G. P., in which $r = 1.06$, $n = 4$, and $s = \$1500$.

Substituting the known values in (I) and (III), §§ 521, 525,

$$l = a\ 1.06^{4-1}, \text{ or } l = 1.06^3 a; \quad (1)$$

and $$\$1500 = \frac{1.06\,l - a}{1.06 - 1}, \text{ or } l = \frac{a + \$1500 \times .06}{1.06}. \quad (2)$$

From (1) and (2), $$1.06^3 a = \frac{a + \$1500 \times .06}{1.06}.$$

$$1.06^4 a = a + \$1500 \times .06.$$

$$a = \frac{\$1500 \times .06}{(1.06)^4 - 1}.$$

Solving, $a = \$342.88$.

That is, the first portion of the principal paid $= \$342.88$; but the first year's interest $= 6\%$ of \$1500, or \$90; hence, the entire first payment $= \$342.88 + \$90 = \$432.88$, which is the sum to be paid each year.

25. By reasoning similar to that in Ex. 23, the portions of the principal paid form a G. P., in which $r = 1.04$, $n = 6$, and $s = \$10,000$.

Substituting the known values in (I) and (III), §§ 521, 525,

$$l = a\ 1.04^{6-1}, \text{ or } l = 1.04^5 a; \quad (1)$$

and $$\$10,000 = \frac{1.04\,l - a}{1.04 - 1}, \text{ or } l = \frac{a + \$10,000 \times .04}{1.04}. \quad (2)$$

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From (1) and (2), $1.04^5 a = \dfrac{a + \$10{,}000 \times .04}{1.04}.$

$$1.04^6 a = a + \$10{,}000 \times .04.$$

$$a = \frac{\$10{,}000 \times .04}{(1.04)^6 - 1}.$$

Solving, $a = \$1507.62.$

That is, the first portion of the principal paid = \$1507.62; but the first year's interest = 4 % of \$10,000, or \$400; hence, the entire first payment = \$1507.62 + \$400 = \$1907.62, which is the sum to be paid each year.

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3. $(b-n)^7 = b^7 - 7\,b^6n + 21\,b^5n^2 - 35\,b^4n^3 + 35\,b^3n^4 - 21\,b^2n^5 + 7\,bn^6 - n^7.$

4. $(1+a^{-1})^4 = 1^4 + 4\cdot 1^3\cdot a^{-1} + 6\cdot 1^2\cdot (a^{-1})^2 + 4\cdot 1\cdot (a^{-1})^3 + (a^{-1})^4$
$= 1 + 4\,a^{-1} + 6\,a^{-2} + 4\,a^{-3} + a^{-4}.$

5. $(2-3\,x)^6 = 2^6 - 6\cdot 2^5\cdot 3\,x + 15\cdot 2^4(3\,x)^2 - 20\cdot 2^3(3\,x)^3 + 15\cdot 2^2(3\,x)^4 - 6\cdot 2(3\,x)^5 + (3\,x)^6$
$= 64 - 576\,x + 2160\,x^2 - 4320\,x^3 + 4860\,x^4 - 2916\,x^5 + 729\,x^6.$

6. $(x^2-x)^8 = [x(x-1)]^8 = x^8(x-1)^8$
$= x^8(x^8 - 8\,x^7 + 28\,x^6 - 56\,x^5 + 70\,x^4 - 56\,x^3 + 28\,x^2 - 8\,x + 1)$
$= x^{16} - 8\,x^{15} + 28\,x^{14} - 56\,x^{13} + 70\,x^{12} - 56\,x^{11} + 28\,x^{10} - 8\,x^9 + x^8.$

7. $(x+x^{-1})^6 = [x^{-1}(x^2+1)]^6 = x^{-6}(x^2+1)^6$
$= x^{-6}(x^{12} + 6\,x^{10} + 15\,x^8 + 20\,x^6 + 15\,x^4 + 6\,x^2 + 1)$
$= x^6 + 6\,x^4 + 15\,x^2 + 20 + 15\,x^{-2} + 6\,x^{-4} + x^{-6}.$

8. $(2\,a+\sqrt{x})^3 = (2\,a)^3 + 3(2\,a)^2\sqrt{x} + 3(2\,a)(\sqrt{x})^2 + (\sqrt{x})^3$
$= 8\,a^3 + 12\,a^2\sqrt{x} + 6\,ax + x\sqrt{x}.$

9. $(a+a\sqrt{a})^4 = [a(1+\sqrt{a})]^4 = a^4(1+\sqrt{a})^4$
$= a^4(1^4 + 4)\cdot 1^3\cdot\sqrt{a} + 6\cdot 1^2(\sqrt{a})^2 + 4\cdot 1(\sqrt{a})^3 + (\sqrt{a})^4$
$= a^4(1 + 4\sqrt{a} + 6\,a + 4\,a\sqrt{a} + a^2)$
$= a^4 + 4\,a^4\sqrt{a} + 6\,a^5 + 4\,a^5\sqrt{a} + a^6.$

10. $\left(1+\dfrac{2}{x^2}\right)^5 = 1^5 + 5\cdot 1^4\cdot\dfrac{2}{x^2} + 10\cdot 1^3\left(\dfrac{2}{x^2}\right)^2 + 10\cdot 1^2\left(\dfrac{2}{x^2}\right)^3 + 5\cdot 1\left(\dfrac{2}{x^2}\right)^4 + \left(\dfrac{2}{x^2}\right)^5$
$= 1 + \dfrac{10}{x^2} + \dfrac{40}{x^4} + \dfrac{80}{x^6} + \dfrac{80}{x^8} + \dfrac{32}{x^{10}}.$

11. $\left(\dfrac{a}{x}-\dfrac{x}{a}\right)^5 = \left(\dfrac{a}{x}\right)^5 - 5\left(\dfrac{a}{x}\right)^4\dfrac{x}{a} + 10\left(\dfrac{a}{x}\right)^3\left(\dfrac{x}{a}\right)^2 - 10\left(\dfrac{a}{x}\right)^2\left(\dfrac{x}{a}\right)^3 + 5\,\dfrac{a}{x}\left(\dfrac{x}{a}\right)^4 - \left(\dfrac{x}{a}\right)^5$
$= \dfrac{a^5}{x^5} - 5\,\dfrac{a^3}{x^3} + 10\,\dfrac{a}{x} - 10\,\dfrac{x}{a} + 5\,\dfrac{x^3}{a^3} - \dfrac{x^5}{a^5}.$

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12. $\left(\frac{1}{x}-\frac{a}{y}\right)^3=\left(\frac{1}{x}\right)^3-3\left(\frac{1}{x}\right)^2\frac{a}{y}+3\frac{1}{x}\left(\frac{a}{y}\right)^2-\left(\frac{a}{y}\right)^3$

$$=\frac{1}{x^3}-3\frac{a}{x^2y}+3\frac{a^2}{xy^2}-\frac{a^3}{y^3}.$$

13. $(\sqrt[3]{a^2}+\sqrt[4]{b^3})^3=(\sqrt[3]{a^2})^3+3(\sqrt[3]{a^2})^2\sqrt[4]{b^3}+3\sqrt[3]{a^2}(\sqrt[4]{b^3})^2+(\sqrt[4]{b^3})^3$

$$=a^2+3a\sqrt[3]{a}\sqrt[4]{b^3}+3\sqrt[3]{a^2}\sqrt{b^3}+b^2\sqrt[4]{b}$$

$$=a^2+3a\sqrt[12]{a^4b^9}+3b\sqrt[6]{a^4b^3}+b^2\sqrt[4]{b}.$$

14. $(2\sqrt{2}-\sqrt[3]{3})^6=(2^{\frac{3}{2}}-3^{\frac{1}{3}})^6$

$$=(2^{\frac{3}{2}})^6-6(2^{\frac{3}{2}})^5\cdot 3^{\frac{1}{3}}+15(2^{\frac{3}{2}})^4(3^{\frac{1}{3}})^2-20(2^{\frac{3}{2}})^3(3^{\frac{1}{3}})^3+15(2^{\frac{3}{2}})^2(3^{\frac{1}{3}})^4$$

$$-6\cdot 2^{\frac{3}{2}}(3^{\frac{1}{3}})^5+(3^{\frac{1}{3}})^6$$

$$=2^9-6\cdot 2^7\cdot 2^{\frac{1}{2}}\cdot 3^{\frac{1}{3}}+15\cdot 2^6\cdot 3^{\frac{2}{3}}-20\cdot 2^4\cdot 2^{\frac{1}{2}}\cdot 3+15\cdot 2^3\cdot 3\cdot 3^{\frac{1}{3}}$$

$$-6\cdot 2\cdot 2^{\frac{1}{2}}\cdot 3\cdot 3^{\frac{2}{3}}+3^2$$

$$=512-768\sqrt[6]{72}+960\sqrt[3]{9}-960\sqrt{2}+360\sqrt[3]{3}-36\sqrt[6]{648}+9$$

$$=521-768\sqrt[6]{72}+960\sqrt[3]{9}-960\sqrt{2}+360\sqrt[3]{3}-36\sqrt[6]{648}.$$

15. $\left(\sqrt{2}+\frac{1}{\sqrt{x}}\right)^3=(\sqrt{2})^3+3(\sqrt{2})^2\cdot\frac{1}{\sqrt{x}}+3\sqrt{2}\left(\frac{1}{\sqrt{x}}\right)^2+\left(\frac{1}{\sqrt{x}}\right)^3$

$$=2\sqrt{2}+\frac{6}{\sqrt{x}}+\frac{3\sqrt{2}}{x}+\frac{1}{x\sqrt{x}}.$$

16. $(x^{\frac{n-1}{n}}-x^{\frac{1}{n}})^4=(x^{\frac{n-1}{n}})^4-4(x^{\frac{n-1}{n}})^3\cdot x^{\frac{1}{n}}+6(x^{\frac{n-1}{n}})^2(x^{\frac{1}{n}})^2$

$$-4x^{\frac{n-1}{n}}(x^{\frac{1}{n}})^3+(x^{\frac{1}{n}})^4$$

$$=x^{\frac{4n-4}{n}}-4x^{\frac{3n-2}{n}}+6x^2-4x^{\frac{n+2}{n}}+x^{\frac{4}{n}}.$$

17. $(ax^{-2}-b\sqrt{x})^6=(ax^{-2})^6-6(ax^{-2})^5b\sqrt{x}+15(ax^{-2})^4(b\sqrt{x})^2$

$$-20(ax^{-2})^3(b\sqrt{x})^3+15(ax^{-2})^2(b\sqrt{x})^4-6ax^{-2}(b\sqrt{x})^5+(b\sqrt{x})^6$$

$$=a^6x^{-12}-6a^5bx^{-\frac{19}{2}}+15a^4b^2x^{-7}-20a^3b^3x^{-\frac{9}{2}}+15a^2b^4x^{-2}$$

$$-6ab^5x^{\frac{1}{2}}+b^6x^3.$$

18. $\left(\frac{\sqrt{a}}{\sqrt[3]{b}}-\frac{\sqrt{b}}{\sqrt{a^3}}\right)^6=\left(\frac{a^2-b^{\frac{5}{6}}}{a\sqrt[6]{a^3b^2}}\right)^6=\frac{1}{a^6\cdot a^3b^2}(a^2-b^{\frac{5}{6}})^6$

$$=\frac{1}{a^9b^2}[(a^2)^6-6(a^2)^5b^{\frac{5}{6}}+15(a^2)^4(b^{\frac{5}{6}})^2-20(a^2)^3(b^{\frac{5}{6}})^3+15(a^2)^2(b^{\frac{5}{6}})^4$$

$$-6a^2(b^{\frac{5}{6}})^5+(b^{\frac{5}{6}})^6]$$

$$=\frac{1}{a^9b^2}(a^{12}-6a^{10}b^{\frac{5}{6}}+15a^8b^{\frac{5}{3}}-20a^6b^{\frac{5}{2}}+15a^4b^{\frac{10}{3}}-6a^2b^{\frac{25}{6}}+b^5)$$

$$=\frac{a^3}{b^2}-\frac{6a}{b^2}\sqrt[6]{b^5}+\frac{15}{ab}\sqrt[3]{b^2}-\frac{20}{a^3}\sqrt{b}+\frac{15b}{a^5}\sqrt[3]{b}-\frac{6b^2}{a^7}\sqrt[6]{b}+\frac{b^3}{a^9}.$$

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19. $\left(\frac{x}{y}\sqrt{\frac{x}{y}}+\frac{2}{3}\sqrt{\frac{2}{3}}\right)^3=\left[\left(\frac{x}{y}\right)^{\frac{3}{2}}+\left(\frac{2}{3}\right)^{\frac{3}{2}}\right]^3$

$$=\left(\frac{x}{y}\right)^{\frac{9}{2}}+3\left(\frac{x}{y}\right)^3\left(\frac{2}{3}\right)^{\frac{3}{2}}+3\left(\frac{x}{y}\right)^{\frac{3}{2}}\left(\frac{2}{3}\right)^3+\left(\frac{2}{3}\right)^{\frac{9}{2}}$$

$$=\frac{x^4}{y^4}\sqrt{\frac{x}{y}}+3\cdot\frac{x^3}{y^3}\cdot\frac{2}{3}\sqrt{\frac{2}{3}}+3\,\frac{x}{y}\sqrt{\frac{x}{y}}\cdot\frac{8}{27}+\left(\frac{2}{3}\right)^4\sqrt{\frac{2}{3}}$$

$$=\frac{x^4\sqrt{xy}}{y^5}+\frac{2\,x^3}{3\,y^3}\sqrt{6}+\frac{8\,x\sqrt{xy}}{9\,y^2}+\frac{16\sqrt{6}}{243}.$$

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2. 4th term $=\frac{10\cdot 9\cdot 8}{1\cdot 2\cdot 3}a^{10-3}2^3=960\,a^7.$

3. 8th term $=\frac{11\cdot 10\cdot 9\cdot 8\cdot 7\cdot 6\cdot 5}{1\cdot 2\cdot 3\cdot 4\cdot 5\cdot 6\cdot 7}x^{11-7}(-y)^7=-330\,x^4y^7.$

4. 5th term $=\frac{12\cdot 11\cdot 10\cdot 9}{1\cdot 2\cdot 3\cdot 4}x^{12-4}(-2\,y)^4=7920\,x^8y^4.$

5. 20th term = coef. 6th term times $1^{24-19}\,x^{19}$

$$=\frac{24\cdot 23\cdot 22\cdot 21\cdot 20}{1\cdot 2\cdot 3\cdot 4\cdot 5}x^{19}=42{,}504\,x^{19}.$$

6. 18th term = coef. 4th term times $1^{20-17}(-2\,x)^{17}$

$$=\frac{20\cdot 19\cdot 18}{1\cdot 2\cdot 3}\times -131{,}072\,x^{17}=-149{,}422{,}080\,x^{17}.$$

7. 13th term = coef. 4th term times $(a^2)^{15-12}(-a^{-2})^{12}$

$$=\frac{15\cdot 14\cdot 13}{1\cdot 2\cdot 3}a^6\cdot a^{-24}=455\,a^{-18}.$$

8. Since there are $6+1$ terms, the middle term is the $(3+1)$th.

4th term $=\frac{6\cdot 5\cdot 4}{1\cdot 2\cdot 3}a^{6-3}(3\,b)^3=540\,a^3b^3.$

9. 6th term $=\frac{10\cdot 9\cdot 8\cdot 7\cdot 6}{1\cdot 2\cdot 3\cdot 4\cdot 5}x^{10-5}\left(\frac{1}{x}\right)^5=252\,x^0=252.$

10. Since there are $8+1$ terms, the middle term is the $(4+1)$th.

5th term $=\frac{8\cdot 7\cdot 6\cdot 5}{1\cdot 2\cdot 3\cdot 4}\left(\frac{x}{y}\right)^{8-4}\left(-\frac{y}{x}\right)^4=70.$

11. Since there are $9+1$ terms, the middle terms are the 5th and 6th. They have numerically equal coefficients.

5th term $=\frac{9\cdot 8\cdot 7\cdot 6}{1\cdot 2\cdot 3\cdot 4}\left(\frac{a}{b}\right)^{9-4}\left(-\frac{b}{a}\right)^4=126\,\frac{a}{b}=\frac{126\,a}{b}.$

6th term $=126\left(\frac{a}{b}\right)^{9-5}\left(-\frac{b}{a}\right)^5=-126\,\frac{b}{a}=-\frac{126\,b}{a}.$

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13. $(a^3 + a)^5 = a^5(a^2 + 1)^5$.

The term sought corresponds to that term of the expansion of $(a^2 + 1)^5$ which contains a^4, or $(a^2)^2$. This term is the third from the last, or from the 6th term. Hence, the term sought is the 4th.

Coef. 4th term $= \dfrac{5 \cdot 4 \cdot 3}{1 \cdot 2 \cdot 3} = 10$.

14. Since $(a - b)^{27} = \left[a\left(1 - \dfrac{b}{a}\right)\right]^{27} = a^{27}\left(1 - \dfrac{b}{a}\right)^{27}$, every term of the series expanded from $\left(1 - \dfrac{b}{a}\right)^{27}$ will be multiplied by a^{27}.

Hence, the term sought is that which contains $\left(\dfrac{-b}{a}\right)^9$, or $\dfrac{-b^9}{a^9}$; that is, the $(9 + 1)$th, or 10th term.

$$10\text{th term} = a^{27}\,\frac{27 \cdot 26 \cdot 25 \cdot 24 \cdot 23 \cdot 22 \cdot 21 \cdot 20 \cdot 19}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7 \cdot 8 \cdot 9}\left(-\frac{b}{a}\right)^9$$
$$= -\,4{,}686{,}825\,a^{18}b^9. \qquad (1)$$

Simplifying $b = 15^{-\frac{1}{9}}(143^{\frac{2}{9}}\,a^4)^{-\frac{1}{2}}$, $b = \dfrac{1}{2145^{\frac{1}{9}}\,a^2}$. (2)

Substituting (2) in (1), 10th term $= -\,4{,}686{,}825\,a^{18} \times \dfrac{1}{(2145^{\frac{1}{9}}\,a^2)^9}$.

$\therefore$ 10th term $= -\,2185$.

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16. $$(a + x)^{\frac{1}{2}} = a^{\frac{1}{2}} + \tfrac{1}{2}a^{\frac{1}{2}-1}x + \frac{\frac{1}{2}(\frac{1}{2} - 1)}{1 \cdot 2}a^{\frac{1}{2}-2}x^2 + \frac{\frac{1}{2}(\frac{1}{2} - 1)(\frac{1}{2} - 2)}{1 \cdot 2 \cdot 3}a^{\frac{1}{2}-3}x^3 + \frac{\frac{1}{2}(\frac{1}{2} - 1)(\frac{1}{2} - 2)(\frac{1}{2} - 3)}{1 \cdot 2 \cdot 3 \cdot 4}a^{\frac{1}{2}-4}x^4$$
$$= a^{\frac{1}{2}} + \tfrac{1}{2}a^{-\frac{1}{2}}x - \tfrac{1}{8}a^{-\frac{3}{2}}x^2 + \tfrac{1}{16}a^{-\frac{5}{2}}x^3 - \tfrac{5}{128}a^{-\frac{7}{2}}x^4.$$

10th term
$$= \frac{\frac{1}{2}(\frac{1}{2} - 1)(\frac{1}{2} - 2)(\frac{1}{2} - 3)(\frac{1}{2} - 4)(\frac{1}{2} - 5)(\frac{1}{2} - 6)(\frac{1}{2} - 7)(\frac{1}{2} - 8)}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7 \cdot 8 \cdot 9}a^{\frac{1}{2}-9}x^9$$
$$= \frac{\frac{1}{2} \times -\frac{1}{2} \times -\frac{3}{2} \times -\frac{5}{2} \times -\frac{7}{2} \times -\frac{9}{2} \times -\frac{11}{2} \times -\frac{13}{2} \times -\frac{15}{2}}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7 \cdot 8 \cdot 9}a^{-\frac{17}{2}}x^9$$
$$= \frac{715}{65{,}536}a^{-\frac{17}{2}}x^9.$$

17. $$(1 - a)^{-1} = 1^{-1} - (-1)1^{-2}a + \frac{-1(-2)}{1 \cdot 2}1^{-3}a^2 - \frac{-1(-2)(-3)}{1 \cdot 2 \cdot 3}1^{-4}a^3$$
$$= 1 + a + a^2 + a^3.$$

18. $$(1 + a)^{-1} = 1^{-1} + (-1)1^{-2}a + \frac{-1(-2)}{1 \cdot 2}1^{-3}a^2 + \frac{-1(-2)(-3)}{1 \cdot 2 \cdot 3}1^{-4}a^3$$
$$= 1 - a + a^2 - a^3.$$

19. $$(a - b)^{\frac{1}{2}} = a^{\frac{1}{2}} - \tfrac{1}{2}a^{-\frac{1}{2}}b + \frac{\frac{1}{2}(-\frac{1}{2})}{1 \cdot 2}a^{-\frac{3}{2}}b^2 - \frac{\frac{1}{2}(-\frac{1}{2})(-\frac{3}{2})}{1 \cdot 2 \cdot 3}a^{-\frac{5}{2}}b^3$$
$$= a^{\frac{1}{2}} - \tfrac{1}{2}a^{-\frac{1}{2}}b - \tfrac{1}{8}a^{-\frac{3}{2}}b^2 - \tfrac{1}{16}a^{-\frac{5}{2}}b^3.$$

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20. $\sqrt{4+x}=(4+x)^{\frac{1}{2}}=4^{\frac{1}{2}}+\frac{1}{2}\cdot 4^{-\frac{1}{2}}x+\frac{\frac{1}{2}(-\frac{1}{2})}{1\cdot 2}4^{-\frac{3}{2}}x^2+\frac{\frac{1}{2}(-\frac{1}{2})(-\frac{3}{2})}{1\cdot 2\cdot 3}4^{-\frac{5}{2}}x^3$
$=2+\frac{1}{4}x-\frac{1}{64}x^2+\frac{1}{512}x^3.$

21. $(a+b)^{\frac{2}{3}}=a^{\frac{2}{3}}+\frac{2}{3}a^{-\frac{1}{3}}b+\frac{\frac{2}{3}(-\frac{1}{3})}{1\cdot 2}a^{-\frac{4}{3}}b^2+\frac{\frac{2}{3}(-\frac{1}{3})(-\frac{4}{3})}{1\cdot 2\cdot 3}a^{-\frac{7}{3}}b^3$
$=a^{\frac{2}{3}}+\frac{2}{3}a^{-\frac{1}{3}}b-\frac{1}{9}a^{-\frac{4}{3}}b^2+\frac{4}{81}a^{-\frac{7}{3}}b^3.$

22. $\sqrt[4]{(a-b)^3}=(a-b)^{\frac{3}{4}}$
$=a^{\frac{3}{4}}-\frac{3}{4}a^{-\frac{1}{4}}b+\frac{\frac{3}{4}(-\frac{1}{4})}{1\cdot 2}a^{-\frac{5}{4}}b^2-\frac{\frac{3}{4}(-\frac{1}{4})(-\frac{5}{4})}{1\cdot 2\cdot 3}a^{-\frac{9}{4}}b^3$
$=a^{\frac{3}{4}}-\frac{3}{4}a^{-\frac{1}{4}}b-\frac{3}{32}a^{-\frac{5}{4}}b^2-\frac{5}{128}a^{-\frac{9}{4}}b^3.$

23. $\sqrt{(9-x)^3}=(9-x)^{\frac{3}{2}}=9^{\frac{3}{2}}-\frac{3}{2}\cdot 9^{\frac{1}{2}}x+\frac{\frac{3}{2}\cdot\frac{1}{2}}{1\cdot 2}9^{-\frac{1}{2}}x^2-\frac{\frac{3}{2}\cdot\frac{1}{2}(-\frac{1}{2})}{1\cdot 2\cdot 3}9^{-\frac{3}{2}}x^3$
$=27-\frac{9}{2}x+\frac{1}{8}x^2+\frac{1}{432}x^3.$

24. $(a+b)^{-\frac{1}{2}}=a^{-\frac{1}{2}}+(-\frac{1}{2})a^{-\frac{3}{2}}b+\frac{-\frac{1}{2}(-\frac{3}{2})}{1\cdot 2}a^{-\frac{5}{2}}b^2+\frac{-\frac{1}{2}(-\frac{3}{2})(-\frac{5}{2})}{1\cdot 2\cdot 3}a^{-\frac{7}{2}}b^3$
$=a^{-\frac{1}{2}}-\frac{1}{2}a^{-\frac{3}{2}}b+\frac{3}{8}a^{-\frac{5}{2}}b^2-\frac{5}{16}a^{-\frac{7}{2}}b^3.$

25. $(1+x)^{\frac{2}{5}}=1^{\frac{2}{5}}+\frac{2}{5}\cdot 1^{-\frac{3}{5}}x+\frac{\frac{2}{5}(-\frac{3}{5})}{1\cdot 2}1^{-\frac{8}{5}}x^2+\frac{\frac{2}{5}(-\frac{3}{5})(-\frac{8}{5})}{1\cdot 2\cdot 3}1^{-\frac{13}{5}}x^3$
$=1+\frac{2}{5}x-\frac{3}{25}x^2+\frac{8}{125}x^3.$

26. $(1-x)^{-3}=1^{-3}-(-3)1^{-4}x+\frac{(-3)(-4)}{1\cdot 2}1^{-5}x^2-\frac{(-3)(-4)(-5)}{1\cdot 2\cdot 3}1^{-6}x^3$
$=1+3x+6x^2+10x^3.$

27. $(a^{\frac{2}{3}}-x^{-1})^{\frac{3}{2}}=(a^{\frac{2}{3}})^{\frac{3}{2}}-\frac{3}{2}(a^{\frac{2}{3}})^{\frac{1}{2}}x^{-1}+\frac{\frac{3}{2}\cdot\frac{1}{2}}{1\cdot 2}(a^{\frac{2}{3}})^{-\frac{1}{2}}(x^{-1})^2$
$-\frac{\frac{3}{2}\cdot\frac{1}{2}(-\frac{1}{2})}{1\cdot 2\cdot 3}(a^{\frac{2}{3}})^{-\frac{3}{2}}(x^{-1})^3$
$=a-\frac{3}{2}a^{\frac{1}{3}}x^{-1}+\frac{3}{8}a^{-\frac{1}{3}}x^{-2}+\frac{1}{16}a^{-1}x^{-3}.$

28. $(a^{\frac{1}{2}}-x^{\frac{1}{3}})^{-6}=(a^{\frac{1}{2}})^{-6}-(-6)(a^{\frac{1}{2}})^{-7}x^{\frac{1}{3}}+\frac{(-6)(-7)}{1\cdot 2}(a^{\frac{1}{2}})^{-8}(x^{\frac{1}{3}})^2$
$-\frac{(-6)(-7)(-8)}{1\cdot 2\cdot 3}(a^{\frac{1}{2}})^{-9}(x^{\frac{1}{3}})^3$
$=a^{-3}+6a^{-\frac{7}{2}}x^{\frac{1}{3}}+21a^{-4}x^{\frac{2}{3}}+56a^{-\frac{9}{2}}x.$

30. $\sqrt{5}=\sqrt{4+1}=\sqrt{4}\sqrt{1+\frac{1}{4}}=2(1+\frac{1}{4})^{\frac{1}{2}}$
$=2[1^{\frac{1}{2}}+\frac{1}{2}\cdot 1^{-\frac{1}{2}}\cdot\frac{1}{4}+\frac{\frac{1}{2}(-\frac{1}{2})}{1\cdot 2}1^{-\frac{3}{2}}(\frac{1}{4})^2+\frac{\frac{1}{2}(-\frac{1}{2})(-\frac{3}{2})}{1\cdot 2\cdot 3}1^{-\frac{5}{2}}(\frac{1}{4})^3+\cdots]$
$=2[1+\frac{1}{8}-\frac{1}{128}+\frac{1}{1024}-\cdots]=\frac{1145}{512}=2.236.$

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31. $\sqrt{17} = \sqrt{16+1} = \sqrt{16}\sqrt{1+\frac{1}{16}} = 4(1+\frac{1}{16})^{\frac{1}{2}}$

$$= 4\left[1^{\frac{1}{2}} + \tfrac{1}{2}\cdot 1^{-\frac{1}{2}}\cdot\tfrac{1}{16} + \frac{\frac{1}{2}(-\frac{1}{2})}{1\cdot 2}1^{-\frac{3}{2}}(\tfrac{1}{16})^2 + \frac{\frac{1}{2}(-\frac{1}{2})(-\frac{3}{2})}{1\cdot 2\cdot 3}1^{-\frac{5}{2}}(\tfrac{1}{16})^3 + \cdots\right]$$

$$= 4[1 + \tfrac{1}{32} - \tfrac{1}{2048} + \tfrac{1}{65536} - \cdots] = 4 + \tfrac{2017}{16384} = 4.123.$$

32. $\sqrt{26} = \sqrt{25+1} = \sqrt{25}\sqrt{1+\frac{1}{25}} = 5(1+\frac{1}{25})^{\frac{1}{2}}$

$$= 5\left[1^{\frac{1}{2}} + \tfrac{1}{2}\cdot 1^{-\frac{1}{2}}\cdot\tfrac{1}{25} + \frac{\frac{1}{2}(-\frac{1}{2})}{1\cdot 2}1^{-\frac{3}{2}}(\tfrac{1}{25})^2 + \frac{\frac{1}{2}(-\frac{1}{2})(-\frac{3}{2})}{1\cdot 2\cdot 3}1^{-\frac{5}{2}}(\tfrac{1}{25})^3 + \cdots\right]$$

$$= 5 + .1 - .001 + .00002 = 5.099.$$

33. $\sqrt[3]{25} = \sqrt[3]{27-2} = \sqrt[3]{27}\sqrt[3]{1-\frac{2}{27}} = 3(1-\frac{2}{27})^{\frac{1}{3}}$

$$= 3\left[1^{\frac{1}{3}} - \tfrac{1}{3}\cdot 1^{-\frac{2}{3}}\cdot\tfrac{2}{27} + \frac{\frac{1}{3}(-\frac{2}{3})}{1\cdot 2}1^{-\frac{5}{3}}(\tfrac{2}{27})^2 - \frac{\frac{1}{3}(-\frac{2}{3})(-\frac{5}{3})}{1\cdot 2\cdot 3}1^{-\frac{8}{3}}(\tfrac{2}{27})^3 + \cdots\right]$$

$$= 3 - \frac{2}{27} - \frac{4}{3\cdot 27^2} - \frac{40}{27^4} - \cdots$$

$$= 3 - \tfrac{40378}{531441} = 3 - .0759 = 2.924.$$

34. $\sqrt[3]{9} = \sqrt[3]{8+1} = \sqrt[3]{8}\sqrt[3]{1+\frac{1}{8}} = 2(1+\frac{1}{8})^{\frac{1}{3}}$

$$= 2\left[1^{\frac{1}{3}} + \tfrac{1}{3}\cdot 1^{-\frac{2}{3}}\cdot\tfrac{1}{8} + \frac{\frac{1}{3}(-\frac{2}{3})}{1\cdot 2}1^{-\frac{5}{3}}(\tfrac{1}{8})^2 + \frac{\frac{1}{3}(-\frac{2}{3})(-\frac{5}{3})}{1\cdot 2\cdot 3}1^{-\frac{8}{3}}(\tfrac{1}{8})^3 + \cdots\right]$$

$$= 2\left[1 + \frac{1}{3\cdot 8} - \frac{1}{9\cdot 8^2} + \frac{5}{81\cdot 8^3} - \cdots\right]$$

$$= 2 + \frac{3322}{9^2\cdot 8^3} = 2.080.$$

35. $\sqrt[5]{30} = \sqrt[5]{32-2} = \sqrt[5]{32}\sqrt[5]{1-\frac{1}{16}} = 2(1-\frac{1}{16})^{\frac{1}{5}}$

$$= 2\left[1^{\frac{1}{5}} - \tfrac{1}{5}\cdot 1^{-\frac{4}{5}}\cdot\tfrac{1}{16} + \frac{\frac{1}{5}(-\frac{4}{5})}{1\cdot 2}1^{-\frac{9}{5}}(\tfrac{1}{16})^2 - \cdots\right]$$

$$= 2[1 - \tfrac{1}{80} - \tfrac{1}{3200} - \cdots] = 2 - .0256 = 1.974.$$

LOGARITHMS

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16.

log 1050	=	3.0212
log 1060	=	3.0253
Tab. Diff.	=	41
		.4
Tab. Diff. × .4	=	16.4
log 1050	=	3.0212
log 1054	=	3.0228

17.

log 1270	=	3.1038
log 1280	=	3.1072
Tab. Diff.	=	34
		.2
Tab. Diff. × .2	=	6.8
log 1270	=	3.1038
log 1272	=	3.1045

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18.
$$\begin{array}{ll} \log .0165 & = \bar{2}.2175 \\ \text{Adding } 10-10, & = 8.2175-10 \end{array}$$

19.
$$\begin{array}{lr} \log 1900 & = 3.2788 \\ \log 1910 & = 3.2810 \\ \text{Tab. Diff.} & = 22 \\ & .6 \\ \text{Tab. Diff.} \times .6 & = 132 \\ \log 1900 & = 3.2788 \\ \log 1906 & = 3.2801 \end{array}$$

20.
$$\begin{array}{lr} \log 21.00 & = 1.3222 \\ \log 21.10 & = 1.3243 \\ \text{Tab. Diff.} & = 21 \\ & .9 \\ \text{Tab. Diff.} \times .9 & = 189 \\ \log 21.00 & = 1.3222 \\ \log 21.09 & = 1.3241 \end{array}$$

22.
$$\begin{array}{lr} \log 441.0 & = 2.6444 \\ \log 442.0 & = 2.6454 \\ \text{Tab. Diff.} & = 10 \\ & .1 \\ \text{Tab. Diff.} \times .1 & = 1 \\ \log 441.0 & = 2.6444 \\ \log 441.1 & = 2.6445 \end{array}$$

23.
$$\begin{array}{lr} \log .7850 & = \bar{1}.8949 \\ \log .7860 & = \bar{1}.8954 \\ \text{Tab. Diff.} & = 5 \\ & .4 \\ \text{Tab. Diff.} \times .4 & = 2 \\ \log .7850 & = \bar{1}.8949 \\ \log .7854 & = \bar{1}.8951 \\ \text{Adding } 10-10, & = 9.8951-10 \end{array}$$

24.
$$\begin{array}{lr} \log .09090 & = \bar{2}.9586 \\ \log .09100 & = \bar{2}.9590 \\ \text{Tab. Diff.} & = 4 \\ & .5 \\ \text{Tab. Diff.} \times .5 & = 2 \\ \log .09090 & = \bar{2}.9586 \\ \log .09095 & = \bar{2}.9588 \\ \text{Adding } 10-10, & = 8.9588-10 \end{array}$$

25.
$$\begin{array}{lr} \log .10100 & = \bar{1}.0043 \\ \log .10200 & = \bar{1}.0086 \\ \text{Tab. Diff.} & = 43 \\ & .25 \\ \text{Tab. Diff.} \times .25 & = 1075 \\ \log .10100 & = \bar{1}.0043 \\ \log .10125 & = \bar{1}.0054 \\ \text{Adding } 10-10, & = 9.0054-10 \end{array}$$

26.
$$\begin{array}{lr} \log 54.600 & = 1.7372 \\ \log 54.700 & = 1.7380 \\ \text{Tab. Diff.} & = 8 \\ & .75 \\ \text{Tab. Diff.} \times .75 & = 6 \\ \log 54.600 & = 1.7372 \\ \log 54.675 & = 1.7378 \end{array}$$

27.
$$\begin{array}{lr} \log .09880 & = \bar{2}.9948 \\ \log .09890 & = \bar{2}.9952 \\ \text{Tab. Diff.} & = 4 \\ & .5 \\ \text{Tab. Diff.} \times .5 & = 2 \\ \log .09880 & = \bar{2}.9948 \\ \log .09885 & = \bar{2}.9950 \\ \text{Adding } 10-10, & = 8.9950-10 \end{array}$$

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6.
$$\begin{array}{ll} \text{Given mantissa,} & .6669. \\ \text{Mantissa next less,} & .6665\text{; figures corresponding, 464.} \\ \text{Difference,} & \quad 4 \\ \text{Tabular difference,} & 10 \,\rfloor\, 4 \,\lfloor .4 \end{array}$$

Figures corresponding to given mantissa, 4644.
Hence, the number corresponding to 1.6669 is 46.44.

7.
$$\begin{array}{ll} \text{Given mantissa,} & .7971. \\ \text{Mantissa next less,} & .7966\text{; figures corresponding, 626.} \\ \text{Difference,} & \quad 5 \\ \text{Tabular difference,} & 7 \,\rfloor\, 5 \,\lfloor .7 \end{array}$$

Figures corresponding to given mantissa, 6267.
Hence, the number corresponding to 2.7971 is 626.7.

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8. Given mantissa, .9545.
Mantissa next less, .9542; figures corresponding, 900.
Difference, 3
Tabular difference, 5| 3 |.6
Figures corresponding to given mantissa, 9006.
Hence, the number corresponding to 3.9545 is 9006.

9. Given mantissa, .8794.
Mantissa next less, .8791; figures corresponding, 757.
Difference, 3
Tabular difference, 6| 3 |.5
Figures corresponding to given mantissa, 7575.
Hence, the number corresponding to 0.8794 is 7.575.

10. Given mantissa, .9371.
Mantissa next less, .9370; figures corresponding, 865.
Difference, 1
Tabular difference, 5| 1 |.2
Figures corresponding to given mantissa, 8652.
Hence, the number corresponding to 2.9371 is 865.2.

11. Given mantissa, .8294.
Mantissa next less, .8293; figures corresponding, 675.
Difference, 1
Tabular difference, 6| 1 |.2
Figures corresponding to given mantissa, 6752.
Hence, the number corresponding to 0.8294 is 6.752.

12. Given mantissa, .9039.
Mantissa next less, .9036; figures corresponding, 801.
Difference, 3
Tabular difference, 6| 3 |.5
Figures corresponding to given mantissa, 8015.
Hence, the number corresponding to 1.9039 is 80.15.

13. Given mantissa, .3685.
Mantissa next less, .3674; figures corresponding, 233.
Difference, 11
Tabular difference, 18| 11 |.6
Figures corresponding to given mantissa, 2336.
Since the characteristic is 9 – 10, or – 1, the number corresponding to the logarithm 9.3685 – 10 is .2336.

14. Given mantissa, .9932.
Mantissa next less, .9930; figures corresponding, 984.
Difference, 2
Tabular difference, 4| 2 |.5
Figures corresponding to given mantissa, 9845.
Since the characteristic is 8 – 10, or – 2, the number corresponding to the logarithm 8.9932 – 10 is .09845.

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15. Given mantissa, .9535.
Mantissa next less, .9533; figures corresponding, 898.
Difference, 2
Tabular difference, 5) 2 (.4
Figures corresponding to given mantissa, 8984.

Since the characteristic is 8 − 10, or − 2, the number corresponding to the logarithm 8.9535 − 10 is .08984.

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2.
$$\begin{array}{ll}
\log 3.8 & = 0.5798 \\
\log 56 & = 1.7482 \\ \hline
\text{log of product} & = 2.3280 \\
2.3280 & = \log 212.8 \\
\therefore \quad 3.8 \times 56 & = 212.8.
\end{array}$$

3.
$$\begin{array}{ll}
\log 72 & = 1.8573 \\
\log 39 & = 1.5911 \\ \hline
\text{log of product} & = 3.4484 \\
3.4484 & = \log 2808 \\
\therefore \quad 72 \times 39 & = 2808.
\end{array}$$

4.
$$\begin{array}{ll}
\log 8.5 & = 0.9294 \\
\log 6.2 & = 0.7924 \\ \hline
\text{log of product} & = 1.7218 \\
1.7218 & = \log 52.70 \\
\therefore \quad 8.5 \times 6.2 & = 52.7.
\end{array}$$

5.
$$\begin{array}{ll}
\log 1.64 & = 0.2148 \\
\log 35 & = 1.5441 \\ \hline
\text{log of product} & = 1.7589 \\
1.7589 & = \log 57.40 \\
\therefore \quad 1.64 \times 35 & = 57.4.
\end{array}$$

6.
$$\begin{array}{ll}
\log 2.26 & = 0.3541 \\
\log 85 & = 1.9294 \\ \hline
\text{log of product} & = 2.2835 \\
2.2835 & = \log 192.1 \\
\therefore \quad 2.26 \times 85 & = 192.1.
\end{array}$$

7.
$$\begin{array}{ll}
\log 7.25 & = 0.8603 \\
\log 240 & = 2.3802 \\ \hline
\text{log of product} & = 3.2405 \\
3.2405 & = \log 1740 \\
\therefore \quad 7.25 \times 240 & = 1740.
\end{array}$$

8.
$$\begin{array}{ll}
\log 3272 & = 3.5148 \\
\log 75 & = 1.8751 \\ \hline
\text{log of product} & = 5.3899 \\
5.3899 & = \log 245{,}400 \\
\therefore \quad 3272 \times 75 & = 245{,}400.
\end{array}$$

9.
$$\begin{array}{ll}
\log .892 & = 9.9504 - 10 \\
\log .805 & = 9.9058 - 10 \\ \hline
\text{log of product} & = 19.8562 - 20 \\
 & = \bar{1}.8562 \\
\bar{1}.8562 & = \log .7182 \\
\therefore \quad .892 \times .805 & = .7182.
\end{array}$$

10.
$$\begin{array}{ll}
\log 289 & = 2.4609 \\
\log .7854 & = 9.8951 - 10 \\ \hline
\text{log of product} & = 12.3560 - 10 \\
 & = 2.3560 \\
2.3560 & = \log 227.0 \\
\therefore \quad 289 \times .7854 & = 227.
\end{array}$$

11.
$$\begin{array}{ll}
\log 42.37 & = 1.6271 \\
\log .236 & = 9.3729 - 10 \\ \hline
\text{log of product} & = 11.0000 - 10 \\
 & = 1.0000 \\
1.0000 & = \log 10.00 \\
\therefore \quad 42.37 \times .236 & = 10.
\end{array}$$

12.
$$\begin{array}{ll}
\log 2912 & = 3.4642 \\
\log .7281 & = 9.8622 - 10 \\ \hline
\text{log of product} & = 13.3264 - 10 \\
 & = 3.3264 \\
3.3264 & = \log 2120 \\
\therefore \quad 2912 \times .7281 & = 2120.
\end{array}$$

13.
$$\begin{array}{ll}
\log 1.414 & = 0.1504 \\
\log 2.829 & = 0.4516 \\ \hline
\text{log of product} & = 0.6020 \\
0.6020 & = \log 3.999 \\
\therefore \quad 1.414 \times 2.829 & = 3.999.
\end{array}$$

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3.
$$\begin{array}{ll} \log 3025 & = 3.4807 \\ \log 55 & = 1.7404 \\ \hline \text{log of quotient} & = 1.7403 \\ 1.7403 & = \log 54.99 \\ \therefore \quad 3025 \div 55 & = 54.99. \end{array}$$

4.
$$\begin{array}{ll} \log 4090 & = 3.6117 \\ \log 32 & = 1.5051 \\ \hline \text{log of quotient} & = 2.1066 \\ 2.1066 & = \log 127.8 \\ \therefore \quad 4090 \div 32 & = 127.8. \end{array}$$

5.
$$\begin{array}{ll} \log 3250 & = 3.5119 \\ \log 57 & = 1.7559 \\ \hline \text{log of quotient} & = 1.7560 \\ 1.7560 & = \log 57.01 \\ \therefore \quad 3250 \div 57 & = 57.01. \end{array}$$

6.
$$\begin{array}{ll} \log .2601 & = 19.4152 - 20 \\ \log .68 & = 9.8325 - 10 \\ \hline \text{log of quotient} & = 9.5827 - 10 \\ 9.5827 - 10 & = \log .3825 \\ \therefore \quad .2601 \div .68 & = .3825. \end{array}$$

7.
$$\begin{array}{ll} \log 3950 & = 13.5966 - 10 \\ \log .250 & = 9.3979 - 10 \\ \hline \text{log of quotient} & = 4.1987 \\ 4.1987 & = \log 15{,}800 \\ \therefore \quad 3950 \div .250 & = 15{,}800. \end{array}$$

8.
$$\begin{array}{ll} \log 10 & = 1.0000 \\ \log 3.14 & = 0.4969 \\ \hline \text{log of quotient} & = 0.5031 \\ 0.5031 & = \log 3.185 \\ \therefore \quad 10 \div 3.14 & = 3.185 \end{array}$$

9.
$$\begin{array}{ll} \log .6911 & = 19.8396 - 20 \\ \log .7854 & = 9.8951 - 10 \\ \hline \text{log of quotient} & = 9.9445 - 10 \\ 9.9445 - 10 & = \log .8800 \\ \therefore .6911 \div .7854 & = .88. \end{array}$$

10.
$$\begin{array}{ll} \log 2.816 & = 10.4496 - 10 \\ \log 22.5 & = 1.3522 \\ \hline \text{log of quotient} & = 9.0974 - 10 \\ 9.0974 - 10 & = \log .1251 \\ \therefore \quad 2.816 \div 22.5 & = .1251. \end{array}$$

11.
$$\begin{array}{ll} \log 4 & = 10.6021 - 10 \\ \log .00521 & = 7.7168 - 10 \\ \hline \text{log of quotient} & = 2.8853 \\ 2.8853 & = \log 767.8 \\ \therefore \quad 4 \div .00521 & = 767.8. \end{array}$$

12.
$$\begin{array}{ll} \log 26 & = 11.4150 - 10 \\ \log .06771 & = 8.8307 - 10 \\ \hline \text{log of quotient} & = 2.5843 \\ 2.5843 & = \log 384.0 \\ \therefore \quad 26 \div .06771 & = 384. \end{array}$$

13.
$$\begin{array}{ll} \log 1 & = 10.0000 - 10 \\ \log 40 & = 1.6021 \\ \hline \log (1 \div 40) & = 8.3979 - 10 \\ 8.3979 - 10 & = \log .0250 \\ \therefore \quad 1 \div 40 & = .025. \end{array}$$

14.
$$\begin{array}{ll} \log 1 & = 10.0000 - 10 \\ \log 75 & = 1.8751 \\ \hline \text{log of quotient} & = 8.1249 - 10 \\ 8.1249 - 10 & = \log .01333 \\ \therefore \quad 1 \div 75 & = .01333. \end{array}$$

15.
$$\begin{array}{ll} \log 200 & = 12.3010 - 10 \\ \log .5236 & = 9.7190 - 10 \\ \hline \text{log of quotient} & = 2.5820 \\ 2.5820 & = \log 381.9 \\ \therefore \quad 200 \div .5236 & = 381.9. \end{array}$$

16.
$$\begin{array}{ll} \log 300 & = 2.4771 \\ \log 17.32 & = 1.2385 \\ \hline \text{log of quotient} & = 1.2386 \\ 1.2386 & = \log 17.32 \\ \therefore \quad 300 \div 17.32 & = 17.32. \end{array}$$

17.
$$\begin{array}{ll} \log .220 & = 19.3424 - 20 \\ \log .3183 & = 9.5028 - 10 \\ \hline \text{log of quotient} & = 9.8396 - 10 \\ 9.8396 - 10 & = \log .6912. \\ \therefore \quad .220 \div .3183 & = .6912. \end{array}$$

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2.

log 110	= 2.0414
log 3.1	= 0.4914
log .653	= 9.8149 − 10
colog 33	= 8.4815 − 10
colog 7.854	= 9.1049 − 10
colog 1.7	= 9.7696 − 10
log of result	= 39.7037 − 40
∴ result	= .5054.

3.

log 15	= 1.1761
log .37	= 9.5682 − 10
log 26.16	= 1.4176
colog 88	= 8.0555 − 10
colog .18	= 0.7447
colog 6.67	= 9.1759 − 10
log of result	= 30.1380 − 30
	= 0.1380
∴ result	= 1.374.

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4. In accordance with § 578, treat negative numbers as positive.

log 3.04	= 0.4829
log .2608	= 9.4163 − 10
colog 2.046	= 9.6891 − 10
colog .06219	= 1.2063
log of result	= 20.7946 − 20
∴ result	= 6.231.

By law of signs result is negative.

∴ result = − 6.231.

5.

log 600	= 2.7782
log 5	= 0.6990
log 29	= 1.4624
colog .7854	= 0.1049
colog 25000	= 5.6021 − 10
colog 81.7	= 8.0878 − 10
log of result	= 18.7344 − 20
∴ result	= .05425.

6.

log 3.516	= 0.5460
log 485	= 2.6857
log 65	= 1.8129
colog 3.33	= 9.4776 − 10
colog 17	= 8.7696 − 10
colog 18	= 8.7447 − 10
colog 73	= 8.1367 − 10
log of result	= 40.1732 − 40
	= 0.1732
∴ result	= 1.49.

7. In accordance with § 578, treat negative numbers as positive.

log .4051	= 9.6076 − 10
log 12.45	= 1.0952
colog 8.988	= 9.0463 − 10
colog .01442	= 1.8410
log of result	= 21.5901 − 20
∴ result	= 38.92

By law of signs result is positive.

8.

log 78	= 1.8921
log 52	= 1.7160
log 1605	= 3.2055
colog 338	= 7.4711 − 10
colog 767	= 7.1152 − 10
colog 431	= 7.3655 − 10
log of result	= 28.7654 − 30
∴ result	= .05826.

9.

log .5	= 9.6990 − 10
log .315	= 9.4983 − 10
log 428	= 2.6314
colog .317	= 0.4989
colog .973	= 0.0119
colog 43.7	= 8.3595 − 10
log of result	= 30.6990 − 30
	= 0.6990
∴ result	= 5.

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2.

log 7	= 0.8451
2 log 7	= 1.6902
1.6902	= log 49.00
∴ 7^2	= 49.

3.

log 11	= 1.0414
2 log 11	= 2.0828
2.0828	= log 121.0
∴ 11^2	= 121.

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4. In accordance with § 578, treat negative numbers as positive.

$\log 47 = 1.6721$
$2 \log 47 = 3.3442$
$3.3442 = \log 2209$
$\therefore (-47)^2 = 2209.$

By law of signs result is positive.

5. $\log 4.9 = 0.6902$
$2 \log 4.9 = 1.3804$
$1.3804 = \log 24.01.$
$\therefore 4.9^2 = 24.01.$

6. $\log 5.2 = 0.7160$
$2 \log 5.2 = 1.4320$
$1.4320 = \log 27.04$
$\therefore 5.2^2 = 27.04.$

7. $\log .78 = 9.8921 - 10$
$2 \log .78 = 19.7842 - 20$
$19.7842 - 20 = \log .6084$
$\therefore .78^2 = .6084.$

8. $\log 8.05 = 0.9058$
$2 \log 8.05 = 1.8116$
$1.8116 = \log 64.80$
$\therefore 8.05^2 = 64.8.$

9. $\log 8.33 = 0.9206$
$2 \log 8.33 = 1.8412$
$1.8412 = \log 69.37$
$\therefore 8.33^2 = 69.37.$

10. $\log 6.61 = 0.8202$
$3 \log 6.61 = 2.4606$
$2.4606 = \log 288.8$
$\therefore 6.61^3 = 288.8.$

11. $\log .714 = 9.8537 - 10$
$2 \log .714 = 19.7074 - 20$
$19.7074 - 20 = \log .5098$
$\therefore .714^2 = .5098.$

12. $\log 4.07 = 0.6096$
$3 \log 4.07 = 1.8288$
$1.8288 = \log 67.42$
$\therefore 4.07^3 = 67.42.$

13. $\log .543 = 9.7348 - 10$
$3 \log .543 = 29.2044 - 30$
$29.2044 - 30 = \log .1601$
$\therefore .543^3 = .1601.$

14. In accordance with § 578, treat negative numbers as positive.

$\log 7 = 0.8451$
$4 \log 7 = 3.3804$
$3.3804 = \log 2401$
$\therefore (-7)^4 = 2401.$

By law of signs result is positive.

15. $\log 1.02 = 0.0086$
$5 \log 1.02 = 0.0430$
$0.0430 = \log 1.104$
$\therefore 1.02^5 = 1.104.$

16. $\log 1.738 = 0.2400$
$3 \log 1.738 = 0.7200$
$0.7200 = \log 5.248$
$\therefore 1.738^3 = 5.248.$

17. $\log \frac{3}{20} = \log .15 = 9.1761-10$
$2 \log \frac{3}{20} = 18.3522-20$
$18.3522 - 20 = \log .0225$
$\therefore (\frac{3}{20})^2 = .0225.$

18. $\log \frac{1}{7} = \text{colog } 7 = 9.1549-10$
$3 \log \frac{1}{7} = 27.4647-30$
$27.4647 - 30 = \log .002915$
$\therefore (\frac{1}{7})^3 = .002915.$

19. $\log 64 = 1.8062$
$\text{colog } 869 = 7.0610-10$

$\log \frac{128}{1738} = \log \frac{64}{869} = 8.8672-10$
$2 \log \frac{128}{1738} = 17.7344-20$
$17.7344 - 20 = \log .005425$
$\therefore (\frac{128}{1738})^2 = .005425.$

20. $\log 675 = 2.8293$
$\text{colog } 4121 = 6.3850-10$

$\log \frac{675}{4121} = 9.2143-10$
$3 \log \frac{675}{4121} = 27.6429-30$
$27.6429 - 30 = \log .004394$
$\therefore (\frac{675}{4121})^3 = .004394.$

21. $\log \frac{1}{243} = \text{colog } 243 = 7.6144-10$
$.4 \log \frac{1}{243} = 3.0458- 4$
$= 9.0458-10$
$9.0458 - 10 = \log .1111$
$\therefore (\frac{1}{243})^{.4} = .1111.$

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2.
$$\begin{aligned} \log 225 &= 2.3522 \\ \tfrac{1}{2}\log 225 &= 1.1761 \\ 1.1761 &= \log 15.00 \\ \therefore 225^{\frac{1}{2}} &= 15. \end{aligned}$$

3.
$$\begin{aligned} \log 12.25 &= 1.0882 \\ \tfrac{1}{2}\log 12.25 &= 0.5441 \\ 0.5441 &= \log 3.500 \\ \therefore 12.25^{\frac{1}{2}} &= 3.5. \end{aligned}$$

4.
$$\begin{aligned} \log .2023 &= 19.3060-20 \\ \tfrac{1}{2}\log .2023 &= 9.6530-10 \\ 9.6530-10 &= \log .4498 \\ \therefore .2023^{\frac{1}{2}} &= .4498. \end{aligned}$$

5.
$$\begin{aligned} \log 326 &= 2.5132 \\ \tfrac{1}{2}\log 326 &= 1.2566 \\ 1.2566 &= \log 18.05 \\ \therefore 326^{\frac{1}{2}} &= 18.05. \end{aligned}$$

6.
$$\begin{aligned} \log .512 &= 29.7093-30 \\ \tfrac{1}{3}\log .512 &= 9.9031-10 \\ 9.9031-10 &= \log .8000 \\ \therefore .512^{\frac{1}{3}} &= .8. \end{aligned}$$

7.
$$\begin{aligned} \log .1182 &= 19.0726-20 \\ \tfrac{1}{2}\log .1182 &= 9.5363-10 \\ 9.5363-10 &= \log .3438 \\ \therefore .1182^{\frac{1}{2}} &= .3438. \end{aligned}$$

8. In accordance with § 578, treat negative numbers as positive.
$$\begin{aligned} \log 1331 &= 3.1242 \\ \tfrac{1}{3}\log 1331 &= 1.0414 \\ 1.0414 &= \log 11.00 \\ \therefore 1331^{\frac{1}{3}} &= 11. \end{aligned}$$
By law of signs result is negative.
$$\therefore (-1331)^{\frac{1}{3}} = -11.$$

9.
$$\begin{aligned} \log 1024 &= 3.0103 \\ .7\log 1024 &= 2.1072 \\ 2.1072 &= \log 128.0 \\ \therefore 1024^{\frac{7}{10}} &= 128. \end{aligned}$$

10.
$$\begin{aligned} \log .6724 &= 19.8276-20 \\ \tfrac{1}{2}\log .6724 &= 9.9138-10 \\ 9.9138-10 &= \log .8200 \\ \therefore .6724^{\frac{1}{2}} &= .82. \end{aligned}$$

11.
$$\begin{aligned} \log 5.929 &= 0.7730 \\ \tfrac{1}{2}\log 5.929 &= 0.3865 \\ 0.3865 &= \log 2.435 \\ \therefore 5.929^{\frac{1}{2}} &= 2.435. \end{aligned}$$

12.
$$\begin{aligned} \log .4624 &= 19.6650-20 \\ \tfrac{1}{2}\log .4624 &= 9.8325-10 \\ 9.8325-10 &= \log .6800 \\ \therefore .4624^{\frac{1}{2}} &= .68. \end{aligned}$$

13.
$$\begin{aligned} \log 1.4641 &= 0.1656 \\ \tfrac{1}{4}\log 1.4641 &= 0.0414 \\ 0.0414 &= \log 1.100 \\ \therefore 1.4641^{\frac{1}{4}} &= 1.1. \end{aligned}$$

14.
$$\begin{aligned} \log 2 &= 0.3010 \\ \tfrac{1}{2}\log 2 &= 0.1505 \\ 0.1505 &= \log 1.414 \\ \therefore \sqrt{2} &= 1.414. \end{aligned}$$

15.
$$\begin{aligned} \log 3 &= 0.4771 \\ \tfrac{1}{2}\log 3 &= 0.2386 \\ 0.2386 &= \log 1.732 \\ \therefore \sqrt{3} &= 1.732. \end{aligned}$$

16.
$$\begin{aligned} \log 5 &= 0.6990 \\ \tfrac{1}{2}\log 5 &= 0.3495 \\ 0.3495 &= \log 2.236 \\ \therefore \sqrt{5} &= 2.236. \end{aligned}$$

17.
$$\begin{aligned} \log 6 &= 0.7782 \\ \tfrac{1}{2}\log 6 &= 0.3891 \\ 0.3891 &= \log 2.449 \\ \therefore \sqrt{6} &= 2.449. \end{aligned}$$

18.
$$\begin{aligned} \log 2 &= 0.3010 \\ \tfrac{1}{3}\log 2 &= 0.1003 \\ 0.1003 &= \log 1.260 \\ \therefore \sqrt[3]{2} &= 1.26. \end{aligned}$$

19.
$$\begin{aligned} \log 4 &= 0.6021 \\ \tfrac{1}{4}\log 4 &= 0.1505 \\ 0.1505 &= \log 1.414 \\ \therefore \sqrt[4]{4} &= 1.414. \end{aligned}$$

20. In accordance with § 578, treat negative numbers as positive.
$$\begin{aligned} \log 2 &= 0.3010 \\ \tfrac{1}{5}\log 2 &= 0.0602 \\ 0.0602 &= \log 1.149 \end{aligned}$$
By law of signs result is negative.
$$\therefore \sqrt[5]{-2} = -1.149.$$

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21. log .027 = 28.4314 − 30
$\frac{1}{3}$ log .027 = 9.4771 − 10
9.4771 − 10 = log .3000
$\therefore \sqrt[3]{.027}$ = .3.

22. $\sqrt{30\frac{4}{5}} = \sqrt{\frac{154}{5}}$
log 154 = 2.1875
colog 5 = 9.3010 − 10
log of result = 11.4885 − 10
= 1.4885
$\frac{1}{2}$ log of result = .7443
.7443 = log 5.55
$\therefore \sqrt{30\frac{4}{5}}$ = 5.55.

23. log .90 = 19.9542 − 20
$\frac{1}{2}$ log .90 = 9.9771 − 10
9.9771 − 10 = log .9486
$\therefore \sqrt{.90}$ = .9486.

24. log .52 = 19.7160 − 20
$\frac{1}{2}$ log .52 = 9.8580 − 10
9.8580 − 10 = log .7212
$\therefore \sqrt{.52}$ = .7212.

25. log .032 = 48.5051 − 50
$\frac{1}{5}$ log .032 = 9.7010 − 10
9.7010 − 10 = log .5023
$\therefore \sqrt[5]{.032}$ = .5023.

26. $\dfrac{176}{15 \times 3.1416} = \dfrac{4}{15 \times .0714}$
log 4 = 0.6021
colog 15 = 8.8239 − 10
colog .0714 = 1.1463
log result = 10.5723 − 10
= 0.5723
$\therefore$ result = 3.735.

27. In accordance with § 578, treat negative numbers as positive.
log 100^2 = 4.0000
colog 48 = 8.3188 − 10
colog 64 = 8.1938 − 10
colog 11 = 8.9586 − 10
log result = 29.4712 − 30
$\therefore$ result = .2959.
By law of signs result is positive.

28. log 52 = 1.7160
log 52 = 1.7160
log 300 = 2.4771
colog 12 = 8.9208 − 10
colog .31225 = 0.5055
colog 400000 = 4.3979 − 10
log result = 19.7333 − 20
$\therefore$ result = .5411.

29. log 400 = 2.6021
colog 55 = 8.2596 − 10
colog 3.1416 = 9.5029 − 10
log result^2 = 0.3646
log result = 0.1823
$\therefore$ result = 1.522.

30. log $2^{3.5}$ = 0.3010 × 3.5
= 1.0535 (1)
log $8^{1.63}$ = 0.9031 × 1.63
= 1.4721 (2)
Subtracting (2) from (1),
log $\dfrac{2^{3.5}}{8^{1.63}}$ = 9.5814 − 10
log 50 = 1.6990
log result = 1.2804
$\therefore$ result = 19.07.

31. In accordance with § 578, treat negative numbers as if positive.
log 1.6 = 0.2041
$\frac{1}{3}$ log 1.6 = 0.0680
log 14.5 = 1.1614
colog 11 = 8.9586 − 10
log result = 10.1880 − 10
= 0.1880
$\therefore$ result = 1.542.
By law of signs result is negative.
$\therefore$ result = − 1.542.

32. $\sqrt{\dfrac{.434 \times 96^4}{64 \times 1500}} = \dfrac{96^2}{80}\sqrt{\dfrac{.434}{15}}$
log .434 = 9.6375 − 10
colog 15 = 8.8239 − 10
2 | 18.4614 − 20
log $\sqrt{\dfrac{.434}{15}}$ = 9.2307 − 10
log 96^2 = 3.9646
colog 80 = 8.0969 − 10
log result = 21.2922 − 20
= 1.2922
$\therefore$ result = 19.6.

33. $\dfrac{.32 \times 5000 \times 18}{3.14 \times .1222 \times 8} = \dfrac{1800}{3.14 \times .0611}$
log 1800 = 3.2553
colog 3.14 = 9.5031 − 10
colog .0611 = 1.2140
log result = 13.9724 − 10
= 3.9724
$\therefore$ result = 9384.

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34.

log 11	= 1.0414
log 2.63	= 0.4200
log 4.263	= 0.6297
colog 48	= 8.3188 − 10
colog 3.263	= 9.4864 − 10
log result	= 19.8963 − 20
∴ result	= .7876.

35. In accordance with § 578, treat negative numbers as if positive.

log 1.06	= 0.0253
4 log 1.06	= 0.1012
colog 1.06^4	= 9.8988 − 10
log 3500	= 3.5441
$\log \frac{3500}{1.06^4}$	= 13.4429 − 10
	= 3.4429
$\frac{1}{2} \log \frac{3500}{1.06^4}$	= 1.7215
∴ result	= 52.66.

By law of signs, result is positive.

36.

$$2^{\frac{1}{2}} \times (\tfrac{1}{2})^{\frac{2}{3}} \times \sqrt[3]{\tfrac{3}{2}} \times \sqrt{.1}$$

$$= 2^{\frac{1}{2}} \times (\tfrac{1}{4})^{\frac{1}{3}} \times (1.5)^{\frac{1}{3}} \times (.1)^{\frac{1}{2}}$$

$\frac{1}{2}$ log 2	= 0.1505
$\frac{1}{3}$ log .25	= 9.7993 − 10
$\frac{1}{3}$ log 1.5	= 0.0587
$\frac{1}{2}$ log .1	= 9.5000 − 10
log result	= 19.5085 − 20
∴ result	= .3225.

37. Given

$$A = \pi r^2.$$

Substituting the given values,

$$A = 3.1416 \times (12.35)^2.$$

Then,

$$\log A = \log 3.1416 + 2 \log 12.35$$

$$\log 3.1416 = 0.4971$$

$$2 \log 12.35 = 2.1834$$

$$\therefore \log A = 2.6805$$

$$A = 479.2.$$

Hence, the area of the circle is 479.2 square meters.

38. Given

$$V = \tfrac{4}{3} \pi r^3$$

Substituting the given values,

$$V = \tfrac{4}{3} \times 3.1416 \times (40.11)^3.$$

Then,

$$\log V = \log 4 + \text{colog } 3 + \log 3.1416 + 3 \log 40.11.$$

$$\log 4 = 0.6021$$

$$\text{colog } 3 = 9.5229 - 10$$

$$\log 3.1416 = 0.4971$$

$$3 \log 40.11 = 4.8096$$

$$\therefore \log V = 15.4317 - 10$$

$$\log V = 5.4317$$

$$V = 270{,}200.$$

Hence, the volume of the sphere is 270,200 cubic centimeters.

39. Given

$$V = .7854\, d^2 l. \qquad (1)$$

Expressing the length of the diameter in feet,

.3648 inches = .0304 feet.

Substituting the given values in (1),

$$1 = .7854 \times (.0304)^2 l.$$

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Then,

$$\log l = \log 1 + \text{colog}\ .7854 + 2\ \text{colog}\ .0304.$$

$$\begin{aligned}
\log 1 &= 0.0000\\
\text{colog}\ .7854 &= \ \ .1049\\
2\ \text{colog}\ .0304 &= 3.0342\\
\hline
\log l &= 3.1391\\
\therefore l &= 1378.
\end{aligned}$$

Hence, 1378 feet of No. 00 wire can be made from a cubic foot of copper.

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6. $3^x = 81 = 3^4.$
$\therefore x = 4.$

7. $4^x = 10.$
$x \log 4 = \log 10 = 1.$
$\therefore x = \dfrac{1}{\log 4}$
$= \dfrac{1}{0.6021}.$
$\therefore \log x = \text{colog}\ .6021$
$= 0.2203.$
$\therefore x = 1.660.$

8. $2^x = 80.$
$x \log 2 = \log 80$
$\therefore x = \dfrac{\log 80}{\log 2}$
$= \dfrac{1.9031}{0.3010}.$

$$\begin{aligned}
\log 1.9031 &= 0.2795\\
\log\ \ .3010 &= \bar{1}.4786\\
\hline
\therefore \log x &= 0.8009\\
\therefore x &= 6.323.
\end{aligned}$$

9. $3^{x^2} = 9^{2x} = (3^2)^{2x}.$
$\therefore 3^{x^2} = 3^{4x}.$
$\therefore x^2 = 4x.$
$\therefore x = 4 \text{ or } 0.$

10. $2^{3^y} = 512 = 2^9.$
$\therefore 3^y = 9 = 3^2.$
$\therefore y = 2.$

11. $5^{x^6} = 625 = 5^4.$
$\therefore x^6 = 4.$
$\therefore x = \sqrt[6]{4} = \sqrt[3]{2}.$

12. $2^{x^2} = 512 = 2^9.$
$\therefore x^2 = 9.$
$\therefore x = \pm 3.$

13. $(2^x)^2 = 256 = 16^2.$
$\therefore 2^x = 16 = 2^4.$
$\therefore x = 4.$

14. $\begin{cases} 3^x = 2y, & (1)\\ 4^x = 20y. & (2)\end{cases}$
$4^x = 10 \cdot 2y = 10 \cdot 3^x.$
$\therefore x \log 4 = \log 10 + x \log 3.$
$\therefore x = \dfrac{1}{\log 4 - \log 3}$
$= \dfrac{1}{0.1250} = 8.$
By (1), $\log y = x \log 3 - \log 2$
$= 8 \log 3 - \log 2$
$= 3.5158.$
$\therefore y = 3279.$

15. $3^{2x} - 36 \cdot 3^x + 243 = 0.$
$(3^x - 9)(3^x - 27) = 0.$
$\therefore 3^x = 9 \text{ or } 27;$ that is,
$3^x = 3^2 \text{ or } 3^3.$
$\therefore x = 2 \text{ or } 3.$

16. $\log \log x = \log 2.$
$\therefore \log x = 2.$
$\therefore x = 10^2 = 100.$

17. $\begin{cases} 2^{x+y} = 6, & (1)\\ 2^{x+1} = 3^y. & (2)\end{cases}$

By (1), $2^{x+y} = 2 \cdot 3.$ (3)

Dividing (3) by (2), $2^{y-1} = 2 \cdot 3^{1-y},$

or $2^{y-2} = 3^{1-y}.$

$$\therefore (y-2)\log 2 = (1-y)\log 3.$$
$$(\log 2 + \log 3)y = \log 3 + 2\log 2 = \log(3 \cdot 2^2).$$
$$(\log 6)y = \log 12,$$

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$$\therefore y = \frac{\log 12}{\log 6} = \frac{1.0792}{0.7782}.$$

$$\log y = \log 1.0792 - \log .7782 = 0.1420.$$

$$\therefore y = 1.387.$$

By (1), $(x+y)\log 2 = \log 6.$

$$x = \frac{\log 6 - y\log 2}{\log 2} = \frac{\log 6}{\log 2} - y$$

$$= \frac{0.7782}{0.3010} - 1.387$$

by logarithms, $= 2.585 - 1.387 = 1.198.$

18.
$$\begin{cases} 4^{x+y} = 32, & (1) \\ 2^{2x-y} = 4. & (2) \end{cases}$$

By (1), $2^{2x+2y} = 2^5.$ (3)

By (2), $2^{2x-y} = 2^2.$ (4)

By (3), $2x + 2y = 5.$ (5)

By (4), $2x - y = 2.$ (6)

Solving (5) and (6), $x = \frac{3}{2},\ y = 1.$

19.
$$\begin{cases} 2^x = y, & (1) \\ x = 1 + \log y. & (2) \end{cases}$$

By (1) and (2), $x = 1 + \log 2^x = 1 + x\log 2.$

$$\therefore x = \frac{1}{1-\log 2} = \frac{1}{.6990}.$$

$$\log x = \text{colog } .6990 = 0.1555.$$

$$\therefore x = 1.4306.$$

By (2), $\log y = x - 1 = 0.4306.$

$$\therefore y = 2.695.$$

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2. $A = P(1+r)^n = 225 \times 1.08^5.$

$\log 225$	$= 2.3522$
$\log 1.08^5$	$= 0.1670$
$\log A$	$= 2.5192$
$\therefore$ amount	$= \$330.50.$

3. $A = P(1+r)^n = 700 \times 1.06^5.$

$\log 700$	$= 2.8451$
$\log 1.06^5$	$= 0.1265$
$\log A$	$= 2.9716$
$\therefore$ amount	$= \$936.70.$

4. $A = P(1+r)^n = 400 \times 1.03^{10}.$

$\log 400$	$= 2.6021$
$\log 1.03^{10}$	$= 0.1280$
$\log A$	$= 2.7301$
$\therefore$ amount	$= \$537.10.$

5. $A = P(1+r)^n = 1200 \times 1.04^{20}.$

$\log 1200$	$= 3.0792$
$\log 1.04^{20}$	$= 0.3400$
$\log A$	$= 3.4192$
$\therefore$ amount	$= \$2625.$

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6. $\log P = \log A - n\log(1+r)$
$= \log 1000 - 10\log 1.05.$

$\log 1000$	$= 3.0000$
$10\log 1.05$	$= 0.2120$
$\log P$	$= 2.7880$
$\therefore$ principal	$= \$613.70.$

7. $\log P = \log A - n\log(1+r)$
$= \log 743 - 20\log 1.02.$

$\log 743$	$= 2.8710$
$20\log 1.02$	$= 0.1720$
$\log P$	$= 2.6990$
$\therefore$ principal	$= \$500.$

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8. $\log P = \log A - n\log(1+r)$
$= \log 1500 - 10\log 1.04.$

log 1500	= 3.1761
10 log 1.04	= 0.1700
log P	= 3.0061
∴ principal	= $1014.

9. $\log P = \log A - n\log(1+r)$
$= \log 1000 - 21\log 1.04$

log 1000	= 3.0000
21 log 1.04	= 0.3570
log P	= 2.6430
∴ principal	= $439.50.

10. $n = \dfrac{\log A - \log P}{\log(1+r)}$
$= \dfrac{\log 1834.5 - \log 800}{\log 1.05}.$

log 1834.5	= 3.2635
log 800	= 2.9031
Diff. of logs	= 0.3604
log 1.05	= 0.0212

$.3604 \div .0212 = 17.$
Hence, the time is 17 years.

11. $A = P(1+r)^n.$
$\therefore r = \sqrt[n]{\dfrac{A}{P}} - 1.$

log 402	= 2.6042
log 300	= 2.4771
log (402 ÷ 300)	= 0.1271
$\frac{1}{6}$ log (402 ÷ 300)	= 0.0212
0.0212	= log 1.05
∴ r = 1.05 − 1	= .05;

i.e. rate = 5%.

12. $n = \dfrac{\log A - \log P}{\log(1+r)}$
$= \dfrac{\log 2000 - \log 1000}{\log 1.06}.$

log 2000	= 3.3010
log 1000	= 3.0000
Diff. of logs	= 0.3010
log 1.06	= 0.0253

$.3010 \div .0253 = 11.9$, nearly.
∴ time = 11.9 years, nearly.

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2. $A = \dfrac{a}{r}[(1+r)^n - 1]$
$= \dfrac{25}{.04}[1.04^{20} - 1]$
$= 625[1.04^{20} - 1].$

By logarithms,

1.04^{20}	= 2.188
∴ $1.04^{20} - 1$	= 1.188
log 625	= 2.7959
log 1.188	= 0.0748
∴ log A	= 2.8707

∴ amount = $742.50.

3. $A = \dfrac{a}{r}[(1+r)^n - 1]$
$= \dfrac{17.76}{.035}[1.035^{25} - 1].$

By logarithms,

1.035^{25}	= 2.358
∴ $1.035^{25} - 1$	= 1.358
log 17.76	= 1.2494
log 1.358	= 0.1329
colog .035	= 1.4559
∴ log A	= 2.8382

∴ amount = $689.

4. From $A = \dfrac{a}{r}[(1+r)^n - 1]$,
$a = \dfrac{Ar}{(1+r)^n - 1}$
$= \dfrac{1000 \times .05}{1.05^{10} - 1} = \dfrac{50}{1.05^{10} - 1}.$

By logarithms,

1.05^{10}	= 1.629
∴ $1.05^{10} - 1$	= .629
log 50	= 1.6990
colog .629	= 0.2013
∴ log a	= 1.9003

∴ annuity = $79.48.

5. From $A = \dfrac{a}{r}[(1+r)^n - 1]$,
$a = \dfrac{Ar}{(1+r)^n - 1}$
$= \dfrac{5000 \times .03}{1.03^{12} - 1} = \dfrac{150}{1.03^{12} - 1}.$

By logarithms,

1.03^{12}	= 1.424
∴ $1.03^{12} - 1$	= .424
log 150	= 2.1761
colog .424	= .3726
∴ log a	= 2.5487
∴ annuity	= $353.80.

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2. $P = \frac{a}{r} \cdot \frac{(1+r)^n - 1}{(1+r)^n}$
$= \frac{300}{.04} \cdot \frac{1.04^5 - 1}{1.04^5}$
$= 7500 \cdot \frac{1.04^5 - 1}{1.04^5}.$

By logarithms,

1.04^5	$= 1.216$
$\therefore 1.04^5 - 1$	$= .216$
$\log 7500$	$= 3.8751$
$\log .216$	$= 9.3345 - 10$
$\text{colog } 1.04^5$	$= 9.9150 - 10$
$\therefore \log P$	$= 3.1246$

$\therefore$ p. v. $= \$1332$.

3. $P = \frac{a}{r} \cdot \frac{(1+r)^n - 1}{(1+r)^n}$
$= \frac{1000}{.045} \cdot \frac{1.045^{20} - 1}{1.045^{20}}$

By logarithms,

1.045^{20}	$= 2.41$
$\therefore 1.045^{20} - 1$	$= 1.41$
$\log 1000$	$= 3.0000$
$\log 1.41$	$= 0.1492$
$\text{colog } .045$	$= 1.3468$
$\text{colog } 1.045^{20}$	$= 9.6180 - 10$
$\therefore \log P$	$= 4.1140$

$\therefore$ p. v. $= \$13000$.

4. $P = \frac{a}{r} \cdot \frac{(1+r)^n - 1}{(1+r)^n} = \frac{2000}{.03} \cdot \frac{1.03^{10} - 1}{1.03^{10}}.$

By logarithms,

1.03^{10}	$= 1.343$
$\therefore 1.03^{10} - 1$	$= .343$
$\log 2000$	$= 3.3010$
$\text{colog } .03$	$= 1.5229$
$\log .343$	$= 9.5353 - 10$
$\text{colog } 1.03^{10}$	$= 9.8720 - 10$
$\therefore \log P$	$= 24.2312 - 20$
	$= 4.2312$
$\therefore$ p.v.	$=$ £17,030.

PERMUTATIONS AND COMBINATIONS

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4. The number of ways is equal to the number of permutations of 4 things taken all together.

$$P_4^4 = \lfloor 4 = 4 \cdot 3 \cdot 2 \cdot 1 = 24.$$

5. $$P_6^6 = \lfloor 6 = 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 720.$$

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6. The first prize may be awarded to any one of the 10 athletes, and after that the second prize may be awarded to any one of the 9 athletes remaining. Hence, the number of ways is 10×9, or 90.

7. $$P_7^7 = \lfloor 7 = 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 5040.$$

8. At the bottom of the mountain the traveler has the choice of 5 routes, and having reached the top by any one of them, he may return by any one of the four remaining routes, since he may return by any of the routes except the one he has taken to go up. Hence, the number of ways is 5×4, or 20.

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3. $C^{12}_9 = C^{12}_3 = \frac{12 \cdot 11 \cdot 10}{1 \cdot 2 \cdot 3} = 220.$

4. $C^{52}_5 = \frac{52 \cdot 51 \cdot 50 \cdot 49 \cdot 48}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5} = 2,598,960.$

5. $C^{10}_8 = C^{10}_2$, which is less than C^{10}_3, since $C^{10}_2 = \frac{10 \cdot 9}{1 \cdot 2}$ while

$$C^{10}_3 = \frac{10 \cdot 9}{1 \cdot 2} \times \frac{8}{3};\; C^{10}_4 = \frac{10 \cdot 9 \cdot 8 \cdot 7}{1 \cdot 2 \cdot 3 \cdot 4} \text{ and } C^{10}_5 = \frac{10 \cdot 9 \cdot 8 \cdot 7}{1 \cdot 2 \cdot 3 \cdot 4} \times \frac{6}{5}.$$

$$C^{12}_8 = \frac{12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7 \cdot 8} \text{ and } C^{12}_9 = \frac{12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7 \cdot 8 \cdot 9}$$

$$\therefore C^{12}_8 = \frac{12 \cdot 11 \cdot 10 \cdot 9}{1 \cdot 2 \cdot 3 \cdot 4} \text{ and } C^{12}_9 = \frac{12 \cdot 11 \cdot 10}{1 \cdot 2 \cdot 3}.$$

Hence, $C^{10}_3 > C^{10}_8$, $C^{10}_5 > C^{10}_4$, and $C^{12}_8 > C^{12}_9$.

6. From 11 Republicans, 6 Republicans may be selected in C^{11}_6 ways; from 10 Democrats, 5 Democrats may be selected in C^{10}_5 ways.

Since each combination of 6 Republicans may be associated with each combination of 5 Democrats to form a committee, the number of committees that may be selected is equal to

$$C^{11}_6 \times C^{10}_5 = C^{11}_5 \times C^{10}_5 = \frac{11 \cdot 10 \cdot 9 \cdot 8 \cdot 7}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5} \times \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5} = 116,424.$$

7. He may have to make 10 trials to hit upon the right mark on the first wheel, 13 trials, the second wheel, and 13 trials, the third wheel. In order to get the right combination, therefore, he may have to make $10 \times 13 \times 13$, or 1690, trials.

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10. From 20 consonants, 3 consonants may be selected in C^{20}_3 ways; from 5 vowels, 3 vowels may be selected in C^5_3 ways.

Since each combination of 3 consonants may be associated with each combination of 3 vowels to form a word of six letters, the number of words of six letters differing in the letters composing them is equal to

$$C^{20}_3 \times C^5_3 = C^{20}_3 \times C^5_2 = \frac{20 \cdot 19 \cdot 18}{1 \cdot 2 \cdot 3} \times \frac{5 \cdot 4}{1 \cdot 2} = 11,400.$$

Finally, since from each of these 11,400 words of six letters $\lfloor 6$ words may be formed by permuting the six letters in their places, the whole number of words that may be formed under the conditions of the problem is equal to

$$11,400 \times \lfloor 6 = 11,400 \times 720 = 8,208,000.$$

11. Since there are 5 different coins and these may be taken 1, 2, 3, 4, or 5 at a time, the total number of different sums that may be paid is equal to

$$C^5_1 + C^5_2 + C^5_3 + C^5_4 + C^5_5 = \frac{5}{1} + \frac{5 \cdot 4}{1 \cdot 2} + \frac{5 \cdot 4 \cdot 3}{1 \cdot 2 \cdot 3} + \frac{5 \cdot 4 \cdot 3 \cdot 2}{1 \cdot 2 \cdot 3 \cdot 4} + 1$$
$$= 5 + 10 + 10 + 5 + 1 = 31.$$

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12. The number of boys selected may be 2 or 3 or 4 or 5, and the corresponding number of girls is therefore 4 or 3 or 2 or 1.

From 5 boys 2 boys may be selected in C_2^5 ways and from 5 girls 4 girls may be selected in C_4^5 ways.

Since those committees of 6 which are composed of 2 boys and 4 girls may be formed by associating each group of 2 boys with each group of 4 girls, the number of these committees is $C_2^5 \times C_4^5$.

Similarly, the number of committees of 6 composed of 3 boys and 3 girls is $C_3^5 \times C_3^5$; of 4 boys and 2 girls is $C_4^5 \times C_2^5$; of 5 boys and 1 girl is $C_5^5 \times C_1^5$.

Hence, the whole number of committees of 6 is equal to

$$C_2^5 \times C_4^5 + C_3^5 \times C_3^5 + C_4^5 \times C_2^5 + C_5^5 \times C_1^5 = 10 \cdot 5 + 10 \cdot 10 + 5 \cdot 10 + 1 \cdot 5 = 205.$$

14. Let $$3\,C_3^n = 2\,C_4^{n+1}.$$

Then, $$3\frac{n(n-1)(n-2)}{1 \cdot 2 \cdot 3} = 2\frac{(n+1)(n)(n-1)(n-2)}{1 \cdot 2 \cdot 3 \cdot 4}.$$

Dividing both members by $\dfrac{n(n-1)(n-2)}{1 \cdot 2 \cdot 3}$,

$$3 = \frac{2(n+1)}{4} = \frac{n+1}{2}$$

$$\therefore n = 5.$$

Since $n = 5$, $$C_3^n = \frac{5 \cdot 4 \cdot 3}{1 \cdot 2 \cdot 3} = 10,$$

and $$C_4^{n+1} = \frac{6 \cdot 5 \cdot 4 \cdot 3}{1 \cdot 2 \cdot 3 \cdot 4} = 15.$$

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2. In the word *characteristic* there are three c's, two a's, two r's, two t's, and two i's, and in all fourteen letters. Hence, the number of permutations is

$$\frac{\lfloor 14}{\lfloor 3 \lfloor 2 \lfloor 2 \lfloor 2 \lfloor 2} = 908{,}107{,}200.$$

In the word *coefficient* there are two c's, two e's, two f's, and two i's, and in all eleven letters. Hence, the number of permutations is

$$\frac{\lfloor 11}{\lfloor 2 \lfloor 2 \lfloor 2 \lfloor 2} = 2{,}494{,}800.$$

In the word *ecclesiastical* there are two e's, three c's, two l's, two s's, two i's, and two a's, and in all fourteen letters. Hence, the number of permutations is

$$\frac{\lfloor 14}{\lfloor 2 \lfloor 3 \lfloor 2 \lfloor 2 \lfloor 2 \lfloor 2} = 454{,}053{,}600.$$

In the word *divisibility* there are five i's and in all twelve letters. Hence, the number of permutations is

$$\frac{\lfloor 12}{\lfloor 5} = 3{,}991{,}680.$$

3. The number is $$\frac{\lfloor 11}{\lfloor 4 \lfloor 5 \lfloor 2} = 6930.$$

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2. $C^{10}_{\text{total}} = 2^{10} - 1 = 1023.$

3. $C^{5}_{\text{total}} = 2^{5} - 1 = 31.$

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1. n is an even number, $\therefore r = \dfrac{n}{2} = 5.$

$$C^{10}_{5} = \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5} = 252.$$

2. n is an odd number, $\therefore r = \dfrac{n-1}{2} = 4.$

$$C^{n}_{r} = C^{9}_{4} = \frac{9 \cdot 8 \cdot 7 \cdot 6}{1 \cdot 2 \cdot 3 \cdot 4} = 126.$$

3. (*a*) As many as the number of permutations of the letters in *count*, *er* being attached to each permutation.

$$P^{5}_{5} = \underline{|5} = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120.$$

(*b*) With n as the middle letter the six other letters may be permuted in $P^{6}_{6} = \underline{|6} = 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 720$ ways.

(*c*) If each vowel keeps its position the four consonants may be permuted in $P^{4}_{4} = \underline{|4} = 4 \cdot 3 \cdot 2 \cdot 1 = 24$ ways.

(*d*) The first letter may be taken in 4 ways, since any of the four consonants may be taken. The six letters remaining may be permuted in P^{6}_{6}, or $\underline{|6}$, ways, and each of these permutations may be attached to any one of the 4 initial letters.

Hence, the number of permutations beginning with a consonant is $4\underline{|6} = 4 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 2880.$

4. Since with any of the eight pairs of trousers any of the six vests and any of the five coats may be taken, the number of different suits is $8 \cdot 6 \cdot 5$, or 240.

5. When the flags are taken singly C^{7}_{1} (§ 606, Prin. 3) gives the number of combinations possible, and P^{1}_{1} (§ 604, Prin. 2) gives the number of permutations possible in each combination. Hence, the number of signals that may be made by taking the flags singly is equal to $C^{7}_{1} \times P^{1}_{1}$. When the flags are taken two at a time C^{7}_{2} (Prin. 3) gives the number of combinations possible, and P^{2}_{2} (Prin. 2) gives the number of permutations possible in each combination. Hence, $C^{7}_{2} \times P^{2}_{2}$ gives the number of signals that may be made taking two flags at a time. Similarly, $C^{7}_{3} \times P^{3}_{3}$ gives the number of signals that may be made taking three flags at a time, etc. Then, the whole number of signals is equal to the sum of the signals possible, when taken singly, two at a time, three at a time, etc., as follows,

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$$
\begin{aligned}
C_1^7 \times P_1^1 &= 7 \times 1 &&= 7\\
C_2^7 \times P_2^2 &= \frac{7\cdot 6}{1\cdot 2}\times 1\cdot 2 &&= 42\\
C_3^7 \times P_3^3 &= \frac{7\cdot 6\cdot 5}{1\cdot 2\cdot 3}\times 1\cdot 2\cdot 3 &&= 210\\
C_4^7 \times P_4^4 &= \frac{7\cdot 6\cdot 5\cdot 4}{1\cdot 2\cdot 3\cdot 4}\times 1\cdot 2\cdot 3\cdot 4 &&= 840\\
C_5^7 \times P_5^5 &= \frac{7\cdot 6\cdot 5\cdot 4\cdot 3}{1\cdot 2\cdot 3\cdot 4\cdot 5}\times 1\cdot 2\cdot 3\cdot 4\cdot 5 &&= 2520\\
C_6^7 \times P_6^6 &= \frac{7\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2}{1\cdot 2\cdot 3\cdot 4\cdot 5\cdot 6}\times 1\cdot 2\cdot 3\cdot 4\cdot 5\cdot 6 &&= 5040\\
C_7^7 \times P_7^7 &= \frac{7\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1}{1\cdot 2\cdot 3\cdot 4\cdot 5\cdot 6\cdot 7}\times 1\cdot 2\cdot 3\cdot 4\cdot 5\cdot 6\cdot 7 &&= 5040
\end{aligned}
$$

Hence, the number of signals $= 13699.$

6. Since the vowels are always given the even places, P_3^3 gives the number of permutations possible among three vowels. Then, as the consonants occupy the remaining three places, P_3^3 gives the number of permutations possible among three consonants. Hence, $P_3^3 \times P_3^3$ gives the entire number of permutations possible.

$$P_3^3 \times P_3^3 = 1\cdot 2\cdot 3 \times 1\cdot 2\cdot 3 = 36.$$

7. There are 4 odd places and 3 even places. To fill the odd places two different digits, 1 and 3, are to be selected, and this can be done in C_2^4 ways, since there are two 1's and two 3's. To fill the even places two different digits, 2 and 4, are to be selected, and this can be done in C_2^3 ways since there are two 2's and one 4.

Hence, the whole number of ways is equal to

$$C_2^4 \times C_2^3 = \frac{4\cdot 3}{1\cdot 2}\times\frac{3\cdot 2}{1\cdot 2} = 18.$$

8.

$$
\begin{aligned}
P_5^n &= 24\,P_2^n = P_2^n \times 4 \times 3 \times 2.\\
n(n-1)(n-2)(n-3)(n-4) &= n(n-1)\times 4\times 3\times 2.\\
(n-2)(n-3)(n-4) &= 4\times 3\times 2 = (6-2)(6-3)(6-4).\\
\therefore n &= 6.
\end{aligned}
$$

COMPLEX NUMBERS

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12. $(2+3\sqrt{-1})(1+\sqrt{-1}) = 2 + (2+3)\sqrt{-1} + 3(-1)$
$= 2 - 3 + 5\sqrt{-1} = -1 + 5\sqrt{-1}.$

13. $(5-\sqrt{-1})(1-2\sqrt{-1}) = 5-(10+1)\sqrt{-1} + 2(-1)$
$= 5 - 2 - 11\sqrt{-1} = 3 - 11\sqrt{-1}.$

14. $(\sqrt{2}+\sqrt{-2})(\sqrt{8}-\sqrt{-8}) = \sqrt{2}(1+\sqrt{-1}) \times \sqrt{8}(1-\sqrt{-1})$
$= 4(1+\sqrt{-1})(1-\sqrt{-1}) = 4\times 2 = 8.$

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15. $(2 + 3\,i)^2 = 2^2 + 2 \cdot 6\,i + 3^2 i^2 = 4 + 12\,i - 9 = -5 + 12\,i.$

16. $(2 - 3\,i)^2 = 2^2 - 2 \cdot 6\,i + 3^2 i^2 = 4 - 12\,i - 9 = -5 - 12\,i.$

17. $(a - bi)^2 = a^2 - 2\,abi + b^2 i^2 = a^2 - 2\,abi - b^2.$

18.
$$\begin{aligned}(1 + \sqrt{-3})(1 + \sqrt{-3})(1 + \sqrt{-3}) &= (1 + \sqrt{-3})^2(1 + \sqrt{-3}) \\ &= (1 + 2\sqrt{-3} - 3)(1 + \sqrt{-3}) \\ &= -2(1 - \sqrt{-3})(1 + \sqrt{-3}) \\ &= -2[1^2 - (\sqrt{-3})^2] \\ &= -2(1 + 3) = -8.\end{aligned}$$

19.
$$\begin{aligned}(-1 + \sqrt{-3})(-1 + \sqrt{-3})(-1 + \sqrt{-3}) &= (-1 + \sqrt{-3})^2(-1 + \sqrt{-3}) \\ &= (1 - 2\sqrt{-3} - 3)(-1 + \sqrt{-3}) \\ &= 2(-1 - \sqrt{-3})(-1 + \sqrt{-3}) \\ &= 2[(-1)^2 - (\sqrt{-3})^2] \\ &= 2(1 + 3) = 8.\end{aligned}$$

20. If each factor given in Ex. 19 is divided by 2, the result must be divided by 2^3, or by 8. Therefore,

$$(-\tfrac{1}{2} + \tfrac{1}{2}\sqrt{-3})(-\tfrac{1}{2} + \tfrac{1}{2}\sqrt{-3})(-\tfrac{1}{2} + \tfrac{1}{2}\sqrt{-3}) = 1.$$

22. $$\frac{3}{1 - \sqrt{-2}} = \frac{3(1 + \sqrt{-2})}{(1 - \sqrt{-2})(1 + \sqrt{-2})} = \frac{3(1 + \sqrt{-2})}{1 + 2} = 1 + \sqrt{-2}.$$

23. $$\frac{2}{1 + \sqrt{-1}} = \frac{2(1 - \sqrt{-1})}{(1 + \sqrt{-1})(1 - \sqrt{-1})} = \frac{2(1 - \sqrt{-1})}{1 + 1} = 1 - \sqrt{-1}.$$

24. $$\frac{4 + \sqrt{4}}{2 - \sqrt{-2}} = \frac{(4 + \sqrt{4})(2 + \sqrt{-2})}{(2 - \sqrt{-2})(2 + \sqrt{-2})} = \frac{6(2 + \sqrt{-2})}{4 + 2} = 2 + \sqrt{-2}.$$

25. $$\frac{a^2 + b^2}{a - b\sqrt{-1}} = \frac{a^2 - b^2(\sqrt{-1})^2}{a - b\sqrt{-1}} = \frac{(a + b\sqrt{-1})(a - b\sqrt{-1})}{a - b\sqrt{-1}}$$
$$= a + b\sqrt{-1}.$$

Or
$$\begin{array}{rr|l}
a^2 & + b^2 & a - b\sqrt{-1} \\ \cline{3-3}
\underline{a^2 - ab\sqrt{-1}} & & a + b\sqrt{-1} \\ \cline{3-3}
ab\sqrt{-1} & + b^2 & \\
\underline{ab\sqrt{-1}} & \underline{+ b^2} &
\end{array}$$

26. $$\frac{a - bi}{ai + b} = \frac{(a - bi)(ai - b)}{(ai + b)(ai - b)} = \frac{a^2 i - ab - abi^2 + b^2 i}{a^2 i^2 - b^2}$$
$$= \frac{(a^2 + b^2)i - ab(1 + i^2)}{-(a^2 + b^2)} = \frac{(a^2 + b^2)i - ab(1 - 1)}{-(a^2 + b^2)} = -i.$$

27. $$\frac{(1 + i)^2}{1 - i} = \frac{1 + 2\,i - 1}{1 - i} = \frac{2\,i}{1 - i} = \frac{2\,i(1 + i)}{(1 - i)(1 + i)} = \frac{2(i - 1)}{1 + 1} = i - 1.$$

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29. $4+2\sqrt{-21}=7+2\sqrt{7(-3)}-3=(\sqrt{7}+\sqrt{-3})^2.$

$\therefore \sqrt{4+2\sqrt{-21}}=\sqrt{7}+\sqrt{-3}.$

30. $1+2\sqrt{-6}=3+2\sqrt{3(-2)}-2=(\sqrt{3}+\sqrt{-2})^2.$

$\therefore \sqrt{1+2\sqrt{-6}}=\sqrt{3}+\sqrt{-2}.$

31. $6-2\sqrt{-7}=7-2\sqrt{7(-1)}-1=(\sqrt{7}-\sqrt{-1})^2.$

$\therefore \sqrt{6-2\sqrt{-7}}=\sqrt{7}-\sqrt{-1}.$

32. $9+2\sqrt{-22}=11+2\sqrt{11(-2)}-2=(\sqrt{11}+\sqrt{-2})^2.$

$\therefore \sqrt{9+2\sqrt{-22}}=\sqrt{11}+\sqrt{-2}.$

33. $4\sqrt{-3}-1=-4+3+4\sqrt{-3}=3+4\sqrt{-3}-4=(\sqrt{3}+2\sqrt{-1})^2.$

$\therefore \sqrt{4\sqrt{-3}-1}=\sqrt{3}+2\sqrt{-1}.$

34. $12\sqrt{-1}-5=2\sqrt{-36}+4-9=4+2\sqrt{4(-9)}-9.$

$\therefore \sqrt{12\sqrt{-1}-5}=\sqrt{4+2\sqrt{4(-9)}-9}=\sqrt{4}+\sqrt{-9}=2+3\sqrt{-1}.$

35. $24\sqrt{-1}-7=2\sqrt{-144}+9-16=9+2\sqrt{9(-16)}-16.$

$\therefore \sqrt{24\sqrt{-1}-7}=\sqrt{9+2\sqrt{9(-16)}-16}=\sqrt{9}+\sqrt{-16}=3+4\sqrt{-1}.$

36. $b^2+2ab\sqrt{-1}-a^2=b^2+2b\cdot a\sqrt{-1}+(a\sqrt{-1})^2=(b+a\sqrt{-1})^2.$

$\therefore \sqrt{b^2+2ab\sqrt{-1}-a^2}=b+a\sqrt{-1}.$

37. Substituting $-1+\sqrt{-1}$ for x in $x^2+2x+2=0$,

$$(-1+\sqrt{-1})^2+2(-1+\sqrt{-1})+2=1-2\sqrt{-1}-1-2+2\sqrt{-1}+2$$
$$=0, \text{ or } 0=0.$$

Substituting $-1-\sqrt{-1}$ for x in $x^2+2x+2=0$,

$$(-1-\sqrt{-1})^2+2(-1-\sqrt{-1})+2=1+2\sqrt{-1}-1-2-2\sqrt{-1}+2$$
$$=0, \text{ or } 0=0.$$

Hence, $-1+\sqrt{-1}$ and $-1-\sqrt{-1}$ are roots of the equation $x^2+2x+2=0$.

38. $(\frac{1}{2}+\frac{1}{2}\sqrt{-3})^3=(\frac{1}{2})^3+3(\frac{1}{2})^2\frac{1}{2}\sqrt{-3}+3(\frac{1}{2})(\frac{1}{2}\sqrt{-3})^2+(\frac{1}{2}\sqrt{-3})^3$

$=\frac{1}{8}+\frac{3}{8}\sqrt{-3}-\frac{9}{8}-\frac{3}{8}\sqrt{-3}=-1.$

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KEY TO

STANDARD ALGEBRA REVISED

NOTE. — *References are to pages in the Key.*

DEFINITIONS AND NOTATION

Page 20

2–17. See Ex. 2–17, p. 3. **19–30**. See Ex. 19–30, p. 3.

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31–34. See Ex. 31–34, pp. 3–4. **36–46**. See Ex. 36–46, p. 4.

47. See Ex. 51, p. 4. **49**. See Ex. 53, p. 4. **52**. See Ex. 56, p. 4.

48. See Ex. 52, p. 4. **51**. See Ex. 55, p. 4. **53**. See Ex. 57, p. 4.

54. $$4\,sy^2 \div \tfrac{5}{8}\,x^2r^2 - \tfrac{1}{12}\,x^3y^2z = 4 \cdot 10 \cdot 3^2 \div \tfrac{5}{8} \cdot 6^2 \cdot 2^2 - \tfrac{1}{12} \cdot 6^3 \cdot 3^2 \cdot 0$$
$$= 360 \div 90 - 0 = 4.$$

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1. $A = bh = 12 \cdot 5 = 60$, the number of square feet in the area.

2. $A = bh = 20 \cdot 10 = 200$, the number of square feet in the area.

3. See Ex. 1, p. 5. **5**. See Ex. 3, p. 5. **7**. See Ex. 5, p. 5.

4. See Ex. 2, p. 5. **6**. See Ex. 4, p. 5. **8**. See Ex. 6, p. 5.

SUBTRACTION

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18–19. See Ex. 18–19, p. 5.

20. $$\begin{aligned} & a + y - \{5 + 4\,a - (6\,y + 3)\} - (7\,y - 4\,a - 1) \\ &= a + y - \{5 + 4\,a - 6\,y - 3\} - 7\,y + 4\,a + 1 \\ &= 5\,a - 6\,y - \{4\,a - 6\,y + 2\} + 1 \\ &= 5\,a - 6\,y - 4\,a + 6\,y - 2 + 1 = a - 1. \end{aligned}$$

21. See Ex. 21, p. 5.

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22. $a+2b+(14a-5b)-\{6a+6b-(5a-4a+4b)\}$
$=a+2b+14a-5b-\{6a+6b-a-4b\}$
$=15a-3b-\{5a+2b\}$
$=15a-3b-5a-2b=10a-5b.$

23. $12a-\{4-3b-(6b+3c)+b-8-(5a-2b-6)\}$
$=12a-\{4-3b-6b-3c+b-8-5a+2b+6\}$
$=12a-\{2-6b-3c-5a\}$
$=12a-2+6b+3c+5a=17a+6b+3c-2.$

24. $a+b-\{-[a+b-c-x]-[3a-c+x-a-b]+4a\}$
$=a+b-\{-a-b+c+x-3a+c-x+a+b+4a\}$
$=a+b-\{a+2c\}$
$=a+b-a-2c=b-2c.$

25. $x^3-[x^2-(1-x)]-\{1+x^2-(1-x)+x^3\}$
$=x^3-x^2+(1-x)-1-x^2+(1-x)-x^3$
$=(1-x)-1-2x^2+(1-x)$
$=1-x-1-2x^2+1-x=1-2x-2x^2.$

26. $4-\{5y-3+\overline{2x-2}-x-(5y-x-3)\}$
$=4-\{5y-3+2x-2-x-5y+x+3\}$
$=4-\{-2+2x\}=4+2-2x=6-2x.$

27. $ab-\{5+x-(b+c-ab+x)\}+x-(b-c-7)$
$=ab-\{5+x-b-c+ab-x\}+x-b+c+7$
$=ab-\{5-b-c+ab\}+x-b+c+7$
$=ab-5+b+c-ab+x-b+c+7=2+2c+x.$

28. $-\{3ax-[5xy-3z]+z-(4xy+[6z+7ax]+3z)\}$
$=-\{3ax-5xy+3z+z-(4xy+6z+7ax+3z)\}$
$=-\{3ax-5xy+4z-4xy-9z-7ax\}$
$=-\{-4ax-9xy-5z\}=4ax+9xy+5z.$

29. $1-x-\{1-x-[1-x-\overline{1-x}-\overline{x-1}]-x+1\}$
$=1-x-\{1-x-[1-x-1+x-x+1]-x+1\}$
$=1-x-\{1-x-[1-x]-x+1\}$
$=1-x-\{1-x-1+x-x+1\}$
$=1-x-\{1-x\}=1-x-1+x=0.$

30–31. See Ex. 30–31, p. 6.

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27. Let $x=$ number of feet in width of rectangle.
Then, $x+5=$ number of feet in length of rectangle.
$\therefore x+x+x+5+x+5=46.$
Solving, $x=9,$
and $x+5=14.$
Hence, the rectangle is 14 feet long and 9 feet wide.

28. Let $x=$ number of rods in width of lawn.
Then, $x+7=$ number of rods in length of lawn.
$\therefore x+x+x+7+x+7=62.$
Solving, $x=12,$
and $x+7=19.$
Hence, the lawn is 19 rods long and 12 rods wide.

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29. Let $x =$ number of inches in width of desk top.
Then, $x + 15 =$ number of inches in length of desk top.
$\therefore x+x+x+15+x+15 = 170.$
Solving, $x = 35.$
Hence, the width of the desk top is 35 inches.

30. Let $x =$ number of years in man's present age.
$\therefore x + 16 = 2x - 4.$
Solving, $x = 20.$
Hence, the man's present age is 20 years.

31. Let $x =$ number of years in sister's age.
Then, $x - 8 =$ number of years in boy's age,
$x + 4 =$ number of years in sister's age 4 years hence,
and $(x - 8) + 4 =$ number of years in boy's age 4 years hence.
$\therefore x + 4 + (x - 8) + 4 = 26.$
Solving, $x = 13,$
and $x - 8 = 5.$
Hence, the boy is 5 years old and his sister 13 years old.

32. See Ex. 13, p. 6. **34.** See Ex. 15, p. 7. **36.** See Ex. 19, p. 7.
33. See Ex. 14, p. 7. **35.** See Ex. 17, p. 7.

37. Let $x =$ number of feet in width of tunnel.
Then, $22\frac{1}{2}x =$ number of feet in length of tunnel.
$\therefore 22\frac{1}{2}x - 50 = 20x.$
Solving, $x = 20,$
and $22\frac{1}{2}x = 450.$
Hence, the tunnel is 450 feet long and 20 feet wide.

38. See Ex. 20, p. 7. **39.** See Ex. 21, p. 8.

REVIEW

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15. See Ex. 14, p. 9. **17.** See Ex. 16, p. 9.
16. See Ex. 15, p. 9. **18.** See Ex. 17, p. 9.

MULTIPLICATION

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8–12. See Ex. 8–12, p. 9.

Page 62

13–17. See Ex. 13–17, p. 10.

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2. $x + (y + m) = 3 + (-4 + 6) = 5.$
3. $(m - n)^2 + y = (6 - 2)^2 - 4 = 12.$
4. $n(y - m) + z = 2(-4 - 6) + 0 = -20.$
5. $(m + y + x)^2 - n = (6 - 4 + 3)^2 - 2 = 23.$
6. $m(x + y) + z^2 = 6(3 - 4) + 0^2 = -6.$

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7. See Ex. 3, p. 10. 8. See Ex. 4, p. 10.

9. $(n-m)+3xy=(2-6)+3\cdot 3(-4)=-4-36=-40.$

10. $(m+n)^2-(y+z)^2=(6+2)^2-(-4+0)^2=64-16=48.$

11. $(m+y)^3+xz-n^2=(6-4)^3+3\cdot 0-2^2=8+0-4=4.$

12. See Ex. 5, p. 10. 13. See Ex. 6, p. 10.

14. $xyz-n(x-m)^3-(nx)^2=3(-4)0-2(3-6)^3-(2\cdot 3)^2$
$=0+54-36=18.$

15. $3(x+z)-\frac{2}{3}mn+5y=3(3+0)-\frac{2}{3}\cdot 6\cdot 2+5(-4)$
$=9-8-20=-19.$

16. $\frac{1}{2}(y-2n)-\frac{3}{4}(n-2y)(3y-4n)$
$=\frac{1}{2}(-4-4)-\frac{3}{4}(2+8)(-12-8)=-4+150=146.$

17. $(x+y)^2-xy(x-y)+(x+y)(x^2-y^2)$
$=(3-4)^2-3(-4)(3+4)+(3-4)(9-16)=1+84+7=92.$

18. $3m(x-y-n)^2-(y-n-x)(n-x-y)$
$=18(3+4-2)^2-(-4-2-3)(2-3+4)=450+27=477.$

19. $(2x+y)^n-(x^2+2y)-(m+n)^2(x+y+z)^3$
$=(6-4)^2-(9-8)-(6+2)^2(3-4+0)^3=4-1+64=67.$

20. See Ex. 16, p. 11. 21. See Ex. 17, p. 11.

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37. See Ex. 21, p. 11.

38. $(-x-2y)(-x+2y)=(-x)^2-(2y)^2=x^2-4y^2.$

39. $(3x^m+7y^n)(3x^m-7y^n)=(3x^m)^2-(7y^n)^2=9x^{2m}-49y^{2n}.$

40. $(mx^a+2y^b)(mx^a-2y^b)=(mx^a)^2-(2y^b)^2=m^2x^{2a}-4y^{2b}.$

41. $(a^nb^m+a^mb^n)(a^nb^m-a^mb^n)=(a^nb^m)^2-(a^mb^n)^2=a^{2n}b^{2m}-a^{2m}b^{2n}.$

42. $(x^{m-1}+y^{n+1})(x^{m-1}-y^{n+1})=(x^{m-1})^2-(y^{n+1})^2=x^{2m-2}-y^{2n+2}.$

43. $(5a^3b^2+2x^s)(5a^3b^2-2x^s)=(5a^3b^2)^2-(2x^s)^2=25a^6b^4-4x^{2s}.$

45. See Ex. 28, p. 11. 50. See Ex. 33, p. 11.

46. See Ex. 29, p. 11. 51. See Ex. 34, p. 11.

47. See Ex. 30, p. 11. 52. See Ex. 35, p. 11.

48. See Ex. 31, p. 11. 53. See Ex. 36, p. 11.

49. See Ex. 32, p. 11. 54. See Ex. 37, p. 11.

55. $(n^4-2n^2+1)(n^4+2n^2+1)=[(n^4+1)-2n^2][(n^4+1)+2n^2]$
$=(n^4+1)^2-(2n^2)^2=n^8+2n^4+1-4n^4=n^8-2n^4+1.$

56. $(x^2+xy+y^2)(x^2-xy+y^2)=[(x^2+y^2)+xy][(x^2+y^2)-xy]$
$=(x^2+y^2)^2-(xy)^2=x^4+2x^2y^2+y^4-x^2y^2=x^4+x^2y^2+y^4.$

57. $(2x+3y-4)(2x+3y+4)=[(2x+3y)-4][(2x+3y)+4]$
$=(2x+3y)^2-4^2=4x^2+12xy+9y^2-16.$

58. $(r^2-rs+3s)(r^2+rs-3s)=[r^2-(rs-3s)][r^2+(rs-3s)]$
$=r^4-(r^2s^2-6rs^2+9s^2)=r^4-r^2s^2+6rs^2-9s^2.$

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29–31. See Ex. 29–31, p. 12.

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19. Let $x =$ number of feet in width of rectangle.
Then, $2x =$ number of feet in length of rectangle.
$\therefore 2(x + 2x) = 240.$
Solving, $x = 40,$
and $2x = 80.$
Hence, the rectangle is 80 feet long and 40 feet wide.

20. Let $x =$ number of feet in width of lawn.
Then, $x + 45 =$ number of feet in depth of lawn.
$\therefore 2(x + x + 45) = 270.$
Solving, $x = 45,$
and $x + 45 = 90.$
Hence, the lawn is 90 feet deep and 45 feet wide.

21. Let $x =$ the number.
Then, $x + 1 =$ the next higher number.
$\therefore x(x + 1) - x^2 = 15.$
Solving, $x = 15.$
Hence, the number is 15.

22. Let $x =$ the first number.
Then, $x + 1 =$ the second number.
$\therefore (x + 1)^2 - x^2 = 33.$
Solving, $x = 16,$
and $x + 1 = 17.$
Hence, the two consecutive numbers are 16 and 17.

23. Let $x =$ number of seniors in the school.
Then, $210 - x =$ number of the rest of the pupils,
$1.50x =$ number of dollars given by seniors,
and $.50(210 - x) =$ number of dollars given by rest of pupils.
$\therefore 1.50x + .50(210 - x) = 175.$
Solving, $x = 70.$
Hence, there were 70 seniors in the school.

24. Let $x =$ number of reserved seat tickets.
Then, $150 - x =$ number of other tickets,
$75x =$ number of cents received for reserved seat tickets,
and $50(150 - x) =$ number of cents received for other tickets.
$\therefore 75x + 50(150 - x) = 8750.$
Solving, $x = 50,$
and $150 - x = 100.$
Hence, 50 reserved seat tickets and 100 other tickets were sold.

25. Let $x =$ number of plums Carl has.
Then, $3x =$ number of plums Leo has.
$\therefore 3x + 5 = 2(x + 5).$
Solving, $x = 5,$
and $3x = 15.$
Hence, Carl has 5 plums and Leo has 15 plums.

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26. Let $x =$ number of green apples.
Then, $350 - x =$ number of red apples,
$3x =$ number of cents received for green apples,
and $5(350 - x) =$ number of cents received for red apples.
$\therefore 3x + 5(350 - x) = 1160.$
Solving, $x = 295,$
and $350 - x = 55.$
Hence, the boys sold 295 green apples and 55 red apples.

27. Let $x =$ number of feet in width of classroom.
Then, $2x + 4 =$ number of feet in length of classroom.
$\therefore 2(x + 2) + 2(2x + 4) = 120.$
Solving, $x = 18,$
and $2x + 4 = 40.$
Hence, the classroom is 40 feet long and 18 feet wide.

28. Let $x =$ number of feet in width of house.
Then, $x + 16 =$ number of feet in depth of house.
$\therefore 2x + 2(x + 16 + 6) = 140.$
Solving, $x = 24,$
and $x + 16 = 40.$
Hence, the house is 40 feet deep and 24 feet wide.

29. See Ex. 23, p. 13.

DIVISION

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23. Rearranging terms, dividend $= a^2m^4 - 4am^3 + 3m^2.$

$$\begin{array}{l|l} a^2m^4 - 4am^3 + 3m^2 & am - 1 \\ \cline{2-2} \underline{a^2m^4 - \quad am^3} & am^3 - 3m^2 \\ \quad -3am^3 + 3m^2 & \\ \quad \underline{-3am^3 + 3m^2} & \end{array}$$

TEST. — Let $a = 2$, $m = 1$. Then, the dividend becomes $4 - 8 + 3$, or -1; the divisor $2 - 1$, or 1; and the quotient $2 - 3$, or -1. Since $-1 \div 1 = -1$, it may be assumed that the quotient is correct.

24. See Ex. 18, p. 14.

25. Rearranging terms, dividend $= -25x^3 + 20x^2y + 27xy^2 - 18y^3$; divisor $= -5x + 6y$.

$$\begin{array}{l|l} -25x^3 + 20x^2y + 27xy^2 - 18y^3 & -5x + 6y \\ \cline{2-2} \underline{-25x^3 + 30x^2y} & 5x^2 + 2xy - 3y^2 \\ \qquad -10x^2y + 27xy^2 & \\ \qquad \underline{-10x^2y + 12xy^2} & \\ \qquad\qquad 15xy^2 - 18y^3 & \\ \qquad\qquad \underline{15xy^2 - 18y^3} & \end{array}$$

TEST.—Let $x = 1$, $y = 5$. Then, the dividend becomes $-25 + 100 + 675 - 2250$, or -1500; the divisor $-5 + 30$, or 25; and the quotient $5 + 10 - 75$, or -60. Since $-1500 \div 25 = -60$, it may be assumed that the quotient is correct.

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26.

$$\begin{array}{l|l} r^3 + 3r^2s + 3rs^2 + s^3 & r^2 + 2rs + s^2 \\ \cline{2-2} \underline{r^3 + 2r^2s + \quad rs^2} & r + s \\ \qquad\quad r^2s + 2rs^2 + s^3 & \\ \qquad\quad \underline{r^2s + 2rs^2 + s^3} & \end{array}$$

Test. — Let $r = 1$, $s = -2$. Then, the dividend becomes $1 - 6 + 12 - 8$, or -1; the divisor $1 - 4 + 4$, or 1; and the quotient $1 - 2$, or -1. Since $-1 \div 1 = -1$, it may be assumed that the quotient is correct.

27.

$$\begin{array}{l|l} x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4 & x + y \\ \cline{2-2} \underline{x^4 + \quad x^3y} & x^3 + 3x^2y + 3xy^2 + y^3 \\ \qquad 3x^3y + 6x^2y^2 & \\ \qquad \underline{3x^3y + 3x^2y^2} & \\ \qquad\qquad 3x^2y^2 + 4xy^3 & \\ \qquad\qquad \underline{3x^2y^2 + 3xy^3} & \\ \qquad\qquad\qquad xy^3 + y^4 & \\ \qquad\qquad\qquad \underline{xy^3 + y^4} & \end{array}$$

Test. — Let $x = 1$, $y = 1$. Then, the dividend becomes $1 + 4 + 6 + 4 + 1$, or 16; the divisor $1 + 1$, or 2; and the quotient $1 + 3 + 3 + 1$, or 8. Since $16 \div 2 = 8$, it may be assumed that the quotient is correct.

28.

$$\begin{array}{l|l} a^3 + 5a^2x + 5ax^2 + x^3 & a^2 + 4ax + x^2 \\ \cline{2-2} \underline{a^3 + 4a^2x + \quad ax^2} & a + x \\ \qquad\quad a^2x + 4ax^2 + x^3 & \\ \qquad\quad \underline{a^2x + 4ax^2 + x^3} & \end{array}$$

Test. — Let $a = 2$, $x = 3$. Then, the dividend becomes $8 + 60 + 90 + 27$, or 185; the divisor $4 + 24 + 9$, or 37; and the quotient $2 + 3$, or 5. Since $185 \div 37 = 5$, it may be assumed that the quotient is correct.

29. See Ex. 20, p. 14.

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32. See Ex. 23, p. 14.
33. See Ex. 24, p. 14.
34. See Ex. 25, p. 15.
35. See Ex. 26, p. 15.
36. See Ex. 27, p. 15.
37. See Ex. 28, p. 15.
41. See Ex. 29, p. 16.
42. See Ex. 30, p. 16.
43. See Ex. 31, p. 16.
44. See Ex. 32, p. 17.
45. See Ex. 33, p. 17.
46. See Ex. 34, p. 17.
47. See Ex. 35, p. 17.

48.

$$\begin{array}{l|l} x^4 - a^2x^2 - 2bx^2 + b^2 & x^2 + ax - b \\ \cline{2-2} \underline{x^4 + ax^3 - \quad bx^2} & x^2 - ax - b \\ \quad -ax^3 - a^2x^2 - bx^2 & \\ \quad \underline{-ax^3 - a^2x^2 \qquad + abx} & \\ \qquad\qquad -bx^2 - abx + b^2 & \\ \qquad\qquad \underline{-bx^2 - abx + b^2} & \end{array}$$

49. See Ex. 36, p. 18.
50. See Ex. 37, p. 18.
51. See Ex. 38, p. 18.
52. See Ex. 39, p. 18.
53. See Ex. 40, p. 19.
54. See Ex. 41, p. 19.
55. See Ex. 42, p. 19.
56. See Ex. 43, p. 19.
57. See Ex. 45, p. 19.
58. See Ex. 46, p. 20.
59. See Ex. 47, p. 20.
60. See Ex. 48, p. 20.

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61.
$$\begin{array}{l|l}
-x^{2r+1}y^{2s}-2x^{2r+3}y^{2s+1}-x^{2r+5}y^{2s+2} & -x^ry^{s-1}-x^{r+2}y^s \\
\cline{2-2}
\underline{-x^{2r+1}y^{2s}-x^{2r+3}y^{2s+1}} & x^{r+1}y^{s+1}+x^{r+3}y^{s+2} \\
\qquad\quad -x^{2r+3}y^{2s+1}-x^{2r+5}y^{2s+2} & \\
\qquad\quad \underline{-x^{2r+3}y^{2s+1}-x^{2r+5}y^{2s+2}} &
\end{array}$$

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63. See Ex. 53, p. 21.
64. See Ex. 54, p. 21.
65. See Ex. 55, p. 21.
66. See Ex. 56, p. 22.
67. See Ex. 57, p. 22.
68. See Ex. 58, p. 22.
69. See Ex. 59, p. 22.
70. See Ex. 60, p. 23.
71. See Ex. 61, p. 23.
72. See Ex. 62, p. 23.
73. See Ex. 63, p. 23.
74. See Ex. 64, p. 24.
75. See Ex. 65, p. 24.
76. See Ex. 66, p. 24.
77. See Ex. 67, p. 24.
78. See Ex. 68, p. 24.
79. See Ex. 69, p. 24.
80. See Ex. 70, p. 25.
81. See Ex. 71, p. 25.
82. See Ex. 72, p. 25.

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36. See Ex. 23, p. 25.
37. See Ex. 24, p. 25.
38. See Ex. 25, p. 25.
39. See Ex. 26, p. 25.
40. See Ex. 27, p. 25.
41. See Ex. 28, p. 25.
42. See Ex. 29, p. 25.
43. See Ex. 30, p. 25.
44. See Ex. 31, p. 26.
45. See Ex. 32, p. 26.
46. See Ex. 33, p. 26.
47. See Ex. 34, p. 26.
48. See Ex. 35, p. 26.

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18–19. See Ex. 18–19, p. 26.

20. Let $3x =$ number of years in boy's present age.
Then, $3x - 12 =$ number of years in his age 12 years ago.
$$\therefore 3x - 12 = \tfrac{1}{3}(3x).$$
Solving, $x = 6,$
and $3x = 18.$
Hence, the boy is 18 years old.

VERIFICATION. $(18 - 12)$ years $= \frac{1}{3}$ of 18 years.

21. See Ex. 20, p. 27.

22. Let $x =$ number of years in son's age.
Then, $4x =$ number of years in father's age.
$$\therefore 4x - 6 = 7(x - 6).$$
Solving, $x = 12,$
and $4x = 48.$
Hence, the son is 12 years old and the father 48 years old.

VERIFICATION. — 1st condition: 48 years is 4 times 12 years.
2d condition: $(48 - 6)$ years $= 7$ times $(12 - 6)$ years.

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23. Let $x =$ number of dimes.
Then, $2x =$ number of quarters.
$$\therefore 25(2x) + 10x = 180.$$
Solving, $x = 3,$
and $2x = 6.$
Hence, the man has 3 dimes and 6 quarters.

VERIFICATION. — 1st condition: 6, the number of quarters, is 2 times 3, the number of dimes.
2d condition: 6 times 25 cents + 3 times 10 cents = $1.80.

24. Let $x =$ number of 30-cent books.
Then, $15 - x =$ number of 60-cent books.
$$\therefore 30x + 60(15 - x) = 660.$$
Solving, $x = 8,$
and $15 - x = 7.$
Hence, I bought 8 30-cent books and 7 60-cent books.

VERIFICATION. — 1st condition: 8 books + 7 books = 15 books.
2d condition: 8 times 30 cents + 7 times 60 cents = $6.60.

25. Let $x =$ number of nickels John has.
Then, $3x =$ number of dimes John has,
and $4x =$ number of quarters John has.
$$\therefore 5x + 10(3x) + 25(4x) = 675.$$
Solving, $x = 5,$
$3x = 15,$
and $4x = 20.$
Hence, John has 5 nickels, 15 dimes, and 20 quarters.

VERIFICATION. — 1st condition: 5 times 5 cents + 15 times 10 cents + 20 times 25 cents = $6.75.
2d condition: 15, the number of dimes, is 3 times 5, the number of nickels, and 20, the number of quarters, is equal to 15 + 5, the sum of nickels and dimes.

26. Let $x =$ number of quarters.
Then, $5x =$ number of half dollars.
$$\therefore .25x + .50(5x) = 27.50.$$
Solving, $x = 10,$
and $5x = 50.$
Hence, the man has 10 quarters and 50 half dollars.

VERIFICATION. — 1st condition: 10 times 25 cents + 50 times 50 cents = $27.50.
2d condition: 50, the number of half dollars, is 5 times 10, the number of quarters.

27. See Ex. 28, p. 28.

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28. Let $x =$ number of years in B's age.
Then, $x + 25 =$ number of years in A's age.
$$\therefore x + 25 + 20 = 2(x + 20).$$
Solving, $x = 5,$
and $x + 25 = 30.$
Hence, A is 30 years old and B is 5 years old.

VERIFICATION. — 1st condition: 30 years is 25 years more than 5 years.
2d condition: $(5 + 25 + 20)$ years is 2 times $(5 + 20)$ years.

29. Let $x =$ number of years in Frank's age.
Then, $x + 15 =$ number of years in John's age.
$$\therefore x + 15 + 5 = 2(x + 5).$$
Solving, $x = 10,$
and $x + 15 = 25.$
Hence, Frank is 10 years old and John is 25 years old.

VERIFICATION. — 1st condition: 25 years is 15 years more than 10 years.
2d condition: $(10 + 5)$ years is $\frac{1}{2}$ of $(10 + 15 + 5)$ years.

30. See Ex. 29, p. 28.

31. Let $x =$ number of years in brother's age.
Then, $nx =$ number of years in Harold's age.
$$\therefore m(x - r) = nx - r.$$
Solving, $x = \dfrac{mr - r}{m - n},$
and $nx = \dfrac{mnr - nr}{m - n}.$
Hence, Harold is $\dfrac{mnr - nr}{m - n}$ years old and his brother $\dfrac{mr - r}{m - n}$ years old.

VERIFICATION. — 1st condition: $\dfrac{mnr - nr}{m - n}$ years is n times $\dfrac{mr - r}{m - n}$ years.
2d condition: $m\left(\dfrac{mr - r}{m - n} - r\right)$ years $= \left(\dfrac{mnr - nr}{m - n} - r\right)$ years.

32. Let $x =$ number of marbles B had.
Then, $3x =$ number of marbles A had.
$$\therefore x + 50 = 2(3x - 50).$$
Solving, $x = 30,$
and $3x = 90.$
Hence, A had 90 marbles and B 30 marbles.

VERIFICATION. — 1st condition: 90 marbles are 3 times 30 marbles.
2d condition: $(30 + 50)$ marbles $= 2$ times $(90 - 50)$ marbles.

33. See Ex. 26, p. 27.

34. See Ex. 27, p. 28.

35. Let $x =$ number of feet in width of shed.
Then, $\frac{1}{2}(4x + 52)$, or $2x + 26 =$ number of feet in length of shed.
$$\therefore 2x + 2(2x + 26) = 544.$$
Solving, $x = 82,$
and $2x + 26 = 190.$
Hence, the shed is 82 feet wide and 190 feet long.

VERIFICATION. — 1st condition: 2 times 82 feet + 2 times 190 feet = 544 feet.
2d condition: 2 times 190 feet = 4 times 82 feet + 52 feet.

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REVIEW

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5–8. See Ex. 5–8, pp. 28–29.

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20–39. See Ex. 20–39, pp. 29–30.

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45–63. See Ex. 45–63, pp. 30–32.

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78–85. See Ex. 78–85, pp. 32–33.

FACTORING

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4–31. See Ex. 4–31, pp. 33–34.

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38. See Ex. 31, p. 34. **40.** See Ex. 34, p. 34. **42.** See Ex. 35, p. 34.
39. See Ex. 32, p. 34. **41.** See Ex. 33, p. 34. **43.** See Ex. 36, p. 34.
44. See Ex. 37, p. 35.

45. $(a-x)^2+4(a-x)(x-b)+4(x-b)^2$
$$=[a-x+2(x-b)][a-x+2(x-b)]$$
$$=(a-x+2x-2b)(a-x+2x-2b)$$
$$=(a-2b+x)(a-2b+x).$$

46. See Ex. 39, p. 35. **47.** See Ex. 40, p. 35. **48.** See Ex. 42, p. 35.

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13. $a^4-81=(a^2+9)(a^2-9)=(a^2+9)(a+3)(a-3).$

22. See Ex. 15, p. 35.

23. $16a^4-81b^4=(4a^2+9b^2)(4a^2-9b^2)$
$$=(4a^2+9b^2)(2a+3b)(2a-3b).$$

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27. $2x^4-2y^4=2(x^2+y^2)(x^2-y^2)=2(x^2+y^2)(x+y)(x-y).$

31. See Ex. 23, p. 35. **32.** See Ex. 24, p. 35.

33. $18c^2-50=2(9c^2-25)=2(3c+5)(3c-5).$

34. See Ex. 28, p. 35. **35.** See Ex. 18, p. 35. **36.** See Ex. 25, p. 35.

37. $3m^5-3m=3m(m^4-1)=3m(m^2+1)(m^2-1)$
$$=3m(m^2+1)(m+1)(m-1).$$

38. $4a^4-4b^4=4(a^4-b^4)=4(a^2+b^2)(a^2-b^2)$
$$=4(a^2+b^2)(a+b)(a-b).$$

39. See Ex. 29, p. 35. **40.** See Ex. 12, p. 35.

41. $2x^8-2y^8=2(x^8-y^8)=2(x^4+y^4)(x^4-y^4)$
$$=2(x^4+y^4)(x^2+y^2)(x^2-y^2)$$
$$=2(x^4+y^4)(x^2+y^2)(x+y)(x-y).$$

43. See Ex. 32, p. 36.

44. $x^4-\frac{1}{16}=(x^2+\frac{1}{4})(x^2-\frac{1}{4})=(x^2+\frac{1}{4})(x+\frac{1}{2})(x-\frac{1}{2}).$

46. $4a^2-.25=(2a+.5)(2a-.5).$

47. $x^{4a}-y^{4b}=(x^{2a}+y^{2b})(x^{2a}-y^{2b})=(x^{2a}+y^{2b})(x^a+y^b)(x^a-y^b).$

48. $8a^{2n}-18b^{2s}=2(4a^{2n}-9b^{2s})=2(2a^n+3b^s)(2a^n-3b^s).$

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49. See Ex. 35, p. 36.
50. See Ex. 36, p. 36.
52. See Ex. 38, p. 36.
53. See Ex. 40, p. 36.
54. See Ex. 39, p. 36.
55. See Ex. 41, p. 36.
56. See Ex. 42, p. 36.
57. See Ex. 43, p. 36.
58. See Ex. 44, p. 36.
59. See Ex. 45, p. 36.

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61. See Ex. 47, p. 36.
62. See Ex. 48, p. 36.
63. See Ex. 49, p. 36.
64. See Ex. 50, p. 36.
65. See Ex. 51, p. 36.
66. See Ex. 52, p. 36.
67. See Ex. 53, p. 36.
68. See Ex. 54, p. 36.
69. See Ex. 55, p. 37.
70. See Ex. 56, p. 37.
73. See Ex. 59, p. 37.
74. See Ex. 60, p. 37.
75. See Ex. 61, p. 37.
76. See Ex. 62, p. 37.
77. See Ex. 64, p. 37.
78. See Ex. 65, p. 37.
79. See Ex. 66, p. 37.
80. See Ex. 67, p. 37.
81. See Ex. 69, p. 37.
82. See Ex. 70, p. 37.
83. See Ex. 71, p. 37.
84. See Ex. 72, p. 37.
85. See Ex. 74, p. 37.
86. See Ex. 73, p. 37.

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13. $30 + r - r^2 = -(r^2 - r - 30) = -(r - 6)(r + 5) = (6 - r)(5 + r).$

14. $15 - a^2 + 2a = -(a^2 - 2a - 15) = -(a - 5)(a + 3) = (5 - a)(3 + a).$

15. See Ex. 16, p. 37.

16. $-a^2 - 9a + 52 = -(a^2 + 9a - 52) = -(a - 4)(a + 13)$
$= (4 - a)(13 + a).$

17. See Ex. 18, p. 37.

18. See Ex. 20, p. 38.

19. $-x^2 + 25x - 100 = -(x^2 - 25x + 100) = -(x - 5)(x - 20)$
$= (5 - x)(x - 20).$

20. See Ex. 22, p. 38.

21. $4ax - 2ax^2 + 48a = -2a(x^2 - 2x - 24) = -2a(x - 6)(x + 4)$
$= 2a(6 - x)(4 + x).$

22. See Ex. 23, p. 38.

23. $150n - 5nx^2 - 65nx = -5n(x^2 + 13x - 30) = -5n(x - 2)(x + 15)$
$= 5n(2 - x)(15 + x).$

24. See Ex. 27, p. 38.
25. See Ex. 25, p. 38.
26. See Ex. 28, p. 38.
27. See Ex. 29, p. 38.
28. See Ex. 30, p. 38.
29. See Ex. 31, p. 38.

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2. See Ex. 2, p. 38.
3. See Ex. 5, p. 38.
4. See Ex. 6, p. 38.
5. See Ex. 3, p. 38.
6. See Ex. 13, p. 39.
7. See Ex. 4, p. 38.
8. See Ex. 7, p. 38.

9.
$$\begin{array}{lllll} \text{First factor, try} & 9b^2 + 2, & 9b^2 - 2, & 9b^2 + 7, & 9b^2 - 7, \cdots \\ \text{Second factor, try} & \underline{3b^2 - 7,} & \underline{3b^2 + 7,} & \underline{3b^2 - 2,} & \underline{3b^2 + 2,} \cdots \\ \text{Products, 2d terms,} & -57b^2, & +57b^2, & +3b^2, & -3b^2, \cdots \end{array}$$
$\therefore\ 27b^4 - 3b^2 - 14 = (9b^2 - 7)(3b^2 + 2).$

10. $10x^6 + 42x^3 + 44 = 2(5x^6 + 21x^3 + 22).$

$$\begin{array}{lll} \text{To factor } 5x^6 + 21x^3 + 22, \text{ try} & 5x^3 + 22, & 5x^3 + 11, \cdots \\ \text{multiplied by} & \underline{x^3 + 1}\ , & \underline{x^3 + 2}\ , \cdots \\ \text{Products, 2d terms,} & +27x^3, & +21x^3\ , \cdots \end{array}$$
$\therefore\ 10x^6 + 42x^3 + 44 = 2(5x^3 + 11)(x^3 + 2).$

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11. $16x^2-68x+66=2(8x^2-34x+33)$.

$$\begin{array}{llll} \text{To factor } 8x^2-34x+33\text{, try} & 2x-11, & 2x-3, & \cdots \\ \text{multiplied by} & \underline{4x-3,} & \underline{4x-11,} & \cdots \\ \text{Products, 2d terms,} & -50x, & -34x, & \cdots \end{array}$$

$\therefore\ 16x^2-68x+66=2(4x-11)(2x-3)$.

12. See Ex. 11, p. 39.

13. $6x^2-10x+4=2(3x^2-5x+2)$.

$$\begin{array}{lll} \text{To factor } 3x^2-5x+2\text{, try} & 3x-1, & 3x-2. \\ \text{multiplied by} & \underline{x-2,} & \underline{x-1.} \\ \text{Products, 2d terms,} & -7x, & -5x. \end{array}$$

$\therefore\ 6x^2-10x+4=2(3x-2)(x-1)$.

14.
$$\begin{array}{llllll} \text{First factor, try} & 7a+2, & 7a-2, & 7a+5, & 7a-5, & \cdots \\ \text{Second factor, try} & \underline{3a-5,} & \underline{3a+5,} & \underline{3a-2,} & \underline{3a+2,} & \cdots \\ \text{Products, 2d terms,} & -29a, & +29a, & +a, & -a, & \cdots \end{array}$$

$\therefore\ 21a^2-a-10=(7a-5)(3a+2)$.

15.
$$\begin{array}{lllll} \text{First factor, try} & 5x^2+3, & 5x^2-3, & 5x^2+2, & \cdots \\ \text{Second factor, try} & \underline{2x^2-2,} & \underline{2x^2+2,} & \underline{2x^2-3,} & \cdots \\ \text{Products, 2d terms,} & -4x^2, & +4x^2, & -11x^2, & \cdots \end{array}$$

$\therefore\ 10x^4-11x^2-6=(5x^2+2)(2x^2-3)$.

16.
$$\begin{array}{lllll} \text{First factor, try} & 5x+2, & 5x+8, & 5x+4, & \cdots \\ \text{Second factor, try} & \underline{3x+4,} & \underline{3x+1,} & \underline{3x+2,} & \cdots \\ \text{Products, 2d terms,} & +26x, & +29x, & +22x, & \cdots \end{array}$$

$\therefore\ 15x^2+22x+8=(5x+4)(3x+2)$.

17.
$$\begin{array}{llllll} \text{First factor, try} & 5x+4, & 5x-4, & 5x+1, & 5x-1, & \cdots \\ \text{Second factor, try} & \underline{3x-1,} & \underline{3x+1,} & \underline{3x-4,} & \underline{3x+4,} & \cdots \\ \text{Products, 2d terms,} & +7x, & -7x, & -17x, & +17x, & \cdots \end{array}$$

$\therefore\ 15x^2+17x-4=(5x-1)(3x+4)$.

18. $18x^2-51x+36=3(6x^2-17x+12)$.

$$\begin{array}{lllll} \text{To factor } 6x^2-17x+12\text{, try} & 3x-2, & 3x-6, & 3x-4, & \cdots \\ \text{multiplied by} & \underline{2x-6,} & \underline{2x-2,} & \underline{2x-3,} & \cdots \\ \text{Products, 2d terms,} & -22x, & -18x, & -17x, & \cdots \end{array}$$

$\therefore\ 18x^2-51x+36=3(3x-4)(2x-3)$.

19. $12x^2+14x-40=2(6x^2+7x-20)$.

$$\begin{array}{llllll} \text{To factor } 6x^2+7x-20\text{, try} & 3x+5, & 3x-5, & 3x+4, & 3x-4, & \cdots \\ \text{multiplied by} & \underline{2x-4,} & \underline{2x+4,} & \underline{2x-5,} & \underline{2x+5,} & \cdots \\ \text{Products, 2d terms,} & -2x, & +2x, & -7x, & +7x, & \cdots \end{array}$$

$\therefore\ 12x^2+14x-40=2(3x-4)(2x+5)$.

20. See Ex. 24, p. 40. **21.** See Ex. 25, p. 40.

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25. See Ex. 12, p. 39. **27.** See Ex. 16, p. 39. **29.** See Ex. 19, p. 39.

26. See Ex. 14, p. 39. **28.** See Ex. 18, p. 39.

30.
$$3x^2-10xy+3y^2=\frac{9x^2-30xy+9y^2}{3}=\frac{(3x)^2-10y(3x)+9y^2}{3}$$
$$=\frac{(3x-y)(3x-9y)}{3}=(3x-y)(x-3y).$$

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31. $9x^2+43x-10=\dfrac{81x^2+387x-90}{9}=\dfrac{(9x)^2+43(9x)-90}{9}$

$=\dfrac{(9x-2)(9x+45)}{9}=(9x-2)(x+5).$

32. $18x^2-9x-35=\dfrac{36x^2-18x-70}{2}=\dfrac{(6x)^2-3(6x)-70}{2}$

$=\dfrac{(6x+7)(6x-10)}{2}=(6x+7)(3x-5).$

33. See Ex. 21, p. 40. **34.** See Ex. 20, p. 39.

35. $32n^2+28n-15=\dfrac{64n^2+56n-30}{2}=\dfrac{(8n)^2+7(8n)-30}{2}$

$=\dfrac{(8n+10)(8n-3)}{2}=(4n+5)(8n-3).$

36. $5x^{2n}+9x^ny-2y^2=\dfrac{25x^{2n}+45x^ny-10y^2}{5}=\dfrac{(5x^n)^2+9y(5x^n)-10y^2}{5}$

$=\dfrac{(5x^n+10y)(5x^n-y)}{5}=(x^n+2y)(5x^n-y).$

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8. $r^9+s^9=(r^3)^3+(s^3)^3=(r^3+s^3)(r^6-r^3s^3+s^6)$
$=(r+s)(r^2-rs+s^2)(r^6-r^3s^3+s^6).$

9–20. See Ex. 9–20, p. 41.

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2–10. See Ex. 2–10, pp. 41–42.

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13. See Ex. 12, p. 42. **14.** See Ex. 13, p. 42.

15. $x^3-7x^2+7x+15$

Substituting -1 for x, $=-1-7-7+15=0$; $\therefore x+1$ is a factor,

and $x^3-7x^2+7x+15=(x+1)(x^2-8x+15)$

§ 154, $=(x+1)(x-3)(x-5).$

16. See Ex. 14, p. 42.

17. $2x^3-3x^2-17x-12$

Substituting -1 for x, $=-2-3+17-12=0$; $\therefore x+1$ is a factor,

and $2x^3-3x^2-17x-12=(x+1)(2x^2-5x-12)$

§ 156, $=(x+1)(x-4)(2x+3).$

18. $x^3-13x^2+46x-48$

Substituting 2 for x, $=8-52+92-48=0$; $\therefore x-2$ is a factor,

and $x^3-13x^2+46x-48=(x-2)(x^2-11x+24)$

§ 154, $=(x-2)(x-3)(x-8).$

19. See Ex. 17, p. 42.

20. $2x^3-9x^2-2x+24$

Substituting 2 for x, $=16-36-4+24=0$; $\therefore x-2$ is a factor,

and $2x^3-9x^2-2x+24=(x-2)(2x^2-5x-12)$

§ 156, $=(x-2)(x-4)(2x+3).$

21. See Ex. 33, p. 44.

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22. $x^3 + 2x^2y - xy^2 - 2y^3$

Substituting y for x, $= y^3 + 2y^3 - y^3 - 2y^3 = 0$; $\therefore x - y$ is a factor,

and $x^3 + 2x^2y - xy^2 - 2y^3 = (x-y)(x^2 + 3xy + 2y^2)$

§ 154, $= (x-y)(x+y)(x+2y)$.

23. $x^3 + 4x^2y + 5xy^2 + 2y^3$

Substituting $-y$ for x, $= -y^3 + 4y^3 - 5y^3 + 2y^3 = 0$; $\therefore x + y$ is a factor,

and $x^3 + 4x^2y + 5xy^2 + 2y^3 = (x+y)(x^2 + 3xy + 2y^2)$

§ 154, $= (x+y)(x+y)(x+2y)$.

24. See Ex. 22, p. 43. **27.** See Ex. 24, p. 43. **30.** See Ex. 21, p. 43.
25. See Ex. 23, p. 43. **28.** See Ex. 19, p. 43. **31.** See Ex. 26, p. 43.
26. See Ex. 25, p. 43. **29.** See Ex. 20, p. 43. **32.** See Ex. 27, p. 43.

33. $2n^3 - 7n^2 - 7n + 30$

Substituting -2 for n, $= -16 - 28 + 14 + 30 = 0$; $\therefore n + 2$ is a factor,

and $2n^3 - 7n^2 - 7n + 30 = (n+2)(2n^2 - 11n + 15)$

§ 156, $= (n+2)(n-3)(2n-5)$.

34. See Ex. 29, p. 43. **38.** See Ex. 34, p. 44. **42.** See Ex. 39, p. 44.
35. See Ex. 30, p. 43. **39.** See Ex. 41, p. 45. **43.** See Ex. 43, p. 45.
36. See Ex. 31, p. 44. **40.** See Ex. 42, p. 45. **44.** See Ex. 44, p. 45.
37. See Ex. 32, p. 44. **41.** See Ex. 38, p. 44. **45.** See Ex. 45, p. 45.

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2–6. See Ex. 2–6, pp. 45–46.

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4–24. See Ex. 4–24, pp. 46–47. **2–8.** See Ex. 2–8, pp. 47–48.

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52. See Ex. 2, p. 48. **55.** See Ex. 3, p. 48. **57.** See Ex. 7, p. 48.
53. See Ex. 4, p. 48. **56.** See Ex. 6, p. 48. **58.** See Ex. 8, p. 48.
54. See Ex. 5, p. 48. **59.** $a^9 - b^3 = (a^3)^3 - b^3 = (a^3 - b)(a^6 + a^3b + b^2)$.
60. See Ex. 21, p. 48. **62.** See Ex. 13, p. 48. **64.** See Ex. 11, p. 48.
61. See Ex. 12, p. 48. **63.** See Ex. 19, p. 48.
65. $7n^7 + 7n = 7n(n^6 + 1) = 7n(n^2+1)(n^4 - n^2 + 1)$.
66. See Ex. 16, p. 48. **68.** See Ex. 20, p. 48. **70.** See Ex. 24, p. 49.
67. See Ex. 17, p. 48. **69.** See Ex. 22, p. 48. **71.** See Ex. 23, p. 48.

72. First factor, try $2x + 1$, $2x - 1$.
Second factor, try $\underline{x - 1}$, $\underline{x + 1}$.
Products, 2d terms, $-x$, $+x$.
$\therefore 2x^2 + x - 1 = (2x-1)(x+1)$.

73. $x^2 + 9x - 90 = x^2 + (+15 - 6)x + (+15)(-6) = (x+15)(x-6)$.

74. $$3x^2 - 2x - 8 = \frac{9x^2 - 6x - 24}{3} = \frac{(3x)^2 - 2(3x) - 24}{3}$$
$$= \frac{(3x+4)(3x-6)}{3} = (3x+4)(x-2).$$

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75. $15 + 6x - 9x^2 = -3(3x^2 - 2x - 5)$

By trial, $= -3(3x-5)(x+1) = 3(5-3x)(1+x).$

76. $17 - 16a - a^2 = -(a^2 + 16a - 17)$

$= -(a-1)(a+17) = (1-a)(17+a).$

78. $6x + 5x^2 - x^3 = -x(x^2 - 5x - 6)$

$= -x(x-6)(x+1) = x(6-x)(1+x).$

82. See Ex. 25, p. 49. **83.** See Ex. 28, p. 49. **84.** See Ex. 31, p. 49.

85. $4a - 3ax - ax^2 = -a(x^2 + 3x - 4) = -a(x-1)(x+4)$

$= a(1-x)(4+x).$

86. See Ex. 32, p. 49.
87. See Ex. 46, p. 50.
88. See Ex. 53, p. 50.
90. See Ex. 55, p. 50.
91. See Ex. 33, p. 49.
93. See Ex. 60, p. 50.
94. See Ex. 57, p. 50.
95. See Ex. 29, p. 49.
96. See Ex. 87, p. 52.
97. See Ex. 44, p. 49.
98. See Ex. 56, p. 50.

99. $77 - 30x^2 - 37x = -(30x^2 + 37x - 77)$

By trial, $= -(10x - 11)(3x + 7) = (11 - 10x)(7 + 3x).$

100. See Ex. 30, p. 49.
101. See Ex. 45, p. 50.
102. See Ex. 48, p. 50.
103. See Ex. 50, p. 50.
104. See Ex. 54, p. 50.
105. See Ex. 51, p. 50.
106. See Ex. 39, p. 49.
107. See Ex. 41, p. 49.
108. See Ex. 42, p. 49.
109. See Ex. 63, p. 51.

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110. See Ex. 111, p. 54.
111. See Ex. 100, p. 53.
112. See Ex. 88, p. 52.
113. See Ex. 83, p. 52.
114. See Ex. 108, p. 54.
115. See Ex. 109, p. 54.
116. See Ex. 84, p. 52.
117. See Ex. 85, p. 52.
118. See Ex. 116, p. 54.
119. See Ex. 61, p. 51.
121. See Ex. 107, p. 53.
122. See Ex. 112, p. 54.
123. See Ex. 47, p. 50.
124. See Ex. 103, p. 53.
125. See Ex. 115, p. 54.
126. See Ex. 89, p. 52.

127. $4x^3 + x^2 - 8x - 2 = x^2(4x+1) - 2(4x+1) = (4x+1)(x^2-2).$

128. $x^2 + 5x + ax + 5a = x(x+5) + a(x+5) = (x+5)(x+a).$

129. See Ex. 117, p. 54.
130. See Ex. 113, p. 54.
131. See Ex. 118, p. 54.
132. See Ex. 101, p. 53.
133. See Ex. 119, p. 54.
134. See Ex. 114, p. 54.
136. See Ex. 110, p. 54.
137. See Ex. 105, p. 53.
138. See Ex. 98, p. 53.
139. See Ex. 65, p. 51.
140. See Ex. 66, p. 51.
141. See Ex. 67, p. 51.
142. See Ex. 68, p. 51.
143. See Ex. 74, p. 51.
144. See Ex. 75, p. 51.
145. See Ex. 76, p. 51.
146. See Ex. 69, p. 51.
147. See Ex. 70, p. 51.
148 See Ex. 123, p. 55.
149. See Ex. 124, p. 55.
150. See Ex. 125, p. 55.
151. See Ex. 126, p. 55.
152. See Ex. 127, p. 55.
153. See Ex. 128, p. 55.

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93–99. See Ex. 93–99, p. 56.

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HIGHEST COMMON FACTOR

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30. See Ex. 30, p. 56.

31. See Ex. 32, p. 56.

32. See Ex. 34, p. 57.

33. See Ex. 35, p. 57.

34.
$$\begin{aligned} 6x^2-54 &= 6(x^2-9) = 3\cdot 2(x+3)(x-3)\\ 9(x+3) &= 3\cdot 3(x+3)\\ 30(x^2-x-12) &= 3\cdot 2\cdot 5(x+3)(x-4) \end{aligned}$$
$$\therefore \text{H.C.F.} = 3(x+3).$$

35.
$$\begin{aligned} 8a-8a^2 &= -8a(a-1) = -2\cdot 2\cdot 2\cdot a(a-1)\\ 12a(a^2-1)^2 &= 2\cdot 2\cdot 3\cdot a(a-1)(a+1)(a-1)(a+1)\\ 18a^2-36a+18 &= 2\cdot 3\cdot 3(a-1)(a-1) \end{aligned}$$
$$\therefore \text{H.C.F.} = 2(a-1).$$

36.
$$\begin{aligned} 9a^2(x^2-8x+16) &= 3\cdot 3\cdot a\cdot a(x-4)(x-4)\\ 3a^2x+6ax-12a^2-24a &= 3a(x-4)(a+2) \end{aligned}$$
$$\therefore \text{H.C.F.} = 3a(x-4).$$

37. See Ex. 36, p. 57.

38–39. See Ex. 38–39, p. 57.

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40–62. See Ex. 40–62, pp. 57–59.

LOWEST COMMON MULTIPLE

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26–40. See Ex. 26–40, pp. 59–60.

FRACTIONS

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28–39. See Ex. 28–39, pp. 60–61.

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40. $\dfrac{y^2-81}{y^2+7y-18} = \dfrac{(y+9)(y-9)}{(y+9)(y-2)} = \dfrac{y-9}{y-2}.$

41. See Ex. 40, p. 61.

42. $\dfrac{x^2-6x-7}{x^2-11x+28} = \dfrac{(x-7)(x+1)}{(x-7)(x-4)} = \dfrac{x+1}{x-4}.$

43. $\dfrac{2x^2-2x-12}{6x^2-10x-44} = \dfrac{2(x-3)(x+2)}{2(3x-11)(x+2)} = \dfrac{x-3}{3x-11}.$

44. See Ex. 41, p. 61.

45. $\dfrac{7x-2x^2-3}{2x^2+11x-6} = \dfrac{(3-x)(2x-1)}{(x+6)(2x-1)} = \dfrac{3-x}{x+6} = \dfrac{3-x}{6+x}.$

46. $\dfrac{b^2+b-12}{3b^2+9b-54} = \dfrac{(b+4)(b-3)}{3(b+6)(b-3)} = \dfrac{b+4}{3(b+6)}.$

47. See Ex. 45, p. 61.

48. $\dfrac{a^2b(a+2b)^4}{ab(a^2-4b^2)^2} = \dfrac{a^2b(a+2b)(a+2b)(a+2b)(a+2b)}{ab(a+2b)(a-2b)(a+2b)(a-2b)} = \dfrac{a(a+2b)^2}{(a-2b)^2}.$

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49. See Ex. 46, p. 61. **50.** See Ex. 44, p. 61.

51. $\dfrac{(a+b)^2-1}{a^2c+abc+ac}=\dfrac{(a+b+1)(a+b-1)}{ac(a+b+1)}=\dfrac{a+b-1}{ac}.$

52. See Ex. 54, p. 62. **53.** See Ex. 55, p. 62. **54.** See Ex. 47, p. 61.

55. $\dfrac{x^4+x^2y^2+y^4}{x^3+y^3}=\dfrac{(x^2+xy+y^2)(x^2-xy+y^2)}{(x+y)(x^2-xy+y^2)}=\dfrac{x^2+xy+y^2}{x+y}.$

56. See Ex. 48, p. 61.

57. $\dfrac{cd^3-c}{c^2d^4+c^2d^2+c^2}=\dfrac{c(d-1)(d^2+d+1)}{c^2(d^2+d+1)(d^2-d+1)}=\dfrac{d-1}{c(d^2-d+1)}.$

58. See Ex. 51, p. 62. **59.** See Ex. 49, p. 62.

60. $\dfrac{5r^2-26rs+5s^2}{r^3-5r^2s+r-5s}=\dfrac{(5r-s)(r-5s)}{(r^2+1)(r-5s)}=\dfrac{5r-s}{r^2+1}.$

61. $\dfrac{a^3-ab-a^2b+b^2}{a^4-a^2b-a^2b^2+b^3}=\dfrac{(a^2-b)(a-b)}{(a^2-b)(a-b)(a+b)}=\dfrac{1}{a+b}.$

62. $\dfrac{x^5-x^2-x^4+x^3}{x^4-1}=\dfrac{x^2(x^2+1)(x-1)}{(x^2+1)(x+1)(x-1)}=\dfrac{x^2}{x+1}.$

63. See Ex. 50, p. 62. **65.** See Ex. 53, p. 62.

64. See Ex. 52, p. 62. **66.** See Ex. 56, p. 62.

67. $\dfrac{9a^4-13a^2b^2+4b^4}{24a^2d-40abd+16b^2d}=\dfrac{(3a+2b)(3a-2b)(a+b)(a-b)}{8d(3a-2b)(a-b)}$

$=\dfrac{(3a+2b)(a+b)}{8d}.$

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7–12. See Ex. 7–12, pp. 63–64.

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11–17. See Ex. 11–17, pp. 64–65.

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27–32. See Ex. 27–32, pp. 65–66. **34–46.** See Ex. 34–46, pp. 66–67.

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47–52. See Ex. 47–52, p. 67. **54–57.** See Ex. 54–57, p. 68.

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59–61. See Ex. 59–61, p. 69. **63–67.** See Ex. 63–67, pp. 69–70.

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25. $\dfrac{x-4}{x+2}\cdot\dfrac{4-x^2}{16-x^2}=\dfrac{x-4}{x+2}\cdot\dfrac{-(x+2)(x-2)}{-(x+4)(x-4)}=\dfrac{x-2}{x+4}.$

27. See Ex. 12, p. 70. **28.** See Ex. 18, p. 71.

29. $\dfrac{x^2+3x+2}{x^2-3x-10}\cdot\dfrac{x^2-6x+5}{x^2+8x+7}=\dfrac{(x+2)(x+1)}{(x-5)(x+2)}\cdot\dfrac{(x-5)(x-1)}{(x+7)(x+1)}=\dfrac{x-1}{x+7}.$

30. See Ex. 13, p. 70. **32.** See Ex. 17, p. 71. **34.** See Ex. 20, p. 71.

31. See Ex. 16, p. 70. **33.** See Ex. 19, p. 71.

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35. $$\frac{a^2+ab+2a+2b}{ax-2ay+2x-4y}\cdot\frac{x^2-2xy}{(a+b)^2}=\frac{(a+2)(a+b)}{(a+2)(x-2y)}\cdot\frac{x(x-2y)}{(a+b)(a+b)}=\frac{x}{a+b}.$$

36. See Ex. 15, p. 70. **37.** See Ex. 14, p. 70.

38 $$\left(\frac{2}{3x}-1\right)\cdot\frac{9x^2y}{4-9x^2}\cdot\frac{2+3x}{4y^3}=\frac{2-3x}{3x}\cdot\frac{9x^2y}{(2+3x)(2-3x)}\cdot\frac{2+3x}{4y^3}=\frac{3x}{4y^2}.$$

39. See Ex. 21, p. 71.

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21. See Ex. 15, p. 72. **24.** See Ex. 19, p. 72. **27.** See Ex. 21, p. 72.
22. See Ex. 16, p. 72. **25.** See Ex. 18, p. 72. **28.** See Ex. 22, p. 72.
23. See Ex. 17, p. 72. **26.** See Ex. 20, p. 72. **29.** See Ex. 23, p. 72.

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30. $$\frac{a^3+27}{a^3-27}\div\frac{a+3}{a^2+3a+9}=\frac{(a+3)(a^2-3a+9)}{(a-3)(a^2+3a+9)}\times\frac{a^2+3a+9}{a+3}=\frac{a^2-3a+9}{a-3}.$$

31. $$\frac{a^2+b^2-c^2+2ab}{a^2-b^2-c^2+2bc}\div\frac{a+b+c}{a-b+c}=\frac{(a+b+c)(a+b-c)}{(a+b-c)(a-b+c)}\times\frac{a-b+c}{a+b+c}=1.$$

32. See Ex. 26, p. 73. **34.** See Ex. 28, p. 73. **36.** See Ex. 30, p. 73.
33. See Ex. 27, p. 73. **35.** See Ex. 29, p. 73. **37.** See Ex. 31, p. 73.

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2–10. See Ex. 2–10, pp. 73–74. **12–27.** See Ex. 12–27, pp. 74–76.

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29. See Ex. 29, p. 76. **30.** See Ex. 33, p. 77. **31.** See Ex. 31, p. 77.

32. $$\frac{x}{x+1-\dfrac{x}{x+\dfrac{x}{x-1}}}=\frac{x}{x+1-\dfrac{x}{\dfrac{x^2}{x-1}}}=\frac{x}{x+1-\dfrac{x-1}{x}}=\frac{x}{\dfrac{x^2+1}{x}}=\frac{x^2}{x^2+1}.$$

33. See Ex. 32, p. 77. **34.** See Ex. 30, p. 77. **35.** See Ex. 34, p. 77.

36. $$a+\frac{1}{a-1-\dfrac{2a}{2+\dfrac{1}{a}}}=a+\frac{1}{a-1-\dfrac{2a}{\dfrac{2a+1}{a}}}=a+\frac{1}{a-1-\dfrac{2a^2}{2a+1}}$$
$$=a+\frac{1}{-\dfrac{a+1}{2a+1}}=a-\frac{2a+1}{a+1}=\frac{a^2-a-1}{a+1}.$$

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REVIEW

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18. $\dfrac{5x^2-11x-12}{10x^2+23x+12}=\dfrac{(5x+4)(x-3)}{(5x+4)(2x+3)}=\dfrac{x-3}{2x+3}.$

19. $\dfrac{x^3+3x^2-x-3}{x^3+3x^2+x+3}=\dfrac{x^2(x+3)-1(x+3)}{x^2(x+3)+1(x+3)}=\dfrac{(x^2-1)(x+3)}{(x^2+1)(x+3)}=\dfrac{x^2-1}{x^2+1}.$

20. $\dfrac{a^3+b^3}{a^4+a^2b^2+b^4}=\dfrac{(a+b)(a^2-ab+b^2)}{(a^2+ab+b^2)(a^2-ab+b^2)}=\dfrac{a+b}{a^2+ab+b^2}.$

21. $\dfrac{cx-cd}{cx+3x-3d-cd}=\dfrac{c(x-d)}{(c+3)(x-d)}=\dfrac{c}{c+3}.$

22–32. See Ex. 22–32, pp. 78–79.

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33–47. See Ex. 33–47, pp. 80–81.

SIMPLE EQUATIONS

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19–29. See Ex. 19–29, pp. 81–83.

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30–34. See Ex. 30–34, pp. 83–84.

36–38. See Ex. 36–38, pp. 84–85.

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39–51. See Ex. 39–51, pp. 85–88.

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2–14. See Ex. 2–14, pp. 88–91.

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15–24. See Ex. 15–24, pp. 91–93.

Page 163

1. See Ex. 1, p. 93.

2. Let $x =$ number of logs that each of three wagons held.
Then, $x + 2 =$ number of logs the fourth wagon held.
$\therefore\ 3x + x + 2 = 38.$
Solving, $x = 9,$
and $x + 2 = 11.$

Hence, three of the wagons held 9 logs each, and the other wagon, 11 logs.

3. Let $x =$ the cost of the harness in dollars.
Then, $2x =$ the cost of the carriage in dollars,
and $3x =$ the cost of the horse in dollars.
$\therefore\ x + 2x + 3x = 300.$
Solving, $x = 50,$
$2x = 100,$
and $3x = 150.$

Hence, the cost of the harness was \$50; of the carriage, \$100; of the horse, \$150.

4. See Ex. 3, p. 94. **5.** See Ex. 5, p. 94. **6.** See Ex. 6, p. 95.

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7. Let $x =$ number of eggs in all the baskets.
Then, $576 - x =$ number of eggs in all the pails.
$\therefore 576 - x = \frac{1}{3}x.$
Solving, $x = 432$, number of eggs in all the baskets,
and $576 - x = 144$, number of eggs in all the pails.

Since there are 3 pails and 6 baskets, each pail contains $\frac{1}{3}$ of 144 eggs, or 48 eggs, and each basket contains $\frac{1}{6}$ of 432 eggs, or 72 eggs.

8. Let $x =$ the number.
Then, $\frac{1}{4}x + \frac{1}{5}x = 90.$
Solving, $x = 200.$
Hence, the number is 200.

9. Let $x =$ the number.
Then, $x - (\frac{1}{2}x + \frac{1}{3}x) = 25.$
Solving, $x = 150.$
Hence, the number is 150.

10. Let $x =$ the number.
Then, $\frac{1}{3}x + \frac{1}{6}x = a.$
Solving, $x = 2a.$
Hence, the number is $2a$.

11. Let $x =$ the number.
Then, $3 \cdot \frac{1}{2}x - \frac{1}{6}x = 24.$
Solving, $x = 18.$
Hence, the number is 18.

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12. See Ex. 11, p. 96.

13. Let $x =$ number of feet in the width of the rectangle.
Then, $x + 9 =$ number of feet in the length of the rectangle,
and $x + 3 =$ number of feet in the side of the square.
$\therefore x(x + 9) = (x + 3)(x + 3).$
Solving, $x = 3,$
and $x + 9 = 12.$
Hence, the rectangle is 12 feet long and 3 feet wide.

14. See Ex. 14, p. 96.
15. See Ex. 9, p. 95.
16. See Ex. 13, p. 96.
17. See Ex. 15, p. 97.
18. See Ex. 16, p. 97.
19. See Ex. 17, p. 97.
20. See Ex. 18, p. 97.

21. Let $x =$ the number.
Then, $4(\frac{1}{5}x + \frac{1}{6}x) = 88.$
Solving, $x = 60.$
Hence, the number is 60.

22. Let $x =$ one part.
Then, $54 - x =$ other part.
$\therefore \frac{1}{12}[x - (54 - x)] = \frac{1}{3}.$
Solving, $x = 29,$
and $54 - x = 25.$
Hence, 29 is one of the parts into which 54 is separated, and 25 is the other part.

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23. Let $x =$ one part.

Then, $m - x =$ other part.

$$\therefore \frac{1}{n}[x - (m-x)] = \frac{1}{r}.$$

Solving, $x = \dfrac{mr+n}{2r}$,

and $m - x = \dfrac{mr-n}{2r}$.

Hence, $\dfrac{mr+n}{2r}$ is one part and $\dfrac{mr-n}{2r}$ is the other part.

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25. See Ex. 21, p. 97. **26.** See Ex. 22, p. 98.

27. Let $x =$ the required number of days.

Then, $\dfrac{1}{x} =$ the part of the work both can do in 1 day.

$$\therefore \frac{1}{x} = \frac{1}{2\frac{1}{2}} + \frac{1}{3\frac{1}{3}}.$$

Solving, $x = 1\frac{3}{7}$.

Hence, A and B together can do the work in $1\frac{3}{7}$ days.

28. Let $x =$ the required number of days.

Then, $\dfrac{1}{x} =$ the part of the barn all can paint in 1 day.

$$\therefore \frac{1}{x} = \frac{1}{12} + \frac{1}{4}.$$

Solving, $x = 3$.

Hence, A, B, and C together can paint the barn in 3 days.

29. See Ex. 24, p. 98. **30.** See Ex. 25, p. 98.

31. Let $x =$ the required number of minutes.

Then, $\dfrac{1}{x} =$ the part of the tank both pipes can fill in 1 minute.

$$\therefore \frac{1}{x} = \frac{1}{45} + \frac{1}{55}.$$

Solving, $x = 24\frac{3}{4}$.

Hence, the two pipes can fill the tank in $24\frac{3}{4}$ minutes.

32. Let $x =$ the required number of hours.

Then, $\dfrac{1}{x} =$ the part of the tank that can be filled in 1 hour.

$$\therefore \frac{1}{x} = \frac{1}{a} + \frac{1}{c} - \frac{1}{b}.$$

Solving, $x = \dfrac{abc}{ab + bc - ac}$.

Hence, if all the pipes are open, the tank can be filled in $\dfrac{abc}{ab + bc - ac}$ hours.

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34. See Ex. 27, p. 99.

35. See Ex. 28, p. 99.

36. Let $x =$ digit in units' place.
Then, $x + 3 =$ digit in tens' place,
and $10(x + 3) + x$, or $11x + 30 =$ the number.

$$\therefore \frac{11x + 30 - 6}{x + x + 3} = 6.$$

Solving, $x = 6$,
and $x + 3 = 9$.
Hence, the number is 96.

37. Let $x =$ digit in units' place.
Then, $x + 5 =$ digit in tens' place,
$10(x + 5) + x$, or $11x + 50 =$ the number,
and $10x + (x + 5)$, or $11x + 5 =$ the number with its digits reversed.

$$\therefore 11x + 5 = \tfrac{3}{8}(11x + 50).$$

Solving, $x = 2$,
and $x + 5 = 7$.
Hence, the number is 72.

38. Let $x =$ digit in tens' place.
Then, $x + 3 =$ digit in units' place,
$10x + (x + 3)$, or $11x + 3 =$ the number,
and $10(x + 3) + x$, or $11x + 30 =$ the number with its digits reversed.

$$\therefore 8(11x + 30) = 14(11x + 3).$$

Solving, $x = 3$,
and $x + 3 = 6$.
Hence, the number is 36.

39. Let $x =$ digit in tens' place.
Then, $11 - x =$ digit in units' place,
$10x + (11 - x)$, or $9x + 11 =$ the number,
and $10(11 - x) + x$, or $110 - 9x =$ the number with its digits reversed.

$$\therefore 9x + 11 + 63 = 110 - 9x.$$

Solving, $x = 2$,
and $11 - x = 9$.
Hence, the number is 29.

40. Let $x =$ digit in tens' place.
Then, $5 - x =$ digit in units' place,
$10x + (5 - x)$, or $9x + 5 =$ the number,
and $10(5 - x) + x$, or $50 - 9x =$ the number with its digits reversed.

$$\therefore 3(9x + 5) - 1 = 50 - 9x.$$

Solving, $x = 1$,
and $5 - x = 4$.
Hence, the number is 14.

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42. See Ex. 33, p. 99.

44. See Ex. 35, p. 100.

43. See Ex. 34, p. 100.

45. See Ex. 36, p. 100.

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46. Let $x =$ number of dollars invested at 6 %.
Then, $15{,}000 - x =$ number of dollars invested at 3 %.
$\therefore \frac{6}{100}x + \frac{3}{100}(15{,}000 - x) = \frac{5}{100} \times 15{,}000.$
Solving, $x = 10{,}000,$
and $15{,}000 - x = 5000.$
Hence, Mr. Johnson invested \$ 10,000 at 6 %, and \$ 5000 at 3 %.

47. Let $x =$ number of dollars invested at 4 %.
Then, $5650 - x =$ number of dollars invested at 6 %.
$\therefore \frac{4}{100}x + \frac{6}{100}(5650 - x) = 298.$
Solving, $x = 2050,$
and $5650 - x = 3600.$
Hence, he invested \$ 2050 at 4 %, and \$ 3600 at 6 %.

48. Let $x =$ number of dollars invested in 6 % bonds.
Then, $12{,}000 - x =$ number of dollars invested in 4 % bonds.
$$\therefore \frac{6}{100}x + \frac{4}{100}(12{,}000 - x) = \frac{4\frac{1}{2}}{100} \times 12{,}000.$$
Solving, $x = 3000,$
and $12{,}000 - x = 9000.$
Hence, he invests \$ 3000 in 6 % bonds, and \$ 9000 in 4 % bonds.

49. Let $x =$ number of dollars invested at 6 %.
Then, $s - x =$ number of dollars invested at 5 %.
$\therefore \frac{6}{100}x + \frac{5}{100}(s - x) = m.$
Solving, $x = 100\,m - 5\,s,$
and $s - x = 6\,s - 100\,m.$
Hence, the bank invests $(100\,m - 5\,s)$ dollars at 6 %, and $(6\,s - 100\,m)$ dollars at 5 %.

50. Let $x =$ number of dollars of capital.

Then, $\frac{x}{n} =$ number of dollars invested at 5 %,

and $\frac{(n-1)x}{n} =$ number of dollars invested at 6 %.

$$\therefore \frac{5}{100} \cdot \frac{x}{n} + \frac{6}{100} \cdot \frac{(n-1)x}{n} = m.$$

Solving, $x = \frac{100\,mn}{6\,n - 1}.$

Hence, my capital is $\frac{100\,mn}{6\,n - 1}$ dollars.

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52. See Ex. 41, p. 101.

53. See Ex. 42, p. 101.

54. See Ex. 43, p. 101.

55. See Ex. 44, pp. 101–102.

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56. Let $x =$ number of minute spaces the minute hand travels after 2 o'clock before the hands are at right angles.

Then, $\frac{x}{12} =$ number of minute spaces the hour hand travels in the same time.

Since the minute hand gains 10 + 15, or 25, minute spaces,

$$x - \frac{x}{12} = 25.$$

Solving, $x = 27\frac{3}{11}$, the number of minutes after 2 o'clock.

Hence, at $2:27\frac{3}{11}$ o'clock the hands will be at right angles to each other.

57. Let $x =$ number of minute spaces the minute hand travels after 6 o'clock before it is 15 minute spaces *behind* the hour hand.

Then, $\frac{x}{12} =$ number of minute spaces the hour hand travels in the same time.

Since the minute hand gains 30 − 15, or 15, minute spaces,

$$x - \frac{x}{12} = 15.$$

Solving, $x = 16\frac{4}{11}$, the number of minutes after 6 o'clock.

Again, let $x =$ number of minute spaces the minute hand travels before it is 15 minute spaces *ahead* of the hour hand.

Then, $\frac{x}{12} =$ number of minute spaces the hour hand travels in the same time.

Since the minute hand gains 30 + 15, or 45, minute spaces,

$$x - \frac{x}{12} = 45.$$

Solving, $x = 49\frac{1}{11}$, the number of minutes after 6 o'clock.

Hence, the required times are $6:16\frac{4}{11}$ o'clock and $6:49\frac{1}{11}$ o'clock.

58. Let $x =$ number of minute spaces the minute hand travels after 1 o'clock before the hands form a straight line.

Then, $\frac{x}{12} =$ number of minute spaces the hour hand travels in the same time.

Since the minute hand gains 5 + 30, or 35, minute spaces,

$$x - \frac{x}{12} = 35.$$

Solving, $x = 38\frac{2}{11}$, the number of minutes after 1 o'clock.

Hence, the required time is $1:38\frac{2}{11}$ o'clock.

59. Let $x =$ number of miles I may ride.

Then, $\frac{x}{9} =$ number of hours riding,

and $\frac{x}{3\frac{1}{2}} =$ number of hours walking.

$$\therefore \frac{x}{9} + \frac{x}{3\frac{1}{2}} = 6\frac{1}{4}.$$

Solving, $x = 15\frac{3}{4}$.

Hence, I may ride $15\frac{3}{4}$ miles.

60. See Ex. 45, p. 102.

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61. Let $x =$ the velocity of the stream.

Then, $12 + x =$ the rate of the steamboat downstream,

and $12 - x =$ the rate of the steamboat upstream.

$$\therefore \frac{16}{12 - x} = \frac{32}{12 + x}.$$

Solving, $x = 4.$

Hence, the velocity of the stream is 4 miles per hour.

62. See Ex. 46, p. 102.

63. Let $x =$ number of miles the yacht sailed up the river.

Then, $\frac{x}{s} =$ number of hours going,

and $\frac{x}{t} =$ number of hours returning.

$$\therefore \frac{x}{s} + \frac{x}{t} = r.$$

Solving, $x = \frac{rst}{s + t}.$

Hence, the yacht sailed $\frac{rst}{s + t}$ miles up the river.

64. Let $x =$ number of hours required.

Then, $20(x + \frac{1}{2}) =$ number of miles first train travels,

and $50x =$ number of miles second train travels.

$$\therefore 20(x + \tfrac{1}{2}) = 50x.$$

Solving, $x = \frac{1}{3}.$

Hence, it takes the second train $\frac{1}{3}$ of an hour, or 20 minutes, to overtake the first train.

65. See Ex. 47, p. 102.

66. Let $x =$ the number of minutes required.

$$\therefore 15(x + 1) = 105.$$

Solving, $x = 6.$

Hence, it took the airship 6 minutes to go a mile.

67. Let $x =$ the number of minutes required.

$$\therefore d(x + m) = 60t.$$

Solving, $x = \frac{60t - dm}{d}.$

Hence, it took the automobile $\frac{60t - dm}{d}$ minutes to go a mile.

68. Let $x =$ number of pounds the pods weighed.

Then, $\frac{x}{4} =$ number of pounds the beans weighed.

$$\therefore x + \frac{x}{4} = 1000.$$

Solving, $x = 800,$

and $\frac{x}{4} = 200.$

Hence, the beans weighed 200 pounds, and the pods 800 pounds.

69. See Ex. 53, p. 104.

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70. Let $x =$ number of pounds of zinc.
Then, $mx =$ number of pounds of tin,
and $r(mx + n) =$ number of pounds of copper.
$\therefore x + mx + r(mx + n) = p.$

Solving, $x = \dfrac{p - nr}{mr + m + 1}.$

Hence, there were $\dfrac{p - nr}{mr + m + 1}$ pounds of zinc in the bronze.

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71. See Ex. 55, p. 104.

72. See Ex. 56, p. 104.

73. See Ex. 57, p. 104.

74. See Ex. 58, p. 104.

75. Let $x =$ number of pounds of copper to be added.
Then, $100 + x =$ number of pounds new alloy weighs,
and $\frac{10}{100}(100 + x) =$ number of pounds of zinc in new alloy.
$\therefore \frac{10}{100}(100 + x) = 100 - 75.$
Solving, $x = 150.$
Hence, 150 pounds of copper must be added to make the alloy 10 % zinc.

76. Let $x =$ number of pounds of water to be evaporated.
Then, $60 - x =$ number of pounds new solution weighs,
and $\frac{10}{100}(60 - x) =$ number of pounds of salt in new solution.
$\therefore \frac{10}{100}(60 - x) = 3.$
Solving, $x = 30.$
Hence, 30 pounds of water must be evaporated to make the solution 10 % salt.

77. See Ex. 60, p. 105.

78. See Ex. 61, p. 105.

79. See Ex. 62, p. 105.

80. Let $x =$ number of ounces of pure gold to be added.
Then, $\frac{14}{24}(180) + x =$ number of ounces of pure gold after the addition,
but $\frac{16}{24}(180 + x) =$ number of ounces of pure gold after the addition.
$\therefore \frac{14}{24}(180) + x = \frac{16}{24}(180 + x).$
Solving, $x = 45.$
Hence, 45 ounces of pure gold must be added to make the alloy 16 carats fine.

81. Let $x =$ number of ounces of pure gold to be added.
Then, $\frac{18}{24}w + x =$ number of ounces of pure gold after the addition,
but $\frac{22}{24}(w + x) =$ number of ounces of pure gold after the addition.
$\therefore \frac{18}{24}w + x = \frac{22}{24}(w + x).$
Solving, $x = 2w.$
Hence, $2w$ ounces of pure gold must be added to make the alloy 22 carats fine.

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82. See Ex. 63, p. 105.

83. See Ex. 64, p. 105.

84. See Ex. 65, p. 105.

85. See Ex. 66, p. 106.

86. See Ex. 67, p. 106.

87. See Ex. 30, p. 99.

88. See Ex. 31, p. 99.

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2–4. See Ex. 2–4, pp. 106–107.

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5. See Ex. 6, p. 107.

6. See Ex. 7, p. 107.

7. See Ex. 8, p. 107.

8. See Ex. 9, p. 107.

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19. See Ex. 20, p. 108.

20. See Ex. 21, p. 108.

21. See Ex. 22, p. 108.

22. See Ex. 23, p. 108.

23. See Ex. 24, p. 108.

24. Substituting the given values in

$$pd = WD,$$

we have $$50(5 - D) = 200\,D.$$

Solving, $$D = 1.$$

Hence, the weight is 1 foot from the fulcrum.

25–26. See Ex. 25–26, p. 108.

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27. See Ex. 35, p. 108.

28. See Ex. 36, p. 108.

29. See Ex. 37, p. 108.

30. See Ex. 38, p. 109.

31. See Ex. 39, p. 109.

32. See Ex. 40, p. 109.

33. See Ex. 41, p. 109.

SIMULTANEOUS SIMPLE EQUATIONS

Pages 179–187

As these pages are identical in the Standard Algebra and the Standard Algebra Revised, see pages 109–123 of the Key.

Page 188

1–3. See Ex. 1–3, pp. 123–124.

4. Let $x =$ one number,

and $y =$ the other number.

Then, $$x - y = 4, \quad (1)$$

and $$\tfrac{1}{4}(x + y) = 9. \quad (2)$$

Multiplying (2) by 4, $$x + y = 36. \quad (3)$$

Adding (1) and (3), $2x = 40$; $\therefore x = 20$, one number.

Substituting 20 for x in (1), $20 - y = 4$; $\therefore\ y = 16$, the other number.

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5. See Ex. 4, p. 124.

6. See Ex. 5, p. 124.

7. See Ex. 6, p. 124.

8. See Ex. 7, p. 124.

9. See Ex. 8, p. 124.

10. See Ex. 9, p. 125.

11. Let x = number of 2-dollar bills,
and y = number of 5-dollar bills.

Then, $x + y = 100$, (1)
and $2x + 5y = 275$. (2)
Multiplying (1) by 2, $2x + 2y = 200$. (3)
Subtracting (3) from (2), $3y = 75$; $\therefore\ y = 25$, the number of 5-dollar bills.

Substituting 25 for y in (1), $x + 25 = 100$; $\therefore\ x = 75$, the number of 2-dollar bills.

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12. Let x = the number of quarters,
and y = the number of half dollars.

Then, $x + y = 110$, (1)
and $\frac{1}{4}x + \frac{1}{2}y = 40$. (2)
Multiplying (2) by 2, $\frac{1}{2}x + y = 80$. (3)
Subtracting (3) from (1), $\frac{1}{2}x = 30$; $\therefore\ x = 60$, the number of quarters.

Substituting 60 for x in (1), $60 + y = 110$; $\therefore\ y = 50$, the number of half dollars.

13. Let x = the number of Lombard trees,
and y = the number of Gage trees.

Then, $x + y = 133$, (1)
and $x - \frac{5}{9}y = 7$. (2)
Subtracting (2) from (1), $1\frac{5}{9}y = 126$; $\therefore\ y = 81$, the number of Gage trees.

Substituting 81 for y in (1), $x + 81 = 133$; $\therefore\ x = 52$, the number of Lombard trees.

14. Let x = number of cents sugar cost per pound,
and y = number of cents coffee cost per pound.

Then, $5x + 8y = 270$, (1)
and $9x + 12y = 414$. (2)
Multiplying (1) by 3, $15x + 24y = 810$. (3)
Multiplying (2) by 2, $18x + 24y = 828$. (4)
Subtracting (3) from (4), $3x = 18$; $\therefore x = 6$.
Substituting 6 for x in (1), $30 + 8y = 270$; $\therefore\ y = 30$.
Hence, sugar costs 6¢ per pound, and coffee 30¢ per pound.

15. See Ex. 12, p. 125.

16. See Ex. 13, p. 125.

17. Let x = the number of months sea duty,
and y = the number of months shore duty.

Then, $x + y = 12$, (1)
and $150x + 127\frac{1}{2}y = 1620$. (2)
Multiplying (2) by 2, $300x + 255y = 3240$. (3)
Multiplying (1) by 300, $300x + 300y = 3600$. (4)
Subtracting (3) from (4), $45y = 360$; $\therefore y = 8$.
Hence, the lieutenant was 8 months on shore duty.

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18. Let $x =$ number of acres at \$60 per acre,
and $y =$ number of acres at \$20 per acre.
Then, $x + y = 80$, (1)
and $60x + 20y = 4500$. (2)
Multiplying (1) by 20, $20x + 20y = 1600$. (3)
Subtracting (3) from (2), $40x = 2900$; $\therefore x = 72\frac{1}{2}$.
Substituting $72\frac{1}{2}$ for x in (1), $72\frac{1}{2} + y = 80$; $\therefore y = 7\frac{1}{2}$.
Hence, he bought $72\frac{1}{2}$ acres at \$60 each, and $7\frac{1}{2}$ acres at \$20 each.

19. Let $x =$ number of bushels a basket holds,
and $y =$ number of bushels a crate holds.
Then, $8x + 4y = 8$, (1)
and $6x + 8y = 9\frac{3}{4}$. (2)
Multiplying (1) by 2, $16x + 8y = 16$. (3)
Subtracting (2) from (3), $10x = 6\frac{1}{4}$; $\therefore x = \frac{5}{8}$.
Substituting $\frac{5}{8}$ for x in (1), $5 + 4y = 8$; $\therefore y = \frac{3}{4}$.
Hence, the capacity of a basket is $\frac{5}{8}$ of a bushel, and of a crate $\frac{3}{4}$ of a bushel.

20. Let $x =$ number of Troy grains a 5-dollar gold piece weighs,
and $y =$ number of Troy grains a 10-dollar gold piece weighs.
Then, $x = \frac{1}{2}y$, (1)
and $3x + 2y = 903$. (2)
From (1), $y = 2x$. (3)
Substituting $2x$ for y in (2), $3x + 4x = 903$; $\therefore x = 129$.
Substituting 129 for x in (3), $y = 258$.
Hence, a 5-dollar gold piece weighs 129 Troy grains, and a 10-dollar gold piece, 258 Troy grains.

21. Let $x =$ number of pounds of Rio coffee,
and $y =$ number of pounds of Java coffee.
Then, $x + y = 120$, (1)
and $20x + 32y = 120 \cdot 28$. (2)
From (1), $x = 120 - y$. (3)
Substituting (3) in (2), $2400 - 20y + 32y = 3360$; $\therefore y = 80$.
Substituting 80 for y in (1), $x + 80 = 120$; $\therefore x = 40$.
Hence, 40 pounds of Rio coffee and 80 pounds of Java coffee are required for the blend.

22. Let $x =$ number of bushels of corn,
and $y =$ number of bushels of wheat.
Then, $x + y = a$, (1)
and $rx + sy = ab$. (2)
From (1), $x = a - y$. (3)
Substituting (3) in (2), $ar - ry + sy = ab$; $\therefore y = \dfrac{a(b-r)}{s-r}$.
Substituting $\dfrac{a(b-r)}{s-r}$ for y in (3), $x = a - \dfrac{a(b-r)}{s-r}$, or $\dfrac{a(s-b)}{s-r}$.
Hence, $\dfrac{a(s-b)}{s-r}$ bushels of corn and $\dfrac{a(b-r)}{s-r}$ bushels of wheat are mixed.

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23. See Ex. 18, p. 126.

24. See Ex. 19, p. 126.

25. Let $x =$ one digit,
and $y =$ the other.
Then, $x + y = 9$, (1)
and $x - y = 3$. (2)
Adding (1) and (2), $2x = 12$; $\therefore x = 6$.
Substituting 6 for x in (1), $6 + y = 9$; $\therefore y = 3$.
Hence, the number may be either 63 or 36.

26. See Ex. 20, p. 127.

27. Let $x =$ the tens' digit,
and $y =$ the units' digit;
whence, $10x + y =$ the number.
Then, $x + y = 12$, (1)
and $11x - (10x + y) = 2$. (2)
Simplifying (2), $x - y = 2$. (3)
Adding (1) and (3), $2x = 14$; $\therefore x = 7$.
Substituting 7 for x in (1), $7 + y = 12$; $\therefore y = 5$.
Hence, the number is 75.

28. See Ex. 22, p. 127.

29. See Ex. 23, p. 127.

30. See Ex. 24, pp. 127–128.

31. See Ex. 25, p. 128.

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32. Let $x =$ number of ounces of silver,
and $y =$ number of ounces of gold.
Then, $x + y = 12$, (1)
and $.095x + .051y = .788$. (2)
Multiplying (1) by .051,
$.051x + .051y = .612$. (3)
Subtracting (3) from (2), $.044x = .176$; $\therefore x = 4$.
Substituting 4 for x in (1), $4 + y = 12$; $\therefore y = 8$.
Hence, there are 4 ounces of silver and 8 ounces of gold in the piece.

33. Let $x =$ number of pounds of tin,
and $y =$ number of pounds of copper.
Then, $x + y = 18$, (1)
and $.137x + .112y = 2.316$. (2)
Multiplying (1) by .112,
$.112x + .112y = 2.016$. (3)
Subtracting (3) from (2), $.025x = .3$; $\therefore x = 12$.
Substituting 12 for x in (1), $12 + y = 18$; $\therefore y = 6$.
Hence, there were 12 pounds of tin and 6 pounds of copper in the piece.

34. Let $x =$ number of pounds of tin,
and $y =$ number of pounds of lead.
Then, $x + y = 14$, (1)
and $.137x + .089y = 1.594$. (2)
Multiplying (1) by .089,
$.089x + .089y = 1.246$. (3)
Subtracting (3) from (2), $.048x = .348$; $\therefore x = 7\frac{1}{4}$.
Substituting $7\frac{1}{4}$ for x in (1), $7\frac{1}{4} + y = 14$; $\therefore y = 6\frac{3}{4}$.
Hence, there were $7\frac{1}{4}$ pounds of tin and $6\frac{3}{4}$ pounds of lead in the piece.

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35. Let $x =$ capacity of first pump in gallons per minute,
and $y =$ capacity of second pump in gallons per minute.

Then, $5x + 3y = 2260,$ (1)

and $4x + 7y = 3280.$ (2)

Multiplying (1) by 4, $20x + 12y = 9040.$ (3)

Multiplying (2) by 5, $20x + 35y = 16{,}400.$ (4)

Subtracting (3) from (4), $23y = 7360;\ \therefore y = 320.$

Substituting 320 for y in (1),

$$5x + 960 = 2260;\ \therefore x = 260.$$

Hence, the capacity of the first pump per minute is 260 gallons, and of the second pump 320 gallons.

36. See Ex. 31, p. 129.

37. See Ex. 33, p. 130.

38. See Ex. 34, p. 131.

39. Let $x =$ number of days in which A can do the work.
and $y =$ number of days in which B can do the work.

Then, $$\frac{1}{x} + \frac{1}{y} = \frac{1}{a}, \quad (1)$$

and $$\frac{m}{x} + \frac{n}{a} = 1. \quad (2)$$

From (2), $$\frac{m}{x} = \frac{a-n}{a};\ \therefore x = \frac{am}{a-n}.$$

Multiplying (1) by m,

$$\frac{m}{x} + \frac{m}{y} = \frac{m}{a}. \quad (3)$$

Substituting $\frac{a-n}{a}$ for $\frac{m}{x}$ in (3),

$$\frac{a-n}{a} + \frac{m}{y} = \frac{m}{a};\ \therefore y = \frac{am}{m+n-a}.$$

Hence, A can do the work in $\frac{am}{a-n}$ days, and B in $\frac{am}{m+n-a}$ days.

40. See Ex. 35, p. 131.

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41. See Ex. 36, p. 131.

42. See Ex. 37, p. 132.

43. See Ex. 38, p. 132.

44. See Ex. 39, p. 132.

45. See Ex. 40, p. 133.

46. See Ex. 41, p. 133.

47. See Ex. 42, pp. 133–134.

48. See Ex. 43, p. 134.

Pages 194–196

2–34. See Ex. 2–34, pp. 135–141.

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1–3. See Ex. 1–3, pp. 141–142.

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4. Let $x =$ number of days it will take A,
$y =$ number of days it will take B,
and $z =$ number of days it will take C.

Then, $$\frac{1}{x}+\frac{1}{y}=\frac{1}{r}, \quad (1)$$

$$\frac{1}{x}+\frac{1}{z}=\frac{1}{s}, \quad (2)$$

and $$\frac{1}{y}+\frac{1}{z}=\frac{1}{t}. \quad (3)$$

Adding the given equations and dividing the result by 2,
$$\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=\frac{st+rt+rs}{2\,rst}. \quad (4)$$

Subtracting (3), (2), and (1) successively from (4), and solving,
$$x=\frac{2\,rst}{st+rt-rs},\quad y=\frac{2\,rst}{st-rt+rs},\quad \text{and}\quad z=\frac{2\,rst}{rs-st+rt}.$$

Hence, it will take A $\frac{2\,rst}{st+rt-rs}$ days, B $\frac{2\,rst}{st-rt+rs}$ days, and C $\frac{2\,rst}{rs-st+rt}$ days.

5. See Ex. 5, p. 142.

6. Let $x =$ the first number,
$y =$ the second number,
and $z =$ the third number.

Then, $$x+y+z=162, \quad (1)$$

$$\frac{y}{x}=2, \quad (2)$$

and $$\frac{z}{x}=3. \quad (3)$$

From (2), $y = 2x$. (4)
From (3), $z = 3x$. (5)
Substituting (4) and (5) in (1), $x + 2x + 3x = 162$; $\therefore\ x = 27$.
Substituting 27 for x in (4), $y = 54$.
Substituting 27 for x in (5), $z = 81$.
Hence, the three numbers are 27, 54, and 81.

7. See Ex. 6, p. 143.

8. See Ex. 7, p. 143.

9. From problem, $3a + 3b + 2c = 3230$, (1)
$2a + 2b + 3c = 2820$, (2)
and $3a + 2b + 2c = 2870$. (3)
Multiplying (1) by 3, $9a + 9b + 6c = 9690$. (4)
Multiplying (2) by 2, $4a + 4b + 6c = 5640$. (5)
Subtracting (5) from (4), $5a + 5b = 4050$;
whence, $a + b = 810$. (6)
Subtracting (3) from (1), $b = 360$. (7)
Substituting (7) in (6), $a = 450$. (8)
Substituting (7) and (8) in (1), $c = 400$.
Hence, the capacity of scow a is 450 cubic yards, of scow b 360 cubic yards, and of scow c 400 cubic yards.

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GRAPHIC SOLUTIONS

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16–24. See Ex. 16–24, pp. 144–146.

1–21. See Ex. 1–21, pp. 147–150.

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2–23. See Ex. 2–23, pp. 150–153.

INVOLUTION

Page 212

16–20. See Ex. 16–20, p. 154.

Page 213

28–31. See Ex. 28–31, p. 154.

32. $(3a + b^2)^4 = (3a)^4 + 4(3a)^3b^2 + 6(3a)^2b^4 + 4(3a)b^6 + b^8$
$= 81a^4 + 108a^3b^2 + 54a^2b^4 + 12ab^6 + b^8.$

33. See Ex. 32, p. 154.

34. $(x^2 + 5y)^3 = (x^2)^3 + 3(x^2)^2(5y) + 3(x^2)(5y)^2 + (5y)^3$
$= x^6 + 15x^4y + 75x^2y^2 + 125y^3.$

35. See Ex. 34, p. 154.

36. See Ex. 33, p. 154.

37. See Ex. 35, p. 154.

38. See Ex. 36, p. 154.

39. See Ex. 37, p. 154.

40. $(a^2x + 4)^5$
$= (a^2x)^5 + 5(a^2x)^4(4) + 10(a^2x)^3(4)^2 + 10(a^2x)^2(4)^3 + 5(a^2x)(4)^4 + (4)^5$
$= a^{10}x^5 + 20a^8x^4 + 160a^6x^3 + 640a^4x^2 + 1280a^2x + 1024.$

41. See Ex. 38, p. 154.

42. See Ex. 39, p. 154.

43. $(\frac{1}{3} + x)^3 = (\frac{1}{3})^3 + 3(\frac{1}{3})^2x + 3(\frac{1}{3})x^2 + x^3 = \frac{1}{27} + \frac{1}{3}x + x^2 + x^3.$

44. See Ex. 40, p. 154.

45. See Ex. 41, p. 154.

46. See Ex. 42, p. 155.

47. See Ex. 43, p. 155.

48. $(\frac{3}{4} - 2x)^4 = (\frac{3}{4})^4 - 4(\frac{3}{4})^3(2x) + 6(\frac{3}{4})^2(2x)^2 - 4(\frac{3}{4})(2x)^3 + (2x)^4$
$= \frac{81}{256} - \frac{27x}{8} + \frac{27x^2}{2} - 24x^3 + 16x^4.$

49. See Ex. 44, p. 155.

50. See Ex. 45, p. 155.

51. $\left(\frac{1}{a} - a\right)^5 = \left(\frac{1}{a}\right)^5 - 5\left(\frac{1}{a}\right)^4 a + 10\left(\frac{1}{a}\right)^3 a^2 - 10\left(\frac{1}{a}\right)^2 a^3 + 5\left(\frac{1}{a}\right)a^4 - a^5$
$= \frac{1}{a^5} - \frac{5}{a^3} + \frac{10}{a} - 10a + 5a^3 - a^5.$

52. $\left(x + \frac{1}{x}\right)^6$
$= x^6 + 6x^5\left(\frac{1}{x}\right) + 15x^4\left(\frac{1}{x}\right)^2 + 20x^3\left(\frac{1}{x}\right)^3 + 15x^2\left(\frac{1}{x}\right)^4 + 6x\left(\frac{1}{x}\right)^5 + \left(\frac{1}{x}\right)^6$
$= x^6 + 6x^4 + 15x^2 + 20 + \frac{15}{x^2} + \frac{6}{x^4} + \frac{1}{x^6}.$

53. See Ex. 46, p. 155.

54. $\left(2a^2 - \frac{b^3}{2}\right)^3 = (2a^2)^3 - 3(2a^2)^2\left(\frac{b^3}{2}\right) + 3(2a^2)\left(\frac{b^3}{2}\right)^2 - \left(\frac{b^3}{2}\right)^3$
$= 8a^6 - 6a^4b^3 + \frac{3a^2b^6}{2} - \frac{b^9}{8}.$

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56. See Ex. 50, p. 155.

57. See Ex. 53, p. 156.

58. See Ex. 54, p. 156.

59. See Ex. 55, p. 156.

60. See Ex. 57, p. 156.

61. See Ex. 58, p. 156.

62. See Ex. 59, p. 156.

EVOLUTION

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10–35. See Ex. 10–35, pp. 157–162.

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37–42. See Ex. 37–42, pp. 162–163.

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6–11. See Ex. 6–11, pp. 163–164.

Page 224

12–14. See Ex. 12–14, p. 164.

23–30. See Ex. 23–30, pp. 164–166.

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3.

$$\begin{array}{l|l}
 & b^3 + 6\,b^2 + 12\,b + 8 \;\underline{|\,b + 2} \\
 & b^3 \\ \hline
3\,b^2 & + 6\,b^2 + 12\,b + 8 \\
3\,b^2 + 6\,b + 4 & + 6\,b^2 + 12\,b + 8 \\ \hline
\end{array}$$

4. See Ex. 3, p. 166.

5. See Ex. 4, p. 166.

6.

$$\begin{array}{l|l}
 & a^3x^6 + 12\,a^2x^4 + 48\,ax^2 + 64 \;\underline{|\,ax^2 + 4} \\
 & a^3x^6 \\ \hline
3\,a^2x^4 & + 12\,a^2x^4 + 48\,ax^2 + 64 \\
3\,a^2x^4 + 12\,ax^2 + 16 & + 12\,a^2x^4 + 48\,ax^2 + 64 \\ \hline
\end{array}$$

7. See Ex. 5, p. 166.

8. See Ex. 6, p. 166.

9. See Ex. 8, p. 166.

10. See Ex. 7, p. 166.

11.

$$\begin{array}{l|l}
 & 64\,a^3b^3 - 240\,a^2b^2c + 300\,abc^2 - 125\,c^3 \;\underline{|\,4\,ab - 5\,c} \\
 & 64\,a^3b^3 \\ \hline
48\,a^2b^2 & - 240\,a^2b^2c + 300\,abc^2 - 125\,c^3 \\
48\,a^2b^2 - 60\,abc + 25\,c^2 & - 240\,a^2b^2c + 300\,abc^2 - 125\,c^3 \\ \hline
\end{array}$$

12. See Ex. 10, p. 167.

13. See Ex. 11, p. 167.

14. See Ex. 9, p. 166.

15.

$$\begin{array}{l|l}
 & y^6 + 3\,y^5 + 12\,y^4 + 19\,y^3 + 36\,y^2 + 27\,y + 27 \;\underline{|\,y^2 + y + 3} \\
 & y^6 \\ \hline
3\,y^4 & + 3\,y^5 + 12\,y^4 + 19\,y^3 \\
3\,y^4 + 3\,y^3 + y^2 & + 3\,y^5 + \;\;3\,y^4 + \;\;\;\;y^3 \\ \hline
3\,y^4 + 6\,y^3 + 3\,y^2 & + 9\,y^4 + 18\,y^3 + 36\,y^2 + 27\,y + 27 \\
3\,y^4 + 6\,y^3 + 12\,y^2 + 9\,y + 9 & + 9\,y^4 + 18\,y^3 + 36\,y^2 + 27\,y + 27 \\ \hline
\end{array}$$

16. See Ex. 12, p. 167.

17. See Ex. 13, p. 167.

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18.

$$
\begin{array}{l|l}
 & a^6+9a^5+21a^4-9a^3-42a^2+36a-8 \;\underline{|\, a^2+3a-2} \\
 & a^6 \\
3a^4 & +9a^5+21a^4-9a^3 \\
3a^4+9a^3+9a^2 & +9a^5+27a^4+27a^3 \\
\hline
3a^4+18a^3+27a^2 & -6a^4-36a^3-42a^2+36a-8 \\
3a^4+18a^3+21a^2-18a+4 & -6a^4-36a^3-42a^2+36a-8 \\
\hline
\end{array}
$$

19.

$$
\begin{array}{l|l}
 & b^9+3b^8+6b^7+7b^6+6b^5+3b^4+b^3 \;\underline{|\, b^3+b^2+b} \\
 & b^9 \\
3b^6 & +3b^8+6b^7+7b^6 \\
3b^6+3b^5+b^4 & +3b^8+3b^7+b^6 \\
\hline
3b^6+6b^5+3b^4 & +3b^7+6b^6+6b^5+3b^4+b^3 \\
3b^6+6b^5+6b^4+3b^3+b^2 & +3b^7+6b^6+6b^5+3b^4+b^3 \\
\hline
\end{array}
$$

20.

$$
\begin{array}{l|l}
 & 8c^6-60c^5+198c^4-365c^3+396c^2-240c+64 \;\underline{|\, 2c^2-5c+4} \\
 & 8c^6 \\
12c^4 & -60c^5+198c^4-365c^3 \\
12c^4-30c^3+25c^2 & -60c^5+150c^4-125c^3 \\
\hline
12c^4-60c^3+75c^2 & +48c^4-240c^3+396c^2-240c+64 \\
12c^4-60c^3+99c^2-60c+16 & +48c^4-240c^3+396c^2-240c+64 \\
\hline
\end{array}
$$

21. See Ex. 14, p. 167. **22.** See Ex. 15, p. 167.

23.

$$
\begin{array}{l|l}
 & x^6-3x^5y+6x^4y^2-7x^3y^3+6x^2y^4-3xy^5+y^6 \;\underline{|\, x^2-xy+y^2} \\
 & x^6 \\
3x^4 & -3x^5y+6x^4y^2-7x^3y^3 \\
3x^4-3x^3y+x^2y^2 & -3x^5y+3x^4y^2-x^3y^3 \\
\hline
3x^4-6x^3y+3x^2y^2 & +3x^4y^2-6x^3y^3+6x^2y^4-3xy^5+y^6 \\
3x^4-6x^3y+6x^2y^2-3xy^3+y^4 & +3x^4y^2-6x^3y^3+6x^2y^4-3xy^5+y^6 \\
\hline
\end{array}
$$

24. See Ex. 21, p. 169. **25.** See Ex. 16, p. 167.

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26. See Ex. 17, p. 168.
27. See Ex. 18, p. 168.
28. See Ex. 19, p. 168.
29. See Ex. 26, p. 170.
30. See Ex. 27, p. 170.
31. See Ex. 28, p. 170.

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3–28. See Ex. 3–28, pp. 170–175.

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1–22. See Ex. 1–22, pp. 175–178.

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THEORY OF EXPONENTS

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2–13. See Ex. 2–13, pp. 178–179.

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36–37. See Ex. 36–37, p. 179.

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16–23. See Ex. 16–23, p. 179.

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34–68. See Ex. 34–68, pp. 179–181.

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69–72. See Ex. 69–72, p. 182.

74–99. See Ex. 74–99, pp. 182–183.

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100. $\left(\frac{2^{-2}}{2^{-3}}\right)^2 = 2^2 = 4.$

101. $\left(\frac{2^{\frac{1}{3}}}{4^{\frac{1}{4}}}\right)^{-12} = \left(\frac{2^{\frac{1}{3}}}{2^{\frac{1}{2}}}\right)^{-12} = (2^{-\frac{1}{6}})^{-12} = 2^2 = 4.$

102. See Ex. 100, p. 183.

103. See Ex. 101, p. 183.

104. $\left(\frac{27^{-\frac{1}{3}}}{a}\right)^{-1} = \frac{27^{\frac{1}{3}}}{a^{-1}} = 3a.$

105. $\left(\frac{a^4b^{-3}}{a^3b^{-2}}\right)^2 = \left(\frac{a}{b}\right)^2 = \frac{a^2}{b^2}.$

106. See Ex. 102, p. 184.

107. See Ex. 103, p. 184.

108. $\left(\frac{x^{-3} \div x^{-5}}{4x^{-4}}\right)^{-\frac{1}{2}} = \left(\frac{x^6}{4}\right)^{-\frac{1}{2}} = \left(\frac{4}{x^6}\right)^{\frac{1}{2}} = \frac{2}{x^3}.$

109. See Ex. 104, p. 184.

110. See Ex. 105, p. 184.

111. $\frac{6x^{-2} \div 3x^{-\frac{1}{2}}}{2x^{-3} \times \sqrt{x}} = \frac{2x^{-\frac{3}{2}}}{2x^{-\frac{5}{2}}} = x.$

112. $\frac{2^{-1} \times 2^{-2}}{4^{-2} \times 4^{-3}} = \frac{2^{-3}}{4^{-5}} = \frac{2^{-3}}{2^{-10}} = 2^7 = 128.$

113. $\frac{32^{\frac{3}{5}} + 125^{\frac{2}{3}}}{81^{\frac{3}{4}} + 216^{\frac{1}{3}}} = \frac{8 + 25}{27 + 6} = \frac{33}{33} = 1.$

114. See Ex. 106, p. 184.

115. See Ex. 107, p. 184.

116. $\frac{x^2 - y^2}{(x^{\frac{1}{2}} - y^{\frac{1}{2}})(x^{\frac{1}{2}} + y^{\frac{1}{2}})} = \frac{x^2 - y^2}{x - y} = x + y.$

117. $\frac{a^{-\frac{1}{2}} \times a^{-\frac{5}{2}} \times a^3}{(a + b)^{-2}} = \frac{a^0}{(a + b)^{-2}} = \frac{1}{(a + b)^{-2}} = (a + b)^2.$

118. See Ex. 108, p. 184.
119. See Ex. 109, p. 184.
120. See Ex. 110, p. 184.
121. See Ex. 111, p. 184.
122. See Ex. 112, p. 184.
123. See Ex. 113, p. 185.
124. See Ex. 114, p. 185.

RADICALS

Pages 243–267

As these pages are identical in the Standard Algebra and the Standard Algebra Revised, see pages 186–217 of the Key.

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IMAGINARY NUMBERS

Pages 269–271

As these pages are identical in the Standard Algebra and the Standard Algebra Revised, see pages 217–219 of the Key.

REVIEW

Page 272

6. See Ex. 6, p. 220.

Page 273

16. See Ex. 16, p. 220.
27. See Ex. 27, p. 220.
30. See Ex. 30, p. 220.
31. See Ex. 31, p. 220.

Page 274

1–19. See Ex. 1–19, pp. 221–223.

Page 275

20–52. See Ex. 20–52, pp. 223–226.

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53. See Ex. 53, p. 226.

54.

$$\begin{array}{lr|l} & & 405'224 \;\underline{|74} \\ & & 343 \\ 70^2 \times 3 & = 14700 & 62224 \\ 70 \times 4 \times 3 = & 840 & \\ 4^2 & = 16 & \\ \hline & 15556 & 62224 \end{array}$$

55. See Ex. 54, p. 226.
56–82. See Ex. 56–82, pp. 227–230.

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83–84. See Ex. 83–84, p. 230.

85. $$\frac{7-2x}{10}-\frac{7x-4}{15}=\frac{1-6x}{30}.$$

Multiplying by 30, $$21-6x-14x+8=1-6x.$$
$$-14x=-28.$$
$$\therefore x=2.$$

86. See Ex. 85, p. 231.

87. See Ex. 87, p. 231.

88. $$\begin{cases} \dfrac{3x-5y}{3}-\dfrac{4x-3y}{12}=\dfrac{7}{12}, & (1) \\ \frac{1}{2}(x+y+3)-\frac{2}{3}(x+y)=0. & (2) \end{cases}$$

Reducing (1), $$8x-17y=7. \quad (3)$$
Reducing (2), $$x+y=9. \quad (4)$$
From (4), $$x=9-y. \quad (5)$$
Substituting (5) in (3), $$72-8y-17y=7;\ \therefore y=2\tfrac{3}{5}.$$
Substituting $2\frac{3}{5}$ for y in (5), $$x=9-2\tfrac{3}{5};\ \therefore x=6\tfrac{2}{5}.$$

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89. See Ex. 89, p. 231.

90. $\dfrac{16^{\frac{3}{4}} \times 81^{\frac{1}{4}}}{25^{\frac{1}{2}} \times 64^{\frac{5}{6}}} \div 125^{-\frac{2}{3}} = \dfrac{8 \times 3}{5 \times 32} \times 25 = \dfrac{15}{4} = 3\tfrac{3}{4}.$

91. See Ex. 91, p. 232.

92. $5^0 - \sqrt[5]{-32} + \sqrt[4]{256} - 8^{-\frac{2}{3}} = 1 - (-2) + 4 - \frac{1}{4} = 6\frac{3}{4}.$

93. See Ex. 92, p. 232.

94. See Ex. 93, p. 232.

95. See Ex. 94, p. 232.

96. See Ex. 95, p. 232.

97. See Ex. 90, p. 231.

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99. See Ex. 98, p. 232.

100. See Ex. 99, p. 232.

101. See Ex. 100, p. 233.

102. See Ex. 101, p. 233.

103. See Ex. 102, p. 233.

104. See Ex. 103, p. 233.

105. See Ex. 104, p. 233.

106. See Ex. 105, p. 233.

107. See Ex. 106, p. 233.

108. See Ex. 107, p. 233.

109. See Ex. 108, p. 233.

110. See Ex. 109, p. 233.

111. See Ex. 110, p. 234.

112. See Ex. 111, p 234.

113. See Ex. 112, p. 234.

114. See Ex. 113, p. 234.

115. See Ex. 114, p. 235.

QUADRATIC EQUATIONS

Page 281

3–31. See Ex. 3–31, pp. 235–239.

Page 282

2–5. See Ex. 2–5, p. 239.

6. Let $x =$ the number.
Then, $\frac{1}{4}x \cdot x = 16.$
Solving, $x = \pm 8.$
Hence, the number is either $+8$ or -8.

7–8. See Ex. 7–8, p. 240.

9. See Ex. 10, p. 240.

10. See Ex. 11, p. 240.

11. See Ex. 12, p. 240.

12. See Ex. 13, p. 241.

13. See Ex. 14, p. 241.

14. Let $x + 2 =$ one number,
and $x - 2 =$ the other number.
Then, $(x+2)^2 + (x-2)^2 = 208.$
Solving, $x = \pm 10,$
$x + 2 = 12$ or $-8,$
and $x - 2 = 8$ or $-12.$
Hence, the numbers are 12 and 8, or -8 and -12.

Page 283

15. See Ex. 15, p. 241.

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16. Let $x =$ number of inches in width.

Then, $1\frac{1}{3}x =$ number of inches in length.

$\therefore 1\frac{1}{3}x^2 = 48.$

Solving, $x = \pm 6,$

and $1\frac{1}{3}x = \pm 8.$

Hence, rejecting negative values, the length of the sheet of mica is 8 inches, and its width is 6 inches.

17. Let $\frac{15}{2} + x =$ number of feet in length.

Then, $\frac{15}{2} - x =$ number of feet in width.

$\therefore (\frac{15}{2} + x)(\frac{15}{2} - x) = 54.$

Solving, $x = \pm \frac{3}{2},$

$\frac{15}{2} + x = 9$ or $6,$

and $\frac{15}{2} - x = 6$ or $9.$

Hence, the rug is 9 feet long and 6 feet wide.

18. Let $x + 7 =$ number of inches in length.

Then, $x - 7 =$ number of inches in width.

$\therefore (x + 7)(x - 7) = 912.$

Solving, $x = \pm 31,$

and $x + 7 = 38$ or $-24.$

Hence, the length of the sheet of paper is 38 inches.

19. See Ex. 16, p. 241.

20. See Ex. 17, p. 241.

21. See Ex. 19, p. 242.

22. See Ex. 18, p. 241.

1–6. See Ex. 1–6, pp. 242–243.

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7–19. See Ex. 7–19, pp. 243–244.

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12–20. See Ex. 12–20, pp. 244–245.

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21–25. See Ex. 21–25, pp. 245–246.

Page 289

2–22. See Ex. 2–22, pp. 246–249.

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23–26. See Ex. 23–26, p. 249.

2–9. See Ex. 2–9, p. 250.

10. Writing the equation $5x^2 - 18x = 72$, in the general form, we have $5x^2 - 18x - 72 = 0.$

$$\therefore x = \frac{18 \pm \sqrt{324 - 4 \cdot 5(-72)}}{2 \cdot 5} = \frac{18 \pm 42}{10} = 6 \text{ or } -\tfrac{12}{5}.$$

11. See Ex. 10, p. 250.

12. See Ex. 11, p. 250.

13. See Ex. 12, p. 250.

14. See Ex. 13, p. 250.

15. See Ex. 14, p. 250.

16. See Ex. 15, p. 250.

17. See Ex. 16, p. 250.

18. See Ex. 17, p. 250.

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19. Writing the equation $x(3x + 4) = -2$ in the general form, we have $3x^2 + 4x + 2 = 0$.

$$\therefore x = \frac{-4 \pm \sqrt{16 - 4 \cdot 3 \cdot 2}}{2 \cdot 3} = \frac{-4 \pm 2\sqrt{-2}}{6} = \tfrac{1}{3}(-2 \pm \sqrt{-2}).$$

Page 292

3–34. See Ex. 3–34, pp. 251–254.

Page 293

1–22. See Ex. 1–22, pp. 254–256.

Page 294

23–39. See Ex. 23–39, pp. 256–259.

Page 296

3–8. See Ex. 3–8, pp. 259–261.

Page 297

9–28. See Ex. 9–28, pp. 261–269.

Page 298

2–7. See Ex. 2–7, pp. 269–270.

8. Let x = number of inches in width.

Then, $x + 23$ = number of inches in length.

$$\therefore x(x + 23) = 2838.$$

Solving, $x = 43$ or -66,

and $x + 23 = 66$ or -43.

Hence, rejecting negative values, the tablet is 66 inches long and 43 inches wide.

9. Let x = number of cents paid per cake.

Then, $x - 4$ = number of cakes used.

$$\therefore x(x - 4) = 480.$$

Solving, $x = 24$ or -20,

and $x - 4 = 20$ or -24.

Hence, 20 cakes of ice were used.

10. Let x = number of feet in height of box.

Then, $x + 5$ = number of feet in length of box,

and $x - \frac{1}{6}$ = number of feet in width of box.

$$\therefore (x + 5)(x - \tfrac{1}{6}) = 8\tfrac{2}{3}.$$

Solving, $x = 1\frac{1}{2}$ or $-6\frac{1}{3}$,

$x + 5 = 6\frac{1}{2}$ or $-1\frac{1}{3}$,

and $x - \frac{1}{6} = 1\frac{1}{3}$ or $-6\frac{1}{2}$.

Hence, rejecting the negative values, the box is $6\frac{1}{2}$ feet long, $1\frac{1}{3}$ feet wide, and $1\frac{1}{2}$ feet high.

Page 299

12. See Ex. 11, p. 270. **13.** See Ex. 12, p. 270.

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14. Let $x =$ number of feet in width.
Then, $x =$ number of feet in height,
and $35 - 2x =$ number of feet in length.

$$\therefore x(35 - 2x) - x^2 = 50.$$

Solving, $x = 10$ or $1\frac{2}{3}$,
and $35 - 2x = 15$ or $31\frac{2}{3}$.

Hence, the block is 15 feet long, 10 feet wide, and 10 feet high, or it is $31\frac{2}{3}$ feet long, $1\frac{2}{3}$ feet wide, and $1\frac{2}{3}$ feet high.

15. Let $x =$ number of inches in length.
Then, $x =$ number of inches in width,
and $58 - 2x =$ number of inches in height.

$$\therefore x^2 - x(58 - 2x) = 176.$$

Solving, $x = 22$ or $-\frac{8}{3}$,
and $58 - 2x = 14$ or $63\frac{1}{3}$.

The second values are inadmissible, since the length could not be a negative number of inches. Hence, the height of the box is 14 inches.

16. Let $x =$ number of feet in width.
Then, $x + \frac{1}{3} =$ number of feet in thickness.

$$\therefore 4\tfrac{1}{2}\, x\left(x + \tfrac{1}{3}\right) = 21.$$

Solving, $x = 2$ or $-\frac{7}{3}$,
and $x + \frac{1}{3} = 2\frac{1}{3}$ or -2.

Hence, rejecting negative values, the width of the bale of cotton is 2 feet and its thickness is $2\frac{1}{3}$ feet.

17. Let $x =$ number of tons of raisins sold.
Then, $\dfrac{480}{x} =$ number of dollars received per ton.

$$\therefore (x + 2)\left(\frac{480}{x} - 20\right) = 480.$$

Solving, $x = 6$ or -8.
Hence, he sold 6 tons of raisins.

Page 300

18. See Ex. 13, p. 271.

19. See Ex. 14, p. 271.

20. See Ex. 16, p. 271.

21. See Ex. 17, pp. 271–272.

22. See Ex. 18, p. 272.

23. See Ex. 19, p. 272.

24. See Ex. 20, p. 272.

25. Let $x =$ number of knots traveled per hour.
Then, $\dfrac{150}{x} =$ number of hours required to travel 150 knots.

$$\therefore \frac{150}{x} - \frac{150}{x + 1\frac{1}{4}} = \frac{1}{2}.$$

Solving, $x = 18\frac{3}{4}$ or -20.
Hence, the rate of the sailing vessel is $18\frac{3}{4}$ knots per hour.

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26. Let $x =$ number of miles the man rode per hour.

Then, $\frac{90}{x} =$ number of hours the journey lasted.

$$\therefore \frac{90}{x} - \frac{90}{x + \frac{2}{11}} = \frac{1}{6}.$$

Solving, $x = 9\frac{9}{11}$ or -10,

and $\frac{90}{x} = 9\frac{1}{6}$ or -9.

Hence, rejecting the negative value, the journey lasted $9\frac{1}{6}$ hours.

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28. See Ex. 22, p. 272.

29. Let $x =$ number of hours it takes larger pipe.

Then, $x + 1\frac{1}{2} =$ number of hours it takes smaller pipe.

$$\therefore \frac{1}{x} + \frac{1}{x + 1\frac{1}{2}} = \frac{1}{3\frac{1}{3}}.$$

Solving, $x = 6$ or $-\frac{5}{6}$,

and $x + 1\frac{1}{2} = 7\frac{1}{2}$ or $\frac{2}{3}$.

The second value is inadmissible, since it could not take a negative number of hours.

Hence, the larger pipe takes 6 hours, and the smaller pipe, $7\frac{1}{2}$ hours.

30. See Ex. 23, pp. 272–273.

31. See Ex. 25, p. 273.

32. See Ex. 26, p. 273.

33. See Ex. 27, p. 273.

34. See Ex. 28, p. 274.

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1–6. See Ex. 1–6, pp. 274–276.

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3–18. See Ex. 3–18, pp. 276–277.

20–25. See Ex. 20–25, p. 278.

Page 305

27–28. See Ex. 27–28, p. 278.

Page 306

30–33. See Ex. 30–33, p. 279.

Page 307

35–56. See Ex. 35–56, pp. 279–283.

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57–78. See Ex. 57–78, pp. 283–289.

Page 309

2–5. See Ex. 2–5, pp. 289–290.

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6–9. See Ex. 6–9, p. 290.

2. See Ex. 2, p. 290.

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4–13. See Ex. 4–13, pp. 291–293.

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14–15. See Ex. 14–15, pp. 293–294.

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2–9. See Ex. 2–9, pp. 294–296.

Page 314

2–3. See Ex. 2–3, p. 297.

Page 315

4–9. See Ex. 4–9, pp. 297–299.

Page 320

10–19. See Ex. 10–19, pp. 299–302. **1–8.** See Ex. 1–8, pp. 302–303.

Page 321

9–34. See Ex. 9–34, pp. 303–310.

Page 322

35–50. See Ex. 35–50, pp. 310–315.

Page 623

1. See Ex. 1, p. 315.

2. Let $x =$ one number,
and $y =$ the other number.
Then, $x - y = 4,$
and $xy = 77.$
Solving, $x = 11$ or $-7,$
and $y = 7$ or $-11.$
Hence, the numbers are 11 and 7, or -7 and -11.

3. Let $x =$ one number,
and $y =$ the other number.
Then, $xy = 108,$
and $\frac{x}{y} = 1\frac{1}{3}.$
Solving, $x = \pm 12,$
and $y = \pm 9.$
Hence, the numbers are 12 and 9, or -12 and -9.

4. See Ex. 2, p. 315. **6.** See Ex. 4, p. 316.

5. See Ex. 3, p. 316.

7. Let $x =$ number of inches in length,
and $y =$ number of inches in width.
Then, $2x + 2y = 20,$
and $xy = 24.$
Solving, $x = 6$ or $4,$
and $y = 4$ or $6.$
Hence, the rectangle is 6 inches long and 4 inches wide.

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8. See Ex. 5, p. 316.

9. Let $x =$ number of rods in length,
and $y =$ number of rods in width.
Then, $x - y = 4$,
and $xy = 3 \cdot 160$.
Solving, $x = 24$ or -20,
and $y = 20$ or -24.
Hence, the field is 24 rods long and 20 rods wide.

10. Let $x =$ number of feet in length,
and $y =$ number of feet in width.
Then, $xy = 35$,
and $y - 1 = \frac{1}{2}(x + 1)$.
Solving, $x = 7$ or -10,
and $y = 5$ or $-\frac{7}{2}$.
Hence, the blanket is 7 feet long and 5 feet wide.

11. Let $x =$ the larger number,
and $y =$ the smaller number.
Then, $xy = 10x - 18$, (1)
and $xy = 10y - 8$. (2)
Subtracting (2) from (1), $10x - 10y - 10 = 0$. (3)
From (3), $x = y + 1$. (4)
Substituting (4) in (1), $y^2 - 9y + 8 = 0$. (5)
Solving, $y = 8$ or 1. (6)
Substituting (6) in (4), $x = 9$ or 2.
Hence, the numbers are 9 and 8, or 2 and 1.

12. Let $x =$ the tens' digit,
and $y =$ the units' digit.
Then, $y(10x + y) = 24$, (1)
and $10x + y + x + y = 15$. (2)
From (2), $x = \dfrac{15 - 2y}{11}$. (3)
Substituting (3) in (1), $9y^2 - 150y + 264 = 0$.
Solving, $y = \frac{44}{3}$ or 2.
Rejecting the first value, from (3), $x = 1$.
Hence, the number is 12.

13. See Ex. 7, p. 316.

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14. Let $x =$ one number,
and $y =$ the other number.
Then, $x - y = 2a$,
and $xy = b$.
Solving, $x = a \pm \sqrt{a^2 + b}$,
and $y = -a \pm \sqrt{a^2 + b}$.
Hence, the numbers are $a + \sqrt{a^2 + b}$ and $-a + \sqrt{a^2 + b}$, or $a - \sqrt{a^2 + b}$ and $-a - \sqrt{a^2 + b}$.

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15. Let $x =$ number of inches in length,

and $y =$ number of inches in width.

Then, $xy = 882,$

and $x - 6 = y + 5\frac{1}{2}.$

Solving, $x = 36 \text{ or } -24\frac{1}{2},$

and $y = 24\frac{1}{2} \text{ or } -36.$

Hence, the door mat is 36 inches long and $24\frac{1}{2}$ inches wide.

16. See Ex. 15, p. 318.

17. Let $x =$ number of yards of ribbon bought,

and $y =$ number of cents paid per yard.

Then, $xy = 75,$ (1)

and $(x + 2)(y - 10) = 75.$ (2)

Subtracting (2) from (1),

$10x - 2y + 20 = 0;$ (3)

whence, $y = 5x + 10.$ (4)

Substituting (4) in (1),

$5x^2 + 10x = 75;$ (5)

whence, $x = 3 \text{ or } -5.$ (6)

Substituting (6) in (1), $y = 25 \text{ or } -15.$

Hence, I bought 3 yards of ribbon at 25 cents per yard.

18. See Ex. 8, p. 316.

19. See Ex. 9, p. 317.

20. See Ex. 10, p. 317.

21. Let $x =$ number of bunches of carrots,

and $y =$ number of cents paid per bunch.

Then, $xy = 440,$ (1)

and $(x - 4)(y + 1) = 440.$ (2)

Subtracting (2) from (1),

$4y - x + 4 = 0;$ (3)

whence, $4y = x - 4.$ (4)

From (1), $x \cdot 4y = 1760.$ (5)

Substituting (4) in (5), $x^2 - 4x = 1760;$

whence, $x = 44 \text{ or } -40.$ (6)

Substituting (6) in (1), $y = 10 \text{ or } -11.$

Hence, the price of the carrots was 10 cents per bunch.

22. See Ex. 11, p. 317.

23. Let $x =$ number of tons loaded per hour,

and $y =$ number of hours required.

Then, $xy = 2000,$ (1)

and $(x + 50)(y - 1\frac{1}{3}) = 2000.$ (2)

Subtracting (2) from (1), etc.,

$2x - 75y + 100 = 0;$ (3)

whence, $2x = 75y - 100.$ (4)

From (1), $2x \cdot y = 4000.$ (5)

Substituting (4) in (5),

$75y^2 - 100y = 4000;$

whence, $y = 8 \text{ or } -\frac{20}{3}.$

Hence, it took 8 hours to load the coal.

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24. See Ex. 13, p. 318.

25. See Ex. 12, p. 317.

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26. Let $x =$ number of feet in width of rug,
and $y =$ number of feet in width of border.

Then, $(x + 2y)(x + 2y + 3) = 108,$ (1)

and $x(x + 3) = 54.$ (2)

From (2), $x = 6$ or $-9.$ (3)

Rejecting the negative value and substituting (3) in (1),

$$(6 + 2y)(9 + 2y) = 108.$$

Solving, $y = 1\frac{1}{2}$ or $-9.$

Hence, the width of the border is $1\frac{1}{2}$ feet.

27. See Ex. 16, p. 318.

28. See Ex. 19, p. 319.

29. See Ex. 18, p. 319.

30. Let $x =$ number of dollars in the principal,
and $y =$ number of per cent in the rate.

Then, $\frac{xy}{100} = 60,$ (1)

and $(x - 500)\left(\frac{y + 1}{100}\right) = 60.$ (2)

Subtracting (1) from (2),

$$x - 500y - 500 = 0; \quad (3)$$

whence, $x = 500y + 500.$ (4)

Substituting (4) in (1),

$$\frac{y}{100}(500y + 500) = 60.$$

Solving, $y = 3$ or $-4.$ (5)

Substituting (5) in (1), $x = 2000$ or $-1500.$

Hence, the principal was $2000 and the rate 3 per cent.

31. See Ex. 22, p. 320.

32. See Ex. 20, pp. 319–320.

GRAPHIC SOLUTIONS

Pages 329–338

As these pages are identical in the Standard Algebra and the Standard Algebra Revised, see pages 322–337 of the Key.

PROPERTIES OF QUADRATIC EQUATIONS

Pages 340–349

As these pages are identical in the Standard Algebra and the Standard Algebra Revised, see pages 338–353 of the Key.

GENERAL REVIEW

Page 353

1–6. See Ex. 1–6, p. 354.

8–14. See Ex. 8–14, pp. 354–355.

16. See Ex. 16, p. 355.

19. See Ex. 19, p. 356.

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20–24. See Ex. 20-24, p. 356.

26–36. See Ex. 26–36, pp. 356–357.

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Page 355

37–59. See Ex. 37–59, pp. 358–362.

Page 356

1. Let $x =$ one number.
Then, $72 - x =$ the other number.

$$\therefore \frac{72 - x}{x} = 8.$$

Solving, $x = 8,$
and $72 - x = 64.$
Hence, the numbers are 64 and 8.

2. See Ex. 3, p. 363.

3. Let $x =$ number of cents paid per dozen.

Then, $2x = 96 \div \frac{x}{12}.$

Solving, $x = \pm 24.$
Hence, the price was 24 cents per dozen.

4. See Ex. 1, p. 363.

5. See Ex. 4, p. 363.

6. See Ex. 5, p. 363.

7. See Ex. 12, p. 365.

8. Let $x =$ the larger number,
and $y =$ the smaller number.

Then, $\frac{x-4}{y} = 7,$ (1)

and $\frac{y}{x} = \frac{2}{15}.$ (2)

From (2), $x = \frac{15}{2}y.$ (3)
Substituting (3) in (1), etc.,

$$\tfrac{15}{2}y - 7y = 4; \therefore y = 8.$$

Substituting 8 for y in (3), $x = 60.$
Hence, the two numbers are 60 and 8.

9. Let $x =$ the larger number,
and $y =$ the smaller number.

Then, $\frac{x-s}{y} = r,$ (1)

and $\frac{y}{x} = t.$ (2)

From (2), $y = tx.$ (3)

Substituting (3) in (1), $\frac{x-s}{tx} = r.$

Solving, $x = \frac{s}{1-rt}.$ (4)

Substituting (4) in (3), $y = \frac{st}{1-rt}.$

Hence, the numbers are $\frac{s}{1-rt}$ and $\frac{st}{1-rt}.$

10. See Ex. 7, p. 364.

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11. Let x = number of revolutions made by first machine per minute,
and y = number of revolutions made by second machine per minute.

Then, $x - y = 60$,
and $5x = 8y$.

Solving, $x = 160$, and $y = 100$.

Hence, the rate of the first machine is 160 revolutions per minute, and of the second machine, 100 revolutions per minute.

12. Let x = the numerator of the fraction,
and y = the denominator.

Then, $$\frac{x}{y} = \frac{7}{8},$$

and $$\frac{x-4}{y+4} = \frac{3}{4}.$$

Solving, $x = 49$, and $y = 56$.

Hence, the fraction is $\frac{49}{56}$.

13. See Ex. 10, p. 364.

14. See Ex. 13, p. 365.

15. See Ex. 15, p. 365.

16. Let x = number of cents per dozen in first price,
and $x - 2$ = number of cents per dozen in second price.

Then, $$\frac{60}{x-2} = \frac{60}{x} + 1.$$

Solving, $x = 12$ or -10.

Hence, he sold apples for 12 cents per dozen at first.

17. Let x = number of miles traveled by the train.

Then, $\frac{x}{63}$ = number of miles traveled per hour.

$$\therefore \frac{x - 44}{\frac{x}{63} + 4\frac{2}{3}} = 50.$$

Solving, $x = 1344$.

Hence, the train traveled 1344 miles.

18. See Ex. 16, p. 366.

19. See Ex. 14, p. 365.

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20. See Ex. 9, p. 364.

21. Let x = number of units in length,
and y = number of units in width.

Then, $$2x + 2y = 8m, \quad (1)$$
and $$xy = 2m^2. \quad (2)$$
From (1), $$y = 4m - x. \quad (3)$$
Substituting (3) in (2), $4mx - x^2 = 2m^2$.

Solving, $$x = 2m \pm m\sqrt{2}. \quad (4)$$
Substituting (4) in (3), $$y = 2m \mp m\sqrt{2}.$$

Hence, the length of the rectangle is $2m + m\sqrt{2}$ and its width is $2m - m\sqrt{2}$, or its length is $2m - m\sqrt{2}$ and its width is $2m + m\sqrt{2}$.

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22. Let x = number of inches in edge of larger cube,
and y = number of inches in edge of smaller cube.

Then, $x - y = 2$,
and $x^3 - y^3 = 296$.
Solving, $x = 8$ or -6,
and $y = 6$ or -8.

Hence, the edge of one cube is 8 inches and of the other 6 inches.

23. Let x = number of feet in width.
Then, $x + 1$ = number of feet in height,
and $5x$ = number of feet in length.

$$\therefore 5x^2(x + 1) = 400. \quad (1)$$

Expanding (1), $5x^3 + 5x^2 = 400$;
whence, $x^3 + x^2 - 80 = 0$.

Factoring the first member of the equation by the factor theorem,

$$(x - 4)(x^2 + 5x + 20) = 0.$$

Solving, $x = 4$ or $\dfrac{-5 \pm \sqrt{-55}}{2}$.

Rejecting the last two values of x,

$$x + 1 = 5,$$

and $5x = 20$.

Hence, the tank is 20 feet long, 4 feet wide, and 5 feet high.

24. Let x = number of square feet C and D do per day,
and y = number of square feet A and B do per day.

Then, $x - y = 50$, (1)

and $\dfrac{900}{y} - \dfrac{900}{x} = \dfrac{3}{5}$. (2)

From (2), $xy = 1500x - 1500y$. (3)
Multiplying (1) by 1500, $75{,}000 = 1500x - 1500y$. (4)
Subtracting (4) from (3), $xy = 75{,}000$. (5)
From (5), $4xy = 300{,}000$. (6)
Squaring (1), $x^2 - 2xy + y^2 = 2500$. (7)
Adding (6) and (7), $x^2 + 2xy + y^2 = 302{,}500$;
whence, $x + y = \pm 550$. (8)
Adding (8) and (1), $2x = 600$ or -500; $\therefore x = 300$ or -250.
Substituting 300 for x in (1), $y = 250$.

Hence, A and B can do 250 square feet per day, and C and D, 300 square feet per day.

25. See Ex. 23, p. 368. **26.** See Ex. 21, p. 367.

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1–12. See Ex. 1–12, pp. 368–370.

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13. See Ex. 13, p. 370. **15–25.** See Ex. 15–25, pp 371–374.

Pages 366–442

As these pages are identical in the Standard Algebra and the Standard Algebra Revised, see pages 374–426 of the Key.

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COMPLEX NUMBERS

Page 445

12–20. See Ex. 12–20, pp. 430–431. **22–27.** See Ex. 22–27, p. 431.

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29–38. See Ex. 29–38, p. 432.

SUPPLEMENTARY EXERCISES

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7. Let $x =$ the number of games won.
Then, $x - 2 =$ the number of games lost.
$\therefore x + x - 2 = 26.$
Solving, $x = 14.$
Hence, the team won 14 games.

8. Let $x =$ number of points scored by the captain.
Then, $x + 62 =$ number of points scored by other players.
$\therefore x + x + 62 = 322.$
Solving, $x = 130.$
Hence, the captain scored 130 points in the season.

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11. Let $x =$ the number.
Then, $12x - 6 = 5x + 8.$
Solving, $x = 2.$
Hence, the number is 2.

12. Let $x =$ number of feet in width of rectangle.
Then, $x + 5 =$ number of feet in length of rectangle.
$\therefore x + x + x + 5 + x + 5 = 26.$
Solving, $x = 4,$
and $x + 5 = 9.$
Hence, the rectangle is 9 feet long and 4 feet wide.

13. Let $x =$ number of rods in width of lawn.
Then, $x + 17 =$ number of rods in length of lawn.
$\therefore x + x + x + 17 + x + 17 = 194.$
Solving, $x = 40,$
and $x + 17 = 57.$
Hence, the lawn is 57 rods long and 40 rods wide.

14. Let $x =$ one number.
Then, $5x =$ the other number.
$\therefore x + 5x = 2 \cdot 5x - 28.$
Solving, $x = 7,$
and $5x = 35.$
Hence, one of the numbers is 7 and the other 35.

15. Let $x =$ number of boys who received letters.
Then, $2x + 17 = 47.$
Solving, $x = 15.$
Hence, 15 boys received letters.

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16. Let $x =$ the number.
Then, $2x - 8 = x + 6.$
Solving, $x = 14.$
Hence, the number is 14.

17. Let $x =$ the number of feet in the width of the chasm.
Then, $2x - 80 = 300.$
Solving, $x = 190.$
Hence, the width of the chasm is 190 feet.

18. Let $x =$ number of dollars in initial cost.
Then, $8x + 5.09 = 61.41.$
Solving, $x = 7.04.$
Hence, the initial cost of the 2-year-old heifer is $7.04.

19. Let $x =$ number of dollars in cost of labor.
Then, $5x + 2.42 = 24.67.$
Solving, $x = 4.45.$
Hence, the average cost of labor for the care of a yearling heifer is $4.45.

20. Let $x =$ number of dollars in cost of labor.
Then, $5x + 1.78 = 40.83.$
Solving, $x = 7.81.$
Hence, the average cost of labor for the care of a 2-year-old heifer is $7.81.

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21. Let $x =$ number of dollars the house cost.
Then, $\frac{3}{10}x =$ number of dollars gained.
$$\therefore x + \tfrac{3}{10}x = 6500.$$
Solving, $x = 5000,$
and $\frac{3}{10}x = 1500.$
Hence, the dealer gained $1500 on the sale of the house.

22. Let $x =$ number of pounds in weight of older horse.
Then, $\frac{9}{10}x =$ number of pounds in weight of younger horse.
$$\therefore x + \tfrac{9}{10}x = 1900.$$
Solving, $x = 1000,$
and $\frac{9}{10}x = 900.$
Hence, the older horse weighs 1000 pounds and the younger 900 pounds

23. Let $x =$ number of trout caught the day before.
Then, $2x - 4 = 48.$
Solving, $x = 26.$
Hence, the boys caught 26 trout the day before.

24. Let $x =$ number of dollars a cruise to St. Johns costs.
Then, $2x - 40 = 110.$
Solving, $x = 75.$
Hence, the cost of a cruise to St. Johns is $75.

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25. Let $x =$ number of dollars first boy shall receive.
Then, $2x =$ number of dollars second boy shall receive,
and $4x =$ number of dollars third boy shall receive.

$$\therefore x + 2x + 4x = 168.$$

Solving, $x = 24,$

$$2x = 48,$$

and $4x = 96.$

Hence, the first boy shall receive \$24, the second \$48, and the third \$96.

26. Let $x =$ number of dollars A had.
Then, $\frac{3}{4}x + 15 =$ number of dollars B had,
and $\frac{3}{4}x - 35 =$ number of dollars C had.

$$\therefore x + \tfrac{3}{4}x + 15 + \tfrac{3}{4}x - 35 = 430.$$

Solving, $x = 180,$

$$\tfrac{3}{4}x + 15 = 150,$$

and $\frac{3}{4}x - 35 = 100.$

Hence, A had \$180, B \$150, and C \$100.

27. Let $x =$ number of dollars in average profit per acre.
Then, $2x - 108 = 756.$
Solving, $x = 432.$
Hence, the average annual profit per acre was \$432.

28. Let $x =$ number of crates of raspberries raised the second year.
Then, $\frac{1}{14}x + 12 = 42.$
Solving, $x = 420.$
Hence, the farmer raised 420 crates of raspberries the second year.

29. Let $x =$ number of cents in selling price of a crate of raspberries.
Then, $\frac{1}{6}x + 5 = 25.$
Solving, $x = 120.$
Hence, the raspberries were sold at 120 cents, or \$1.20, per crate.

30. Let $x =$ number of feet in height of boiler stack.
Then, $16x + 26 = 698.$
Solving, $x = 42.$
Hence, the height of the boiler stack is 42 feet.

31. Let $x =$ number of pounds of butter fat in seven-day record.
Then, $5x - 34.5 = 102.5.$
Solving, $x = 27.4.$
Hence, the cow's seven-day record was 27.4 pounds of butter fat.

32. Let $x =$ number of pounds of butter fat in seven-day record.
Then, $4x + 1.4 = 111.8.$
Solving, $x = 27.6.$
Hence, the cow's seven-day record was 27.6 pounds of butter fat.

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33. Let $x =$ number of tons in the weight of the pillar that has been cored.
Then, $2x =$ number of tons in weight of other pillar.
$\therefore x + 2x = 24.$
Solving, $x = 8,$
and $2x = 16.$
Hence, the weight of the pillar that has been cored is 8 tons, and of the other 16 tons.

34. Let $x =$ the number of plum trees.
Then, $2x =$ the number of peach trees,
and $x - 200 =$ the number of pear trees.
$\therefore x + 2x + x - 200 = 1800.$
Solving, $x = 500,$
$2x = 1000,$
and $x - 200 = 300.$
Hence, there were 500 plum trees, 1000 peach trees, and 300 pear trees in the orchard.

35. Let $x =$ number of feet of clay bored through.
Then, $3x - 5 =$ number of feet of rock bored through.
$\therefore x + 3x - 5 = 75.$
Solving, $x = 20.$
Hence, he bored through 20 feet of clay.

36. Let $x =$ number of pounds pressure per square inch on the bicycle tire.
Then, $4x - 12 = 92.$
Solving, $x = 26.$
Hence, the pressure on the bicycle tire is 26 pounds per square inch.

37. Let $x =$ number of dollars the steam yacht cost.
Then, $\frac{1}{2}x + 5000 =$ number of dollars the motor boat cost.
$\therefore x - 10{,}000 = \frac{1}{2}x + 5000 + 15{,}000.$
Solving, $x = 60{,}000,$
and $\frac{1}{2}x + 5000 = 35{,}000.$
Hence, the yacht cost \$60,000 and the motor boat \$35,000.

38. Let $x =$ number of acres in farm.
Then, $\frac{3}{5}x + 2 =$ number of acres sold,
and $\frac{1}{2}x - 4 =$ number of acres left.
$\therefore x - (\frac{3}{5}x + 2) = \frac{1}{2}x - 4.$
Solving, $x = 20.$
Hence, the farm contained 20 acres.

39. Let $x =$ number of feet in diameter at the top.
Then, $x + 10\frac{1}{2} =$ number of feet in diameter at the base.
$\therefore x + 10\frac{1}{2} = 3x - 7\frac{1}{2}.$
Solving, $x = 9,$
and $x + 10\frac{1}{2} = 19\frac{1}{2}.$
Hence, the diameter of the chimney at the base is $19\frac{1}{2}$ feet.

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40. Let $x =$ number of years in son's age.
Then, $3x + 4 =$ number of years in father's age.
$\therefore x + 3x + 4 = 64.$
Solving, $x = 15,$
and $3x + 4 = 49.$
Hence, the son is 15 years old and the father 49 years old.

41. Let $x =$ number of feet in width of ship.
Then, $6x - 7 =$ number of feet in length of ship.
$\therefore 6x - 7 - 345 = x + 108.$
Solving, $x = 92,$
and $6x - 7 = 545.$
Hence, the length of the ship is 545 feet and its width 92 feet.

42. Let $x =$ the smaller number.
Then, $2x =$ the larger number.
$\therefore 32 - x + 11 = 50 - 2x.$
Solving, $x = 7,$
and $2x = 14.$
Hence, one number is 7 and the other 14.

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54. $(\frac{1}{2}a - \frac{2}{3}b)(\frac{1}{2}a + \frac{2}{3}b) = (\frac{1}{2}a)^2 - (\frac{2}{3}b)^2 = \frac{1}{4}a^2 - \frac{4}{9}b^2.$

55. $(2x^2b - y)(2x^2b + y) = (2x^2b)^2 - y^2 = 4x^4b^2 - y^2.$

56. $(4x^a - y^2)(4x^a + y^2) = (4x^a)^2 - (y^2)^2 = 16x^{2a} - y^4.$

57. $(2c^m - d^n)(2c^m + d^n) = (2c^m)^2 - (d^n)^2 = 4c^{2m} - d^{2n}.$

58. $(5ax - 6b^2)(5ax + 6b^2) = (5ax)^2 - (6b^2)^2 = 25a^2x^2 - 36b^4.$

70. $$(\overline{a+b} - 3)(\overline{a+b} + 4) = (a+b)^2 + 1(a+b) - 12$$
$$= a^2 + 2ab + b^2 + a + b - 12.$$

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1. Let $x =$ the smaller number.
Then, $2x + 2 =$ the larger number.
$\therefore x + 2x + 2 = 44.$
Solving, $x = 14,$
and $2x + 2 = 30.$
Hence, the numbers are 14 and 30.

2. Let $x =$ one part.
Then, $3x - 4 =$ other part.
$\therefore x + 3x - 4 = 28.$
Solving, $x = 8,$
and $3x - 4 = 20.$
Hence, the parts of 28 are 8 and 20.

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3. Let $x =$ number of years in son's age.
Then, $3x + 3 =$ number of years in father's age.
$\therefore\ x + 3x + 3 = 47.$
Solving, $x = 11,$
and $3x + 3 = 36.$
Hence, the son is 11 years old and the father 36 years old.

4. Let $x =$ one number.
Then, $x + 1 =$ the other number.
$\therefore\ (x + 1)^2 - x^2 = 121.$
Solving, $x = 60,$
and $x + 1 = 61.$
Hence, the numbers are 60 and 61.

5. Let $x =$ number of inches in each dimension of flag.
Then, $(x + 4)^2 - x^2 = 144.$
Solving, $x = 16.$
Hence, the flag was 16 inches square.

6. Let $x =$ number of marbles third boy had.
Then, $x + 6 =$ number of marbles second boy had,
and $2(x + 6) =$ number of marbles first boy had.
$\therefore\ x + x + 6 + 2(x + 6) = 122.$
Solving, $x = 26,$
$x + 6 = 32,$
and $2(x + 6) = 64.$
Hence, the first boy had 64 marbles, the second 32, and the third 26.

7. Let $x =$ number of grains new bill weighs.
Then, $27x =$ number of grains gold piece weighs.
$\therefore\ 27x - 20x = 140.$
Solving, $x = 20,$
and $27x = 540.$
Hence, a new one-dollar bill weighs 20 grains and a twenty-dollar gold piece 540 grains.

8. Let $x =$ number of kilometers in length of shorter race course.
Then, $x + 2 =$ number of kilometers in length of longer course.
$\therefore\ 20(x + 2) = 21x.$
Solving, $x = 40,$
and $x + 2 = 42.$
Hence, the longer course is 42 kilometers long and the shorter course 40 kilometers long.

9. Let $x =$ number of feet in diameter of tower.
Then, $x - 15 =$ number of feet in diameter of belfrey.
$\therefore\ 2(x - 15) = x + 23.$
Solving, $x = 53,$
and $x - 15 = 38.$
Hence, the diameter of the tower is 53 feet, and of the belfrey 38 feet.

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10. Let $x =$ number of trees in one tract.
Then, $15x - 15 =$ number of trees in the other tract.
$\therefore\ x + 15x - 15 = 1473.$
Solving, $x = 93,$
and $15x - 15 = 1380.$
Hence, there were 93 trees in one tract and 1380 trees in the other.

11. Let $x =$ number of eggs bought.
Then, $\frac{1}{3}x =$ number of eggs each case contained.
$\therefore\ 2(\frac{1}{3}x + 2 \cdot 12) = 768.$
Solving, $x = 1080.$
Hence, the grocer bought 1080 eggs.

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12. Let $x =$ number of years in B's age.
Then, $x + 12 =$ number of years in A's age.
$\therefore\ x + 12 - 4 = 2(x - 4).$
Solving, $x = 16,$
and $x + 12 = 28.$
Hence, A is 28 years old and B is 16 years old.

13. Let $x =$ number of years in Leon's age.
Then, $x + 26 =$ number of years in father's age.
$\therefore\ 3(x - 2) = x + 26.$
Solving, $x = 16.$
Hence, Leon is 16 years old.

14. Let $x =$ one number.
Then, $x + 2 =$ the other number.
$\therefore\ (x + 2)^2 - x^2 = 64.$
Solving, $x = 15,$
and $x + 2 = 17.$
Hence, the numbers are 15 and 17.

15. Let $x =$ the smaller part.
Then, $100 - x =$ the larger part.
$\therefore\ 100 - x - 60 = 2(34 - x).$
Solving, $x = 28,$
and $100 - x = 72.$
Hence, the two parts of 100 are 28 and 72.

16. Let $x =$ number of feet in width of boat.
Then, $9(x - 10) =$ number of feet in length of boat.
$\therefore\ 9(x - 10) = 3(x + 4).$
Solving, $x = 17,$
and $9(x - 10) = 63.$
Hence, the boat is 63 feet long and 17 feet wide.

17. Let $x =$ number of cents child receives per day.
Then, $x + 4 =$ number of cents woman receives per day.
$\therefore\ 6x + 6(x + 4) = 216.$
Solving, $x = 16,$
and $x + 4 = 20.$
Hence, the daily wages of a child are 16¢ and of a woman 20¢.

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18. Let $x =$ number of dollars in present price of article.
Then, $x + 3.75 =$ number of dollars in former price.
$$\therefore x = .40(x + 3.75).$$
Solving, $x = 2.5$.
Hence, the present price of the article is \$2.50.

19. Let $x =$ the number of quarters in the purse.
Then, $18 - x =$ the number of dimes in the purse.
$$\therefore .25x + .10(18 - x) = 2.40.$$
Solving, $x = 4$,
and $18 - x = 14$.
Hence, the purse contained 4 quarters and 14 dimes.

20. Let $x =$ number of trips A made.
Then, $x + 1 =$ number of trips B made,
and $2(x + 1) - x$, or $x + 2 =$ number of trips C made.
$$\therefore x + x + 1 + x + 2 = 30.$$
Solving, $x = 9$,
$x + 1 = 10$,
and $x + 2 = 11$.
Hence, A made 9 trips, B 10 trips, and C 11 trips.

21. Let $x =$ the first number.
Then, $x + 1 =$ the second number,
and $x + 2 =$ the third number.
$$\therefore (x + 1)(x + 2) - x^2 = 47.$$
Solving, $x = 15$,
$x + 1 = 16$,
and $x + 2 = 17$.
Hence, the three consecutive numbers are 15, 16, and 17.

22. Let $x =$ number of inches in length of box.
Then, $x - 4 =$ number of inches in width of box,
and $x - 12 =$ number of inches in height of box.
$$\therefore x(x-4) - (x-4)(x-12) = 264.$$
Solving, $x = 26$,
$x - 4 = 22$,
and $x - 12 = 14$.
Hence, the box is 26 inches long, 22 inches wide, and 14 inches high.

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23. Let $x =$ the first number.
Then, $x - 4 =$ the second number.
$$\therefore x^2 - (x - 4)^2 = 48.$$
Solving, $x = 8$,
and $x - 4 = 4$.
Hence, the numbers are 8 and 4.

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24. Let $x =$ number of dollars paid for each calf.
Then, $12x =$ number of dollars paid for 12 calves.
$$\therefore 12x = 15(x-1).$$
Solving, $x = 5.$
Hence, the farmer paid \$5 for each calf.

25. Let $x =$ number of meters in length of central arch.
Then, $x - 35 =$ number of meters in length of each of the 6 other arches.
$$\therefore x + 6(x-35) = 175.$$
Solving, $x = 55,$
and $x - 35 = 20.$
Hence, the central arch of the bridge is 55 meters long, and the other arches are each 20 meters long.

26. Let $x =$ number of 14-foot cuts.
Then, $2x + 2 =$ number of 12-foot cuts.
$$\therefore x + 2x + 2 + 1 = 15.$$
Solving, $x = 4,$
and $2x + 2 = 10.$
Hence, there were 10 12-foot cuts made from the poplar.

27. Let $x =$ number of feet in diameter of one circle.
Then, $x - 10\frac{1}{2} =$ number of feet in diameter of other circle.
$$\therefore x + 7\tfrac{1}{2} = 3(x - 10\tfrac{1}{2}).$$
Solving, $x = 19\frac{1}{2},$
and $x - 10\frac{1}{2} = 9.$
Hence, the diameter of one circle is 9 feet and of the other $19\frac{1}{2}$ feet.

28. Let $x =$ the number of bushels of wheat.
Then, $\frac{1}{4}x + 5000 =$ the number of bushels of corn or of barley.
$$\therefore x + 2(\tfrac{1}{4}x + 5000) = 90{,}000.$$
Solving, $x = 40{,}000,$
and $\frac{1}{4}x + 5000 = 25{,}000.$
Hence, the steamer carried 40,000 bushels of wheat, 25,000 bushels of corn, and 25,000 bushels of barley.

29. Let $x =$ number of ounces larger nugget weighed.
Then, $4715 - x =$ number of ounces smaller nugget weighed.
$$\therefore 439x = 504(4715 - x).$$
Solving, $x = 2520.$
Hence, the larger nugget weighed 2520 ounces.

30. Let $x =$ number of hits A makes.
Then, $30 - x =$ number of misses A makes,
$2x =$ number of hits B makes,
and $30 - 2x =$ number of misses B makes.
$$\therefore 30 - x = 3(30 - 2x).$$
Solving, $x = 12,$
$30 - x = 18,$
$2x = 24,$
and $30 - 2x = 6.$
Hence, A made 12 hits and 18 misses and B made 24 hits and 6 misses.

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31. Let $x =$ number of pounds of 50-cent tea.
Then, $12 - x =$ number of pounds of 80-cent tea.
$\therefore 50x + 80(12 - x) = 12 \times 60.$
Solving, $x = 8,$
and $12 - x = 4.$
Hence, 8 pounds of 50-cent tea and 4 pounds of 80-cent tea are required.

32. Let $x =$ number of pounds of 20-cent coffee.
Then, $40 - x =$ number of pounds of 28-cent coffee.
$\therefore 20x + 28(40 - x) = 40 \times 25.$
Solving, $x = 15,$
and $40 - x = 25.$
Hence, 15 pounds of 20-cent coffee and 25 pounds of 28-cent coffee are required.

33. Let $x =$ number of feet in height of statue.
Then, $x + 5 =$ number of feet in height of pedestal,
and $2(x + 5) + 5 =$ number of feet in height of shaft.
$\therefore x + x + 5 + 2(x + 5) + 5 = 160.$
Solving, $x = 35,$
$x + 5 = 40,$
and $2(x + 5) + 5 = 85.$
Hence, the height of the statue is 35 feet, of the pedestal 40 feet, and of the shaft 85 feet.

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30.
$$\begin{array}{l|l}
x^{a+3} + x^{a+1} + x^{a-1} & x^2 - x + 1 \\
\underline{x^{a+3} + x^{a+1} - x^{a+2}} & \overline{x^{a+1} + x^a + x^{a-1}} \\
\quad x^{a+2} + x^{a-1} & \\
\quad \underline{x^{a+2} - x^{a+1} + x^a} & \\
\qquad x^{a+1} - x^a + x^{a-1} & \\
\qquad \underline{x^{a+1} - x^a + x^{a-1}} &
\end{array}$$

31.
$$\begin{array}{l|l}
a^6 + b^6 & a + b \\
\underline{a^6 + a^5b} & \overline{a^5 - a^4b + a^3b^2 - a^2b^3 + ab^4} \\
\quad - a^5b + b^6 & \\
\quad \underline{- a^5b - a^4b^2} & \\
\qquad a^4b^2 + b^6 & \\
\qquad \underline{a^4b^2 + a^3b^3} & \\
\qquad\quad - a^3b^3 + b^6 & \\
\qquad\quad \underline{- a^3b^3 - a^2b^4} & \\
\qquad\qquad a^2b^4 + b^6 & \\
\qquad\qquad \underline{a^2b^4 + ab^5} & \\
\qquad\qquad\quad - ab^5 + b^6 &
\end{array}$$

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15. $[(c - d)^2 - 4] \div [c - d - 2] = [(c - d)^2 - 4] \div [(c - d) - 2]$
$= (c - d) + 2, \text{ or } c - d + 2.$

16. $[(a + b)^3 + 1] \div [a + b + 1] = [(a + b)^3 + 1] \div [(a + b) + 1]$
$= (a + b)^2 - (a + b) + 1.$

17. $[8 - (c + d)^3] \div [2 - c - d] = [8 - (c + d)^3] \div [2 - (c + d)]$
$= 4 + 2(c + d) + (c + d)^2.$

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18. $[(x+y)^2-100] \div [x+y+10] = [(x+y)^2-100] \div [(x+y)+10]$
$= (x+y)-10$, or $x+y-10$.

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1. Let $x =$ number of years in man's present age.
Then, $x - 14 = \frac{1}{3}x$.
Solving, $x = 21$.
Hence, the man is 21 years old.

2. Let $x =$ number of games the team played.
Then, $\frac{3}{5}x = 99$.
Solving, $x = 165$.
Hence, the team played 165 games that year.

3. Let $x =$ one number.
Then, $42 - x =$ the other number.
$\therefore \frac{1}{2}[x - (42 - x)] = 4$.
Solving, $x = 25$,
and $42 - x = 17$.
Hence, the numbers are 25 and 17.

4. Let $x =$ number of years in Ruth's age.
Then, $\frac{1}{2}x =$ number of years in Mary's age.
$\therefore \frac{1}{2}x + 6 = \frac{2}{3}(x + 6)$.
Solving, $x = 12$,
and $\frac{1}{2}x = 6$.
Hence, Mary is 6 years old and Ruth is 12 years old.

5. Let $x =$ number of years in son's age.
Then, $mx =$ number of years in father's age.
$\therefore mx + p = n(x + p)$.
Solving, $x = \dfrac{np - p}{m - n}$,
and $mx = \dfrac{m(np - p)}{m - n}$.
Hence, the son's age is $\dfrac{np - p}{m - n}$ years and the father's $\dfrac{m(np - p)}{m - n}$ years.

6. Let $x =$ the larger part.
Then, $25 - x =$ the smaller part.
$\therefore \frac{1}{2}x = 2(25 - x)$.
Solving, $x = 20$,
and $25 - x = 5$.
Hence, the parts of 25 are 20 and 5.

7. Let $x =$ the larger part.
Then, $c - x =$ the smaller part.
$\therefore ax = b(c - x)$.
Solving, $x = \dfrac{bc}{a + b}$,
and $c - x = \dfrac{ac}{a + b}$.
Hence, the parts of c are $\dfrac{bc}{a + b}$ and $\dfrac{ac}{a + b}$.

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8. Let $x =$ number of pounds in latter boy's weight.
Then, $\frac{1}{5}x + 85 = 100.$
Solving, $x = 75.$
Hence, the boy weighed 75 pounds.

9. Let $x =$ number of feet in length of each of the two equal parts.
Then, $\frac{1}{3}x =$ number of feet in length of butt.
$\therefore\ 2x + \frac{1}{3}x = 7.$
Solving, $x = 3,$
and $\frac{1}{3}x = 1.$
Hence, the butt of the rod is 1 foot long, and each of the other two parts is 3 feet long.

10. Let $x =$ number of games won.
Then, $\frac{1}{4}x =$ number of games lost,
and $\frac{1}{6}x =$ number of drawn games.
$\therefore\ x + \frac{1}{4}x + \frac{1}{6}x = 17.$
Solving, $x = 12.$
Hence, he won 12 games.

11. Let $x =$ number of years in son's age.
Then, $3x + 3 =$ number of years in father's age.
$\therefore\ x - 5 = \frac{1}{5}(3x + 3).$
Solving, $x = 14,$
and $3x + 3 = 45.$
Hence, the son is 14 years old and the father 45 years old.

12. Let $x =$ number of feet in height of house.
Then, $\frac{1}{5}x =$ number of feet in length of flagstaff.
$\therefore\ x + \frac{1}{5}x = 180.$
Solving, $x = 150,$
and $\frac{1}{5}x = 30.$
Hence, the house is 150 feet high and the flagstaff 30 feet long.

13. Let $x =$ number of bats that are used.
Then, $x + 30{,}000 =$ number of balls that are used.
$\therefore\ 2(x + 30{,}000) = 5x.$
Solving, $x = 20{,}000.$
Hence, 20,000 bats are used per year.

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14. Let $x =$ number of pages in the index.
Then, $65x + 34 =$ number of pages in the body of the book.
$\therefore\ x + 65x + 34 = 496.$
Solving, $x = 7.$
Hence, there are 7 pages in the index of the book.

15. Let $x =$ number of muffs in storage room.
Then, $2x =$ number of coats in storage room.
$\therefore\ \frac{1}{3}(2x) = x - 5000.$
Solving, $x = 15{,}000,$
and $2x = 30{,}000.$
Hence, the storage room contains 15,000 muffs and 30,000 coats.

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16. Let $x =$ number of pairs of blinds made.
Then, $x + 1000 =$ number of doors made,
and $4x =$ number of windows made.

$$\therefore\ 4x - 500 = x + 1000.$$

Solving, $x = 500,$

$$x + 1000 = 1500,$$

and $4x = 2000.$

Hence, the daily output of the factory was 500 pairs of blinds, 1500 doors, and 2000 windows.

17. Let $x =$ number of inches in width of card.
Then, $x + 4 =$ number of inches in length of card.

$$\therefore\ x + 4 + 2 = 2(x - 1).$$

Solving, $x = 8,$
and $x + 4 = 12.$

Hence, the card is 12 inches long and 8 inches wide.

18. Let $x =$ number of feet in width of ship.
Then, $x + 384 =$ number of feet in length of ship.

$$\therefore\ 1\tfrac{1}{6}x = \tfrac{1}{6}(x + 384) - 10.$$

Solving, $x = 54,$
and $x + 384 = 438.$

Hence, the ship is 438 feet long.

19. Let $x =$ number of feet in width of stadium.
Then, $x + 196 =$ number of feet in length of stadium.

$$\therefore\ x + 26 = \tfrac{5}{6}(x + 196 - 70).$$

Solving, $x = 474,$
and $x + 196 = 670.$

Hence, the stadium is 670 feet long and 474 feet wide.

20. Let $x =$ number of inches in width of first picture.
Then, $x + 4 =$ number of inches in length of first picture,
$x - 6 =$ number of inches in width of second picture,
and $x + 4 + 12$, or $x + 16 =$ number of inches in length of second picture.

$$\therefore\ x(x + 4) = (x - 6)(x + 16).$$

Solving, $x = 16,$

$$x + 4 = 20,$$

$$x - 6 = 10,$$

and $x + 16 = 32.$

Hence, the first picture is 20 inches by 16 inches and the second picture 32 inches by 10 inches.

21. Let $x =$ number of dollars gas cost.
Then, $4x =$ number of dollars balloon cost,
and $x + 25 =$ number of dollars rest of outfit cost.

$$\therefore\ x + 4x + x + 25 = 775.$$

Solving, $x = 125,$
and $4x = 500.$

Hence, the balloon cost \$500.

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22. Let $x =$ number of inches in diameter of projectile.

Then, $4x - 4 =$ number of inches in length of projectile.

$$\therefore\ x - 4 = \tfrac{1}{3}(4x - 4).$$

Solving, $x = 16$,

and $4x - 4 = 60$.

Hence, the projectile was 60 inches long and 16 inches in diameter.

23. Let $x =$ number of vessels that passed through the canal the preceding year.

Then, $34(4116 - x) + 2 = 4116$.

Solving, $x = 3995$.

Hence, 3995 vessels passed through the Suez Canal the preceding year.

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19. $3a + ab + 3b + b^2 = a(3 + b) + b(3 + b) = (a + b)(3 + b)$.

20. $x^2 + xy + xz + yz = x(x + y) + z(x + y) = (x + z)(x + y)$.

21. $x^2 + 3x + xy + 3y = x(x + 3) + y(x + 3) = (x + y)(x + 3)$.

22. $ac - ad + bc - bd = a(c - d) + b(c - d) = (a + b)(c - d)$.

23. $cx - dx - cy + dy = x(c - d) - y(c - d) = (x - y)(c - d)$.

24. $2y^2 + 4y - yz - 2z = 2y(y + 2) - z(y + 2) = (2y - z)(y + 2)$.

25. $2m + am + 2a + a^2 = m(2 + a) + a(2 + a) = (m + a)(2 + a)$.

26. $2a + 3b + 2am + 3bm = 1(2a + 3b) + m(2a + 3b)$
$= (1 + m)(2a + 3b)$.

27. $3ap^2 - aq^2 + 3bp^2 - bq^2 = a(3p^2 - q^2) + b(3p^2 - q^2)$
$= (a + b)(3p^2 - q^2)$.

28. $2x^2y^2 - 2yz + x^2yz^2 - z^3 = 2y(x^2y - z) + z^2(x^2y - z)$
$= (2y + z^2)(x^2y - z)$.

29. $3ab - ac + 6bd - 2cd = a(3b - c) + 2d(3b - c)$
$= (a + 2d)(3b - c)$.

30. $a^3r^3 + 2ar^2s - a^2brs - 2bs^2 = ar^2(a^2r + 2s) - bs(a^2r + 2s)$
$= (ar^2 - bs)(a^2r + 2s)$.

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79.
$$4x^2 - 11x - 3 = (4x^2 - 11x - 3) \times \tfrac{4}{4} = \frac{16x^2 - 44x - 12}{4}$$
$$= \frac{(4x)^2 - 11(4x) - 12}{4} = \frac{(4x - 12)(4x + 1)}{4}$$
$$= \frac{4(x - 3)(4x + 1)}{4} = (x - 3)(4x + 1).$$

80. $6 + 5r - 6r^2 = -1(6r^2 - 5r - 6)$.

To factor $6r^2 - 5r - 6$, try	$6r + 3$,	$6r - 3$,	$3r - 2$,	$3r + 2$,	...
multiplied by	$r - 2$,	$r + 2$,	$2r + 3$,	$2r - 3$,	...
Products, 2d terms,	$-9r$,	$+9r$,	$+5r$,	$-5r$,	...

$\therefore\ 6 + 5r - 6r^2 = -1(3r + 2)(2r - 3) = (2 + 3r)(3 - 2r)$.

81. First factor, try	$6y + 1$,	$6y + 2$,	$4y + 2$,	$4y + 1$,	...
Second factor, try	$2y + 2$,	$2y + 1$,	$3y + 1$,	$3y + 2$,	...
Products, 2d terms,	$+14y$,	$+10y$,	$+10y$,	$+11y$,	...

$\therefore\ 12y^2 + 11y + 2 = (4y + 1)(3y + 2)$.

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82. $4x^2+16x+15=(2x)^2+8(2x)+15=(2x+5)(2x+3)$.

83.

First factor, try	$8x+5$,	$8x+3$,	$4x+5$,	$4x+3$,	⋯
Second factor, try	$x+3$,	$x+5$,	$2x+3$,	$2x+5$,	⋯
Products, 2d terms,	$+29x$,	$+43x$,	$+22x$,	$+26x$,	⋯

$\therefore\ 8x^2+26x+15=(4x+3)(2x+5)$.

84.

First factor, try	$6x^3-7$,	$6x^3-5$,	$6x^3+5$,	$6x^3+7$,	⋯
Second factor, try	x^3+5,	x^3+7,	x^3-7,	x^3-5,	⋯
Products, 2d terms,	$+23x^3$,	$+37x^3$,	$-37x^3$,	$-23x^3$,	⋯

$\therefore\ 6x^6-23x^3-35=(6x^3+7)(x^3-5)$.

85. $8x^2y^2-8xy-6=2(4x^2y^2-4xy-3)=2[(2xy)^2-2(2xy)-3]$
$=2(2xy+1)(2xy-3)$.

86. $-16y^2+20y+24=-(16y^2-20y-24)=-[(4y)^2-5(4y)-24]$
$=-(4y+3)(4y-8)=-4(y-2)(4y+3)$
$=4(2-y)(3+4y)$.

87.

First factor, try	$2a-b$,	$2a+b$,	⋯
Second factor, try	$a+2b$,	$a-2b$,	⋯
Products, 2d terms,	$+3ab$,	$-3ab$,	⋯

$\therefore\ 2a^2-3ab-2b^2=(2a+b)(a-2b)$.

88.

First factor, try	$6a+5c$,	$6a-5c$,	$3a-4c$,	$3a+4c$,	⋯
Second factor, try	$a-4c$,	$a+4c$,	$2a+5c$,	$2a-5c$,	⋯
Products, 2d terms,	$-19ac$,	$+19ac$,	$+7ac$,	$-7ac$,	⋯

$\therefore\ 6a^2-7ac-20c^2=(3a+4c)(2a-5c)$.

89. $5ap-p^2-6a^2=-(6a^2-5ap+p^2)$.

To factor $6a^2-5ap+p^2$, try	$6a-p$,	$3a-p$.
multiplied by	$a-p$,	$2a-p$.
Products, 2d terms,	$-7ap$,	$-5ap$.

$\therefore\ 5ap-p^2-6a^2=-(3a-p)(2a-p)=(p-3a)(2a-p)$.

90.

First factor, try	$3x^n+7$,	$3x^n-7$,	$3x^n+2$,	$3x^n-2$,	⋯
Second factor, try	x^n-2,	x^n+2,	x^n-7,	x^n+7,	⋯
Products, 2d terms,	$+x^n$,	$-x^n$,	$-19x^n$,	$+19x^n$,	⋯

$\therefore\ 3x^{2n}+19x^n-14=(x^n+7)(3x^n-2)$.

91. $6x^2+2xy-48y^2=2(3x^2+xy-24y^2)$.

To factor $3x^2+xy-24y^2$, try	$3x+8y$,	$3x-8y$,	⋯
multiplied by	$x-3y$,	$x+3y$,	⋯
Products, 2d terms,	$-xy$,	$+xy$,	⋯

$\therefore\ 6x^2+2xy-48y^2=2(x+3y)(3x-8y)$.

92. $a^2x^2-3abx-4b^2=(ax)^2-3b(ax)-4b^2=(ax+b)(ax-4b)$.

93. $4x^2y^2+6xy-40=(2xy)^2+3(2xy)-40$
$=(2xy+8)(2xy-5)=2(xy+4)(2xy-5)$.

94. $16q^2+8pq-3p^2=(4q)^2+2p(4q)-3p^2=(4q+3p)(4q-p)$.

95. $9x^2+21dx+10d^2=(3x)^2+7d(3x)+10d^2=(3x+2d)(3x+5d)$.

96. $18b^4+3ab^2-10a^2=\dfrac{36b^4+6ab^2-20a^2}{2}=\dfrac{(6b^2)^2+a(6b^2)-20a^2}{2}$
$=\dfrac{(6b^2+5a)(6b^2-4a)}{2}=(6b^2+5a)(3b^2-2a)$.

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97. $10a^2 - 25ab^2 - 60b^4 = 5(2a^2 - 5ab^2 - 12b^4)$.

$$\begin{array}{lll}
\text{To factor } 2a^2 - 5ab^2 - 12b^4\text{, try} & 2a - 3b^2, & 2a + 3b^2, \cdots \\
\text{multiplied by} & \underline{a + 4b^2,} & \underline{a - 4b^2,} \cdots \\
\text{Products, 2d terms,} & +5ab^2, & -5ab^2, \cdots
\end{array}$$

$\therefore 10a^2 - 25ab^2 - 60b^4 = 5(2a + 3b^2)(a - 4b^2)$.

98.

$$\begin{array}{llll}
\text{First factor, try} & 5ab + cd, & 5ab + 2cd, & 5ab + 4cd, \cdots \\
\text{Second factor, try} & \underline{ab + 4cd,} & \underline{ab + 2cd,} & \underline{ab + cd,} \cdots \\
\text{Products, 2d terms,} & +21abcd, & +12abcd, & +9abcd, \cdots
\end{array}$$

$\therefore 5a^2b^2 + 9abcd + 4c^2d^2 = (5ab + 4cd)(ab + cd)$.

101. $16a^3 - 2 = 2(8a^3 - 1) = 2(2a - 1)(4a^2 + 2a + 1)$.

102. $x^{10} + xy^3 = x(x^9 + y^3) = x(x^3 + y)(x^6 - x^3y + y^2)$.

103. $m^3 - 216 = m^3 - 6^3 = (m - 6)(m^2 + 6m + 36)$.

104. $x^3 + 125 = x^3 + 5^3 = (x + 5)(x^2 - 5x + 25)$.

105. $64 - 8y^3 = 8(8 - y^3) = 8(2 - y)(4 + 2y + y^2)$.

106. $8a^6 - b^3c^3 = (2a^2)^3 - b^3c^3 = (2a^2 - bc)(4a^4 + 2a^2bc + b^2c^2)$.

107. $64c^3 + d^6 = (4c)^3 + (d^2)^3 = (4c + d^2)(16c^2 - 4cd^2 + d^4)$.

108. $27a^3 - x^3y^3 = (3a)^3 - x^3y^3 = (3a - xy)(9a^2 + 3axy + x^2y^2)$.

109. $y^{3n+1} - y = y(y^{3n} - 1) = y(y^n - 1)(y^{2n} + y^n + 1)$.

110. $3a^3 + 81b^3 = 3(a^3 + 27b^3) = 3(a + 3b)(a^2 - 3ab + 9b^2)$.

111. $8x^3 + 27y^3 = (2x)^3 + (3y)^3 = (2x + 3y)(4x^2 - 6xy + 9y^2)$.

112. $64x^3 + 125y^3 = (4x)^3 + (5y)^3 = (4x + 5y)(16x^2 - 20xy + 25y^2)$.

113. $27(a + b)^3 + 8 = [3(a + b) + 2][9(a + b)^2 - 6(a + b) + 4]$
$= (3a + 3b + 2)(9a^2 + 18ab + 9b^2 - 6a - 6b + 4)$.

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19. $y^6 - 8 = (y^2)^3 - 2^3 = (y^2 - 2)(y^4 + 2y^2 + 4)$.

20. $b^8 - 81 = (b^4 + 9)(b^4 - 9) = (b^4 + 9)(b^2 + 3)(b^2 - 3)$.

21. $c^3d^3 - 1 = (cd - 1)(c^2d^2 + cd + 1)$.

22. $x^4y^{12} - z^8 = (x^2y^6 + z^4)(x^2y^6 - z^4) = (x^2y^6 + z^4)(xy^3 + z^2)(xy^3 - z^2)$.

23. $c^4 + 64d^4 = c^4 + 16c^2d^2 + 64d^4 - 16c^2d^2 = (c^2 + 8d^2)^2 - 16c^2d^2$
$= (c^2 + 4cd + 8d^2)(c^2 - 4cd + 8d^2)$.

24. $a^3b^6 - 1 = (ab^2)^3 - 1 = (ab^2 - 1)(a^2b^4 + ab^2 + 1)$.

25. $3x^3 - 12x = 3x(x^2 - 4) = 3x(x + 2)(x - 2)$.

26. $x^4y - y^5 = y(x^4 - y^4) = y(x^2 + y^2)(x^2 - y^2) = y(x^2 + y^2)(x + y)(x - y)$.

27. $27b^4 + bc^3 = b(27b^3 + c^3) = b(3b + c)(9b^2 - 3bc + c^2)$.

28. $x^{2n+1} - xy^{2n} = x(x^{2n} - y^{2n}) = x(x^n + y^n)(x^n - y^n)$.

30. $a^3b - bc^3 = b(a^3 - c^3) = b(a - c)(a^2 + ac + c^2)$.

32. $64p^3 - q^3 = (4p)^3 - q^3 = (4p - q)(16p^2 + 4pq + q^2)$.

33. $bx^4 - 16a^4b = b(x^4 - 16a^4) = b(x^2 + 4a^2)(x^2 - 4a^2)$
$= b(x^2 + 4a^2)(x + 2a)(x - 2a)$.

37. $x^4 + y^4 - 7x^2y^2 = x^4 + 2x^2y^2 + y^4 - 9x^2y^2 = (x^2 + y^2)^2 - 9x^2y^2$
$= (x^2 + 3xy + y^2)(x^2 - 3xy + y^2)$.

39. $4a^2 + 32a + 39 = (2a)^2 + 16(2a) + 39 = (2a + 13)(2a + 3)$.

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40. $d^3-d^2-d+1=d^2(d-1)-1(d-1)=(d^2-1)(d-1)$
$=(d-1)(d+1)(d-1)$.

42. $16\,a^2+8\,ab-15\,b^2=(4\,a)^2+2\,b(4\,a)-15\,b^2=(4\,a+5\,b)(4\,a-3\,b)$.

44. $3\,a^4-30\,a^2b^2+75\,b^4=3(a^4-10\,a^2b^2+25\,b^4)=3(a^2-5\,b^2)(a^2-5\,b^2)$.

45. $28\,x-15-12\,x^2=-(12\,x^2-28\,x+15)$.

To factor $12\,x^2-28\,x+15$, try	$4\,x-3$,	$6\,x-5$,	...
multiplied by	$3\,x-5$,	$2\,x-3$,	...
Products, 2d terms,	$-29\,x$,	$-28\,x$,	...

$\therefore 28\,x-15-12\,x^2=-(6\,x-5)(2\,x-3)=(5-6\,x)(2\,x-3)$.

46. $(m-2\,a)^2-4(a+b)^2=[(m-2\,a)+2(a+b)][(m-2\,a)-2(a+b)]$
$=(m+2\,b)(m-4\,a-2\,b)$.

47.

First factor, try	$4\,a-3\,d$,	$6\,a+5\,d$,	$4\,a+3\,d$,	...
Second factor, try	$3\,a+5\,d$,	$2\,a-3\,d$,	$3\,a-5\,d$,	...
Products, 2d terms,	$+11\,ad$,	$-8\,ad$,	$-11\,ad$,	...

$\therefore 12\,a^2-11\,ad-15\,d^2=(4\,a+3\,d)(3\,a-5\,d)$.

48. $a^2+2\,ac-b^2+c^2=(a^2+2\,ac+c^2)-b^2=(a+c+b)(a+c-b)$.

49. $6\,ax-by+3\,ay-2\,bx=3\,a(2\,x+y)-b(2\,x+y)=(3\,a-b)(2\,x+y)$.

50. $8\,x^2-8\,c^2-32\,x+32=8(x^2-c^2-4\,x+4)=8[(x^2-4\,x+4)-c^2]$
$=8(x-2+c)(x-2-c)$.

51. $81\,p^2-54\,pq+9\,q^2=9(9\,p^2-6\,pq+q^2)=9(3\,p-q)(3\,p-q)$.

52. $3\,cx+9\,cy-4\,dx-12\,dy=3\,c(x+3\,y)-4\,d(x+3\,y)$
$=(3\,c-4\,d)(x+3\,y)$.

53. $4\,x^3-y^3+x^2y-4\,xy^2=x^2(4\,x+y)-y^2(4\,x+y)=(x^2-y^2)(4\,x+y)$
$=(x+y)(x-y)(4\,x+y)$.

54. $36\,c-21\,cx-135\,cx^2=3\,c(12-7\,x-45\,x^2)$.

To factor $12-7\,x-45\,x^2$, try	$6+5\,x$,	$6-5\,x$,	$3-5\,x$,	$3+5\,x$,	...
multiplied by	$2-9\,x$,	$2+9\,x$,	$4+9\,x$,	$4-9\,x$,	...
Products, 2d terms,	$-44\,x$,	$+44\,x$,	$+7\,x$,	$-7\,x$,	...

$\therefore 36\,c-21\,cx-135\,cx^2=3\,c(3+5\,x)(4-9\,x)$.

55. $9\,a^2b^2c+6\,ab^3c+b^4c=b^2c(9\,a^2+6\,ab+b^2)=b^2c(3\,a+b)(3\,a+b)$.

56. $12\,d^2x^2-4\,d^2xy-16\,d^2y^2=4\,d^2(3\,x^2-xy-4\,y^2)$.

To factor $3\,x^2-xy-4\,y^2$, try	$3\,x+4\,y$,	$3\,x-4\,y$,	...
multiplied by	$x-\ \ y$,	$x+\ \ y$,	...
Products, 2d terms,	$+xy$,	$-xy$,	...

$\therefore 12\,d^2x^2-4\,d^2xy-16\,d^2y^2=4\,d^2(x+y)(3\,x-4\,y)$.

57. $48\,x^2-72\,axy+27\,a^2y^2=3(16\,x^2-24\,axy+9\,a^2y^2)$
$=3(4\,x-3\,ay)(4\,x-3\,ay)$.

58. $6\,a^4x^2+a^3x-6\,a^3x^3-a^2x^2=a^2x[6\,a^2x+a-6\,ax^2-x]$
$=a^2x[a(6\,ax+1)-x(6\,ax+1)]$
$=a^2x(a-x)(6\,ax+1)$.

59. $x^4-2\,x^2(a-y)+(a-y)^2=[x^2-(a-y)][x^2-(a-y)]$
$=(x^2-a+y)(x^2-a+y)$.

60. $apq+bcpq-az-bcz=pq(a+bc)-z(a+bc)=(pq-z)(a+bc)$.

61. $(a-x)^2+8(a-x)x+16\,x^2=[(a-x)+4\,x][(a-x)+4\,x]$
$=(a+3\,x)(a+3\,x)$.

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5. The L. C. D. is $c^2 - d^2$.

$$\therefore \frac{3b}{c^2-d^2} = \frac{3b}{c^2-d^2}, \text{ and } \frac{1}{c+d} = \frac{c-d}{c^2-d^2}.$$

6. The L. C. D. is $24x^2y^2z$.

$$\therefore \frac{3z+2}{8xy^2} = \frac{3xz(3z+2)}{24x^2y^2z} = \frac{9xz^2+6xz}{24x^2y^2z},$$

and $$\frac{3y-2}{12x^2z} = \frac{2y^2(3y-2)}{24x^2y^2z} = \frac{6y^3-4y^2}{24x^2y^2z}.$$

7. The L. C. D. is $3a(x+y)^2$.

$$\therefore \frac{2c}{(x+y)^2} = \frac{3a\cdot 2c}{3a(x+y)^2} = \frac{6ac}{3a(x+y)^2},$$

and $$\frac{4}{3a(x+y)} = \frac{4(x+y)}{3a(x+y)^2} = \frac{4x+4y}{3a(x+y)^2}.$$

8. The L. C. D. is $r^3 - s^3$.

$$\therefore \frac{2}{r^3-s^3} = \frac{2}{r^3-s^3}, \text{ and } \frac{2}{r^2+rs+s^2} = \frac{2(r-s)}{r^3-s^3} = \frac{2r-2s}{r^3-s^3}.$$

9. The L. C. D. is $(x-1)(x-2)(x-3)$.

$$\therefore \frac{x-1}{x^2-5x+6} = \frac{(x-1)(x-1)}{(x-1)(x-2)(x-3)} = \frac{x^2-2x+1}{(x-1)(x-2)(x-3)},$$

and $$\frac{x-2}{x^2-4x+3} = \frac{(x-2)(x-2)}{(x-1)(x-2)(x-3)} = \frac{x^2-4x+4}{(x-1)(x-2)(x-3)}.$$

10. The L. C. D. is $b(a^2 - b^2)$.

$$\therefore a = \frac{ab(a^2-b^2)}{b(a^2-b^2)} = \frac{a^3b-ab^3}{b(a^2-b^2)},$$

$$\frac{ab^2}{a^2-b^2} = \frac{ab^3}{b(a^2-b^2)},$$

and $$\frac{a^3}{ab-b^2} = \frac{a^3(a+b)}{b(a^2-b^2)} = \frac{a^4+a^3b}{b(a^2-b^2)}.$$

11. The L. C. D. is $(1+z)(1-z^3)$.

$$\therefore \frac{1}{1-z^2} = \frac{1+z+z^2}{(1+z)(1-z^3)},$$

$$\frac{3}{1-z^3} = \frac{3(1+z)}{(1+z)(1-z^3)} = \frac{3+3z}{(1+z)(1-z^3)},$$

and $$\frac{4a}{z-1} = -\frac{4a(1+z)(1+z+z^2)}{(1+z)(1-z^3)} = -\frac{4a+8az+8az^2+4az^3}{(1+z)(1-z^3)}.$$

12. The L. C. D. is $(x-1)(x-2)(x+2)$.

$$\therefore \frac{5a}{x^2+x-2} = \frac{5a(x-2)}{(x-1)(x-2)(x+2)} = \frac{5ax-10a}{(x-1)(x-2)(x+2)},$$

$$\frac{3a}{x^2-4} = \frac{3a(x-1)}{(x-1)(x-2)(x+2)} = \frac{3ax-3a}{(x-1)(x-2)(x+2)},$$

and $$\frac{ab}{x-1} = \frac{ab(x-2)(x+2)}{(x-1)(x-2)(x+2)} = \frac{abx^2-4ab}{(x-1)(x-2)(x+2)}.$$

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16. $\frac{y+2}{3}+\frac{y+3}{5}=\frac{5y+10+3y+9}{15}=\frac{8y+19}{15}.$

17. $\frac{a-2b}{2a}+\frac{a+7b}{8a}=\frac{4a-8b+a+7b}{8a}=\frac{5a-b}{8a}.$

18. $\frac{x+7a}{2b}-\frac{2x+a}{5b}=\frac{5x+35a-4x-2a}{10b}=\frac{x+33a}{10b}.$

19. $\frac{2x+5}{3x}-\frac{13}{8x^2}-\frac{1}{2x}=\frac{16x^2+40x-39-12x}{24x^2}=\frac{16x^2+28x-39}{24x^2}.$

20. $\frac{a-x}{x}+\frac{a+x}{a}+\frac{a^2-x^2}{2ax}=\frac{2a^2-2ax+2ax+2x^2+a^2-x^2}{2ax}=\frac{3a^2+x^2}{2ax}.$

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37. $\frac{3a^2}{5a-10}\cdot\frac{3a-6}{4a^3}=\frac{3a^2}{5(a-2)}\cdot\frac{3(a-2)}{4a^3}=\frac{9}{20a}.$

38. $\frac{x^2+2x+1}{y}\cdot\frac{4y^2}{x^2-1}=\frac{(x+1)(x+1)}{y}\cdot\frac{4y^2}{(x+1)(x-1)}=\frac{4y(x+1)}{x-1}.$

39. $\frac{x^4-y^4}{8x^3+8xy^2}\cdot\frac{4}{x^2-y^2}=\frac{(x^2+y^2)(x^2-y^2)}{8x(x^2+y^2)}\cdot\frac{4}{x^2-y^2}=\frac{1}{2x}.$

41. $\frac{c+d}{5x}\cdot\frac{10x^2}{c^2-cd-2d^2}\cdot\frac{ac-2ad}{2x}$

$$=\frac{c+d}{5x}\cdot\frac{10x^2}{(c-2d)(c+d)}\cdot\frac{a(c-2d)}{2x}=a.$$

42. $\frac{a-b}{(a+b)^2}\cdot\frac{a^2-b^2}{3}\cdot\frac{4}{a^2-2ab+b^2}$

$$=\frac{a-b}{(a+b)^2}\cdot\frac{(a+b)(a-b)}{3}\cdot\frac{4}{(a-b)^2}=\frac{4}{3(a+b)}.$$

43. $\frac{x^2+x-2}{4x-12}\cdot\frac{3}{x^2-x-6}\cdot\frac{x-3}{x-1}$

$$=\frac{(x+2)(x-1)}{4(x-3)}\cdot\frac{3}{(x-3)(x+2)}\cdot\frac{x-3}{x-1}=\frac{3}{4(x-3)}.$$

50. $\frac{4-x^2}{2-3x+x^2}\div(2+x)=\frac{(2+x)(2-x)}{(2-x)(1-x)}\times\frac{1}{2+x}=\frac{1}{1-x}.$

51. $\frac{xy-x^2y^2}{axy}\div\frac{a-axy}{a^2x+a^2y}=\frac{xy(1-xy)}{axy}\times\frac{a^2(x+y)}{a(1-xy)}=x+y.$

52. $\frac{x^3+y^3}{abx+aby}\div\frac{x^2-xy+y^2}{a^2}=\frac{(x+y)(x^2-xy+y^2)}{ab(x+y)}\times\frac{a^2}{x^2-xy+y^2}=\frac{a}{b}.$

53. $\frac{ab-b^2}{3xy^3}\div\frac{a^2-2ab+b^2}{27x^3y^3}=\frac{b(a-b)}{3xy^3}\times\frac{27x^3y^3}{(a-b)(a-b)}=\frac{9bx^2}{a-b}.$

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54. $\dfrac{1-8x^3}{a-2ax} \div (4x^2+2x+1)$

$$= \frac{(1-2x)(1+2x+4x^2)}{a(1-2x)} \times \frac{1}{4x^2+2x+1} = \frac{1}{a}.$$

55. $(x^2-4y^2z^2) \div \dfrac{3x+6yz}{9a} = (x+2yz)(x-2yz) \times \dfrac{9a}{3(x+2yz)}$

$$= 3a(x-2yz).$$

56. $\dfrac{4a-8b}{(a-b)^2} \div \dfrac{a^2-ab-2b^2}{a^2-b^2} = \dfrac{4(a-2b)}{(a-b)^2} \times \dfrac{(a-b)(a+b)}{(a-2b)(a+b)} = \dfrac{4}{a-b}.$

57. $\dfrac{(r-2s)^2}{r-s} \div \dfrac{r^2-rs-2s^2}{r^2-s^2} = \dfrac{(r-2s)^2}{r-s} \times \dfrac{(r-s)(r+s)}{(r-2s)(r+s)} = r-2s.$

58. $\dfrac{x^2-2xy-3y^2}{x^2+2xy+y^2} \div \dfrac{(x-3y)^2}{x+y} = \dfrac{(x-3y)(x+y)}{(x+y)(x+y)} \times \dfrac{x+y}{(x-3y)^2}$

$$= \frac{1}{x-3y}.$$

59. $\dfrac{2a^2+ab-3b^2}{2a^2+5ab+3b^2} \div \dfrac{a^3-b^3}{a+b}$

$$= \frac{(2a+3b)(a-b)}{(2a+3b)(a+b)} \times \frac{a+b}{(a-b)(a^2+ab+b^2)}$$

$$= \frac{1}{a^2+ab+b^2}.$$

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7. Let $x =$ number of dollars of tax B pays.
Then, $2x =$ number of dollars of tax A pays.
$\therefore x + 2x = 126.$
Solving, $x = 42,$
and $2x = 84.$
Hence, A pays a tax of \$84 and B one of \$42.

8. Let $x =$ number of votes B received.
Then, $x + 72 =$ number of votes A received.
$\therefore x + x + 72 = 584.$
Solving, $x = 256,$
and $x + 72 = 328.$
Hence, 328 votes were cast for A and 256 votes for B.

9. Let $x =$ number of dollars in original price.
Then, $x + 40 =$ number of dollars in present price.
$\therefore 2x - 20 = x + 40.$
Solving, $x = 60.$
Hence, the original price of the typewriter was \$60.

10. Let $x =$ number of dollars mahogany basket costs.
Then, $x - .20x = 8.$
Solving, $x = 10.$
Hence, the cost of the mahogany waste basket is \$10.

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11. Let $x =$ number of years latter kind of shoe had been manufactured.
Then, $2x + 4 = 30.$
Solving, $x = 13.$
Hence, the firm had made the latter kind of shoe for 13 years.

12. Let $x =$ number of miles traveled by trail.
Then, $2x + 5 =$ number of miles traveled by wagon.
$\therefore x + 2x + 5 = 50.$
Solving, $x = 15,$
and $2x + 5 = 35.$
Hence, the distance traveled by trail is 15 miles and by wagon is 35 miles.

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13. Let $x =$ number of dollars in selling price in the United States.
Then, $x + .20x = 3.$
Solving, $x = 2.5.$
Hence, the selling price of the clock in the United States is $2.50.

14. Let $x =$ number of feet in width of rectangle.
Then, $x + 8 =$ number of feet in length of rectangle.
$\therefore 2x + 2(x + 8) = 112.$
Solving, $x = 24,$
and $x + 8 = 32.$
Hence, the rectangle is 32 feet long and 24 feet wide.

15. Let $x =$ number of rods in width of field.
Then, $x + 10 =$ number of rods in length of field.
$\therefore 2x + 2(x + 10) = 132.$
Solving, $x = 28,$
and $x + 10 = 38.$
Hence, the field is 38 rods long and 28 rods wide.

16. Let $x =$ number of feet in side of square.
Then, $x + 120 =$ number of feet in length of rectangle,
and $x - 40 =$ number of feet in width of rectangle.
$\therefore (x + 120)(x - 40) = x^2.$
Solving, $x = 60,$
$x + 120 = 180,$
and $x - 40 = 20.$
Hence, the rectangle is 180 feet by 20 feet and the square 60 feet by 60 feet.

17. Let $x =$ number of rods in width of field.
Then, $x + 5 =$ number of rods in length of field,
$x + 4 =$ number of rods in width when increased,
and $x + 5 + 3$, or $x + 8 =$ number of rods in length when increased.
$\therefore x(x + 5) + 116 = (x + 4)(x + 8).$
Solving, $x = 12,$
and $x + 5 = 17.$
Hence, the field is 17 rods long and 12 rods wide.

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18. Let $x =$ number of feet in width of rectangle.
Then, $x + 4 =$ number of feet in length of rectangle,
$x + 5 =$ number of feet in width when increased,
and $x + 4 - 3$, or $x + 1 =$ number of feet in length when decreased.
$$\therefore x(x+4) + 29 = (x+5)(x+1).$$
Solving, $x = 12$,
and $x + 4 = 16$.
Hence, the rectangle is 16 feet long and 12 feet wide.

19. Let $x =$ number of feet in width of rug.
Then, $x + 3 =$ number of feet in length of rug,
$x + 3 =$ number of feet in width if increased,
and $x + 3 + 3$, or $x + 6 =$ number of feet in length if increased.
$$\therefore x(x+3) + 72 = (x+3)(x+6).$$
Solving, $x = 9$,
and $x + 3 = 12$.
Hence, the rug is 12 feet long and 9 feet wide.

20. Let $x =$ number of yards in width of first field.
Then, $mx =$ number of yards in length of first field,
$x + b =$ number of yards in width of second field,
and $mx + a =$ number of yards in length of second field.
$$\therefore x(mx) + c^2 = (x+b)(mx+a).$$
Solving, $x = \dfrac{c^2 - ab}{a + bm}$,
and $mx = \dfrac{m(c^2 - ab)}{a + bm}$.
Hence, the length of the first field is $\dfrac{m(c^2 - ab)}{a + bm}$ yards and the width $\dfrac{c^2 - ab}{a + bm}$ yards.

21. Let $x =$ number of feet in height of tower.
Then, $x + 69 =$ number of feet in height of main structure.
$$\therefore x + x + 69 = 909.$$
Solving, $x = 420$.
Hence, the height of the tower is 420 feet.

22. Let $x =$ number of miles in length of run.
Then, $\frac{3}{4}x =$ number of extra gallons of gasoline required, if 3 quarts more per mile were used.
$$\therefore 220 + \tfrac{3}{4}x = 352.$$
Solving, $x = 176$.
Hence, the length of the run was 176 miles.

23. Let $x =$ number of feet in width of bridge.
Then, $10x =$ number of feet in length of bridge.
$$\therefore 10x - 50 = 5(x + 5).$$
Solving, $x = 15$,
and $10x = 150$.
Hence, the bridge is 150 feet long.

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24. Let $x =$ number of cents B has.
Then, $\frac{1}{2}x + n =$ number of cents A has.
$\therefore x + \frac{1}{2}x + n = m.$
Solving, $x = \dfrac{2m - 2n}{3},$
and $\dfrac{1}{2}x + n = \dfrac{m + 2n}{3}.$
Hence, A has $\dfrac{m + 2n}{3}$ cents and B has $\dfrac{2m - 2n}{3}$ cents.

25. Let $x =$ number of trees removed.
Then, $x - 200 =$ number of trees left.
$\therefore x + x - 200 = 2868.$
Solving, $x = 1534.$
Hence, 1534 trees were removed from the maple grove.

26. Let $x =$ number of miles of paved streets.
Then, $506 - x =$ number of miles of unpaved streets.
$\therefore x - 20 = 80(506 - x).$
Solving, $x = 500.$
Hence, there were 500 miles of paved streets in the city.

27. Let $x =$ number of miles the train traveled per minute.
Then, $5x =$ number of miles the train traveled in 5 minutes.
$\therefore 5x = 4\frac{1}{2}(x + \frac{1}{5}).$
Solving, $x = 1\frac{4}{5}.$
Hence, the train traveled $1\frac{4}{5}$ miles per minute.

28. Let $x =$ number of feet in original depth of well.
Then, $2x + 4 =$ number of feet in depth of well after it was deepened.
$\therefore 2x + 4 = x + 22.$
Solving, $x = 18,$
and $2x + 4 = 40.$
Hence, the depth of the well after it was deepened was 40 feet.

29. Let $x =$ number of per cent of tin in bronze.
Then, $\frac{3}{7}x =$ number of per cent of zinc in bronze,
and $30(\frac{3}{7}x) =$ number of per cent of copper in bronze.
$\therefore x + \frac{3}{7}x + 30(\frac{3}{7}x) = 100.$
Solving, $x = 7,$
$\frac{3}{7}x = 3,$
and $30(\frac{3}{7}x) = 90.$
Hence, bronze is 90% copper, 3% zinc, and 7% tin.

30. Let $x =$ number of per cent of nickel in a 5-cent piece.
Then, $3x =$ number of per cent of copper in a 5-cent piece.
$\therefore x + 3x = 100.$
Solving, $x = 25,$
and $3x = 75.$
Hence, a 5-cent piece is 25% nickel and 75% copper.

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31. Let $x =$ number of cents in cost of beef.
Then, $2x + 11 = x + 1.01\frac{3}{8}x.$
Solving, $x = 800.$
Hence, the loin of beef cost 800¢, or $8.

32. Let $x =$ number of dollars of capital.
Then, $\frac{4}{5}x =$ number of dollars invested at $4\frac{1}{2}\%$,
and $\frac{1}{5}x =$ number of dollars invested at 4%.

$$\therefore \frac{4\frac{1}{2}}{100}\cdot\frac{4}{5}x + \frac{4}{100}\cdot\frac{1}{5}x = 176.$$

Solving, $x = 4000.$ Hence, his capital was $4000.

33. Let $x =$ number of per cent income.

Then, $\frac{x}{100}\cdot a =$ the annual income from the investment.

$$\therefore \frac{x}{100}\cdot a = 12n.$$

Solving, $x = \frac{1200n}{a}.$

Hence, the annual income is $\frac{1200n}{a}\%$ of the investment.

34. Let $x =$ number of minute spaces the minute hand travels after 7 o'clock before the hands come together.

Then, $\frac{x}{12} =$ number of minute spaces the hour hand travels in the same time.

Since the hands are 35 minute spaces apart at 7 o'clock,

$$x - \frac{x}{12} = 35.$$

Solving, $x = 38\frac{2}{11}.$
Hence, at 7 : $38\frac{2}{11}$ o'clock the hands are together.

35. Let $x =$ number of minute spaces the minute hand travels after 5 o'clock before it is 15 minute spaces *behind* the hour hand.

Then, $\frac{x}{12} =$ number of minute spaces the hour hand travels in the same time.

Since the minute hand gains 25 − 15, or 10, minute spaces,

$$x - \frac{x}{12} = 10.$$

Solving, $x = 10\frac{10}{11}$, the number of minutes after 5 o'clock.
Again, let $x =$ number of minute spaces the minute hand travels before it is 15 minute spaces *ahead* of the hour hand.

Then, $\frac{x}{12} =$ number of minute spaces the hour hand travels in the same time.

Since the minute hand gains 25 + 15, or 40, minute spaces,

$$x - \frac{x}{12} = 40.$$

Solving, $x = 43\frac{7}{11}$, the number of minutes after 5 o'clock.
Hence, the required times are 5 : $10\frac{10}{11}$ o'clock and 5 : $43\frac{7}{11}$ o'clock.

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36. Let $x =$ number of minute spaces the minute hand travels after 7 o'clock before it is 14 minute spaces *behind* the hour hand.

Then, $\frac{x}{12} =$ number of minute spaces the hour hand travels in the same time.

Since the minute hand gains 35 − 14, or 21, minute spaces,

$$x - \frac{x}{12} = 21.$$

Solving, $x = 22\frac{10}{11}$, the number of minutes after 7 o'clock.

Again, let $x =$ number of minute spaces the minute hand travels after 7 o'clock before it is 14 minute spaces *ahead* of the hour hand.

Then, $\frac{x}{12} =$ number of minute spaces the hour hand travels in the same time.

Since the minute hand gains 35 + 14, or 49, minute spaces,

$$x - \frac{x}{12} = 49.$$

Solving, $x = 53\frac{5}{11}$, the number of minutes after 7 o'clock.

Hence, the required times are 7 : $22\frac{10}{11}$ o'clock and 7 : $53\frac{5}{11}$ o'clock.

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37. Let $x =$ the required number of days.

Then, $\frac{1}{x} =$ the part of the work all can do in 1 day.

$$\therefore \frac{1}{x} = \frac{1}{4\frac{1}{2}} + \frac{1}{5} + \frac{1}{7\frac{1}{2}}.$$

Solving, $x = 1\frac{4}{5}$.

Hence, A, B, and C together can do the work in $1\frac{4}{5}$ days.

38. Let $x =$ the required number of days.

Then, $\frac{1}{x} =$ the part of the work all can do in 1 day.

$$\therefore \frac{1}{x} = \frac{1}{c} + \frac{1}{e} + \frac{1}{r}.$$

Solving, $$x = \frac{cer}{er + cr + ce}.$$

Hence, A, B, and C together can do the work in $\frac{cer}{er + cr + ce}$ days.

39. Let $x =$ the required number of hours.

Then, $\frac{1}{x} =$ the part of the cistern that can be filled in 1 hour.

$$\therefore \frac{1}{x} = \frac{1}{m} + \frac{1}{n}.$$

Solving, $$x = \frac{mn}{m + n}.$$

Hence, both pipes flowing together fill the cistern in $\frac{mn}{m + n}$ hours.

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40. Let $x =$ the digit in tens' place.

Then, $x + 2 =$ the digit in units' place,

and $10x + x + 2$, or $11x + 2 =$ the number.

$$\therefore \frac{11x + 2 - 6}{x + x + 2} = 4.$$

Solving, $x = 4$,

and $x + 2 = 6$.

Hence, the number is 46.

41. Let $x =$ the digit in units' place.

Then, $x + 4 =$ the digit in tens' place,

$10(x + 4) + x$, or $11x + 40 =$ the number,

and $10x + x + 4$, or $11x + 4 =$ the number with digits reversed.

$$\therefore 11x + 4 = \tfrac{4}{7}(11x + 40).$$

Solving, $x = 4$,

and $x + 4 = 8$.

Hence, the number is 84.

42. Let $x =$ number of dollars at which property is valued.

Then, $\frac{2}{5}x =$ number of dollars invested at 4 %,

$\frac{1}{3}x =$ number of dollars invested at 5 %,

and $\frac{4}{15}x =$ number of dollars invested at 3 %.

$$\therefore \tfrac{4}{100} \cdot \tfrac{2}{5}x + \tfrac{5}{100} \cdot \tfrac{1}{3}x + \tfrac{3}{100} \cdot \tfrac{4}{15}x = 610.$$

Solving, $x = 15{,}000$.

Hence, the man's property was valued at \$ 15,000.

43. Let $x =$ number of miles first man travels per hour.

Then, $3x =$ number of miles second man travels per hour.

$$\therefore 6(3x) + 6x = 360.$$

Solving, $x = 15$,

and $3x = 45$.

Hence, the first man travels 15 miles per hour, and the second 45 miles per hour.

44. Let $x =$ number of miles in distance to village.

Then, $\frac{x}{4} =$ number of hours required to walk to village.

and $\frac{x}{7} =$ the number of hours required to ride back.

$$\therefore \frac{x}{4} + \frac{x}{7} = 1\tfrac{3}{4}.$$

Solving, $x = 4\tfrac{5}{11}$.

Hence, the village was $4\tfrac{5}{11}$ miles away.

45. Let $x =$ number of ounces of copper to be added.

Then, $40 + x =$ number of ounces in new alloy,

and $\frac{8}{40 + x} =$ part of the new alloy that is silver.

$$\therefore \frac{8}{40 + x} = \frac{4}{60}.$$

Solving, $x = 80$.

Hence, 80 ounces of copper must be added.

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46. Let $x =$ number of ounces of alcohol to be added.
Then, $x + 1 =$ number of ounces of alcohol in new mixture,
and $x + 5 =$ number of ounces new mixture weighs.

$$\therefore \frac{x+1}{x+5} = \frac{40}{100}.$$

Solving, $x = 1\frac{2}{3}$.
Hence, $1\frac{2}{3}$ ounces of alcohol must be added.

47. Let $x =$ number of men on a side at first.
Then, $x^2 =$ number of men at first,
$x - 1 =$ number of men on a side when rearranged,
and $(x-1)^2 + 1 =$ number of men left after the battle.

$$\therefore x^2 - 60 = (x-1)^2 + 1.$$

Solving, $x = 31$,
and $x^2 = 961$.
Hence, there were 961 men in the regiment.

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1. $\begin{cases} 3x + 4y = 29, & (1) \\ 3x - y = 4. & (2) \end{cases}$
(1)−(2), $5y = 25$.
$y = 5$. (3)
Substituting (3) in (2), $x = 3$.

2. $\begin{cases} 2x + y = -2, & (1) \\ 5x + 3y = -3. & (2) \end{cases}$
(1) × 3, $6x + 3y = -6$. (3)
(3)−(2), $x = -3$. (4)
Substituting (4) in (1), $y = 4$.

3. $\begin{cases} 6x + 5y = 13, & (1) \\ 4x + 2y = 10. & (2) \end{cases}$
(2) × $\frac{3}{2}$, $6x + 3y = 15$. (3)
(1)−(3), $2y = -2$.
$y = -1$. (4)
Substituting (4) in (2), $x = 3$.

4. $\begin{cases} 4x + y = 7, & (1) \\ 6x + 2y = 13. & (2) \end{cases}$
(1) × 2, $8x + 2y = 14$. (3)
[(3)−(2)] ÷ 2, $x = \frac{1}{2}$. (4)
Substituting (4) in (1), $y = 5$.

5. $\begin{cases} x + y = -5, & (1) \\ 3x + 2y = -13. & (2) \end{cases}$
(1) × 2, $2x + 2y = -10$. (3)
(2)−(3), $x = -3$. (4)
Substituting (4) in (1), $y = -2$.

6. $\begin{cases} 2x - 4y = 5, & (1) \\ 5x + 8y = 17. & (2) \end{cases}$
(1) × 2, $4x - 8y = 10$. (3)
(2) + (3), $9x = 27$.
$x = 3$. (4)
Substituting (4) in (1), $y = \frac{1}{4}$.

7. $\begin{cases} 3a - 6 = 2b, & (1) \\ 4a - 3 = 3b. & (2) \end{cases}$
(1) × 3, $9a - 18 = 6b$. (3)
(2) × 2, $8a - 6 = 6b$. (4)
(3)−(4), $a = 12$. (5)
Substituting (5) in (1), $b = 15$.

8. $\begin{cases} .5x + .2y = 2, & (1) \\ x - y = 18. & (2) \end{cases}$
(2) × .2, $.2x - .2y = 3.6$. (3)
(1) + (3), $.7x = 5.6$.
$x = 8$. (4)
Substituting (4) in (2), $y = -10$.

9. $\begin{cases} 3m + 6n = 5, & (1) \\ 6m + 4 = 9n. & (2) \end{cases}$
(1) × 2, $6m + 12n = 10$. (3)
(3)−(2), $21n = 14$.
$n = \frac{2}{3}$. (4)
Substituting (4) in (1), $m = \frac{1}{3}$.

10. $\begin{cases} .4r - .3s = -1, & (1) \\ .6r - .3s = 0. & (2) \end{cases}$
(2)−(1), $.2r = 1$.
$r = 5$. (3)
Substituting (3) in (1), $s = 10$.

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11. $\begin{cases} 3p + 4q = 100, & (1) \\ 8p - 3q = 48. & (2) \end{cases}$

$(1) \times 3,\quad 9p + 12q = 300. \quad (3)$
$(2) \times 4,\quad 32p - 12q = 192. \quad (4)$
$(3) + (4),\quad 41p = 492.$
$p = 12. \quad (5)$

Substituting (5) in (1),
$q = 16.$

12. $\begin{cases} 8a + 81 = -5b, & (1) \\ 4a + 23 = b. & (2) \end{cases}$

$(2) \times 2,\quad 8a + 46 = 2b. \quad (3)$
$(1) - (3),\quad -7b = 35.$
$b = -5. \quad (4)$

Substituting (4) in (2),
$a = -7.$

13. $\begin{cases} x + 3y = 9, & (1) \\ 2x + 4y = 14. & (2) \end{cases}$

From (1), $x = 9 - 3y. \quad (3)$
Substituting (3) in (2),
$18 - 6y + 4y = 14.$
$y = 2. \quad (4)$
Substituting (4) in (3),
$x = 3.$

14. $\begin{cases} 3x + 4y = 34, & (1) \\ x - y = -5. & (2) \end{cases}$

From (2), $x = y - 5. \quad (3)$
Substituting (3) in (1),
$3y - 15 + 4y = 34.$
$y = 7. \quad (4)$
Substituting (4) in (3),
$x = 2.$

15. $\begin{cases} x - 5 = 2y, & (1) \\ 2x + 6y = 15. & (2) \end{cases}$

From (1), $x = 2y + 5. \quad (3)$
Substituting (3) in (2),
$4y + 10 + 6y = 15.$
$y = \frac{1}{2}. \quad (4)$
Substituting (4) in (3),
$x = 6.$

16. $\begin{cases} y + 2x = -8, & (1) \\ x + 2y = -7. & (2) \end{cases}$

From (1), $y = -2x - 8. \quad (3)$
Substituting (3) in (2),
$x - 4x - 16 = -7.$
$x = -3. \quad (4)$
Substituting (4) in (3),
$y = -2.$

17. $\begin{cases} 3a - 4b = 0, & (1) \\ 2a = b + 10. & (2) \end{cases}$

From (1), $a = \frac{4}{3}b. \quad (3)$
Substituting (3) in (2),
$\frac{8}{3}b = b + 10.$
$b = 6. \quad (4)$
Substituting (4) in (3),
$a = 8.$

18. $\begin{cases} 4c - 3d = 1, & (1) \\ 2c = 3d. & (2) \end{cases}$

Substituting (2) in (1),
$6d - 3d = 1.$
$d = \frac{1}{3}. \quad (3)$
Substituting (3) in (2),
$c = \frac{1}{2}.$

19. $\begin{cases} y - x = 11, & (1) \\ x + 3y = 1. & (2) \end{cases}$

From (1), $y = x + 11. \quad (3)$
Substituting (3) in (2),
$x + 3x + 33 = 1.$
$x = -8. \quad (4)$
Substituting (4) in (3),
$y = 3.$

20. $\begin{cases} x + y = 0, & (1) \\ 3y + 2x = 4. & (2) \end{cases}$

From (1), $x = -y. \quad (3)$
Substituting (3) in (2),
$3y - 2y = 4.$
$y = 4. \quad (4)$
Substituting (4) in (1),
$x = -4.$

21. $\begin{cases} 48 = 3p + 2q, & (1) \\ 4 = 5p - 3q. & (2) \end{cases}$

From (2), $p = \dfrac{3q + 4}{5}. \quad (3)$
Substituting (3) in (1),
$$48 = 3\left(\frac{3q + 4}{5}\right) + 2q.$$
$q = 12. \quad (4)$
Substituting (4) in (3),
$p = 8.$

22. $\begin{cases} 4x - 13 = -y, & (1) \\ y - 8 = 16x. & (2) \end{cases}$

From (2), $y = 16x + 8. \quad (3)$
Substituting (3) in (1),
$4x - 13 = -16x - 8.$
$x = \frac{1}{4}. \quad (4)$
Substituting (4) in (3),
$y = 12.$

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1. Let $x =$ the larger number,
and $y =$ the smaller number.
Then, $x + y = 24,$ (1)
and $x - y = 12.$ (2)
Adding (1) and (2), $2x = 36$; $\therefore x = 18$, the larger number.
Substituting 18 for x in (1),
$18 + y = 24$; $\therefore y = 6$, the smaller number.

2. Let $x =$ the larger number,
and $y =$ the smaller number.
Then, $x + y = 18,$ (1)
and $\frac{1}{3}(x - y) = 2.$ (2)
Multiplying (2) by 3, $x - y = 6.$ (3)
Adding (1) and (3), $2x = 24$; $\therefore x = 12$, the larger number.
Substituting 12 for x in (1),
$12 + y = 18$; $\therefore y = 6$, the smaller number.

3. Let $x =$ the larger number,
and $y =$ the smaller number.
Then, $x + y = s,$ (1)
and $x - y = d.$ (2)
Adding (1) and (2), $2x = s + d$; $\therefore x = \frac{s + d}{2}$, the larger number.
Subtracting (2) from (1), $2y = s - d$; $\therefore y = \frac{s - d}{2}$, the smaller number.

4. Let $x =$ number of inches in length of rectangle,
and $y =$ number of inches in width of rectangle.
Then, $x - y = 14,$ (1)
and $2x + 2y = 124.$ (2)
Dividing (2) by 2, $x + y = 62.$ (3)
Adding (1) and (3), $2x = 76$; $\therefore x = 38.$
Substituting 38 for x in (1),
$38 - y = 14$; $\therefore y = 24.$
Hence, the rectangle is 38 inches long and 24 inches wide.

5. Let $x =$ number of 2-dollar bills,
and $y =$ number of 5-dollar bills.
Then, $x + y = 35,$ (1)
and $2x + 5y = 100.$ (2)
Multiplying (1) by 2, $2x + 2y = 70.$ (3)
Subtracting (3) from (2), $3y = 30$; $\therefore y = 10.$
Substituting 10 for y in (1), $x + 10 = 35$; $\therefore x = 25.$
Hence, the man deposited 25 2-dollar bills and 10 5-dollar bills.

6. Let $x =$ number of pounds one woman shelled,
and $y =$ number of pounds other woman shelled.
Then, $x + y = 165,$ (1)
and $x + 10 = y - 5.$ (2)
Adding (1) and (2), $2x = 150$; $\therefore x = 75.$
Substituting 75 for x in (1),
$75 + y = 165$; $\therefore y = 90.$
Hence, one woman shelled 75 pounds of hazelnuts and the other woman shelled 90 pounds.

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7. Let $x =$ number of ounces larger apple weighed,
and $y =$ number of ounces smaller apple weighed.
Then, $x + y = 64$, (1)
and $y - 13 = \frac{1}{2}x$. (2)
From (2), $x = 2y - 26$. (3)
Substituting (3) in (1), $2y - 26 + y = 64$; $\therefore y = 30$.
Substituting 30 for y in (1), $x + 30 = 64$; $\therefore x = 34$.
Hence, the larger apple weighed 34 ounces and the smaller 30 ounces.

8. Let $x =$ number of dollars in one man's weekly wage,
and $y =$ number of dollars in other's weekly wage.
Then, $x - y = 6$, (1)
and $5x = 8y$. (2)
From (1), $x = y + 6$. (3)
Substituting (3) in (2), $5y + 30 = 8y$; $\therefore y = 10$.
Substituting 10 for y in (1), $x - 10 = 6$; $\therefore x = 16$.
Hence, one man earned \$16 per week and the other \$10 per week.

9. Let $x =$ one number,
and $y =$ the other number.
Then, $2x + 5y = 20$, (1)
and $3x + 4y = 23$. (2)
Multiplying (1) by 3, $6x + 15y = 60$. (3)
Multiplying (2) by 2, $6x + 8y = 46$. (4)
Subtracting (4) from (3), $7y = 14$; $\therefore y = 2$.
Substituting 2 for y in (1), $2x + 10 = 20$; $\therefore x = 5$.
Hence, one number is 5 and the other 2.

10. Let $x =$ number of knots liner made the first day,
and $y =$ number of knots liner made the second day.
Then, $x + y = 762$, (1)
and $y - x = 152$. (2)
Adding (1) and (2), $2y = 914$; $\therefore y = 457$.
Substituting 457 for y in (1), $x + 457 = 762$; $\therefore x = 305$.
Hence, the liner made 305 knots the first day and 457 knots the second day.

11. Let $x =$ number of feet in length of lock,
and $y =$ number of feet in width of lock.
Then, $x = 7\frac{1}{2}y$, (1)
and $y - 5 = \frac{1}{8}x$. (2)
Substituting (1) in (2), $y - 5 = \frac{1}{8}(7\frac{1}{2}y)$; $\therefore y = 80$.
Substituting 80 for y in (1), $x = 7\frac{1}{2}(80)$; $\therefore x = 600$.
Hence, the lock is 600 feet long and 80 feet wide.

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12. Let $x =$ number of cents in cost of ribbon per yard,
and $y =$ number of cents in cost of lace per yard.
Then, $3x + 2y = 164$, (1)
and $2x - 3y = 1$. (2)
Multiplying (1) by 2, $6x + 4y = 328$. (3)
Multiplying (2) by 3, $6x - 9y = 3$. (4)
Subtracting (4) from (3), $13y = 325$; $\therefore y = 25$.
Substituting 25 for y in (2), $2x - 75 = 1$; $\therefore x = 38$.
Hence, ribbon cost 38¢ per yard and lace 25¢.

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13. Let $x =$ number of bushels a cotton bag holds,
and $y =$ number of bushels a burlap bag holds.

Then, $3x + 2y = 15\frac{1}{2}$, (1)
and $4x + 3y = 22$. (2)
Multiplying (1) by 4, $12x + 8y = 62$. (3)
Multiplying (2) by 3, $12x + 9y = 66$. (4)
Subtracting (3) from (4), $y = 4$.
Substituting 4 for y in (2), $4x + 12 = 22$; $\therefore x = 2\frac{1}{2}$.
Hence, a cotton grain bag holds $2\frac{1}{2}$ bushels and a burlap bag 4 bushels.

14. Let $x =$ number of cents best quality of matches cost per box,
and $y =$ number of cents poorest quality of matches cost.

Then, $10x + 4y = 24$, (1)
and $8x + 8y = 24$. (2)
Multiplying (1) by 2, $20x + 8y = 48$. (3)
Subtracting (2) from (3), $12x = 24$; $\therefore x = 2$.
Substituting 2 for x in (2), $16 + 8y = 24$; $\therefore y = 1$.
Hence, the best quality of matches cost 2 ¢ per box, and the poorest 1 ¢.

15. Let $x =$ rate of bird in miles per hour,
$y =$ rate of wind in miles per hour in first case,
and $2y =$ rate of wind in miles per hour in second case.

Then, $x + y = 55$, (1)
and $x - 2y = 30$. (2)
Subtracting (2) from (1), $3y = 25$; $\therefore y = 8\frac{1}{3}$, and $2y = 16\frac{2}{3}$.
Hence, the rate of the wind was $8\frac{1}{3}$ miles per hour in the first case and $16\frac{2}{3}$ miles per hour in the second case.

16. Let $x =$ rate of rowing in still water in miles per hour,
and $y =$ rate of current in miles per hour.

Then, $x + y = \frac{m}{c}$, (1)

and $x - y = \frac{m}{d}$. (2)

Adding (1) and (2), $2x = \frac{m}{c} + \frac{m}{d}$; $\therefore x = \frac{cm + dm}{2cd}$.

Subtracting (2) from (1), $2y = \frac{m}{c} - \frac{m}{d}$; $\therefore y = \frac{dm - cm}{2cd}$.

Hence, the man's rate of rowing in still water is $\frac{cm+dm}{2cd}$ miles per hour, and the rate of the current is $\frac{dm - cm}{2cd}$ miles per hour.

17. Let $x =$ number of feet in length of pipe line,
and $y =$ number of feet in diameter of pipe line.

Then, $x + 100 = 200y$, (1)
and $x = 100(y + 11)$. (2)
Substituting (2) in (1), $100(y + 11) + 100 = 200y$; $\therefore y = 12$.
Substituting 12 for y in (1), $x + 100 = 2400$; $\therefore x = 2300$.
Hence, the pipe line is 2300 feet long and 12 feet in diameter.

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18. Let x = weight of cast iron in pounds per cubic foot,
and y = weight of wrought iron in pounds per cubic foot.

Then, $y - x = 30,$ (1)
and $16x = 15y.$ (2)
From (2), $x = \frac{15}{16}y.$ (3)
Substituting (3) in (1), $y - \frac{15}{16}y = 30$; $\therefore y = 480.$
Substituting 480 for y in (1), $480 - x = 30$; $\therefore x = 450.$
Hence, cast iron weighs 450 pounds per cubic foot and wrought iron 480 pounds per cubic foot.

19. Let x = number of per cent of water in watermelons,
and y = number of per cent of water in currants.

Then, $x - y = 10,$ (1)
and $17x = 19y.$ (2)
From (2), $x = \frac{19}{17}y.$ (3)
Substituting (3) in (1), $\frac{19}{17}y - y = 10$; $\therefore y = 85.$
Substituting 85 for y in (1), $x - 85 = 10$; $\therefore x = 95.$
Hence, watermelons are 95% water and currants are 85% water.

20. Let x = weight of larger brick in pounds,
and y = weight of smaller brick in pounds.

Then, $x - y = 2\frac{1}{2},$ (1)
and $6x + 5y = 92.$ (2)
Multiplying (1) by 6, $6x - 6y = 15.$ (3)
Subtracting (3) from (2), $11y = 77$; $\therefore y = 7.$
Substituting 7 for y in (1), $x - 7 = 2\frac{1}{2}$; $\therefore x = 9\frac{1}{2}.$
Hence, the larger brick weighs $9\frac{1}{2}$ pounds and the smaller 7 pounds.

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21. Let x = one number,
and y = the other number.

Then, $\frac{1}{x} + \frac{1}{y} = 4\frac{1}{8},$ (1)
and $x = 32y.$ (2)
Substituting (2) in (1), $\frac{1}{32y} + \frac{1}{y} = 4\frac{1}{8}.$
Solving, $y = \frac{1}{4}.$
Substituting $\frac{1}{4}$ for y in (2), $x = 8.$
Hence, the numbers are 8 and $\frac{1}{4}$.

22. Let x = one number,
and y = the other number.

Then, $\frac{1}{x} + \frac{1}{y} = m,$ (1)
and $x = ly.$ (2)
Substituting (2) in (1), $\frac{1}{ly} + \frac{1}{y} = m$; $\therefore y = \frac{l+1}{ml}.$
Substituting $\frac{l+1}{ml}$ for y in (2), $x = \frac{l+1}{m}.$
Hence, the numbers are $\frac{l+1}{m}$ and $\frac{l+1}{ml}.$

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23. Let $x =$ the digit in tens' place,
and $y =$ the digit in units' place.
Then, $x + y = 12,$ (1)
and $y = 2x.$ (2)
Substituting (2) in (1), $x + 2x = 12; \therefore x = 4.$
Substituting 4 for x in (2), $y = 8.$
Hence, the number is 48.

24. Let $x =$ the digit in tens' place,
and $y =$ the digit in units' place.
Then, $10x + y =$ the number,
and $10y + x =$ the number with digits reversed.
$\therefore 10x + y = 8(x + y),$ (1)
and $10x + y - 45 = 10y + x.$ (2)
Reducing (1), $x = \frac{7}{2}y.$ (3)
Reducing (2), $x - y = 5.$ (4)
Substituting (3) in (4), $\frac{7}{2}y - y = 5; \therefore y = 2.$
Substituting 2 for y in (3), $x = 7.$
Hence, the number is 72.

25. Let $x =$ one number,
and $y =$ the other number.
Then, $\frac{3}{x} + \frac{6}{y} = 4,$ (1)
and $x = 1\frac{1}{2}y.$ (2)
Substituting (2) in (1), $\frac{3}{1\frac{1}{2}y} + \frac{6}{y} = 4.$
Solving, $y = 2.$
Substituting 2 for y in (2), $x = 3.$
Hence, the numbers are 3 and 2.

26. Let $\frac{x}{y} =$ the fraction.
Then, $\frac{x - 1}{y} = \frac{5}{6},$ (1)
and $\frac{x}{y + 10} = \frac{1}{2}.$ (2)
Reducing (1), $6x - 5y = 6.$ (3)
Reducing (2), $2x - y = 10.$ (4)
Multiplying (4) by 3, $6x - 3y = 30.$ (5)
Subtracting (3) from (5), $2y = 24; \therefore y = 12.$
Substituting 12 for y in (4), $2x - 12 = 10; \therefore x = 11.$
Hence, the fraction is $\frac{11}{12}$.

27. Let $x =$ number of smallest size of nuts per pound,
and $y =$ number of largest size of nuts per pound.
Then, $x - y = 75,$ (1)
and $2x + 3y = 275.$ (2)
Multiplying (1) by 3, $3x - 3y = 225.$ (3)
Adding (2) and (3), $5x = 500; \therefore x = 100.$
Substituting 100 for x in (1), $100 - y = 75; \therefore y = 25.$
Hence, there are 100 of the smallest size of nuts per pound and 25 of the largest size of nuts per pound.

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28. Let $x =$ number of minutes required to unload a car of grain in bulk,
and $y =$ number of minutes required to unload a car of sacked grain.

Then, $x - y = 50,$ (1)
and $9y = 4x.$ (2)
From (2), $y = \frac{4}{9}x.$ (3)
Substituting (3) in (1), $x - \frac{4}{9}x = 50; \therefore x = 90.$
Substituting 90 for x in (3), $y = 40.$
Hence, it takes 40 minutes to unload a car of sacked grain and 90 minutes to unload a car of grain in bulk.

29. Let $x =$ number of tons of steel in the bridge,
and $y =$ number of tons of iron in the bridge.

Then, $x + y = 5600,$ (1)
and $17x = 11y.$ (2)
From (2), $x = \frac{11}{17}y.$ (3)
Substituting (3) in (1), $\frac{11}{17}y + y = 5600; \therefore y = 3400.$
Substituting 3400 for y in (3), $x = 2200.$
Hence, there are 2200 tons of steel and 3400 tons of iron in the bridge.

30. Let $x =$ number of seats in the bleachers,
and $y =$ the total number of seats at the grounds.

Then, $x - 2000 = \frac{1}{5}y,$ (1)
and $y + 1000 = 4x.$ (2)
From (2), $y = 4x - 1000.$ (3)
Substituting (3) in (1), $x - 2000 = \frac{1}{5}(4x - 1000).$
Solving, $x = 9000.$
Hence, the bleachers will seat 9000 people.

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31. Let $x =$ number of dollars invested at 5%,
and $y =$ number of dollars invested at 3%.

Then, $x + y = 3250,$ (1)
and $.05x + .03y = 117.50.$ (2)
Multiplying (1) by .03, $.03x + .03y = 97.50.$ (3)
Subtracting (3) from (2), $.02x = 20; \therefore x = 1000.$
Substituting 1000 for x in (1),
$$1000 + y = 3250; \therefore y = 2250.$$
Hence, the man invested \$1000 at 5% and \$2250 at 3%.

32. Let $x =$ number of dollars invested at 4%,
and $y =$ number of dollars invested at 3%.

Then, $x + y = 3500,$ (1)
and $.04x = .03y.$ (2)
From (2), $x = \frac{3}{4}y.$ (3)
Substituting (3) in (1), $\frac{3}{4}y + y = 3500; \therefore y = 2000.$
Substituting 2000 for y in (3), $x = 1500.$
Hence, the man invested \$1500 at 4% and \$2000 at 3%.

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33. Let $x =$ the number of bridges,
nd $y =$ the number of tunnels.

Then, $x - 15y = 23,$ (1)
nd $y + 5 = \frac{1}{14}x.$ (2)
From (2), $x = 14y + 70.$ (3)
Substituting (3) in (1), $14y + 70 - 15y = 23; \therefore y = 47.$
Substituting 47 for y in (3), $x = 658 + 70; \therefore x = 728.$
Hence, there were 728 bridges and 47 tunnels on the railroad.

34. Let $x =$ number of pounds of tin,
nd $y =$ number of pounds of lead.

Then, $x + y = 24,$ (1)
nd $.137x + .089y = 2.904.$ (2)
Multiplying (1) by .089, $.089x + .089y = 2.136.$ (3)
Subtracting (3) from (2), $.048x = .768; \therefore x = 16.$
Substituting 16 for x in (1), $16 + y = 24; \therefore y = 8.$
Hence, there were 16 pounds of tin and 8 pounds of lead in the piece.

35. Let $x =$ number of feet in length of pier over land,
nd $y =$ number of feet in length of pier over water.

Then, $x - y = 70,$ (1)
nd $\frac{2}{3}(x + y) = y + 190.$ (2)
Clearing (2) of fractions, etc., $2x - y = 570.$ (3)
From (1), $y = x - 70.$ (4)
Substituting (4) in (3), $2x - (x - 70) = 570; \therefore x = 500.$
Substituting 500 for x in (4), $y = 430.$
Hence, the total length of the pier is 500 feet + 430 feet, or 930 feet.

36. Let $x =$ number of pounds of Canadian bluegrass seed,
and $y =$ number of pounds of Kentucky bluegrass seed.

Then, $x - y = 3,$ (1)
and $5x + 15y = 7(x + y).$ (2)
Reducing (2), $x = 4y.$ (3)
Substituting (3) in (1), $4y - y = 3; \therefore y = 1.$
Substituting 1 for y in (3), $x = 4.$
Hence, the mixture contained 4 pounds of Canadian bluegrass seed and 1 pound of Kentucky bluegrass seed.

37. Let $x =$ capacity of first pipe in gallons per minute,
and $y =$ capacity of second pipe in gallons per minute.

Then, $rx - sy = t,$ (1)
and $sx - uy = v.$ (2)
Multiplying (1) by u, $rux - suy = tu.$ (3)
Multiplying (2) by s, $s^2x - suy = sv.$ (4)
Subtracting (4) from (3), $rux - s^2x = tu - sv; \therefore x = \frac{tu - sv}{ru - s^2}.$
Multiplying (1) by s, $rsx - s^2y = st.$ (5)
Multiplying (2) by r, $rsx - ruy = rv.$ (6)
Subtracting (6) from (5), $ruy - s^2y = st - rv; \therefore y = \frac{st - rv}{ru - s^2}.$

Hence, the capacity of the first pipe is $\frac{tu - sv}{ru - s^2}$ gallons per minute and of the second $\frac{st - rv}{ru - s^2}$ gallons per minute.

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38. Let $x =$ the digit in tens' place,
and $y =$ the digit in units' place.
Then, $10x + y =$ total height of monument in feet,
and $10y + x =$ height of shaft in feet.
Also, $x + y = 15,$ (1)
and $10x + y - (10y + x) = 27.$ (2)
Reducing (2), $x - y = 3.$ (3)
Adding (1) and (3), $2x = 18; \therefore x = 9.$
Substituting 9 for x in (1), $9 + y = 15; \therefore y = 6.$
Hence, the total height of the monument is 96 feet.

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1. Let $x =$ first part,
$y =$ second part,
and $z =$ third part.
Then, $x + y + z = 432,$ (1)
$x + \frac{1}{5}y + \frac{1}{2}z = 257,$ (2)
and $z + \frac{1}{2}y + \frac{1}{2}x = 267.$ (3)
Multiplying (3) by 2, $2z + y + x = 534.$ (4)
Subtracting (1) from (4), $z = 102.$ (5)
Substituting (5) in (1), $x + y = 330.$ (6)
Substituting (5) in (2), $x + \frac{1}{5}y = 206.$ (7)
Subtracting (7) from (6), $\frac{4}{5}y = 124; \therefore y = 155.$
Substituting 155 for y in (6), $x + 155 = 330; \therefore x = 175.$
Hence, the parts of 432 are 175, 155, and 102.

2. Let $x =$ number of cents Henry has,
$y =$ number of cents James has,
and $z =$ number of cents Ralph has.
Then, $x + y + z = 50,$ (1)
$x + z = 35,$ (2)
and $y + z = 40.$ (3)
Subtracting (3) from (1), $x = 10.$ (4)
Subtracting (2) from (1), $y = 15.$ (5)
Substituting (4) and (5) in (1), $z = 25.$
Hence, Henry has 10 ¢, James has 15 ¢, and Ralph has 25 ¢.

3. Let $x =$ number of marbles first boy has,
$y =$ number of marbles second boy has,
and $z =$ number of marbles third boy has.
Then, $x + y + z = 57,$ (1)
$x - 1 - 5 = y + 1,$ (2)
and $x - 1 - 5 = z + 5.$ (3)
Transposing in (2), $x - y = 7.$ (4)
Transposing in (3), $x - z = 11.$ (5)
Adding (4) and (1), $2x + z = 64.$ (6)
Adding (5) and (6), $3x = 75; \therefore x = 25.$
Substituting 25 for x in (4), $25 - y = 7; \therefore y = 18.$
Substituting 25 for x in (5), $25 - z = 11; \therefore z = 14.$
Hence, the first boy had 25 marbles, the second 18, and the third 14.

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4. Let $x =$ daily wages of A in dollars,
$y =$ daily wages of B in dollars,
and $z =$ daily wages of C in dollars.

Then, $x + y + z = 7.50,$ (1)
$x + y = 2z,$ (2)
and $x + z = y + 1.50.$ (3)

Subtracting (2) from (1), $3z = 7.50$; $\therefore z = 2.50.$
Subtracting (3) from (1), $2y = 6$; $\therefore y = 3.$
Substituting 3 for y and 2.50 for z in (1),
$$x + 3 + 2.50 = 7.50; \therefore x = 2.$$
Hence, A earns $2 per day, B $3, and C $2.50.

5. Let $x =$ number of dollars house cost,
$y =$ number of dollars garden cost,
and $z =$ number of dollars stable cost.

Then, $x + y + z = 12{,}000,$ (1)
$x = 2z,$ (2)
and $y + 250 = \frac{1}{2}z.$ (3)
From (3), $y = \frac{1}{2}z - 250.$ (4)
Substituting (2) and (4) in (1),
$$2z + \tfrac{1}{2}z - 250 + z = 12{,}000; \therefore z = 3500.$$
Substituting 3500 for z in (2), $x = 7000.$
Substituting 3500 for z in (4), $y = 1750 - 250 = 1500.$
Hence, the house cost $7000, the garden $1500, and the stable $3500.

6. Let $x =$ first part, $y =$ second part, and $z =$ third part.

Then, $x + y + z = n,$ (1)
$\frac{x}{y} = a,$ (2)
and $\frac{y}{z} = b.$ (3)
From (2), $x = ay.$ (4)
From (3), $z = \frac{y}{b}.$ (5)
(4) and (5) in (1), $ay + y + \frac{y}{b} = n$; $\therefore y = \frac{bn}{ab + b + 1}.$
$\frac{bn}{ab + b + 1}$ in (4) and (5), $x = \frac{abn}{ab + b + 1}$ and $z = \frac{n}{ab + b + 1}.$

7. Let $x =$ digit in hundreds' place,
$y =$ digit in tens' place,
and $z =$ digit in units' place.

Then, $100x + 10y + z =$ the number,
and $100z + 10y + x =$ the number with digits reversed.
Also, $x + y + z = 8,$ (1)
$z - y = 3,$ (2)
and $100x + 10y + z + 396 = 100z + 10y + x.$ (3)
Transposing in (3), etc., $x - z = -4.$ (4)
Adding (1) and (2), $x + 2z = 11.$ (5)
Subtracting (4) from (5), $3z = 15$; $\therefore z = 5.$
Substituting 5 for z in (4), $x - 5 = -4$; $\therefore x = 1.$
Substituting 5 for z in (2), $5 - y = 3$; $\therefore y = 2.$
Hence, the number is 125.

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8. Let $x =$ number of 1-dollar bills,

$y =$ number of 10-dollar bills,

and $z =$ number of 2-dollar bills.

Then, $x + y + z = 20,$ (1)

$x + 10y + 2z = 52,$ (2)

and $x = 4y.$ (3)

Multiplying (1) by 2 and subtracting the result from (2),

$8y - x = 12.$ (4)

Substituting (3) in (4), $8y - 4y = 12;\ \therefore y = 3.$

Substituting 3 for y in (3), $x = 12.$

Substituting 3 for y and 12 for x in (1),

$12 + 3 + z = 20;\ \therefore z = 5.$

Hence, the man deposited 12 1-dollar bills, 3 10-dollar bills, and 5 2-dollar bills.

9. Let $x =$ capacity of smallest box in pounds,

$y =$ capacity of middle-sized box in pounds,

and $z =$ capacity of largest box in pounds.

Then, $3x + 2z = 55,$ (1)

$2y + 4x = 50,$ (2)

and $z + 3y = 65.$ (3)

Multiplying (3) by 2, $2z + 6y = 130.$ (4)

Subtracting (1) from (4), $6y - 3x = 75.$ (5)

Dividing (5) by 3, $2y - x = 25.$ (6)

Subtracting (6) from (2), $5x = 25;\ \therefore x = 5.$

Substituting 5 for x in (2), $2y + 20 = 50;\ \therefore y = 15.$

Substituting 15 for y in (3), $z + 45 = 65;\ \therefore z = 20.$

Hence, the smallest box holds 5 pounds, the middle-sized 15 pounds, and the largest 20 pounds.

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1. $\sqrt{75} + \sqrt{12} + \sqrt{243} = 5\sqrt{3} + 2\sqrt{3} + 9\sqrt{3} = 16\sqrt{3}.$

2. $\sqrt[3]{128} + \sqrt[3]{250} + \sqrt[3]{\frac{1}{4}} = 4\sqrt[3]{2} + 5\sqrt[3]{2} + \frac{1}{2}\sqrt[3]{2} = \frac{19}{2}\sqrt[3]{2}.$

3. $\sqrt{\frac{1}{4}} + \sqrt{\frac{1}{6}} + \sqrt{\frac{2}{3}} + \sqrt{6} = \frac{1}{2} + \frac{1}{6}\sqrt{6} + \frac{1}{3}\sqrt{6} + \sqrt{6} = \frac{1}{2} + \frac{3}{2}\sqrt{6}.$

4. $\sqrt{448} - \sqrt{175} + 3\sqrt{63} = 8\sqrt{7} - 5\sqrt{7} + 9\sqrt{7} = 12\sqrt{7}.$

5. $(\frac{5}{9})^{\frac{1}{2}} - 2\sqrt{\frac{1}{5}} + (5)^{-\frac{1}{2}} - \sqrt{5} = \frac{1}{3}\sqrt{5} - \frac{2}{5}\sqrt{5} + \frac{1}{5}\sqrt{5} - \sqrt{5} = -\frac{13}{15}\sqrt{5}.$

6. $6 \cdot 3^{-\frac{2}{3}} + 9^{\frac{1}{2}} - 3 \cdot 2^{-1} \cdot 3^{\frac{1}{3}} + \sqrt[3]{3} = 6 \cdot \frac{1}{3}\sqrt[3]{3} + 3 - 3 \cdot \frac{1}{2} \cdot \sqrt[3]{3} + \sqrt[3]{3}$

$= 2\sqrt[3]{3} + 3 - \frac{3}{2}\sqrt[3]{3} + \sqrt[3]{3} = \frac{3}{2}\sqrt[3]{3} + 3.$

7. $\sqrt{xyz} - \sqrt[4]{x^2y^2z^2} + \sqrt[6]{8x^3y^3z^3} = \sqrt{xyz} - \sqrt{xyz} + \sqrt{2xyz} = \sqrt{2xyz}.$

8. $\sqrt{2a^3 - 12a^2b + 18ab^2} - \sqrt{2a^3 + 16a^2b + 32ab^2}$

$= (a - 3b)\sqrt{2a} - (a + 4b)\sqrt{2a} = -7b\sqrt{2a}.$

9. $\sqrt{5} \times \sqrt{8} = \sqrt{40} = 2\sqrt{10}.$

10. $3\sqrt{7} \times 2\sqrt{12} = 6\sqrt{84} = 12\sqrt{21}.$

11. $3\sqrt{8} \times 5\sqrt{16} = 15\sqrt{128} = 120\sqrt{2}.$

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12. $2\sqrt{7} \times 3\sqrt{15} = 6\sqrt{105}.$

13. $4\sqrt[3]{9} \times 3\sqrt[3]{21} = 12\sqrt[3]{189} = 36\sqrt[3]{7}.$

14. $\sqrt{xy} \times 7\sqrt{x^2y} = 7\sqrt{x^3y^2} = 7\,xy\sqrt{x}.$

15. $2\sqrt[3]{6} \times 2\sqrt{18} = 2\sqrt[6]{36} \times 6\sqrt[6]{8} = 12\sqrt[6]{288}.$

16. $\sqrt{15} \times 3\sqrt{24} = 3\sqrt{360} = 18\sqrt{10}.$

17. $\sqrt{ab} \times \sqrt[6]{a^3b} = \sqrt[6]{a^3b^3} \times \sqrt[6]{a^3b} = \sqrt[6]{a^6b^4} = a\sqrt[3]{b^2}.$

18. $\sqrt[3]{24} \times 2\sqrt{32} = 2\sqrt[6]{9} \times 8\sqrt[6]{8} = 16\sqrt[6]{72}.$

19. $4\sqrt[5]{2} \times \sqrt[10]{32} = 4\sqrt[10]{4} \times \sqrt[10]{32} = 4\sqrt[10]{128}.$

20. $3\sqrt[3]{6} \times 4\sqrt{12} = 3\sqrt[6]{36} \times 8\sqrt[6]{27} = 24\sqrt[6]{972}.$

21. $\sqrt{\frac{1}{2}} \times \sqrt[3]{\frac{1}{3}} \times \sqrt[6]{\frac{2}{9}} = \sqrt[6]{\frac{1}{8}} \times \frac{1}{2} \times \sqrt[6]{\frac{2}{9}} = \frac{1}{2}\sqrt[6]{\frac{1}{36}} = \frac{1}{2}\sqrt[3]{\frac{1}{6}} = \frac{1}{12}\sqrt[3]{36}.$

22. $\sqrt[3]{\frac{1}{9}} \times \sqrt{\frac{2}{3}} \times \sqrt{\frac{1}{3}} = \sqrt[6]{\frac{1}{81}} \times \sqrt[6]{\frac{8}{27}} \times \sqrt[6]{\frac{1}{27}} = \sqrt[6]{\frac{1}{3^4} \times \frac{2^3}{3^3} \times \frac{1}{3^3}}$

$$= \frac{1}{3}\sqrt[6]{\frac{2^3}{3^4}} = \frac{1}{9}\sqrt[6]{2^3 3^2} = \frac{1}{9}\sqrt[6]{72}.$$

23. $(12)^{\frac{1}{2}} \times (6)^{\frac{2}{3}} \times (8)^{\frac{1}{6}} = 2\sqrt[6]{3^3} \times \sqrt[6]{6^4} \times \sqrt[6]{8} = 2\sqrt[6]{3^3 \times 6^4 \times 8}$

$$= 2\sqrt[6]{3^3 \times 2^4 \times 3^4 \times 2^3} = 2\sqrt[6]{3^7 \times 2^7} = 12\sqrt[6]{6}.$$

24. $(\sqrt{7} + \sqrt{2})(\sqrt{7} - \sqrt{2}) = \sqrt{49 - 2} = \sqrt{47}.$

25. $(2\sqrt{6} + 5\sqrt{2})(3\sqrt{6} - 4\sqrt{2}) = 36 + 7\sqrt{12} - 40 = 14\sqrt{3} - 4.$

26. $\sqrt{27} \div \sqrt{6} = \sqrt{\frac{27}{6}} = \sqrt{\frac{9}{2}} = \frac{3}{2}\sqrt{2}.$

27. $\sqrt{30} \div 2\sqrt{3} = \frac{1}{2}\sqrt{10}.$

28. $5\sqrt{5} \div \sqrt{20} = 5\sqrt{\frac{5}{20}} = 5\sqrt{\frac{1}{4}} = \frac{5}{2}.$

29. $\sqrt{125} \div \sqrt{5} = \sqrt{25} = 5.$

30. $\sqrt[3]{12} \div \sqrt{8} = \sqrt[6]{144} \div 2\sqrt[6]{8} = \frac{1}{2}\sqrt[6]{18}.$

31. $3\sqrt[3]{3} \div \sqrt[6]{12} = 3\sqrt[6]{9} \div \sqrt[6]{12} = 3\sqrt[6]{\frac{9}{12}} = 3\sqrt[6]{\frac{3}{4}} = \frac{3}{2}\sqrt[6]{48}.$

32. $\sqrt{ax} \div \sqrt{ab} = \sqrt{\frac{x}{b}} = \frac{1}{b}\sqrt{bx}.$

33. $\sqrt[3]{xy} \div \sqrt{bx} = \sqrt[6]{x^2y^2} \div \sqrt[6]{b^3x^3} = \sqrt[6]{\frac{x^2y^2}{b^3x^3}} = \sqrt[6]{\frac{y^2}{b^3x}} = \frac{1}{bx}\sqrt[6]{b^3x^5y^2}.$

34. $\sqrt{\frac{2}{3}} \div \sqrt{\frac{3}{4}} = \sqrt{\frac{8}{9}} = \frac{2}{3}\sqrt{2}.$

35. $\sqrt[3]{\frac{1}{2}} \div \sqrt[3]{\frac{2}{3}} = \sqrt[3]{\frac{3}{4}} = \frac{1}{2}\sqrt[3]{6}.$

36. $\sqrt{\frac{1}{3}} \div \sqrt[3]{\frac{4}{9}} = \sqrt[6]{\frac{1}{27}} \div \sqrt[6]{\frac{16}{81}} = \sqrt[6]{\frac{3}{16}} = \frac{1}{2}\sqrt[6]{12}.$

37. $\sqrt{\frac{1}{2}} \div \sqrt[4]{\frac{5}{8}} = \sqrt[4]{\frac{1}{4}} \div \sqrt[4]{\frac{5}{8}} = \sqrt[4]{\frac{2}{5}} = \frac{1}{5}\sqrt[4]{250}.$

38. $(\sqrt{6} - 6\sqrt{3} + 9) \div \sqrt{3} = \sqrt{2} - 6 + 3\sqrt{3}.$

39. $(\sqrt{42} + 6\sqrt{5}) \div (\sqrt{7} + \sqrt{30}) = (\sqrt{42} + \sqrt{180}) \div (\sqrt{7} + \sqrt{30}) = \sqrt{6}.$

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1. Let $x =$ the number.
Then, $\frac{1}{6}x \cdot x = 216.$
Solving, $x = \pm 36.$
Hence, the number is 36 or -36.

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2. Let $x =$ the number.

Then, $\frac{1}{4}x^2 =$ the square of $\frac{1}{2}$ of the number.

$$\therefore x^2 + (\tfrac{1}{2}x)^2 = 80.$$

Solving, $x = \pm 8.$

Hence, the number is 8 or -8.

3. Let $x =$ number of inches in width of blotter.

Then, $2x =$ number of inches in length of blotter.

$$\therefore 2x \cdot x = 32.$$

Solving, $x = \pm 4,$

and $2x = \pm 8.$

Hence, rejecting negative values, the length of the blotter is 8 inches and its width 4 inches.

4. Let $5 + x =$ one number,

and $5 - x =$ the other number.

Then, $(5 + x)(5 - x) = 24.$

Solving, $x = \pm 1,$

whence, $5 + x = 6$ or 4, and $5 - x = 4$ or 6.

Hence, the numbers are 6 and 4.

5. Let $x + 3 =$ one number,

and $x - 3 =$ the other number.

Then, $(x+3)^2 + (x-3)^2 = 68.$

Solving, $x = \pm 5,$

whence, $x + 3 = 8$ or -2, and $x - 3 = 2$ or -8.

Hence, the numbers are 8 and 2, or -2 and -8.

6. Let $\frac{21}{2} + x =$ number of feet in length of room,

and $\frac{21}{2} - x =$ number of feet in width of room.

Then, $(\frac{21}{2} + x)(\frac{21}{2} - x) = 110.$

Solving, $x = \pm \frac{1}{2},$

whence, $\frac{21}{2} + x = 11$ or 10, and $\frac{21}{2} - x = 10$ or 11.

Hence, the room is 11 feet long and 10 feet wide.

7. Let $x =$ cost of seed in cents per pound.

Then, $10x =$ number of pounds of seed in a bushel.

$$\therefore 10x \cdot x = 90.$$

Solving, $x = \pm 3,$

and $10x = \pm 30.$

Hence, rejecting the negative value, the weight of a bushel of the seed was 30 pounds.

8. Let $x =$ number of pounds each box weighed.

Then, $3x =$ the number of boxes.

$$\therefore 3x \cdot x = 192.$$

Solving, $x = \pm 8,$

and $3x = \pm 24.$

Hence, rejecting the negative value, there were 24 boxes.

9. Let $x =$ number of days.

Then, $5x =$ daily output in tons.

$$\therefore x \cdot 5x = 180.$$

Solving, $x = \pm 6,$

and $5x = \pm 30.$

Hence, rejecting the negative value, the daily output was 30 tons.

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10. Let $x =$ cost of butter in cents per pound.
Then, $30x =$ annual yield of butter in pounds.
$$\therefore 30x \cdot x = 3000.$$
Solving, $x = \pm 10,$
and $30x = \pm 300.$
Hence, rejecting the negative value, the annual yield of butter was 300 pounds.

11. Let $x =$ number of needles in each package.
Then, $\frac{2}{5}x =$ number of packages in a bundle.
$$\therefore \tfrac{2}{5}x \cdot x = 250.$$
Solving, $x = \pm 25.$
Hence, rejecting the negative value, there are 25 needles in a package.

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24. $$x(x-5) = 104.$$
Completing the square, $x^2 - 5x + (\frac{5}{2})^2 = 104 + \frac{25}{4} = \frac{441}{4}.$
Extracting the square root, $x - \frac{5}{2} = \pm \frac{21}{2}.$
Taking the upper sign, $x = \frac{5}{2} + \frac{21}{2} = 13.$
Taking the lower sign, $x = \frac{5}{2} - \frac{21}{2} = -8.$

25. $$r^2 - 3r + 2 = 0.$$
Completing the square, $r^2 - 3r + (\frac{3}{2})^2 = -2 + \frac{9}{4} = \frac{1}{4}.$
Extracting the square root, $r - \frac{3}{2} = \pm \frac{1}{2}.$
Taking the upper sign, $r = \frac{3}{2} + \frac{1}{2} = 2.$
Taking the lower sign, $r = \frac{3}{2} - \frac{1}{2} = 1.$

26. $$2x^2 - 5x + 3 = 0.$$
Dividing by 2, etc., $x^2 - \frac{5}{2}x = -\frac{3}{2}.$
Completing the square, $x^2 - \frac{5}{2}x + (\frac{5}{4})^2 = -\frac{3}{2} + \frac{25}{16} = \frac{1}{16}.$
Extracting the square root, $x - \frac{5}{4} = \pm \frac{1}{4}.$
Taking the upper sign, $x = \frac{5}{4} + \frac{1}{4} = \frac{3}{2}.$
Taking the lower sign, $x = \frac{5}{4} - \frac{1}{4} = 1.$

27. $$3x^2 + 8x - 28 = 0.$$
Dividing by 3, etc., $x^2 + \frac{8}{3}x = \frac{28}{3}.$
Completing the square, $x^2 + \frac{8}{3}x + (\frac{4}{3})^2 = \frac{28}{3} + \frac{16}{9} = \frac{100}{9}.$
Extracting the square root, $x + \frac{4}{3} = \pm \frac{10}{3}.$
Taking the upper sign, $x = -\frac{4}{3} + \frac{10}{3} = 2.$
Taking the lower sign, $x = -\frac{4}{3} - \frac{10}{3} = -\frac{14}{3}.$

28. $$5x^2 + 2x - 51 = 0.$$
Dividing by 5, etc., $x^2 + \frac{2}{5}x = \frac{51}{5}.$
Completing the square, $x^2 + \frac{2}{5}x + (\frac{1}{5})^2 = \frac{51}{5} + \frac{1}{25} = \frac{256}{25}.$
Extracting the square root, $x + \frac{1}{5} = \pm \frac{16}{5}.$
Taking the upper sign, $x = -\frac{1}{5} + \frac{16}{5} = 3.$
Taking the lower sign, $x = -\frac{1}{5} - \frac{16}{5} = -\frac{17}{5}.$

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29. $2x^2 + x - 6 = 0.$

$$\therefore\ x = \frac{-1 \pm \sqrt{1 - 4 \cdot 2 \cdot (-6)}}{2 \cdot 2}$$
$$= \frac{-1 \pm 7}{4} = \frac{3}{2} \text{ or } -2.$$

30. $2x^2 - 5x + 2 = 0.$

$$\therefore\ x = \frac{5 \pm \sqrt{25 - 4 \cdot 2 \cdot 2}}{2 \cdot 2}$$
$$= \frac{5 \pm 3}{4} = 2 \text{ or } \frac{1}{2}.$$

31. $15x^2 - x - 2 = 0.$

$$\therefore\ x = \frac{1 \pm \sqrt{1 - 4 \cdot 15 \cdot (-2)}}{2 \cdot 15}$$
$$= \frac{1 \pm 11}{30} = \frac{2}{5} \text{ or } -\frac{1}{3}.$$

32. $12x^2 - 5x - 2 = 0.$

$$\therefore\ x = \frac{5 \pm \sqrt{25 - 4 \cdot 12 \cdot (-2)}}{2 \cdot 12}$$
$$= \frac{5 \pm 11}{24} = \frac{2}{3} \text{ or } -\frac{1}{4}.$$

33. $11x^2 - 10x - 1 = 0.$

$$\therefore\ x = \frac{10 \pm \sqrt{100 - 4 \cdot 11 \cdot (-1)}}{2 \cdot 11}$$
$$= \frac{10 \pm 12}{22} = 1 \text{ or } -\frac{1}{11}.$$

34. $16x(x + 1) = -3.$
$16x^2 + 16x + 3 = 0.$

$$\therefore\ x = \frac{-16 \pm \sqrt{256 - 4 \cdot 16 \cdot 3}}{2 \cdot 16}$$
$$= \frac{-16 \pm 8}{32} = -\frac{1}{4} \text{ or } -\frac{3}{4}.$$

35. $14x^2 = 25(1 - x).$
$14x^2 + 25x - 25 = 0.$

$$\therefore\ x = \frac{-25 \pm \sqrt{625 - 4 \cdot 14 \cdot (-25)}}{2 \cdot 14}$$
$$= \frac{-25 \pm 45}{28} = \frac{5}{7} \text{ or } -\frac{5}{2}.$$

36. $3x^2 = 7x - 2.$
$3x^2 - 7x + 2 = 0.$

$$\therefore\ x = \frac{7 \pm \sqrt{49 - 4 \cdot 3 \cdot 2}}{2 \cdot 3}$$
$$= \frac{7 \pm 5}{6} = 2 \text{ or } \frac{1}{3}.$$

37. $5x^2 + 8x = -3.$
$5x^2 + 8x + 3 = 0.$

$$\therefore\ x = \frac{-8 \pm \sqrt{64 - 4 \cdot 5 \cdot 3}}{2 \cdot 5}$$
$$= \frac{-8 \pm 2}{10} = -\frac{3}{5} \text{ or } -1.$$

38. $x^2 = 3(1 - 2x).$
$x^2 + 6x - 3 = 0.$

$$\therefore\ x = \frac{-6 \pm \sqrt{36 - 4 \cdot 1 \cdot (-3)}}{2 \cdot 1}$$
$$= \frac{-6 \pm 4\sqrt{3}}{2} = -3 \pm 2\sqrt{3}.$$

39. $x^2 + 9x + 4 = 0.$

$$\therefore\ x = \frac{-9 \pm \sqrt{81 - 4 \cdot 1 \cdot 4}}{2 \cdot 1}$$
$$= \frac{-9 \pm \sqrt{65}}{2} = \frac{1}{2}(-9 \pm \sqrt{65}).$$

40. $4x^2 + 7x + 3 = 0.$

$$\therefore\ x = \frac{-7 \pm \sqrt{49 - 4 \cdot 4 \cdot 3}}{2 \cdot 4}$$
$$= \frac{-7 \pm 1}{8} = -\frac{3}{4} \text{ or } -1.$$

41. $2(x^2 + 2x) = 5.$
$2x^2 + 4x - 5 = 0.$

$$\therefore\ x = \frac{-4 \pm \sqrt{16 - 4 \cdot 2 \cdot (-5)}}{2 \cdot 2}$$
$$= \frac{-4 \pm 2\sqrt{14}}{4} = \frac{1}{2}(-2 \pm \sqrt{14}).$$

42. $3(x^2 - 1) + 5x = 0.$
$3x^2 + 5x - 3 = 0.$

$$\therefore\ x = \frac{-5 \pm \sqrt{25 - 4 \cdot 3 \cdot (-3)}}{2 \cdot 3}$$
$$= \frac{-5 \pm \sqrt{61}}{6} = \frac{1}{6}(-5 \pm \sqrt{61}).$$

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1. Let $x =$ one part.
Then, $26 - x =$ the other part.
$\therefore x(26 - x) = 165.$
Solving, $x = 15$ or 11,
and $26 - x = 11$ or 15.
Hence, the parts are 15 and 11.

2. Let $x =$ one integer.
Then, $x + 1 =$ the other integer.
$\therefore x(x + 1) = 272.$
Solving, $x = 16$ or -17,
and $x + 1 = 17$ or -16.
Hence, the two consecutive integers are 16 and 17, or -17 and -16.

3. Let $x =$ one integer.
Then, $x + 1 =$ the other integer.
$\therefore x^2 + (x + 1)^2 = 113.$
Solving, $x = 7$ or -8,
and $x + 1 = 8$ or -7.
Hence, the two consecutive integers are 7 and 8, or -8 and -7.

4. Let $x =$ one integer.
Then, $x + 2 =$ the other integer.
$\therefore x^2 + (x + 2)^2 = 2(a^2 + 1).$
Solving, $x = a - 1$ or $-a - 1$,
and $x + 2 = a + 1$ or $-a + 1$.
Hence, the two consecutive odd integers are $a - 1$ and $a + 1$, or $-a - 1$ and $-a + 1$.

5. Let $x =$ number of feet in width of floor.
Then, $3x + 12 =$ number of feet in length of floor.
$\therefore x(3x + 12) = 180.$
Solving, $x = 6$ or -10,
and $3x + 12 = 30$ or -18.
Hence, rejecting negative values, the floor is 30 feet long and 6 feet wide.

6. Let $x =$ number of feet in side of ventilator.
Then, $(x + 4)(x - 6) = 200.$
Solving, $x = 16$ or -14.
Hence, rejecting the negative value, the ventilator is 16 feet square.

7. Let $x =$ number of boxes of glass.
Then, $x + 2 =$ number of sheets of glass in each box.
$\therefore x(x + 2) = 80.$
Solving, $x = 8$ or -10,
and $x + 2 = 10$ or -8.
Hence, rejecting the negative value, there were 10 sheets of glass in each box.

8. Let $x =$ cost of wood in cents per cord.
Then, $x - 30 =$ number of cords of wood bought.
$\therefore x(x - 30) = 13{,}000.$
Solving, $x = 130$ or -100,
and $x - 30 = 100$ or -130.
Hence, rejecting the negative value, the amount of wood bought was 100 cords.

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9. Let $x =$ number of rows of pins in the paper.

Then, $2x + 6 =$ number of pins per row.

$$\therefore x(2x + 6) = 360.$$

Solving, $x = 12$ or -15,

and $2x + 6 = 30$ or -24.

Hence, rejecting the negative value, there are 30 pins in each row.

10. Let $x =$ length of box in inches.

Then, $5 - x =$ width of box in inches,

and $2x =$ height of box in inches.

$$\therefore x(5 - x) = 6\tfrac{1}{4}.$$

Solving, $x = 2\tfrac{1}{2}$,

$5 - x = 2\tfrac{1}{2}$,

and $2x = 5$.

Hence, the box is $2\frac{1}{2}$ inches long, $2\frac{1}{2}$ inches wide, and 5 inches high.

11. Let $x =$ width of paper in centimeters.

Then, $2x - 10 =$ length of paper in centimeters.

$$\therefore x(2x - 10) = 208.$$

Solving, $x = 13$ or -8,

and $2x - 10 = 16$ or -26.

Hence, rejecting negative values, the length of the sheet of paper is 16 centimeters and its width is 13 centimeters.

12. Let $x =$ number of cars.

Then, $7x + 40 =$ number of barrels in each car.

$$\therefore x(7x + 40) = 12,800.$$

Solving, $x = 40$ or $-\frac{320}{7}$,

and $7x + 40 = 320$ or -280.

Hence, rejecting negative values, there were 40 cars and 320 barrels in each car.

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13. Let $x =$ the number.

Then, $\dfrac{1}{x} =$ the reciprocal of the number.

$$\therefore x + \frac{1}{x} = \frac{13}{6}.$$

Solving, $x = \frac{3}{2}$ or $\frac{2}{3}$.

Hence, the number is $\frac{3}{2}$ or $\frac{2}{3}$.

14. Let $x =$ one integer.

Then, $x + 1 =$ the other integer.

$$\therefore \frac{1}{x} + \frac{1}{x + 1} = \frac{5}{6}.$$

Solving, $x = 2$ or $-\frac{3}{5}$,

and $x + 1 = 3$ or $\frac{2}{5}$.

Hence, the integers are 2 and 3, the other results being inadmissible.

15. Let $x =$ the number.

Then, $x - \dfrac{c}{x} = c - 1.$

Solving, $x = c$ or -1.

Hence, the number is c or -1.

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16. Let x = number of fowls in each crate.
Then, $2x - 8$ = number of crates.
$\therefore x(2x - 8) = 6720.$
Solving, $x = 60$ or -56,
and $2x - 8 = 112$ or -120.
Hence, the negative results being inadmissible, there are 60 fowls in each crate and 112 crates in a car.

17. Let x = number of tons of sand the dredge can lift per minute.
Then, $\frac{10,000}{x} - \frac{10,000}{x + 50} = 10.$
Solving, $x = 200$ or -250.
Hence, the negative value being inadmissible, the dredge can lift 200 tons of sand per minute.

18. Let x = the required number of hours.
Then, $\frac{20}{x} - \frac{20}{x + 2} = \frac{5}{6}.$
Solving, $x = 6$ or -8.
Hence, the negative value being inadmissible, it took the dogs 6 hours.

19. Let x = number of baskets filled.
Then, $\frac{250}{x + 5} - \frac{160}{x} = 2.$
Solving, $x = 20$ or 20.
Hence, the man filled 20 baskets with cherries.

20. Let x = number of inches in width of paper.
Then, $3x + 50$ = number of feet in length of paper.
$\therefore \frac{x}{12}(3x + 50) = 6250.$
Solving, $x = 150$ or $-\frac{500}{3}$,
and $3x + 50 = 500$ or -450.
Hence, the negative value being inadmissible, the length of the paper is 500 feet.

21. Let x = number of persons in the party.
Then, $\frac{12}{x - 2} - \frac{12}{x} = \frac{1}{2}.$
Solving, $x = 8$ or -6.
Hence, rejecting the negative value, there were 8 persons in the party.

22. Let x = number of minutes required by larger pipe.
Then, $x + 24$ = number of minutes required by smaller pipe.
$\therefore \frac{1}{x} + \frac{1}{x + 24} = \frac{1}{35}.$
Solving, $x = 60$ or -14,
and $x + 24 = 84$ or 10.
Hence, the second values being inadmissible, it takes the larger pipe 60 minutes and the smaller 84 minutes to fill the tank.

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23. Let $x =$ number of pineapples in each crate.

Then, $\frac{13,440}{x-4} - \frac{13,440}{x} = 112.$

Solving, $x = 24$ or -20.

Hence, rejecting the negative value, there were 24 pineapples in each crate.

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1. Let $x =$ one number,
and $y =$ the other number.
Then, $x + y = 8,$
and $x^2 + y^2 = 40.$
Solving, $x = 6$ or $2,$
and $y = 2$ or $6.$
Hence, the numbers are 6 and 2.

2. Let $x =$ the larger number,
and $y =$ the smaller number.
Then, $x - y = 2,$
and $x^3 - y^3 = 26.$
Solving, $x = 3$ or $-1,$
and $y = 1$ or $-3.$
Hence, the numbers are 3 and 1, or -1 and -3.

3. Let $x =$ the larger number,
and $y =$ the smaller number.
Then, $xy = s^2,$
and $x - y = 8y.$
Solving, $x = 3s$ or $-3s,$
and $y = \frac{s}{3}$ or $-\frac{s}{3}.$

Hence, the numbers are $3s$ and $\frac{s}{3}$, or $-3s$ and $-\frac{s}{3}$.

4. Let $x =$ length of floor in feet,
and $y =$ width of floor in feet.
Then, $2x + 2y = 44,$
and $xy = 120.$
Solving, $x = 12$ or $10,$
and $y = 10$ or $12.$
Hence, the floor is 12 feet long and 10 feet wide.

5. Let $x =$ length of sign in feet,
and $y =$ width of sign in feet.
Then, $x - y = 10,$
and $xy = 6375.$
Solving, $x = 85$ or $-75,$
and $y = 75$ or $-85.$
Hence, the negative values being inadmissible, the sign is 85 feet long and 75 feet wide.

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6. Let $x =$ base of right triangle in feet,
and $y =$ altitude of right triangle in feet.
Then, $x + 1 =$ hypotenuse of triangle in feet.

$$\therefore x + y + x + 1 = 12, \quad (1)$$

and $$x^2 + y^2 = (x + 1)^2. \quad (2)$$

Transposing in (1), etc., $2x + y = 11.$ (3)
Reducing (2), etc., $2x = y^2 - 1.$ (4)
Substituting (4) in (3), $y^2 + y = 12.$
Solving, $y = 3$ or $-4.$
Substituting the values of y in (3),

$$x = 4 \text{ or } 7\tfrac{1}{2}.$$

Hence, the second value being inadmissible, the base of the triangle is 4 feet.

7. Let $x =$ the digit in tens' place,
and $y =$ the digit in units' place.
Then, $10x + y =$ the number.

$$\therefore (x + y)(10x + y) = 198, \quad (1)$$

and $$\frac{10x + y}{x + y} = 5\tfrac{1}{2}. \quad (2)$$

Reducing (2), $x = y.$ (3)
Substituting (3) in (1), $22y^2 = 198;\ \therefore y = \pm 3.$
Substituting ± 3 for y in (3), $x = \pm 3.$
Hence, rejecting the negative values, the number is 33.

8. Let $x =$ the numerator,
and $y =$ the denonimator.
Then, $y - x = 1,$ (1)

and $$\frac{x}{y}(x + y) = 3\tfrac{1}{3}. \quad (2)$$

From (1), $y = x + 1.$ (3)
Clearing (2), $3x^2 + 3xy = 10y.$ (4)
Substituting (3) in (4), $6x^2 - 7x = 10.$
Solving, $x = 2$ or $-\frac{5}{6}.$
Substituting the values of x in (3),

$$y = 3 \text{ or } \tfrac{1}{6}.$$

Hence, the second values being inadmissible, the fraction is $\frac{2}{3}$.

9. Let $x =$ number of dollars in the principal,
and $y =$ number of per cent in the rate.

Then, $$x + \frac{y}{100} \cdot x = 3990, \quad (1)$$

and $$x + 200 + \frac{y - 1}{100}(x + 200) = 4160. \quad (2)$$

Clearing (1), $100x + xy = 399{,}000.$ (3)
Clearing (2), $99x + xy + 200y = 396{,}200.$ (4)
Subtracting (4) from (3), $x - 200y = 2800.$ (5)
From (5), $x = 200y + 2800.$ (6)
Substituting (6) in (3), etc.,

$$y^2 + 114y = 595.$$

Solving, $y = 5$ or $-119.$ (7)
Substituting (7) in (6), $x = 3800$ or $-21{,}000.$
Hence, the negative values being inadmissible, the principal was \$3800 and the rate 5%.

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10. Let $x =$ number of boxes used,
and $y =$ number of pounds of cherries per box.

Then, $\frac{2000}{y} = x,$ (1)

and $\frac{2000}{y+6} = x - 75.$ (2)

Clearing (1), $xy = 2000.$ (3)

Clearing (2), etc., $6x + xy - 75y = 2450.$ (4)

Subtracting (3) from (4), $6x - 75y = 450.$ (5)

From (5), $x = \frac{25y + 150}{2}.$ (6)

Substituting (6) in (3), $25y^2 + 150y = 4000.$

Solving, $y = 10 \text{ or } -16.$ (7)

Substituting (7) in (1), $x = 200 \text{ or } -125.$

Hence, the negative values being inadmissible, 200 boxes were used and each box contained 10 pounds of cherries.

11. Let $x =$ number of bushels of oats sold.
and $y =$ number of bushels of rye sold.

Then, $\frac{450}{x} =$ number of cents in price of oats per bushel,

and $\frac{420}{y} =$ number of cents in price of rye per bushel.

$\therefore x - y = 3,$ (1)

and $\frac{420}{y} - \frac{450}{x} = 20.$ (2)

From (1), $x = y + 3.$ (3)

Clearing (2), etc., $42x - 45y - 2xy = 0.$ (4)

Substituting (3) in (4), $2y^2 + 9y - 126 = 0.$

Solving, $y = 6 \text{ or } -10\frac{1}{2};$ (5)

whence, $\frac{420}{y} = 70 \text{ or } -40.$

Substituting (5) in (3), $x = 9 \text{ or } -7\frac{1}{2};$

whence, $\frac{450}{x} = 50 \text{ or } -60.$

Hence, the negative values being inadmissible, the farmer sold 6 bushels of rye at 70¢ per bushel and 9 bushels of oats at 50¢ per bushel.

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1. Let $x =$ the larger part,
and $y =$ the smaller part.

Then, $x + y = 60,$ (1)

and $\frac{1}{3}x - \frac{1}{4}y = 6.$ (2)

From (1), $x = 60 - y.$ (3)

Clearing (2), $4x - 3y = 72.$ (4)

Substituting (3) in (4), $240 - 4y - 3y = 72; \therefore y = 24.$

Substituting 24 for y in (3), $x = 60 - 24 = 36.$

Hence, the parts of 60 are 36 and 24.

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2. Let $x =$ the larger number,
and $y =$ the smaller number.
Then, $x + y = 85,$ (1)
and $x - y = 19.$ (2)
Adding (1) and (2), $2x = 104; \therefore x = 52.$
Substituting 52 for x in (1), $52 + y = 85; \therefore y = 33.$
Hence, the numbers are 52 and 33.

3. Let $x =$ the number.
Then, $\frac{9x}{x-6} = \frac{3}{2}x.$
Clearing the equation, $3x^2 - 36x = 0.$
Solving, $x = 0$ or $12.$
Hence, the number is either 0 or 12.

4. Let $x =$ the larger number,
and $y =$ the smaller number.
Then, $x - y = a,$ (1)
and $\sqrt{x} - \sqrt{y} = \sqrt{b}.$ (2)
From (1), $x = y + a.$ (3)
Substituting (3) in (2), $\sqrt{y+a} - \sqrt{y} = \sqrt{b}.$ (4)
Squaring (4), etc., $-2\sqrt{y^2 + ay} = b - a - 2y.$ (5)
Squaring (5), etc., $4by = a^2 - 2ab + b^2; \therefore y = \frac{(a-b)^2}{4b}.$
Substituting $\frac{(a-b)^2}{4b}$ for y in (3), $x = \frac{(a+b)^2}{4b}.$
Hence, the numbers are $\frac{(a+b)^2}{4b}$ and $\frac{(a-b)^2}{4b}.$

5. Let $x =$ side of square in feet.
Then, $x + 6 =$ length of rectangle in feet,
and $x - 3 =$ width of rectangle in feet.
$\therefore x^2 = (x+6)(x-3).$
Solving, $x = 6,$
$x + 6 = 12,$
and $x - 3 = 3.$
Hence, the rectangle is 12 feet long and 3 feet wide.

6. Let $x =$ side of square in feet.
Then, $x + u =$ length of rectangle in feet,
and $x - v =$ width of rectangle in feet.
$\therefore x^2 = (x+u)(x-v).$
Solving, $x = \frac{uv}{u-v}.$
Hence, the side of the square is $\frac{uv}{u-v}$ feet.

7. Let $x =$ weight of seed in pounds.
Then, $.15x = 600.$
Solving, $x = 4000.$
Hence, the cotton seed weighed 4000 pounds.

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8. Let $x =$ number of per cent of copper in the bronze.

Then, $x - 5 =$ number of per cent of iron in the bronze,

$\frac{x}{4\frac{1}{3}} =$ number of per cent of aluminium in the bronze,

and $\frac{x-3}{2} =$ number of per cent of nickel in the bronze.

$\therefore x + x - 5 + \frac{x}{4\frac{1}{3}} + \frac{x-3}{2} = 100.$

Solving, $x = 39,$

$x - 5 = 34,$

$\frac{x}{4\frac{1}{3}} = 9,$

and $\frac{x-3}{2} = 18.$

Hence, the bronze is 39% copper, 34% iron, 9% aluminium, and 18% nickel.

9. Let $x =$ side of square in feet.

Then, $(x + \frac{1}{2})(x - \frac{1}{2}) = 2\frac{2}{9} \times 9 = 20.$

Solving, $x = \pm 4\frac{1}{2}.$

Hence, the negative value being inadmissible, the side of the square is $4\frac{1}{2}$ feet.

10. Let $x =$ number of seconds required.

Then, $\frac{5 \cdot 320}{x - 7} = 10.$

Solving, $x = 167.$

Hence, the time required was 167 seconds, or 2 minutes 47 seconds.

11. Let $x =$ the larger number,

and $y =$ the smaller number.

Then, $\frac{x-2}{y} = 4,$ (1)

and $x - y = 50.$ (2)

From (1), $x = 4y + 2.$ (3)

Substituting (3) in (2), $4y + 2 - y = 50$; $\therefore y = 16.$

Substituting 16 for y in (3), $x = 64 + 2 = 66.$

Hence, the numbers are 66 and 16.

12. Let $x =$ width of block in inches.

Then, $2x =$ length of block in inches,

and $\frac{112}{2x^2} =$ height of block in inches.

But $\frac{140}{2x(x+1)} =$ height of block in inches.

$\therefore \frac{112}{2x^2} = \frac{140}{2x(x+1)}.$

Solving, $x = 4$ or $0.$

Rejecting the second value of x,

$2x = 8,$

and $\frac{112}{2x^2} = 3\frac{1}{2}.$

Hence, the block is 8 inches long, 4 inches wide, and $3\frac{1}{2}$ inches high.

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13. Let $x =$ number of cents in price of rye per bushel,
and $y =$ number of cents in price of corn per bushel.
Then, $20x - 15y = 500,$ (1)
and $15x - 5y = 750.$ (2)
Dividing (1) by 5, $4x - 3y = 100.$ (3)
Multiplying (2) by $\frac{3}{5}$, $9x - 3y = 450.$ (4)
Subtracting (3) from (4), $5x = 350; \therefore x = 70.$
Substituting 70 for x in (3), $280 - 3y = 100; \therefore y = 60.$
Hence, the price of rye was 70¢ per bushel and of corn 60¢ per bushel.

14. Let $x =$ length of viaduct in feet.
Then, $.15x - 6 =$ height of viaduct in feet.
$$\therefore 10(.15x - 6) = x + 1030.$$
Solving, $x = 2180,$
and $.15x - 6 = 321.$
Hence, the viaduct is 2180 feet long and 321 feet high.

15. Let $x =$ the required number of days.
Then, from conditions of problem, $\frac{2}{x} = \frac{1}{l} + \frac{1}{m} + \frac{1}{n}.$
Solving, $x = \frac{2lmn}{mn + ln + lm}.$
Hence, A, B, and C together can do the work in $\frac{2lmn}{mn + ln + lm}$ days.

16. Let $x =$ number of packages in the box.
Then, $x + 24 =$ number of firecrackers in each package.
$$\therefore x(x + 24) = 2560.$$
Solving, $x = 40$ or $-64.$
Hence, rejecting the negative value, there are 40 packages in the box.

17. Let $x =$ width of box in inches,
and $y =$ height of box in inches.
Then, $x + y =$ length of box in inches.
$\therefore xy(x + y) = 120,$ (1)
and $5(x + y) = 8y.$ (2)
From (2), $x = \frac{3}{5}y.$ (3)
Substituting (3) in (1), $\frac{24}{25}y^3 = 120; \therefore y = 5.$
Substituting 5 for y in (3), $x = \frac{3}{5} \times 5; \therefore x = 3.$
$$x + y = 8.$$
Hence, the box is 8 inches long, 3 inches wide, and 5 inches high.

18. Let $x =$ number of days the man worked.
Then, $3x - 4 =$ number of cents earned per day.
$$\therefore x(3x - 4) = 900.$$
Solving, $x = 18$ or $-16\frac{2}{3}.$
Hence, since the negative value is inadmissible, the man worked 18 days.

19. Let $x =$ width of sign in feet.
Then, $3(x + 5) =$ length of sign in feet.
$$\therefore 3x(x + 5) = 2250.$$
Solving, $x = 25$ or $-30.$
Rejecting the second value, $3(x + 5) = 90.$
Hence, the electric sign was 90 feet long and 25 feet wide.

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20. Let $x =$ number of days it takes C alone.

Then, $\frac{1}{x} =$ part of work C can do in 1 day.

Since A and B can do the work in $4\frac{1}{2}$ days, they do $\frac{2}{9}$ of the work in 1 day and $3 \times \frac{2}{9}$, or $\frac{2}{3}$, of the work in 3 days. Hence, there was $\frac{1}{3}$ of the work left to be done in $\frac{7}{8}$ of a day.

$$\therefore \frac{7}{8} \cdot \frac{1}{x} + \frac{7}{8} \cdot \frac{2}{9} = \frac{1}{3}.$$

Solving, $x = 6\frac{3}{10}$.

Hence, C can do the work alone in $6\frac{3}{10}$ days.

21. Let $x =$ number of gallons of water that flowed into the well per minute.

Then, $$\frac{3750}{x - 125} - \frac{3750}{x} = 1.$$

Solving, $x = 750$ or -625.

Hence, since the second value is inadmissible, 750 gallons of water flow into the well per minute.

22. Let $x =$ yield in cratefuls per acre,

and $y =$ number of acres.

Then, $xy = 8500$, (1)

and $(x + 212\frac{1}{2})(y - 2) = 8500$. (2)

Reducing (2),

$$xy + 212\tfrac{1}{2}y - 2x = 8925. \quad (3)$$

Subtracting (3) from (1),

$$2x - 212\tfrac{1}{2}y = -425. \quad (4)$$

From (4), $x = \frac{1}{2}(212\frac{1}{2}y - 425)$. (5)

Substituting (5) in (1) and reducing,

$$y^2 - 2y = 80.$$

Solving, $y = 10$ or -8. (6)

Substituting (6) in (1), $x = 850$ or $-1062\frac{1}{2}$.

Hence, the man had 10 acres of celery from which the yield was 850 cratefuls per acre.

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1. $$\frac{x}{3} - \frac{x-3}{3} = 12 - \frac{x+4}{2} - x.$$

Clearing, $$2x - 2(x - 3) = 72 - 3(x + 4) - 6x.$$
$$2x - 2x + 6 = 72 - 3x - 12 - 6x.$$

Solving, $x = 6$.

Verifying, $\frac{6}{3} - \frac{3}{3} = 12 - \frac{10}{2} - 6$,

which reduces to $1 = 1$.

2.

$$\begin{array}{l|l|l}
 & 4c^4 - 4c^3 + 5c^2 - 2c + 1 & \underline{2c^2 - c + 1} \\
 & \underline{4c^4} & \\
4c^2 & -4c^3 + 5c^2 & \\
\underline{4c^2 - c} & \underline{-4c^3 + c^2} & \\
4c^2 - 2c & +4c^2 - 2c + 1 & \\
\underline{4c^2 - 2c + 1} & \underline{+4c^2 - 2c + 1} &
\end{array}$$

3. $\begin{cases} x + 2y = a, & (1) \\ 2x - y = b. & (2) \end{cases}$

Multiplying (2) by 2, $4x - 2y = 2b.$ (3)

Adding (1) and (3), $5x = a + 2b;\ \therefore x = \dfrac{a+2b}{5}.$

Multiplying (1) by 2, $2x + 4y = 2a.$ (4)

Subtracting (2) from (4), $5y = 2a - b;\ \therefore y = \dfrac{2a-b}{5}.$

4. $\sqrt{\frac{3}{4}} + \sqrt{\frac{1}{3}} = \frac{1}{2}\sqrt{3} + \frac{1}{3}\sqrt{3} = \frac{5}{6}\sqrt{3}.$

$(\sqrt{5} - \sqrt{2})(2\sqrt{5} + 3\sqrt{2}) = 10 + \sqrt{10} - 6 = 4 + \sqrt{10}.$

5. Let $x =$ number of years in A's age,

and $y =$ number of years in B's age.

Then, $x + 5 = 2(y + 5),$ (1)

and $x - 5 = 3(y - 5).$ (2)

Transposing in (1), etc., $x - 2y = 5.$ (3)

Transposing in (2), etc., $x - 3y = -10.$ (4)

Subtracting (4) from (3), $y = 15.$ (5)

Substituting (5) in (3), $x = 35.$

Hence, A is 35 years old and B is 15 years old.

6. Rearranging terms, dividend $= 30a^4 + 11a^3 - 82a^2 - 5a + 12$; divisor $= 3a^2 + 2a - 4$.

$$\begin{array}{r|l}
30a^4 + 11a^3 - 82a^2 - 5a + 12 & 3a^2 + 2a - 4 \\ \cline{2-2}
\underline{30a^4 + 20a^3 - 40a^2 \qquad\qquad} & 10a^2 - 3a - 12 \\
-9a^3 - 42a^2 - 5a \qquad & \\
\underline{-9a^3 - 6a^2 + 12a \qquad} & \\
-36a^2 - 17a + 12 & \\
\underline{-36a^2 - 24a + 48} & \\
7a - 36, \text{ rem.} &
\end{array}$$

8. Let $x =$ number of days it would take B.

Then, $x + 9 =$ number of days it would take A.

$$\therefore \frac{1}{x} + \frac{1}{x+9} = \frac{1}{20}.$$

Clearing, etc., $x^2 - 31x - 180 = 0.$

Solving, $x = 36$ or -5.

Hence, it would take B 36 days to dig the trench.

9. Let $x =$ the first number.

Then, $x + 2 =$ the second number,

and $x + 4 =$ the third number.

$$\therefore x + x + 2 + x + 4 = \tfrac{3}{8} \times x(x+2).$$

Reducing, etc., $x^2 - 6x - 16 = 0.$

Solving, $x = 8$ or -2,

$x + 2 = 10$ or 0,

and $x + 4 = 12$ or 2.

Hence, the numbers are 8, 10, and 12 or -2, 0, and 2.

10. $$\frac{x^2 - \frac{1}{x}}{x + \frac{1}{x} + 1} = \frac{x^3 - 1}{x} \div \frac{x^2 + x + 1}{x}$$

$$= \frac{(x-1)(x^2+x+1)}{x} \times \frac{x}{x^2+x+1} = x - 1.$$

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1. $a^4 + a^2b^2 + b^4 = a^4 + 2a^2b^2 + b^4 - a^2b^2 = (a^2 + b^2)^2 - a^2b^2$
$= (a^2 + ab + b^2)(a^2 - ab + b^2).$
$4a^3 + 4b^3 = 4(a^3 + b^3) = 4(a + b)(a^2 - ab + b^2).$
$2a^2c - 2abc + 2b^2c = 2c(a^2 - ab + b^2).$
$\therefore$ H. C. F. $= a^2 - ab + b^2$,
and L. C. M. $= 4c(a + b)(a^2 + ab + b^2)(a^2 - ab + b^2).$

2. (a) $\sqrt{3} \times \sqrt[6]{2} \times \sqrt[3]{5} = \sqrt[6]{27} \times \sqrt[6]{2} \times \sqrt[6]{25} = \sqrt[6]{1350}.$

(b) $\dfrac{3+\sqrt{-2}}{2-\sqrt{-2}} = \dfrac{(3+\sqrt{-2})(2+\sqrt{-2})}{(2-\sqrt{-2})(2+\sqrt{-2})} = \dfrac{6+5\sqrt{-2}-2}{6} = \dfrac{4+5\sqrt{-2}}{6}.$

(c) $\dfrac{\sqrt{-6}}{-3\sqrt{-2}} = \dfrac{\sqrt{6}\sqrt{-1}}{-3\sqrt{2}\sqrt{-1}} = -\frac{1}{3}\sqrt{3}.$

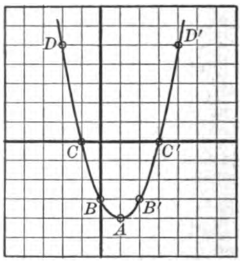
3. $$\frac{2x^3 + x^2 - 25x + 12}{3x^3 + 5x^2 - 34x - 24}.$$

Factoring the numerator by the factor theorem, we have
$$2x^3 + x^2 - 25x + 12 = (x - 3)(x + 4)(2x - 1).$$
By trial, it is found that $x - 3$ and $x + 4$ are factors also of the denominator, whose other factor is $3x + 2$.
$$\therefore \frac{2x^3 + x^2 - 25x + 12}{3x^3 + 5x^2 - 34x - 24} = \frac{(x-3)(x+4)(2x-1)}{(x-3)(x+4)(3x+2)} = \frac{2x-1}{3x+2}.$$

4. (a) Putting $x^2 = 2x + 3$ in the form $x^2 - 2x - 3 = 0$, since the coefficient of x is -2, first substitute 1 for x.

$y = x^2 - 2x - 3$

x	y	POINTS
1	-4	A
0, 2	-3	B, B'
-1, 3	0	C, C'
-2, 4	5	D, D'



Plotting these points and drawing a smooth curve through them, we have the graph of $y = x^2 - 2x - 3$, which crosses the x-axis at 3 and -1.
Hence, the roots of $x^2 = 2x + 3$ are 3 and -1.
(b) $x^2 + x + 1 = 0.$
$$\therefore x = \frac{-1 \pm \sqrt{1 - 4 \cdot 1 \cdot 1}}{2 \cdot 1} = \frac{-1 \pm \sqrt{-3}}{2} = \tfrac{1}{2}(-1 \pm \sqrt{-3}).$$

6. $$ax^2 + bx + c = 3x^2 - 5x + 3 = 0.$$
Since $b^2 - 4ac = 25 - 36 = -11$, a negative number, by § 429, Prin. 3, both roots are imaginary.

7. (a) $$x^{-\frac{2}{3}} : 2 = 1 : x^{\frac{1}{6}}.$$
§ 475, $$x^{-\frac{1}{2}} = 2.$$

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§ 307, $$\frac{1}{x^{\frac{1}{2}}} = 2.$$

Squaring, $$\frac{1}{x} = 4; \ \therefore x = \tfrac{1}{4}.$$

(b) $$\frac{a^{-1}b\sqrt{c}}{a^{\frac{2}{3}}} \div \sqrt{\frac{a^2b^{-1}}{c^{-3}}} = \frac{bc^{\frac{1}{2}}}{a^{\frac{5}{3}}} \div \frac{ac^{\frac{3}{2}}}{b^{\frac{1}{2}}} = \frac{bc^{\frac{1}{2}}}{a^{\frac{5}{3}}} \times \frac{b^{\frac{1}{2}}}{ac^{\frac{3}{2}}} = \frac{b^{\frac{3}{2}}}{a^{\frac{8}{3}}c}.$$

8. (a) The equation is $(x-5)(x+\frac{2}{3}) = 0$, or $x^2 - \frac{13}{3}x - \frac{10}{3} = 0$, or multiplying by 3, $3x^2 - 13x - 10 = 0$.

9. Let $x =$ number of barrels of apples bought.

Then, $\frac{120}{x} =$ number of dollars in cost per barrel,

$x - 2 =$ number of barrels of apples sold,

and $\frac{120}{x} + 2 =$ number of dollars received per barrel.

$$\therefore (x-2)\left(\frac{120}{x} + 2\right) = 154.$$

Clearing, etc., $x^2 - 19x - 120 = 0.$

Solving, $x = 24$ or -5.

Hence, rejecting the negative value, he bought 24 barrels of apples.

10. Let $x =$ rate of train in miles per hour.

Then, $$\frac{273}{x-3} - \frac{273}{x} = \frac{1}{2}.$$

Clearing, etc., $x^2 - 3x - 1638 = 0.$

Solving, $x = 42$ or -39.

Hence, rejecting the negative value, the train traveled 42 miles per hour.

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1. (a)
$$\begin{aligned} 12x^2 + x - 6 &= (3x-2)(4x+3) \\ 6x^2 - 19x + 10 &= (3x-2)(2x-5) \\ 3x^3 - 2x^2 - 12x + 8 &= (3x-2)(x+2)(x-2) \end{aligned}$$

$\therefore$ H. C. F. $= 3x - 2$,

and L. C. M. $= (3x-2)(4x+3)(2x-5)(x+2)(x-2)$.

(b) $$\frac{3}{2c+h} - \frac{8c^2+7h^2}{8c^3+h^3} - \frac{2c-6h}{4c^2-2ch+h^2}$$
$$= \frac{(12c^2 - 6ch + 3h^2) - (8c^2 + 7h^2) - (4c^2 - 10ch - 6h^2)}{8c^3 + h^3}$$
$$= \frac{4ch + 2h^2}{8c^3 + h^3} = \frac{2h(2c+h)}{(2c+h)(4c^2 - 2ch + h^2)} = \frac{2h}{4c^2 - 2ch + h^2}.$$

Substituting $c = 0$ and $h = 1$,
$$\frac{3}{2c+h} - \frac{8c^2+7h^2}{8c^3+h^3} - \frac{2c-6h}{4c^2-2ch+h^2} = \frac{3}{2\cdot 0+1} - \frac{8\cdot 0 + 7\cdot 1}{8\cdot 0 + 1}$$
$$- \frac{2\cdot 0 - 6\cdot 1}{4\cdot 0 - 2\cdot 0\cdot 1 + 1} = 3 - 7 + 6 = 2;$$

and $$\frac{2h}{4c^2 - 2ch + h^2} = \frac{2\cdot 1}{4\cdot 0 - 2\cdot 0\cdot 1 + 1} = 2;$$

hence, the answer is correct.

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2. (a) $$\frac{3x-1}{x+3}-\frac{2x+3}{1-x}=\frac{5x^2-2x-1}{x^2+2x-3}.$$

Clearing of fractions by multiplying by the L. C. D., $(x+3)(x-1)$,

$$3x^2-4x+1+2x^2+9x+9=5x^2-2x-1.$$

Transposing, etc., $7x=-11$; $\therefore x=-\frac{11}{7}$.

(b) $$\frac{x+a-b}{a-b}+\frac{x-a-b}{a+b}=\frac{2a(2c-x)}{a^2-b^2}.$$

Clearing, $ax+a^2+bx-b^2+ax-a^2-bx+b^2=4ac-2ax.$

Transposing, etc., $4ax=4ac$; $\therefore x=c.$

3.
$$\begin{cases} 2x-3y+z=-2, & (1)\\ 4x-4y-3z=2, & (2)\\ 6x+y-4z=6. & (3)\end{cases}$$

Multiplying (1) by 2, $4x-6y+2z=-4.$ (4)

Subtracting (4) from (2), $2y-5z=6.$ (5)

Multiplying (1) by 3, $6x-9y+3z=-6.$ (6)

Subtracting (6) from (3), $10y-7z=12.$ (7)

Multiplying (5) by 5, $10y-25z=30.$ (8)

Subtracting (8) from (7), $18z=-18$; $\therefore z=-1.$

Substituting -1 for z in (5), $2y+5=6$; $\therefore y=\frac{1}{2}.$

Substituting -1 for z and $\frac{1}{2}$ for y in (1), $2x-\frac{3}{2}-1=-2$; $\therefore x=\frac{1}{4}.$

4. (a) $$(3x-11)^{\frac{1}{2}}-2=\tfrac{1}{3}(27x-243)^{\frac{1}{2}}.$$

Squaring, $3x-11-4(3x-11)^{\frac{1}{2}}+4=\frac{1}{9}(27x-243).$

Transposing, etc., $(3x-11)^{\frac{1}{2}}=5.$

Squaring, $3x-11=25.$

Transposing, etc., $x=12.$

(b) Substituting $\dfrac{-1+\sqrt{5}}{2}$ for x,

$$\begin{aligned}x^3-2x+1=\left(\frac{-1+\sqrt{5}}{2}\right)^3-2\left(\frac{-1+\sqrt{5}}{2}\right)+1&=(\sqrt{5}-2)-(-1+\sqrt{5})+1\\&=\sqrt{5}-2+1-\sqrt{5}+1=0.\end{aligned}$$

5. (a) $$\frac{1}{a^{-1}-b^{-1}}+\frac{\sqrt{a^5b^{-1}}(a^{-\frac{1}{2}}b^{\frac{1}{2}})^3}{a-b}=\frac{1}{\frac{1}{a}-\frac{1}{b}}+\frac{(a^{\frac{5}{2}}b^{-\frac{1}{2}})(a^{-\frac{3}{2}}b^{\frac{3}{2}})}{a-b}$$

$$=\frac{ab}{b-a}+\frac{ab}{a-b}=\frac{-ab+ab}{a-b}=\frac{0}{a-b}=0.$$

(b) $8^{-\frac{2}{3}}\times16^{\frac{3}{4}}\times2^0=\frac{1}{4}\times8\times1=2.$

6. Let $x=$ the tens' digit,

and $y=$ the units' digit.

Then, $10x+y=$ the number,

and $10y+x=$ the number with its digits interchanged.

$\therefore 10y+x=2(10x+y)-6,$ (1)

$x+y=\frac{1}{4}(10x+y).$ (2)

Transposing in (1), etc., $8y-19x=-6.$ (3)

From (2), $y=2x.$ (4)

Substituting (4) in (3), $16x-19x=-6$; $\therefore x=2.$

Substituting 2 for x in (4), $y=4.$

Hence, the number is 24.

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7. Let $x =$ number of hours before the passenger train overtakes the freight train.

Then, $x + t =$ number of hours freight travels before being overtaken.

$$\therefore r(x+t) = mx.$$

Transposing, etc., $mx - rx = rt.$

Solving, $x = \dfrac{rt}{m-r}.$

Hence, the passenger train will overtake the freight train in $\dfrac{rt}{m-r}$ hours.

8. Let $x =$ number of A's supporters,

and $y =$ number of B's supporters.

Then, $x + y = 740, \quad (1)$

and $\dfrac{x}{8} + \dfrac{y}{12} = 75. \quad (2)$

From (1), $y = 740 - x. \quad (3)$

Clearing (2), $3x + 2y = 1800. \quad (4)$

Substituting (3) in (4), $3x + 1480 - 2x = 1800; \therefore x = 320.$

Substituting 320 for x in (1), $y = 420.$

Hence, B won the election by a majority of $420 - 320$, or 100.

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1. (a) $\dfrac{3x}{x-2} - \dfrac{2}{x+3} + \dfrac{2}{2-x} = 0.$

Clearing of fractions by multiplying by the L.C.D., $(x-2)(x+3)$,

$$3x^2 + 9x - 2x + 4 - 2x - 6 = 0.$$

Combining, $3x^2 + 5x - 2 = 0.$

Solving by formula, $x = \dfrac{-5 \pm \sqrt{25 - 4 \cdot 3 \cdot (-2)}}{2 \cdot 3} = \dfrac{-5 \pm 7}{6} = \dfrac{1}{3}$ or $-2.$

(b) $$\begin{cases} x - y = 1, & (1) \\ \dfrac{x}{y} - \dfrac{y}{x} = \dfrac{5}{6}. & (2) \end{cases}$$

Clearing (2), $6x^2 - 6y^2 = 5xy. \quad (3)$

From (1), $y = x - 1. \quad (4)$

Substituting (4) in (3), etc., $5x^2 - 17x + 6 = 0.$

Solving, $x = 3$ or $\frac{2}{5}. \quad (5)$

Substituting (5) in (4), $y = 2$ or $-\frac{3}{5}.$

These values of x and y when substituted in the given equations are found to satisfy the equations. Hence, the results are correct.

2. (a) $\sqrt{7x+1} - \sqrt{3x+10} = 1.$

Transposing, $\sqrt{7x+1} = \sqrt{3x+10} + 1.$

Squaring, $7x + 1 = 3x + 10 + 2\sqrt{3x+10} + 1.$

Transposing, etc., $\sqrt{3x+10} = 2x - 5.$

Squaring, etc., $4x^2 - 23x + 15 = 0.$

Solving, $x = 5$ or $\frac{3}{4}.$

Substituting the values of x in the given equation, the value 5 is found to satisfy the equation, but the value $\frac{3}{4}$ does not satisfy the equation according to the convention that only principal roots are to be taken.

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(b) $3(x^2+3x+1)^2 - 7(x^2+3x+1) + 4 = 0.$

Put p for (x^2+3x+1) and p^2 for $(x^2+3x+1)^2$.

Then, $3p^2 - 7p + 4 = 0.$

Solving, $p = \frac{4}{3}$ or 1;

that is, $x^2 + 3x + 1 = \frac{4}{3}$ or 1.

Transposing, etc., $x^2 + 3x - \frac{1}{3} = 0,$ (1)

or $x^2 + 3x = 0.$ (2)

Solving (1), $x = \frac{1}{6}(-9 \pm \sqrt{93}).$

Solving (2), $x = 0$ or -3.

3. Let $x =$ A's rate in miles per hour,

and $y =$ B's rate in miles per hour.

Then, $5x - 5y = 5,$ (1)

and $(5x)^2 + (5y)^2 = 25^2.$ (2)

From (1), $x = y + 1.$ (3)

Reducing (2), $x^2 + y^2 = 25.$ (4)

Substituting (3) in (4), $y^2 + 2y + 1 + y^2 = 25.$

Transposing, etc., $y^2 + y - 12 = 0.$

Solving, $y = 3$ or -4. (5)

Substituting (5) in (3), $x = 4$ or -3.

Hence, since the negative values are inadmissible, A's rate is 4 miles per hour and B's 3 miles per hour.

4. Let $x =$ number of days the man worked.

Then, $\frac{75}{x} =$ number of dollars received per day,

and $\frac{75}{x-5} =$ number of dollars received per day if he had earned \$75 in 5 days less.

$$\therefore \frac{75}{x-5} - \frac{75}{x} = \frac{1}{2}.$$

Clearing, etc., $x^2 - 5x - 750 = 0.$

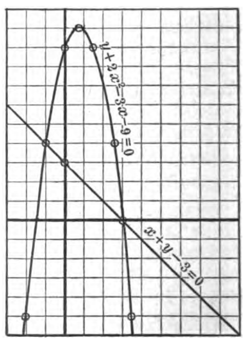
Solving, $x = 30$ or -25.

Hence, the negative value being inadmissible, the man worked 30 days.

5. Solving $y + 2x^2 - 3x - 9 = 0$ for y, we have $y = -2x^2 + 3x + 9$.

Putting $-2x^2 + 3x + 9 = 0$ in the form $x^2 - \frac{3}{2}x - \frac{9}{2} = 0$, § 418, first substitute $\frac{3}{4}$ for x in $y = -2x^2 + 3x + 9$, and then the other values of x as shown in the table, finding the corresponding values of y.

x	y
$\frac{3}{4}$	10.1
$0, 1\frac{1}{2}$	9
$-1, 2\frac{1}{2}$	4
$-2, 3\frac{1}{2}$	-5



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Plotting these points and drawing a smooth curve through them, we have the graph of $y + 2x^2 - 3x - 9 = 0$ which is a parabola.

The graph of $x + y - 3 = 0$ (§ 267) is a straight line which intersects the parabola at the points $(-1, 4)$ and $(3, 0)$.

Hence, the roots of the given equations are $\begin{cases} x = -1, 3; \\ y = 4, 0. \end{cases}$

6. Since the elastic ball is thrown to a height of 15 feet, falls to the ground, bounces to $\frac{3}{4}$ the height it was thrown, and so on until it comes to rest, the distance it travels is twice the sum of an infinite geometrical series in which the first term is 15 and the ratio $\frac{3}{4}$.

Substituting 15 for a and $\frac{3}{4}$ for r in the formula, $s = \dfrac{a}{1-r}$, $s = \dfrac{15}{1-\frac{3}{4}}$ $= \dfrac{15}{\frac{1}{4}} = 60$. Hence, $2s = 2 \times 60 = 120$.

Therefore, the ball traveled 120 feet before coming to rest.

7. (*a*) Since $\left(2x^2 - \dfrac{1}{4x}\right)^8 = \left[x^2\left(2 - \dfrac{1}{4x^3}\right)\right]^8 = x^{16}\left(2 - \dfrac{1}{4x^3}\right)^8$, every term of the series expanded from $\left(2 - \dfrac{1}{4x^3}\right)^8$ will be multiplied by x^{16}.

Hence, the term sought is that which contains $\left(-\dfrac{1}{4x^3}\right)^4$, or $\dfrac{1}{256x^{12}}$; that is, the $(4 + 1)$th, or 5th, term.

$$\text{5th term} = x^{16} \cdot \frac{8 \cdot 7 \cdot 6 \cdot 5}{1 \cdot 2 \cdot 3 \cdot 4}(2^4)\left(\frac{1}{256x^{12}}\right) = \frac{35}{8}x^4.$$

Hence, the coefficient of the term sought is $\frac{35}{8}$.

(*b*) $$x^2 + 5x + d = 0.$$

Solving by formula, $$x = \frac{-5 \pm \sqrt{25 - 4 \cdot 1 \cdot d}}{2 \cdot 1} = \tfrac{1}{2}(-5 \pm \sqrt{25 - 4d}).$$

In order to have the roots imaginary $4d$ must be greater than 25. Hence, the smallest integer which, when substituted for d, makes the roots imaginary is 7.

8. Let the three numbers in geometrical progression be x^2, xy, and y^2.

Then, $x^2 + xy + y^2 = 70$, (1)

and $5xy - 4x^2 = 4y^2 - 5xy$. (2)

Transposing (2), etc., $2x^2 - 5xy + 2y^2 = 0$. (3)

Multiplying (1) by 2, $2x^2 + 2xy + 2y^2 = 140$. (4)

Subtracting (3) from (4), $7xy = 140$.

Dividing by 7, $xy = 20$. (5)

Adding (5) and (1), $x^2 + 2xy + y^2 = 90$.

Extracting the square root, $x + y = \pm 3\sqrt{10}$. (6)

Multiplying (5) by 3, $3xy = 60$. (7)

Subtracting (7) from (1), $x^2 - 2xy + y^2 = 10$.

Extracting the square root, $x - y = \pm\sqrt{10}$. (8)

Adding (6) and (8), $2x = 4\sqrt{10},\ 2\sqrt{10},\ -2\sqrt{10}, \text{ or } -4\sqrt{10}$.

$\therefore x = 2\sqrt{10},\ \sqrt{10},\ -\sqrt{10}, \text{ or } -2\sqrt{10}$.

$x^2 = 40, 10, 10, \text{ or } 40$.

Putting 40 for x^2 and 20 for xy in (1), $40 + 20 + y^2 = 70$; $\therefore y^2 = 10$.

Putting 10 for x^2 and 20 for xy in (1), $10 + 20 + y^2 = 70$; $\therefore y^2 = 40$.

Hence, the three numbers in geometrical progression are 10, 20, and 40.

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1. $$a(x-a)=b(x-b).$$

Expanding, etc., $$ax-bx=a^2-b^2.$$

Dividing by $a-b$, $$x=a+b.$$

2. $$2x^2+5xy+2y^2=\frac{4x^2+10xy+4y^2}{2}=\frac{(2x)^2+5y(2x)+4y^2}{2}$$
$$=\frac{(2x+y)(2x+4y)}{2}=(2x+y)(x+2y).$$

$$(x-y)^2+4(x^2-y^2)-21(x+y)^2$$
$$=[(x-y)+7(x+y)][(x-y)-3(x+y)]$$
$$=(8x+6y)(-2x-4y)=-4(4x+3y)(x+2y).$$

$$(a-b)^3+(c-d)^3=[a-b+c-d][(a-b)^2-(a-b)(c-d)+(c-d)^2].$$

3.
$$12x^3y-12xy^3=12xy(x+y)(x-y)$$
$$2x^2(x+y)^2=2x^2(x+y)(x+y)$$
$$3y^2x^2-6xy^3+3y^4=3y^2(x-y)(x-y)$$
$$\therefore \text{L. C. M.}=12x^2y^2(x+y)^2(x-y)^2.$$

4. $$(\sqrt[3]{a^2}+\sqrt[3]{b^2})(\sqrt[6]{a^8}-\sqrt[3]{a^2b^2}+\sqrt[9]{b^{12}})=(a^{\frac{2}{3}}+b^{\frac{2}{3}})(a^{\frac{4}{3}}-a^{\frac{2}{3}}b^{\frac{2}{3}}+b^{\frac{4}{3}})$$
$$=a^2+b^2.$$

5. $$\sqrt{2x+5}-\sqrt{x-1}=2.$$

Transposing, $$\sqrt{2x+5}=\sqrt{x-1}+2.$$

Squaring, $$2x+5=x-1+4\sqrt{x-1}+4.$$

Transposing, etc., $$x+2=4\sqrt{x-1}.$$

Squaring, $$x^2+4x+4=16x-16.$$

Transposing, etc., $$x^2-12x+20=0.$$

Solving, $$x=10 \text{ or } 2.$$

6. $$\begin{cases} x^3+y^3=19, & (1)\\ x+y+\sqrt{x+y}=2. & (2)\end{cases}$$

Completing the square in (2), $$x+y+\sqrt{x+y}+\tfrac{1}{4}=2+\tfrac{1}{4}=\tfrac{9}{4}.$$

Extracting the square root, $$\sqrt{x+y}+\tfrac{1}{2}=\pm\tfrac{3}{2}.$$

Transposing and squaring, $$x+y=1 \text{ or } 4. \quad (3)$$

Dividing (1) by (3), $$x^2-xy+y^2=19 \text{ or } \tfrac{19}{4}. \quad (4)$$

Squaring (3), $$x^2+2xy+y^2=1 \text{ or } 16. \quad (5)$$

Subtracting (4) from (5), $$3xy=-18 \text{ or } \tfrac{45}{4}.$$
$$xy=-6 \text{ or } \tfrac{15}{4}. \quad (6)$$

Subtracting (6) from (4), $$x^2-2xy+y^2=25 \text{ or } 1.$$

Extracting the square root, $$x-y=\pm 5 \text{ or } \pm 1. \quad (7)$$

From (3) and (7), $$x=3,\ -2,\ \tfrac{5}{2},\ \tfrac{3}{2};$$

and $$y=-2,\ 3,\ \tfrac{3}{2},\ \tfrac{5}{2}.$$

$\begin{cases} x=\tfrac{5}{2},\ \tfrac{3}{2},\\ y=\tfrac{3}{2},\ \tfrac{5}{2},\end{cases}$ do not verify and are rejected.

7. Since there are $12+1$ terms, the middle term is the $(6+1)$th.

$$\text{7th term}=\frac{12\cdot 11\cdot 10\cdot 9\cdot 8\cdot 7}{1\cdot 2\cdot 3\cdot 4\cdot 5\cdot 6}(m^{\frac{1}{3}})^{12-6}(-2m^{-\frac{2}{3}}x)^6=59{,}136\,m^{-2}x^6.$$

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8. Given, $\frac{a}{b}=\frac{c}{d}.$

By alternation, $\frac{a}{c}=\frac{b}{d}.$

By Prin. 9, § 484, $\frac{a^2}{c^2}=\frac{b^2}{d^2}.$

By composition, $\frac{a^2+c^2}{c^2}=\frac{b^2+d^2}{d^2}.$

By alternation, $\frac{a^2+c^2}{b^2+d^2}=\frac{c^2}{d^2}.$

By Prin. 9, § 484, $\frac{\sqrt{a^2+c^2}}{\sqrt{b^2+d^2}}=\frac{c}{d}.$

Ax. 5, $\frac{\sqrt{a^2+c^2}}{\sqrt{b^2+d^2}}=\frac{a}{b}.$

Hence, $\frac{a}{b}=\frac{c}{d}=\frac{\sqrt{a^2+c^2}}{\sqrt{b^2+d^2}}.$

9. Let $\sqrt{x}+\sqrt{y}=\sqrt{29+12\sqrt{5}}.$ (1)

Then, § 361, $\sqrt{x}-\sqrt{y}=\sqrt{29-12\sqrt{5}}.$ (2)

Multiplying (1) by (2), $x-y=\sqrt{841-720}=\sqrt{121},$

or $x-y=11.$ (3)

Squaring (1), $x+2\sqrt{xy}+y=29+12\sqrt{5}.$

Therefore, § 360, $x+y=29.$ (4)

Solving (3) and (4), $x=20,\ y=9.$

$\therefore\ \sqrt{x}=\sqrt{20},\ \sqrt{y}=\sqrt{9}.$

Hence, from (1), $\sqrt{29+12\sqrt{5}}=\sqrt{20}+\sqrt{9}=2\sqrt{5}+3.$

10. $\frac{3\sqrt{2}-4}{3\sqrt{2}+4}=\frac{(3\sqrt{2}-4)(3\sqrt{2}-4)}{(3\sqrt{2}+4)(3\sqrt{2}-4)}=\frac{34-24\sqrt{2}}{2}=17-12\sqrt{2}$

$=17-(12\times 1.414)=17-16.968=.032,$ or $.03.$

11. Let $x=$ the larger number,

and $y=$ the smaller number.

Then, $xy=30,$ (1)

and $x^2-y^2=221.$ (2)

Assume $x=vy.$ (3)

Substituting (3) in (1), $vy^2=30.$ (4)

Substituting (3) in (2), $v^2y^2-y^2=221.$ (5)

From (4) and (5), $y^2=\frac{30}{v}=\frac{221}{v^2-1}.$ (6)

Clearing, etc., $30v^2-221v-30=0.$

Solving, $v=\frac{15}{2}$ or $-\frac{2}{15}.$

Substituting $\frac{15}{2}$ for v in (6), $y=2$ or $-2,$

whence, by (3), $x=15$ or $-15.$

Substituting $-\frac{2}{15}$ for v in (6), $y=15\sqrt{-1}$ or $-15\sqrt{-1},$

whence, by (3), $x=-2\sqrt{-1}$ or $2\sqrt{-1}.$

Hence, $\begin{cases} x=15,\ -15,\ -2\sqrt{-1},\quad 2\sqrt{-1}; \\ y=\ \ 2,\ -\ \ 2,\quad 15\sqrt{-1},\ -15\sqrt{-1}. \end{cases}$

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12. The roots of the equation $x^2 + 2(k+2)x + 9k = 0$ will be equ if the discriminant equals zero, that is, if

$$4(k+2)^2 - 4 \cdot 1 \cdot 9k = 0,$$

or, solving, if $k = 4$ or 1.

13. Let x = number of 25-cent piece

and y = number of 5-cent pieces.

Then, $25x + 5y = 1600,$ (1

and $x + y = 80.$ (2

Dividing (1) by 5, $5x + y = 320.$ (3

Subtracting (2) from (3), $4x = 240$; $\therefore x = 60.$

Substituting 60 for x in (2), $y = 20.$

Hence, the man paid the bill with 60 25-cent pieces and 20 5-cen pieces.

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